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ANNALS

OF THE

CAPE OBSERVATORY.

VOL. XII.

Published by Order of the Lords Commissioners of the Admiralty in obedience to His Majesty's Command.

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ANNALS OF THE CAPE OBSERVATORY.

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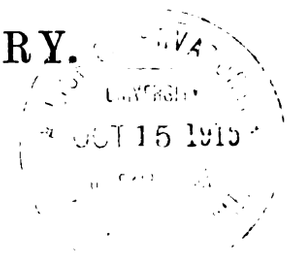

VOL. XII. PART I.

DETERMINATION OF THE MASS OF JUPITER AND ELEMENTS OF THE ORBITS OF ITS SATELLITES

FROM OBSERVATIONS MADE WITH THE CAPE HELIOMETER.

BY

SIR DAVID GILL, K.C.B.,

AND

W. H. FINLAY, M.A.

REDUCED AND DISCUSSED BY

DR W. DE SITTER,

PROFESSOR OF ASTRONOMY IN THE UNIVERSITY OF LEIDEN.

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PREFACE.

THE present work is an account of the heliometer observations of Jupiter's satellites made by the late Sir David Gill in the year 1891. The observations were made at the time when his remarkable and probably unsurpassed powers of observation were at their height, and it may be assumed that they are the best observations which have ever been made with any heliometer.

The publication of this memoir was one of the things which occupied Sir David's thoughts to the last. His sudden illness and lamented death prevented him from completing his personal share in the work, and it is only fit here to relate somewhat in detail the circumstances which have led to the publication of the paper in its present form.

As the result of a correspondence between the late Sir David Gill, Mr Hough (who was then in England), and myself in July and August 1913, it was agreed upon that the account of the observations and discussions were to be written by me, and that Sir David should write an introduction or preface. Accordingly, Mr Hough, immediately after his return to the Cape, sent all the papers relating to the subject to me, and I began at once to write out the memoir. When I had completed the greater part of it, I wrote to Sir David (October 15, 1913) in order further to discuss with him the details relating to the introduction. His reply, delayed by his absence in Paris at the meeting of the Conference of Weights and Measures, was dated October 31, and the following passages are quoted from it:—

"Well now, about this matter of the Part I. of Vol. XII. of the *Cape Annals*; I think it would be well if the introduction could be written entirely by myself, and I can undertake to do it practically without delay, provided that you can send me a few notes about facts, which I may to a great extent have forgotten. I mean if, instead of writing out an introduction as you yourself would have written it, you sent me the heads of what you yourself would have elaborated into the introduction had you written it yourself, together with such matters as dates, etc., to which I have no means of reference here.

"I have now, as you know, got my book* out of hand, and have no immediate pressing work. . . . . .
I do not think the introduction need be a very long one, but I should like also to see your Chapter I., about Details of Observation, Derivations of Scale Values, etc., before I write the introduction, and you might send it to me in MS. at once, because you tell me it is now ready."

* *The History and Description of the Cape Observatory.* (de S.)

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Accordingly, I posted to Sir David on the following day the whole of the MS., together with the notes and information for which he had asked. He wrote to me again on November 8 :—

"I have just looked through the manuscript, and I think I shall have very little difficulty in writing the introduction, with the help of the notes as to dates, etc., which you have sent to me. I think that on pages lxxxiv–lxxxvii of my History you will find something very much like what I will say with regard to the origin of my observations; which, after all, is the matter I have more particularly to deal with. Of course, I shall also state the history of how the reduction of these observations was so long delayed, and how you came to the rescue in the matter. With the aid of the notes, I think I shall have no difficulty in putting the matter together in a not very lengthy way."

The letter from which the above is quoted is the last I received from Sir David. On December 15 he had been working at the introduction all day. In the evening of that day he was attacked by the severe illness from which he never recovered. He died on January 24, not having left his bed since the first day of his illness.

From his secretary I learned afterwards that about ten pages of the MS. of the introduction were complete. These ten pages have been found amongst his papers, and are printed below. Besides, I received through the kindness of Mr A. R. Hinks from Lady Gill a few notes in Sir David's handwriting, which, presumably, are the very last words he ever wrote. These notes enabled me to add a few more lines to what he had already written out for the printers.

But, even so, the introduction was very incomplete. After considering several possibilities, I came to the conclusion that the best course would be to leave the beginning of the introduction as it had been written by Sir David intact, and to continue it to the best of my ability on the lines on which I knew by our correspondence he would have completed it himself, had life been spared him. It has been a sad task for me, still under the impression of the shadow which his unexpected death has cast over the whole scientific world, to write even a short and imperfect account of the origin of this work, recalling to my memory the recollection of so many pleasant and profitable hours spent in his room at the Royal Observatory at the Cape, of so many details of personal intercourse and of daily life in that truly scientific and at the same time intensely human atmosphere which he had created around him, and which always was pervaded by the spirit of his strong and highly sympathetic personality.

W. DE SITTER.

LEIDEN, *March* 1914.

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INTRODUCTION.

(PARTLY BY SIR DAVID GILL.)

THE observations on which this work is based had their origin in a discussion which I had shortly before my departure for the Cape in the year 1879 with the late Professor J. C. Adams on the subject of the kind of observations required to improve the tables of the motions of Jupiter's satellites.

Professor Adams explained that he was then engaged in an endeavour to correct Damoiseau's tables of Jupiter's satellites, with a view to their extension beyond the year 1890, but that there were no existing observations from which the inclinations and nodes could be satisfactorily determined. The jovicentric longitudes of the satellites could be determined with considerable precision from the numerous observed eclipses, but there were no observations from which the corresponding latitudes could be satisfactorily derived. He suggested that measures should be made, with a filar micrometer, of the difference of declination between the limbs of Jupiter and the projected shadows of the satellites in transit. My own view was that the limbs of planets did not lend themselves to very accurate bisection, and I believed that good heliometer observations of position angles and distances of the satellites with respect to each other would better fulfil Professor Adams' object, whilst such observations would also contribute to the determination of other elements of the system, and also give a very accurate determination of the mass of Jupiter, if the scale-value of the heliometer could be determined with sufficient precision.

I promised Professor Adams that, as soon as opportunity offered, I would make a trial of the method; but, as will be presently explained, I was prevented from so doing until the autumn of 1891.

The determination of the mass of Jupiter, since the periodic times of the satellites are known, may be expressed in other words as the determination of the scale-value of any accurate geometrical representation of the orbits of its satellites. Thus, whilst observations of their eclipses determine the longitudes of the satellites with considerable accuracy, and define (although with a much lower accuracy) the planes of their orbits, they obviously afford no data as to the scale of the system, that is to say, they do not contribute to the determination of its mass.

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All the existing tables of Jupiter's satellites have been based solely upon the results of observed eclipses, and the mass of the system had to be derived from the perturbations of other planets produced by the action of Jupiter. It seemed therefore important, not only to make a new determination of the inclinations and nodes of the satellites, but also to base the mass of the planet on the more direct method of determining the mean distances of the satellites from Jupiter.

LAPLACE, in the fourth volume of his *Mécanique Céleste*, developed the mathematical theory of the mutual perturbations of the satellites; his formulæ were reduced to numbers by BOUVARD.

The formidable labour of deriving the values of the elements of Jupiter's system from the observations was first undertaken by DELAMBRE, who in 1792 published tables based on the discussion of more than 2000 eclipses. He afterwards applied small corrections to his coefficients, and his finally corrected tables were based on all the eclipses (about 6000 in number) observed previous to 1804.

From DELAMBRE'S coefficients LAPLACE derived the elements and masses of the satellites and the value of the coefficient J of the compression of Jupiter.

After Delambre, DAMOISEAU took up the subject, and in 1836 published his *Tables Écliptiques des Satellites de Jupiter*, on which, until a very recent date, nearly all computations referring to the satellites (especially the predictions of the eclipses and other phenomena in the national ephemerides) have been based. These tables are, however, very far from perfect, for large discrepancies frequently occur between the predicted and observed time of an eclipse, particularly in the case of the fourth satellite. Unfortunately it is not known on what observations they are based, or how Damoiseau derived the values of the coefficients of the inequalities. But the chief reason why his tables cannot be used as a means of comparing observations with theory is that they do not represent any one consistent system. Some of the important inequalities have values which differ very little from those of Delambre's tables, whilst others are considerably changed; and it seems to be certain that the different inequalities of Damoiseau's tables cannot be represented by one and the same set of elements and masses. Thus, Damoiseau in the introduction to his tables gives values for the masses of the satellites which are inconsistent with the coefficients of the tables; for Professor J. C. Adams (*Scientific Papers*, vol. ii. p. 191) substituted these coefficients in the equations by which Laplace derived the masses from Delambre's coefficients, and the resulting masses do not agree with those given by Damoiseau. In 1842, *only six years after the publication of Damoiseau's tables*, BESSEL, in connection with the reduction of the series of heliometer observations which he made (1833–1839)

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to determine the mass of Jupiter, found himself constrained to state that already new tables were necessary for the proper reduction of his observations.*

The need indicated by Bessel was supplied by the publication in 1880 of SOUILLART's great work, "Théorie analytique des Satellites de Jupiter" (*Memoirs, R.A.S.*, vol. xlv.), which in many respects is more complete than that of Laplace. He describes it as follows:—"Je présente ici, coordonnées dans une rédaction d'ensemble, toutes mes recherches sur cette question, dans le but d'offrir aux astronomes une théorie analytique complète des mouvements de ces satellites, pouvant servir de base à une nouvelle détermination des éléments et à la construction de nouvelles tables."

The data that existed as a basis for deriving the elements of the satellites from this theory were of three classes, viz.:—

(I.) (*a*) The vast number of eclipses observed in the usual way, *i.e.* by noting the times of disappearance into and final emergence from shadow.
(*b*) The Harvard photometric observations, which have recently been reduced by PROFESSOR SAMPSON.

(II.) (*a*) BESSEL's observations (1833–1839) with the Königsberg Heliometer of the distances of all the satellites and the position angles of the Satellites III. and IV. from the planet.
(*b*) E. LUTHER's heliometer observations (1856).
(*c*) SCHUR's heliometer observations (1874–80) of the distances and position angles of all the four satellites from the planet.

(III.) (*a*) AIRY's observations (1832–36) of differences of R.A. between the planet and Satellite IV.
(*b*) VOGEL's observations (1868–70) of differences of R.A. between the planet and all the satellites.

In regard to Class I. of these observations, Professor de Sitter in his DISSERTATION (to be subsequently described) gives the following statement as to their unsuitability for determination of the latitudes of the satellites:

"Amongst the worst determined of the elements are the inclinations and nodes of the orbits. These can be derived from eclipse observations only by the discussion of the duration of the eclipses.

* "Ich fürchte, dass man, um die Formeln für die Bewegungen der Jupitersatelliten wieder theoretisch *ganz zusammenhangend* zu machen, gezwungen sein wird, die unvollständig dargestellten Resultate des Herrn *Damoiseau* zu verlassen und bis zu einer neuen Untersuchung der Elemente zurückzugehen."

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"This duration is given by the formula (Tisserand, iv. p. 76)

$$t' = 2T(1-X)\sqrt{\left\{1+\tfrac{1}{2}X+(1+\kappa')\frac{s_0}{\beta}\right\}\left\{1+\tfrac{1}{2}X-(1+\kappa')\frac{s_0}{\beta}\right\}},$$

from which follows (if the square and higher powers of X are neglected):

$$(1+\kappa')s_0 = \frac{\beta\sqrt{4T^2(1-X)-t'^2}}{2T(1-X)}, \tag{1}$$

where

s_0 = latitude referred to plane of Jupiter's orbit.

$T = \dfrac{\beta}{n-N}$ { n = mean motion of satellite. N = " " " Jupiter.

$\beta = \dfrac{a}{\alpha}$ α = semiaxis major of satellite's orbit.

a = semiaxis major } of section of shadow-cone at distance
κ' = ellipticity } α from Jupiter's centre.
X = a quantity depending on perturbations in longitude.
t' = observed duration.

"It will be seen that $\dfrac{\beta}{T}$ can be considered as rigorously known. T itself, which is half the maximum duration of an eclipse, is however not known.

"If the immersion and emersion of the same eclipse are observed, an approximate knowledge of X is sufficient. In the most general case, however, immersions and emersions of different eclipses, in which X has different values, must be combined. X can then not be considered as known but contains several unknown quantities, which must be determined from the same observations. Consequently, there is a considerable loss of weight in the determination of s_0.

"This is, however, not the most serious objection to the determination of the latitudes from eclipse observations.

"The chief difficulty lies in the determination of the duration itself. For the visual observations, modern as well as ancient, this determination is so mixed up with errors of all kinds, that it is very difficult, if not impossible, to derive any trustworthy results from these observations. The observed disappearance and reappearance of the satellite do not coincide with its immersion in and emersion from the geometrical shadow-cone, but differ from these epochs for a number of reasons, of which it will be sufficient here to mention the most important ones. These are, firstly, the effect of the penumbra, of the refraction of the sun's rays in Jupiter's atmosphere, of the different albedo of the satellites. Further, the aperture and quality of the instrument, the keen-

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ness of sight of the observer, the angular distance from the disc of Jupiter, the transparency of the atmosphere, have a large and unknown influence. All these circumstances make that, to use Laplace's words,* the geometrical shadow must be replaced by a fictitious shadow, which is not only different for different satellites, but also for different instruments and different observers, and even varies with the zenith-distance and the atmospheric definition. To derive a trustworthy result from the visual observations, it would be necessary to determine the form of this fictitious shadow-cone for each satellite, and for each observer and each instrument separately, and also to determine how it varies with the various circumstances of the observation. It is evident from these considerations that the visual eclipse observations are incapable of giving more than a rough approximation to the values of the inclinations and nodes of the orbits of the satellites.

"With the photometric observations the conditions are much better. We can assume that the instant at which the light is reduced to any definite proportion of the light of the uneclipsed satellite will be determined from these observations, free from most of the errors which affect the visual observations.†

"We must now again consider the equation (1). The right-hand member contains the observed duration t' and the quantities X and T, which are supposed to be determined with sufficient accuracy from the observations.

"The left-hand member contains the unknown latitude, and the factor $(1+\kappa')$. This factor has been brought into evidence by transposing it from the denominator of the right-hand member (where it generally appears) to the left-hand member, with the intention to show that the observations of the duration of eclipses can with the same right be considered as determinations of this factor as of the latitude, and they must be considered as determinations of that one of these two quantities of which our present knowledge is least accurate.

"The ellipticity κ' of the section of the shadow-cone is very nearly equal to the ellipticity κ of Jupiter, to which it stands in a known relation (see Tisserand, iv. p. 74). The values for κ which have been determined by different observers have been collected by Dr F. Cohn (*Astronomische Nachrichten*, 3404, pp. 333–336).

* *Mécanique Céleste*, tome iv., page 105.

† The same thing is true for such observations as are now regularly being made by Mr Innes at Johannesburg, who estimates the times when a definite portion of the satellite's disc has disappeared (or reappeared, as the case may be). The advantage is not so much in the *photometric* determination of the loss of light, at any instant, as in the *precise definition* of the phenomenon which is actually observed.

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The values derived by the different observers range from $\frac{1}{13{\cdot}7 \pm 0{\cdot}3}$ (W. STRUVE) to $\frac{1}{18{\cdot}1 \pm 0{\cdot}2}$ (KAISER, 1856), and the final means from observations by all observers are :—

	$\frac{1}{\kappa}$
Heliometer	$16{\cdot}53 \pm 0{\cdot}22$
Double image micrometer . .	$16{\cdot}77 \pm 0{\cdot}17$
Filar micrometer . . .	$15{\cdot}46 \pm 0{\cdot}31$

"(The mean errors given by Cohn have been transformed to probable errors.)

"It is evident that the probable errors given are no measure of the true accuracy, and that the observations must be affected by very large systematic errors. We will certainly not underestimate the real precision of the determination of κ if we adopt ± 1 as the probable error (including systematic errors) of the denominator. The probable error of $1 + \kappa'$ then becomes $\pm \frac{1}{250}$."

This statement demonstrates that from eclipse observations only it is impossible to determine the latitudes with an accuracy greater than that which corresponds with a probable error of $\pm \frac{1}{250}$. Having regard to the uncertainty introduced by the other unknowns, it is safe to increase this estimate to $\pm \frac{1}{150}$, which corresponds with a probable error of $\pm 0^{\circ}{\cdot}02$ in the ultimate determination of the latitudes from observed eclipses. On the other hand, reference to Chapter III. will show that, from the series of observations discussed in the present work, the latitudes of the four satellites are determined by the heliometer observations with respective probable errors of :

I.	II.	III.	IV.
$\pm 0^{\circ}{\cdot}005$	$\pm 0^{\circ}{\cdot}003$	$\pm 0^{\circ}{\cdot}002$	$\pm 0^{\circ}{\cdot}001$.

These facts sufficiently confirm the statement of Professor Adams already quoted, viz., that previous to the year 1879 there were no existing observations from which the latitudes of the satellites could be satisfactorily determined, and they also justify the methods which were adopted to supply the deficiency.

An eclipse observation, on the other hand, especially if made by the photometric method, gives a very strong determination of the longitude. For the determination of the unknowns that depend on the longitudes the need for observations by a new method is not so urgent, and yet, here also, there are some considerations which point to the necessity for longitude determinations other than those which are available from the observation of eclipses.

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As is well known, the near approach to mutual commensurability of their mean motions creates great inequalities in the longitudes of the three inner satellites, which have periods that differ very little from the period of revolution of the satellite. In the eclipses these inequalities present themselves with a common period of 438 days, as has been shown by Laplace (*Mécanique Céleste*, vol. iv., livre viii., chap. 2).

Now in the longitudes of these satellites, there are inequalities of long period, ranging from 406 to 486 days, and which are therefore difficult to separate by eclipse observations from the great inequalities. (See De Sitter, *Proc. Amsterdam Acad.*, vol. x. p. 711.) The coefficients of these long-period inequalities must consequently be regarded as practically unknown until longitude determinations, other than by eclipse observations, have been made at suitable epochs. It is unnecessary to insist here farther on this point, as it has not entered into the considerations which led to the inception of the observations discussed in this work; indeed the present series of observations contributes nothing (at least not directly) to its elucidation. The reader who may be interested in the subject is referred to *The History of the Cape Observatory*, pp. xcvii–ci, for an indication of what remains to be done in the direction above indicated.

With regard to the observations of Class II., they were primarily made for determination of the mass of Jupiter, but none of them are sufficiently refined in plan or execution to yield a result comparable in accuracy with that which can be derived from the perturbations of Saturn or of some of the minor planets by the action of Jupiter.

One fundamental fault in all the series of this class is that sufficient precautions were not taken to determine the *instantaneous* scale-value with the requisite precision.

A heliometer, from its construction, cannot maintain a uniform scale value, because to do so it must be assumed

(1) that all the various parts of the instrument have the same constant temperature;
(2) that the eyepiece is in constant and perfect focal adjustment on the image formed by the object-glass.

Simple as these conditions appear, their realisation is impossible, for the tube, the scale, and the component semi-lenses of the object-glass are exposed to different influences, and, in general, have sensibly different temperatures from each other, whilst also the observer's eye (which is, in fact, a component part of the eyepiece) is subject to change of focus due to fatigue or to change in the zenith distance of the object observed.*

* Here ends the part of the introduction written by Sir David Gill. The rest has been written by me.—W. de S.

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From these considerations it is evident that in order to attain the utmost accuracy which the heliometer is capable of giving, means must be provided by which the instantaneous scale value can be derived from the observations of each night. This end can only be attained by observing on every night, together with the objects whose distance is to be determined, the distance of a pair of standard stars whose true distance is accurately known. If the accuracy within which this standard distance is known is $1/n^{th}$ of the amount, then the accuracy of the resulting value of the reciprocal of the mass can never be greater than $3/n^{th}$ of the amount, entirely apart from accidental errors of observation.

Now, in the observations of Class II. mentioned above, the scale value was not determined from simultaneous observations of a pair of standard stars, but from an empirical formula involving the temperature of the tube. The resulting mass, therefore, is subject to a very large uncertainty—much larger, in fact, than its mean error as derived from the purely accidental errors of observation. Newcomb, accordingly, in his classical determination of the mass of Jupiter* quite correctly assigned a very small weight to these observations.

Beyond this uncertainty of the scale-value the discussion by Schur† has shown that the observations are subject to systematic errors of various origin, which are the reason why they cannot confidently be used as the basis of an accurate new determination of the elements.

The observations of Class III. also were undertaken only with a view to the determination of the mass. Though they are, of course, not subject to the uncertainty of scale-value, which is a fundamental fault in those mentioned under II., their accuracy is too small to give a result that can at all be compared with the values of the mass as derived from perturbations of planets.

A very important source of systematic errors is common to the observations of the Classes II. and III., viz., the great dissimilarity of the two objects whose distance is measured by the instrument: the satellite and the rim of the planet. In order to attain the highest accuracy as well with the heliometer, as in determining differences of R.A. or declination by transits or with the filar micrometer, it is of great importance that the two objects compared shall be as equal as possible in aspect. This consideration led Sir David Gill to observe distances and position-angles of the satellites

* *Astronomical Papers of the American Ephemeris*, vol. v., part 5.
† *Nova Acta der Kais. Leop. Carol. Acad.*, vol. 45, No. 3.

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relatively to each other. The same principle had already been adopted by H. Struve in his classical investigations of the satellites of Saturn.*

These two principles—the observation of the *relative* positions of the satellites, and the strictly differential determination of the *instantaneous* scale-value and index-correction of the position-circle—are the characteristic features of the series of observations discussed in the present work, and it is to the strict adherence to these principles that their excellent quality, which has probably never been surpassed in the history of observational astronomy, is due.

After the erection of the new 7-inch heliometer in the beginning of 1888, the instrument was used for the stellar parallax campaign, the observations of Iris, Victoria, and Sappho in 1888–89, and the triangulation of the Victoria comparison stars in 1890. Thus the observations of the satellites could only be commenced after Gill's return from a trip to England in 1891. The programme was to measure on each night all available combinations of the four satellites, and one or more measures of the standard stars. For these latter a pair of stars were selected near the position of Jupiter and approximately in the same position-angle as the orbital planes of the satellites. In order to derive the mutual distance and position-angle of these two stars at the epoch of observation with all requisite accuracy, observations on the same nights of the standard stars which had served for the triangulation of the Victoria comparison stars were included in the programme.

These observations had, by the pressure of other work, especially the derivation of the solar parallax from the observations of minor planets, remained unreduced till 1898. In the beginning of that year I was charged, in my capacity as a computer at the Cape Observatory, with their reduction, and at the same time Sir David encouraged me to take up the further discussion and the derivation of new elements. In the voluminous computations required for the discussions, I have received much help from various members of the staff of the Cape Observatory and of the astronomical laboratory at Groningen. It is a pleasant duty to express my sincere thanks to all those who assisted me by their help or advice.

The results of my discussion were published in the form of an inaugural dissertation for the doctor's degree †. This publication contained only so much in the way of a description of the observations as was unavoidable for the good understanding of the

* *Beobachtungen der Saturnstrabanten. Observations de Poulcova, Supplément I.*, 1888; and *Publications de Poulcova*, vol. xi., 1898.

† *Discussion of Heliometer Observations of Jupiter's Satellites, made by Sir David Gill, K.C.B., and W. H. Finlay, M.A.*, by W. de Sitter. Groningen, J. B. Wolters, 1901.

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discussions. The intention has existed from the beginning to publish the details of the observations, together with a short account of the discussions and results, in the *Cape Annals*. Accordingly in the present paper, which contains all necessary details of the observations, much matter has been omitted, which either had only a limited interest in connection with the character of the first publication as a dissertation, or has since that time been published elsewhere, or lost its interest for other reasons. On the other hand, several corrections and refinements in the discussions, which have been found necessary after the publication of my dissertation, are here included. The subject-matter which is common to the two publications has been entirely rearranged, so that the present pages must be considered as a practically new and independent work. Thus, *e.g.*, the discussion of the accidental and systematic errors given in Chapter III. is entirely new, and much more complete than it was in my dissertation.

The research on the elements of Jupiter's satellites, which was begun by the discussion of the observations treated in the present work, was brought to a provisional conclusion in 1908 by my paper, "On the Masses and Elements of Jupiter's Satellites and the Mass of the System" (*Proc. Roy. Acad. of Sciences at Amsterdam*, vol. x. pp. 653 and 710). At that date also the substance of the present paper was practically ready for publication. Thus, though the actual writing was done in the last months of 1913, the paper should properly be considered as belonging to the year 1908. For a comprehensive account of the whole complex of investigations relating to the satellites, and for a discussion of the directions in which the research should be continued, the reader may be referred to the *History of the Cape Observatory*, pp. lxxxiv–ci.

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CHAPTER I.

THE OBSERVATIONS.

THE instrument employed was the heliometer belonging to the Cape Observatory, which is fully described in Vol. VII. of the *Cape Annals*.

The observations were made by Sir DAVID GILL, and on a few occasions by Mr W. H. FINLAY, then chief assistant.

For the purpose of determining the scale-value of the heliometer and the index-error of the position-circle, a pair of standard-stars was measured on each occasion when the satellites were observed.

A complete observation of a pair of satellites consists of the following operations :—

1^0 one pointing in position-angle, and turn reversing prism through 90°,
2^0 one pointing in position-angle,
3^0 one pointing in distance, and turn prism,
4^0 one pointing in distance,
5^0–8^0 exchange the semilenses and repeat the same operations in the reverse order.

In this way the mean epoch of the distance and position-angle measures will be approximately the same.

In the observations of standard-stars generally only one pointing in position-angle and one in distance was made in each position of the semilenses.

Sometimes it has occurred, when two pairs of satellites were in approximately the same distance, that after exchanging the semilenses another pair was observed than before. This is the case with the observations 236–237, 238–239, 467–468, 541–542. These observations must, of course, be rejected.

On a few occasions, when two satellites were very near together, the measures of distance and position-angle were combined by placing the images at half or double distance, as is done in measuring a double star.

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Sometimes four pointings were made instead of two in each position of the semilenses.

The instant a pointing is made the observer calls "stop" to an assistant, who then notes the time from a chronometer to the nearest second.

In Table I. are given the dates on which observations were secured. The different columns contain the rotation-numbers of the observations indicated by the headings; ***p*** denotes an observation of position-angle and ***s*** of distance. Rejected observations are inclosed in brackets.

When four pointings were taken the observation is marked with an asterisk (*); when only one pointing was made, by a dagger (†). In observing position-angles, the circle was as a rule read by estimation to the nearest half minute. On some occasions, however, the position-circle microscopes were used. These observations are distinguished by the letter *m* being appended to the rotation number.

TABLE I.

Summary of Dates and Observations.

Date.	Observer.	Victoria Standards.		Standards.		I.–II.		I.–III.		I.–IV.		II.–III.		II.–IV.		III.–IV.		Remarks.
		p	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	
1891.																		
Aug. 17	G					4	3	9	10	2	1	11	12	5	6	7	8	I 2, S 2.
19	G			† 13m	† 14	19	20	25	26	17	18	21	22	23	24	15	16	I 2–3, S 3.
20	G			† 31m	† 32	33	34	27	28	37	38	35	36	29m	30	39	40	I 2-3, S 3 until 20h 30m; after that I 2, S 3.
				† 41m	† 42													
21	G			† 47m	† 48	* 43	* 44									45	46	Definition too bad to measure distances of III. and IV. from I. and II., which are close together: accordingly repeated double distances of I.–II. as before in inte vals of good definition.
						* 49	* 50											I 3, S 3-4, sometimes I 2, S 3. I 2 for last measures of I.–II.
24	G			† 55	† 56	63	64	61	62	59	60	51	52	57	58	53m	54	I 2-3, S 2-3.
				† 65	† 66													
25	G			† 67	† 68	79	80	71	72	82	83	73	74	77	78	69	70	I 3, S 3 in beginning; after second standards I 2-3, S 2-3.
				† 75	† 76		81											
27	F			† 90	† 91					85	84			89	88	87	86	I 3-4, S 3.
29	G		107	† 98	† 99	102	103	100	101	92	93	104	105	94	95	96	97	I 2-3, S 3 up to standards; I 2-3, S 2-3 afterwards.
					106													
31	G		*119	†110	†111	120	121	122	123	124	125	108	109	114	115	112	113	I 3-4, S 3-4 up to standards; I 3, S 3 for standards double set; I 2-3, S 2-3 afterwards.
				†116	'117													
					118													
Sept. 1	G		136	†128	†129	141	[142]	132	133	130	[131]	139	140	137	138	126	127	No note as to definition.
				†134	135													
5	G	*143m	[160]	†148m	149	144	145	155	156	150	151	153	154	157	158	146	147	Till position angles of standards I 3, S 3-4; standards I 2-3, S 3-4; Satellites afterwards I 2, S 2-3.
				*152m	159													
7	G				173	163	164	165	166	161	[162]	171	172	167	168	169	170	I 2-3, S 2-3; though sometimes faint through cloud.

Sept. 1 131 Evident mistake of one scale division. *Rejected.*
142 One pointing not printed. *Rejected* on account of consequent uncertainty of time.
5 160 *Rejected* for discordance.
7 162 Record of pointings not printed.

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TABLE I.—*continued.*

Date.	Observer	Victoria Standards.		Standards.		I.–II.		I.–III.		I.–IV.		II.–III.		II.–IV.		III.–IV.		Remarks.
		p	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	
1891 Sept. 8	G		*184	179m	180 183							174	175	176	177	182m	178 *181	Cloud towards end; last observations very faint and difficult.
10	G		*200	†197	190 {198 199}	193	194	187	188	195	196	185	186	191	192		189	I 2, S 2–3.
13	G			207m	208	203	204	205	206	213	214	201	202	211	212	209	210	I 2, S 3.
14	G	*215m	233	*231m	[†228] 232	220	221	218	219	226	227	216	217	222	223 230	224	225 229	For first three sets of satellites I 3, S 3 and 3–4; for first two sets with IV., I and S 3–4 and 4; for I.–IV., I suddenly good 1–2 or 2. For standards, I 1–2, S 2–3; for repeats of II.–IV. and III.–IV. and afterwards I and S 1–2.
18	F			†246	247	234	[235]	240	241	[236] [238]	[237] [239]	242	243	[236] [238]	[237] [239]	244	245	I 2, S 2 up to I.–IV.; afterwards I 2, S 3; all observations through thin cloud.
19	G		*265	†255	254	248	249	250	251	258	259	252	253	260	261	256	257	Up to first standards I 2, S 3; for first standards I 1–2, S 2–3; remaining satellites except last pair I 1–2, S 2; last pair I 2–3, S 3, Jupiter standards I 2, S 2–3; Victoria standards I 1–2, S 2.
				†262	{263 264}													
20	G		*282	†272 †280	†273 [281]	268	269	270	271	266	267	278	279	276	277	274	275	Generally I 1–2, S 2; sometimes I 1, S 1–2; very satisfactory observations. Last pointings of Victoria standards through cloud.
21	G			†285 [†297]	†286 †298	293	294	291	292	295	296	[283]	284	287	288	289	290	I 1, S 1–2; certainly best definition as yet for Jupiter's satellites. Last standards I 1, S 2–3.
22	G		315	†307 †313	308 314	299	300	301	302	309	310	303	304	311	312	305	306	On the whole I 2, S 2–3; Jupiter standards I 2, S 3–4; Victoria standards I 1–2, S 2.
23	G		327	†316 †325	†317 326							320	321	322	323	318	319 324	I generally 2, S 2–3; last standards I 1–2, S 3; first standards I 2, S 3–4. Satellite I. eclipsed.
24	G					*328	*329											I varied from 2 to 3, but S becomes too bad. Impossible to measure double distance on account of interference of planet.
28	G			†330	331	332	333											When observable I 3, S 3-4. Satellite III. covered by undulations of planet's edge.
29	G			†338	339	334	335	340	341							336	337	For first two pairs I 2, S 3; for standards I 2, S 3–4; after first pointing of last pair I 3-4, S 4. Measures snatched in moments of comparative steadiness. Obliged to give up.
Oct. 4	G			†348	349	346	347	354	355	344	345	342	343	352	353	350	351	For first two pairs I 3, S 3; third pair I 2–3, S 2–3; for remainder I 3, S 3.
6	F			†362	363			364	365	360	361			358	359	356	357	For standards I 3, S 3–4; Satellites I 2–3, S 3; faint through fog.
9	G			†372	373	368	369	366	367	376	377	*378	[379]	374	375	370	371	First pair I 3, S 3; second pair I 2–3, S 2–3; last two pairs I 2, S 2-3; remainder I 2-3, S 3.
10	G							380	381							382	383	I 2–3, S 3. Stopped by cloud.
13	G			†390	†391	384	385	388	389	397	398	386	387	395	*396	393	394	Up to standards I 1–2, S 1–2; afterwards I 2–3, S 2–3; last pair I 3, S 3.
14	F			†411	†412	399	400	405	406	403	404	409	410	401	402	407	408	Up to I.–III. I 2–3, S 3; afterwards I 3, S 3–4.
15	F											417	418	415	416	413	414	First two pairs I 3, S 3; last pair I 3-4, S 3–4 and 4.
16	F			†427	428	423	424	429	430	425	426	421	422	419	420	431	432	Standards I 4, S 4, unobservable. Satellite I. on planet. Up to standards I 2–3, S 3; standards I 2–3, S 3–4; afterwards I 3, S 3–4.
17	F			†439	440			433	434	435	436					437	438	First two pairs I 2–3, S 3–4; third pair I 3, S 3; standards I 3, S 3–4. Satellite II. not visible.

Sept. 14 228 *Rejected* for discordance.
18 235 Evident mistake of one scale division. *Rejected.*
236–7 and 238–9 Of both these observations the first half is an observation of II.–IV.; the second half of I.–IV. *Rejected.*
20 281 *Rejected* for discordance.
21 283 Evident mistake of 30′ in possible angle. *Rejected.*
297 Incomplete. Interrupted by cloud. *Rejected.*
Oct. 9 379 One pointing not printed. *Rejected* on account of consequent uncertainty of time.
13 392 In assigning rotation-numbers to the observations, the number 392 was inadvertedly omitted.

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TABLE I.—*continued.*

Date.	Observer.	Victoria Standards.		Standards.		I.–II.		I.–III.		I.–IV.		II.–III.		II.–IV.		III–IV.		Remarks.
		p	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	
1891.																		
Oct. 19	F			†447	†448	445	446	[441]	442	449	450	443	444	453	454	451	452	On the whole I 2, S 3; standards I 1–2, S 3.
23	F			†461	462							457	458	455	456	[459]	[460]	I 2, S 3. I. occulted; III. and IV. very close and difficult.
Nov. 5	G			†469	470	465	466	463	464	473	474	[467]	[468]	471	472	[467]	[468]	Standards I 3, S 3–4; Satellites I 2–3, S 3–4.
																475	476	
6	G			†483	484	481	482	479	480	485	486	[477]	478			487	488	Up to and including standards I 2, S 2–3; afterwards suddenly I and S 3 and 3–4. Stopped by cloud.
8	G			†493	494							489	490	491	492	495	496	Previous to standards I 2, S 2–3; after I and S 2–3 and 3.
9	G					497	498										499	Clouded up.
19	G															[500]	[501]	Bad definition; cloud.
22	G			†508	509	502	503				512	504	505	510	511	506	507	Up to standards I 2, S 2–3; afterwards I 3, S 3–4. Stopped because getting worse.
27	G			†519	520			515	516	513	514					517	518	First two pairs I 2–3, S 2–3; last two I 2, S 2–3.
30	F					[521]	[522]	[523]	[524]									Definition gone utterly bad, impossible to finish.
Dec. 8	F			†531	532	527	528	525	526	529	530	537	538	535	536	533	534	Up to standards I 2, S 3; standards and afterwards I 3, S 3–4.
14	G			†545	†546			549	550	547	548	[541]	[542]	[541]	[542]	539	540	At first I 2–3, S 3; third pair, standards and pair after I 2, S 3. Last pair I 3, S 3–4.
														543	544			

Oct. 19	441	Evident mistake of 1°. *Rejected.*
23	459–60	Distance of less than 5″, measured by "crossing through." *Rejected* on account of unreliability of this method of observing so short a distance.
Nov. 5	467–68	Second half of observation (after reversing of segments) is of pair III.–IV. *Rejected.*
6	477	Evident mistake of 2°. *Rejected.*
19		All observations on this date are *rejected* on account of bad definition, and uncertainty of chronometer correction.
30		All observations on this date are *rejected* on account of bad definition.
Dec. 14	541–2	The first half is an observation of II.–III.; the second half of II.–IV. *Rejected.*

In Table II. are given for each date the details of the observations of the satellites. At the beginning of the record of each date the observer's name is given, and at the end the readings of the meteorological instruments. B is the reading of the barometer, *t* and T are those of the attached and external thermometer respectively. For each observation of a pair of satellites the name of the pair is given together with the rotation numbers of the observations of position-angle (***p***) and of distance (***s***). The chronometer-times of each pointing are then given together with the readings of the scale-microscopes or of the position-circles. In a few cases (*e.g.* the first observation, No. 1) the position-circle was read at the moment of making the separate pointings in distance. These readings are added in parentheses. The use that is made of these readings will be explained below.

The observations of the standard-stars are not given in detail, but only referred to by mentioning the rotation numbers.

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TABLE II.—Details of Pointings.

Chronometer Times.	Readings of Scales or Position-Circle.		
August 17.			Gill.
	I.–IV.		p : 2 s : 1
h m s			r
20 31 3	100	300	4·598 (170° 12′·5)
34 56·5			4·490 (193 43)
36 53	99	300	4·926 (192 18)
38 34·5			4·920 (177 34)
	°	′	
41 45·5	188	57	
42 35	189	20	
43 49·5	193	10	
44 56·5	194	10	
	I.–II.		p : 4 s : 3
			r
20 49 44	98	301	5·776
55 13			5·579
57 17	101	299	3·816
21 0 42			3·866
	°	′	
2 5	247	50·5	
5 38		47	
6 19		24	
6 54		11	
	II.–IV.		p : 5 s : 6
	°	′	
21 8 35	238	33	
10 19·5	238	28	
			r
11 42	98	302	3·716
13 37·5			3·710
20 8	101	298	5·582
21 28			5·544
	°	′	
22 10	239	2	
23 20·5	239	7	
	III.–IV.		p : 7 s : 8
	°	′	
21 26 18	239	12	
28 49	239	12	
			r
30 32	102	297	5·655
31 33			5·628
34 2	97	303	3·743
37 14			3·850
	°	′	
38 19	239	4	
39 2·5	239	8	

Chronometer Times.	Readings of Scales or Position-Circle.		
August 17—*continued*.			
	I.–III.		p : 9 s : 10
h m s	°	′	
21 41 18·5	243	53	
42 59	243	53	
			r
44 23·5	97	302	4·121
45 57			4·090
47 14	102	298	5·250
50 15			5·211
	°	′	
51 47	244	13	
53 42	244	13	
	II.–III.		p : 11 s : 12
	°	′	
21 57 5·5	239	28	
22 3 15	239	24	
			r
6 50	99	302	3·402
8 43			3·429
10 57	101	299	5·801
12 13			5·780
	°	′	
13 4	240	16	
14 22	240	24	

h m	B in.	t °	T °
20 25	30·29	56·5	56·2
22 15	30·30	56·8	57·0

August 19.			Gill.
	Standards.		p : 13 s : 14
	III.–IV.		p : 15 s : 16
h m s	°	′	′
20 25 30·5	65	14	14·5
27 1	65	16	17
			r
29 1	93	307	5·818
31 45·5			5·827
33 27	107	294	3·500
35 15			3·523
	°	′	′
36 44	65	27	28
37 45	65	23·5	24

Chronometer Times.	Readings of Scales or Position-Circle.		
August 19—*continued*.			
	I.–IV.		p : 17 s : 18
h m s	°	′	′
20 38 48	64	22	23
39 52	64	21·5	22·5
			r
42 4·5	106	294	2·923
44 9·5			2·880
46 59	93	307	2·490
48 2			2·489
	°	′	′
49 45	64	13·5	14·5
51 13	64	14	15
	I.–II.		p : 19 s : 20
	°	′	′
20 55 9	65	57	58
58 6	66	1	1·5
			r
21 2 18	96	304	5·936
3 45·5			5·936
18 22	104	297	3·757
20 13			3·782
	°	′	′
21 28	66	23·5	24
22 48	66	21·5	22
	II.–III.		p : 21 s : 22
	°	′	′
21 24 2·5	67	50	50·5
24 52	67	51·5	52
			r
26 37	104	296	5·279
28 26			5·317
39 20	96	305	3·560
40 57·5			3·467
	°	′	′
42 48	67	53	54
44 30	67	46	47
	II.–IV.		p : 23 s : 24
	°	′	′
21 47 35	61	44	45
48 29	61	49·5	50
			r
49 56·5	97	303	4·970
54 38			5·125
56 13·5	102	297	4·164
57 22·5			4·066
	°	′	′
58 19	62	4	5
59 48	62	4·5	5

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
August 19—*continued.*				
	I.–III.		*p* : 25	*s* : 26
h m s	°	′	′	
22 1 37	86	8·5	9	
2 37	86	28	29	
			r	
4 10	99	300	4·956	(91 43)
5 31			4·880	(79 12)
6 56	100	300	4·420	(82 44)
8 12			4·461	(96 12)
	°	′	′	
9 10	89	32·5	33	
10 6	90	7·5	8·5	

h m	B in.	*t* °	T °
20 0	30·46	55·7	54·6
22 11	30·46	54·2	54·0

August 20.			GILL.	
	I.–III.		*p* : 27	*s* : 28
h m s	°	′	′	
19 30 16·5	96	32·5	33	
32 32	98	19	19·5	
			r	
35 55	100	300	4·391	(104 41)
38 24			4·444	(94 45)
40 5	99	300	4·939	(101 31)
42 42			4·828	(87 4)
	°	′	′	
44 15	97	11	11·5	
45 31	97	39	39·5	
	II.–IV.		*p* : 29	*s* : 30
	°	′	′ ″	′ ″
19 53 57	64	32	8 18	9 56
57 1	64	31	7 5	7 55
			r	
58 39	91	309	6·028	
20 0 27			6·019	
2 16	109	292	3·360	
3 50			3·279	
	°	′	′ ″	′ ″
5 18	64	47	12 11	12 55
7 32	64	45	9 50	10 45

Chronometer Times.	Readings of Scales or Position-Circle.			
August 20—*continued.*				
	Standards.		*p* : 31	*s* : 32
	I.–II.		*p* : 33	*s* : 34
h m s	°	′	′	
20 36 41	64	51·5	52	
38 40	64	49·5	50·5	
			r	
40 37·5	96	304	6·251	
44 13			6·220	
45 55·5	104	297	3·162	
47 3·5			3·169	
	°	′	′	
47 48	65	20·5	21	
49 45	65	29	29·5	
	II.–III.		*p* : 35	*s* : 36
	°	′	′	
20 51 31	63	10·5	11	
52 24	63	4·5	5·5	
			r	
57 7	104	296	6·330	
58 7			6·305	
21 0 6	96	305	3·200	
1 23			3·230	
	°	′	′	
2 16	62	44·5	45·5	
2 56	62	45·5	46	
	I.–IV.		*p* : 37	*s* : 38
	°	′	′	
21 4 53·5	64	8·5	9	
5 58	64	10·5	11·5	
			r	
6 47·5	94	305	4·299	
9 10			4·353	
10 54·5	105	295	5·031	
13 44			4·959	
	°	′	′	
15 58	64	34	35	
17 16	64	34	35	

Chronometer Times.	Readings of Scales or Position Circle.			
August 20—*continued.*				
	III.–IV.		*p* : 39	*s* : 40
h m s	°	′	′	
21 19 26	66	20·5	21	
20 6·5	66	22	22·5	
			r	
20 57	105	296	2·459	
22 35·5			2·496	
24 19·5	95	306	2·996	
25 12			2·945	
	°	′	′	
26 28	66	0	0·5	
27 19	65	57	58	
	Standards.		*p* : 41	*s* : 42

h m	B in.	*t* °	T °
19 28	30·44	49·5	45·0
20 30	30·44	48·3	44·2
21 40	30·42	47·0	43·2

August 21.			GILL.		
	Double distance.		I.–II.	*p* : 43	*s* : 44
h m s	°	′			r
19 34 44	83	29	100	301	3·784
38 23	85	39			3·710
39 52	85	27	100	300	5·789
45 27	86	32			5·820
46 41	86	24	100	300	5·861
51 0	86	50			5·880
52 40·5	82	48	100	301	3·608
55 48	82	45			3·640
	III.–IV.		*p* : 45		*s* : 46
	°	′	′		
20 0 23	66	12	13		
1 13	66	11	12		
			r		
3 43·5	96	304	3·127		
5 36			3·179		
7 0	103	296	6·348		
9 10			6·400		
	°	′	′		
10 25	66	24·5	25·5		
11 15·5	66	21·5	22·5		

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.
August 21—*continued.*	
	Standards. p : 47 s : 48
Double	distance. I.–II. p : 49 s : 50
h m s	° ′ r
20 38 32	82 31 100 300 5·830
40 31·5	85 4 5·889
42 55	81 18 100 301 3·672
44 10	81 18 3·637
46 38	81 42 100 301 3·665
48 3	81 18 3·666
48 55	81 50 100 300 5·782
50 27·5	83 23 *
	B t T
h m	in. ° °
19 32	30·16 53·5 50·0
20 24	30·15 53·3 50·5
August 24.	GILL.
Double	distance. II.–III. p : 51 s : 52
h m s	° ′ ′ r
19 52 14·5	28 58 59 100 300 4·040
53 59	29 57 57·5 4·002
55 29·5	29 30·5 31 99 300 5·391
57 15	29 40·5 41 5·491
	III.–IV. p : 53 s : 54
20 0 46·5	66 3 8 30 8 5
3 42·5	66 3 7 55 8 10
	r
6 21·5	92 308 4·790
7 42	4·769
11 9·5	107 292 4·665
14 39	4·585
16 17	66 11 5 43 7 59
20 11	66 12 12·5
	Standards. p : 55 s : 56

* Last pointing not printed.

Chronometer Times.	Readings of Scales or Position-Circle.
August 24—*continued.*	
	II.–IV. p : 57 s : 58
h m s	° ′ ′
20 34 53	67 0·5 1·5
35 47·5	66 59·5 60·0
	r
37 59·5	107 293 2·507
39 8	2·560
40 9	92 308 2·863
41 42	2·850
42 23	66 56 57
42 52	66 55 56
	I.–IV. p : 59 s : 60
20 45 19·5	67 1 2
45 57	67 1 2
	r
47 28	105 295 2·819
48 5·5	2·840
49 51·5	94 306 2·622
51 37·5	2·661
52 33	66 54·5 55·5
53 19	66 57 58
	I.–III. p : 61 s : 62
20 56 11·5	64 1 2
56 49	63 57 58
	r
58 12	97 302 4·889
59 26	4·862
21 0 48·5	102 298 4·560
1 59·5	4·554
2 47	64 13 14
3 55	64 9·5 10

Chronometer Times.	Readings of Scales or Position-Circle.
August 24—*continued.*	
	I.–II. p : 63 s : 64
h m s	° ′ ′
21 5 33·5	66 58 59
6 8·5	67 4 5
	r
6 57·5	102 299 3·242
8 9	3·281
11 37·5	98 302 5·980
13 22·5	5·930
14 29·5	66 47 47·5
15 58	66 50·5 51·5
	Standards. p : 65 s : 66
	B t T
h m	in. ° °
19 46	30·45 48·0 46·0
20 31	30·48 48·2 47·9
21 24	30·46 48·0 47·3
August 25.	GILL.
	Standards. p : 67 s : 68
	III.–IV. p : 69 s : 70
h m s	° ′ ′
19 50 55	67 28·5 29
51 45	67 29 29·5
	r
53 47	106 293 6·022
55 40	6·026
56 20·5	93 307 3·396
57 10·5	3·348
58 3·5	67 14 15
20 0 21·5	67 12 12·5

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
August 25—*continued.*				
	I.–III.		p : 71	s : 72
h m s	°	′	′	
20 2 21·5	65	7	7·5	
3 51	65	7	8	
			r	
5 3	94	306	5·977	
7 25			6·016	
8 24·5	106	295	3·369	
8 27·5			3·360	
10 23·5	*66	22	22·5	
11 3·5	*66	23	24	
	II.–III.		p : 73	s : 74
20 12 30·5	64	55	56	
14 14·5	64	56	57	
			r	
15 37	105	295	3·238	
16 33·5			3·250	
18 29	94	305	6·290	
19 47			6·297	
21 2	64	42	43	
22 12·5	64	43	44	
	Standards.		p : 75	s : 76
	I.–IV.		p : 77	s : 78
20 35 35	80	34	35	
36 38	80	24·5	25	
			r	
39 0·5	99	302	2·829	(78 28)
40 7			2·763	(82 36)
42 14	101	300	2·731	(83 48)
43 38·5			2·768	(80 21)
44 52	82	30	30·5	
45 59	82	24	25	

* Altered to 65°. The observation should have been rejected, but was included by mistake.

Chronometer Times.	Readings of Scales or Position-Circle.			
August 25—*continued.*				
	I.–II.		p : 79	s : 80, 81
h m s	°	′	′	
20 49 3·5	71	48·5	49	
49 39	71	39·5	40	
			r	
50 25	100	300	2·791	(66 21)
51 45			2·857	(66 21)
53 17	99	301	2·607	(65 13)
54 57			2·611	(73 42)
57 53·5	68	59·5	60	
58 46·5	69	3	4	
			r	
21 0 1	99	301	2·580	(64 32)
1 36			2·570	(73 53)
4 2	100	300	2·858	(67 44)
6 47			2·900	(77 8)
	II.–IV.		p : 82	s : 83
21 10 13	102	13	13·5	
11 32	102	7	8	
			r	
12 36·5	100	300	3·540	(98 0)
13 44			3·576	(107 40)
15 17·5	99	300	5·808	(107 40)
17 1·5			5·720	(94 46)
18 25	102	11	12	
19 26·5	101	59	59·5	

	B	t	T
h m	in.	°	°
19 38	30·56	52·5	52·0
21 26	30·55	50·3	49·2

Chronometer Times.	Readings of Scales or Position-Circle.			
August 27.			FINLAY.	
	I.–IV.		s : 84	p : 85
h m s			r	
19 40 59	94	306	4·775	
42 19·5			4·820	
47 12	63	54	54·5	
48 32·5	63	53·5	54	
50 44·5	64	10	10·5	
52 44	64	15	15·5	
			r	
54 0·5	105	294	4·494	
20 0 47·5			4·349	
	III.–IV.		s : 86	p : 87
			r	
20 3 53	105	296	3·220	
5 50			3·080	
7 52	62	22·5	23	
10 58	62	21	21·5	
12 59	62	3·5	4	
14 34	62	10	10	
			r	
17 52	95	306	2·940	
19 14·5			2·967	
	II.–IV.		s : 88	p : 89
			r	
20 25 35·5	102	298	5·721	
27 12			5·691	
28 37·5	63	25	25·5	
30 25	63	27·5	28	
			r	
33 23	98	303	3·780	
35 13·5			3·750	
37 2	62	44·5	45	
38 35	62	44	44	
	Standards.		p : 90	s : 91

	B	t	T
h m	in.	°	°
19 39	30·05	59·5	57·0
21 2	30·02	55·1	53·9

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.	
August 29.		GILL.
	I.–IV.	*p* : 92 *s* : 93
h m s	° ′	′
19 56 22	65 40	40·5
59 41	65 40·25	41
		r
20 0 54	91 310	2·969
2 4		2·922
4 19	109 292	2·519
5 27		2·530
	° ′	′
7 1	65 49·75	50·5
9 6	65 50·25	50·75
	II.–IV.	*p* : 94 *s* : 95
20 10 19·5	65 18	19
12 8·9	65 18·5	19·5
		r
14 56	110 290	4·072
15 50		4·017
16 58	89 310	5·361
19 53		5·399
	° ′	′
21 59	65 10	10·5
24 5	65 12	12·5
	III.–IV.	*p* : 96 *s* : 97
20 26 11	65 43	44
28 43·5	65 43	44
		r
30 27	88 313	2·911
31 44		2·939
34 47	112 289	2·450
35 48		2·460
	° ′	′
38 18	65 50	50·5
40 5	65 50	50·5
	Standards.	*p* : 98 *s* : 99

Chronometer Times.	Readings of Scales or Position-Circle.	
August 29—*continued.*		
	I.–III.	*p* : 100 *s* : 101
h m s	° ′	′
20 53 0	65 32	32·5
53 46	65 29·5	30·25
		r
54 37	96 303	3·710
55 42·5		3·772
56 54	103 297	5·528
58 23		5·492
	° ′	′
59 4·5	66 1	1·5
21 1 1·5	65 54	54·5
	I.–II.	*p* : 102 *s* : 103
	° ′	′
21 3 13	62 4	5
4 30	61 58	59
		r
5 42	98 302	3·481
6 41		3·496
7 54	101 298	5·790
9 16		5·732
	° ′	′
11 55·5	62 47·5	48
13 43	62 57	58
	II.–III.	*p* : 104 *s* : 105
	° ′	′
21 17 1	69 1·5	2
18 11	68 57	57·5
		r
20 25	98 302	2·923
21 45		2·900
24 6	101 299	2·539
25 48		2·580
	° ′	′
26 37	69 50	50·5
28 40	69 37	37·5
	Standards.	*s* : 106
	Victoria Standards.	*s* : 107

h m	B in.	*t* °	T °
19 53	30·48	51·5	46·3
20 50	30·47	49·7	45·8
21 51	30·45	49·0	45·9

Chronometer Times.	Readings of Scales or Position-Circle.	
August 31.		GILL.
	II.–III.	*p* : 108 *s* : 109
h m s	° ′	′
19 39 22	81 13	13·5
41 0	81 11·5	12
		r
42 28	100 300	3·590 (76 17)
45 16·5		3·657 (87 35)
46 8	99 300	5·680 (87 35)
48 2		5·654 (71 34)
	° ′	′
50 41·5	75 38	39
51 13·5	79 19	19·5
	Standards.	*p* : 110 *s* : 111
	III.–IV.	*p* : 112 *s* : 113
20 10 6	68 32·5	33
11 27	68 31·5	32
		r
12 49·5	105 296	3·909
17 30		4·024
19 34	95 305	5·330
22 7		5·290
	° ′	′
22 40	68 21·5	22·5
25 24	68 24·5	25·5
	II.–IV.	*p* : 114 *s* : 115
	° ′	′
20 27 27	67 16	17
28 19	67 20·25	20·75
		r
31 25	95 305	2·810
32 13		2·740
33 20	104 296	2·671
34 27		2·641
	° ′	′
36 18	67 26	27
37 45	67 31	31·5
	Standards.	*p* : 116 *s* : 117, 118
	Victoria Standards.	*s* : 119

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
August 31—*continued.*				
	I.–II.		p : 120	s : 121
h m s	°	′	′	
21 19 12	66	24·5	25	
19 56	66	19·5	20·25	
			r	
21 22	97	303	2·432	
23 41			2·385	
24 53	102	298	3·046	
25 54			3·097	
26 38	66	51·5	52	
27 33·5	66	47	47·5	
	I.–III.		p : 122	s : 123
21 28 44	64	24	24·5	
29 21	64	12	13	
			r	
30 0	102	298	4·437	
30 57			4·473	
32 6	97	302	4·896	
32 59			4·881	
33 45	63	53	54	
34 28	63	54	55	
	I.–IV.		p : 124	s : 125
21 36 23·5	67	3·5	4	
37 15·5	67	3	4	
			r	
38 38	93	307	5·314	
39 34·5			5·331	
41 4	106	293	4·150	
42 49			4·210	
43 27	67	12	12·5	
44 45	67	11·5	12	

h m	B in.	t °	T °
19 38	30·17	57·2	55·5
21 46	30·16	54·1	51·5

Chronometer Times.	Readings of Scales or Position-Circle.			
September 1.		GILL.		
	III.–IV.		p : 126	s : 127
h m s	°	′	′	
19 38 57	75	2	2·5	
39 34	74	54	55	
			r	
41 8	99	302	3·591	(72° 58′)
42 23·5			3·539	(77 42)
43 39	101	299	5·870	(78 33)
45 29			5·880	(73 34)
47 28	76	4·5	5·5	
48 29	76	25	26	
50 21·5	76	12	13	
51 11	76	32	33	
	Standards.		p : 128	s : 129
	I.–IV.		p : 130	s : 131
20 3 10	66	52	53	
4 12	66	50·75	51	
			r	
6 2	106	294	2·852	
6 57			2·848	
8 20	93	306*	2·604	
9 17·5			2·649	
10 27·5	66	39·75	40·25	
11 27	66	38	39	
	I.–III.		p : 132	s : 133
20 12 33	64	52	53	
15 0	64	53	54	
			r	
16 58·5	94	306	2·558	
19 13·5			2·601	
20 36	105	295	2·748	
21 22			2·734	
22 9	65	8·5	9	
23 16	65	7	8	
	Standards.		p : 134	s : 135

* Altered to 307. (The combination 106 294, 93 306 does not give the same result for the two positions of the semilenses). *Rejected.*

Chronometer Times.	Readings of Scales or Position-Circle.			
September 1—*continued.*				
	Victoria Standards			s : 136
	II.–IV.		p : 137	s : 138
h m s	°	′	′	
20 58 25	66	32	32·5	
59 21	66	31	32	
			r	
21 0 32	94	307	2·611	
1 25·5			2·637	
4 39	106	395	2·720	
5 48·5			2·720	
7 3	66	43	44	
7 57	66	43	44	
	II.–III.		p : 139	s : 140
21 9 15	64	52·5	53	
9 57	64	50	50·5	
			r	
11 34	105	295	6·030	
13 56			5·995	
15 7	95	306	3·420	
16 13			3·470	
17 1	64	35·5	36·5	
18 17	64	37	37·5	
	I.–II.		p : 141	s : 142
21 20 17·5	67	27	27·5	
20 56	67	14	15	
			r	
21 47·5	99	301	3·681	(64° 20′)
23 31			3·678	(71 7)
25 49	100	299	5·709	(72 49)
27 17·5			*	(66 44)
28 28	69	21	22	
29 11	69	33·5	34·5	

h m	B in.	t °	T °
19 52	30·04	58·0	57·2
21 30	30·01	56·5	54·8

* One pointing not printed. *Rejected.*

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
September 5.			GILL.	
	Victoria Standards.		**p** : 143	
	I.–II.		**p** : 144	**s** : 145
h m s	°	′	′	
19 33 33·5	53	40	40·5	
36 12·5	54	33	34	
			r	
37 18	99	301	2·434	(48° 32′)
39 54			2·530	(59 52)
44 42·5	100	300	2·650	(62 16)
45 50			2·606	(51 31)
48 16	57°	23′	24′	
49 7	58	3	4	
	III.–IV.		**p** : 146	**s** : 147
19 51 21	56°	48′	49′	
52 44·5	56	53	53·5	
			r	
54 4	99	302	2·929	(54° 11′)
55 24			2·957	(59 49)
57 23·5	101	300	2·399	(59 49)
20 0 15			2·340	(55 31)
2 24·5	58°	2′·5	3′	
4 44·5	57	50·5	51	
	Standards.		**p** : 148	**s** : 149
	I.–IV.		**p** : 150	**s** : 151
20 27 5·5	63°	31′	31′·5	
28 13	63	32	33	
			r	
29 48·5	96	305	2·782	
31 6			2·849	
34 14	104	297	2·464	
35 47			2·382	
36 52	63°	44′	45′	
38 20	63	34	35	
	Standards.		**p** : 152	

Chronometer Times.	Readings of Scales or Position-Circle.			
September 5—*continued.*				
	II.–III.		**p** : 153	**s** : 154
h m s	°	′	′	
21 4 51	67°	50′·5	51′·5	
5 40·5	67	52	53	
			r	
6 17·5	98	303	3·442	
8 16·5			3·369	
11 46·5	102	298	6·039	
12 52·5			6·051	
13 29	68°	20′·5	21′	
14 22	68	25·5	26·5	
	I.–III.		**p** : 155	**s** : 156
21 16 46·5	65°	40′	40′·5	
18 12·5	65	36·5	37·5	
			r	
18 58·5	97	304	2·630	
20 31			2·656	
21 20	103	298	2·680	
22 4			2·650	
22 47·5	65°	48′·5	49′	
23 39	65	47·5	48·5	
	II.–IV.		**p** : 157	**s** : 158
21 25 10	64°	30′·5	31′	
27 23	64	31·5	32·5	
			r	
28 59	96	303	4·839	
29 44			4·807	
32 33	103	297	4·491	
33 30			4·486	
34 47·5	64°	38′·5	39′·5	
35 26	64	43	43·5	
	Standards.		**s** : 159	
	Victoria Standards.		**s** : 160	

h m	B in.	*t* °	T °
19 30	30·16	57·1	57·0
21 0	30·15	56·2	55·9
22 0	30·14	56·5	55·7

Chronometer Times.	Readings of Scales or Position-Circle.			
September 7.			GILL.	
	I.–IV.		**p** : 161	**s** : 162
h m s	°	′	′	
19 50 11	65°	40′	40′·5	
50 57	65	41	41·5	
51 32	107	292	*	
52 48			*	
53 33	92	308	*	
54 40			*	
55 20	65°	27′·5	28′·5	
56 54 5	65	28·5	29	
	I.–II.		**p** : 163	**s** : 164
19 59 33	65°	59′	59′·5	
20 0 19·5	65	50	50·5	
			r	
2 9·5	96	303	4·747	
2 59			4·700	
3 53	103	297	4·520	
5 15·5			4·539	
6 10·5	66°	19′	19′·5	
12 30	66	18·5	19	
	I.–III.		**p** : 165	**s** : 166
20 15 5·5	63°	52′·5	53′	
15 50	63	50·5	51	
			r	
17 51	97	302	6·322	
18 34·5			6·331	
19 26	102	298	2·888	
20 29·5			2·886	
21 18·5	64°	21′·5	22′	
21 56	64	26	27	

* Micrometer-head readings not printed.

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.		
September 7—*continued.*			
	II.–IV.	p : 167	s : 168
h m s			
20 24 37	65° 40′	40′·5	
25 21	65 40·5	41	
		r	
27 6	89 311	5·670	
28 17		5·691	
29 15·5	111 290	3·500	
30 54		3·462	
	65° 50′·5	51′	
33 4	65 50	50·5	
33 48·5			
	III.–IV.	p : 169	s : 170
20 35 0	65° 18′	19′	
36 6·5	65 19	19·5	
		r	
37 22	110 290	5·173	
39 4·5		5·108	
40 44	90 311	4·129	
42 57		4·154	
	65° 10′·5	11′	
44 2	65 10	11	
44 58			
	II.–III.	p : 171	s : 172
20 48 42	72° 27′·5	28′·5	
49 22·5	72 9	9·5	
		r	
50 14	99 301	4·195	(69° 24′)
51 16·5		4·192	(75 36)
52 24	100 299	5·010	(77 37)
53 7·5		5·031	(71 23)
	74° 19′·5	20′·5	
54 22·5	74 30·5	31	
55 13			
	Standards.	s : 173	

h m	B in.	t °	T °
20 9	30·22	58·5	57·1
21 7	30·24	58·0	57·1

Chronometer Times.	Readings of Scales or Position-Circle.		
September 8.		GILL.	
	II.–III.	p : 174	s : 175
h m s			
19 31 34·5	64° 5′	6′	
32 6	64 4	4·5	
		r	
33 5·5	96 305	2·695	
33 59		2·751	
36 49	104 297	2·420	
37 52·5		2·409	
	64° 24′	24′·5	
38 22	64 22·5	23	
39 10·5			
	II.–IV.	p : 176	s : 177
	66° 50′	51′	
19 41 36·5	66 50·5	51	
42 54			
		r	
47 17	92 309	3·448	
48 7		3·401	
50 15	108 292	5·940	
51 39		5·979	
	67° 0′·5	1′·5	
53 30·5	67 0	0·5	
54 31			
	III.–IV.	s : 178	
		r	
19 58 47	88 313	4·001	
20 1 33		3·995	
4 49	112 288	5·279	
6 18		5·272	
	Standards.	p : 179	s : 180

Chronometer Times.	Readings of Scales or Position-Circle.		
September 8—*continued.*			
	III.–IV.	s : 181	p : 182
h m s		r	
20 23 27	87 312	3·981	
24 51		3·986	
25 59·5	112 288	5·282	
26 49·5		5·265	
28 33		5·260	
30 4		5·254	
30 56	88 313	3·959	
31 56		3·994	
	66° 1′	6′ 54″	7′ 50″
32 43·5	66 2	8 12	9 12
35 34	66 8	13 10	14 5
37 10	66 7	11 48	12 50
40 6·5			
	Standards.	s : 183	
	Victoria Standards.	s : 184	

h m	B in.	t °	T °
19 55	30·10	56·7	56·0
21 12	30·10	56·5	56·0

September 10.	GILL.		
	II.–III.	p : 185	s : 186
h m s			
19 32 5·5	32° 14′·5	15′·5	
32 59·5	32 35	35·5	
		r	
33 57	99 300	5·860	(37° 0′)
35 23·5		5·898	(28 55)
37 4	100 300	3·349	(33 13)
37 59·5		3·302	(43 50)
	40° 36′	37′	
40 20	40 55	56	
41 47			

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.
September 10—*continued.*	
	I.–III. **p** : 187 **s** : 188
h m s	° ′ ′
19 43 52	68 33 34
44 57	68 39 39·5
	r
45 57	98 302 4·800
47 0	4·798
48 32	101 298 4·501
49 57	4·500
50 33	° ′ ′ 69 41 41·5
52 6·5	69 43 44
	III.–IV. **s** : 189
	r
19 53 34·5	95 304 3·620
54 37·5	3·580
55 54	104 296 5·818
57 2	5·799
	Standards. **s** : 190
	II.–IV. **p** : 191 **s** : 192
	° ′ ′
20 13 52	67 9·5 10
15 31	67 5 5·5
	r
16 19·5	95 305 3·546
17 5	3·536
18 26·5	104 295 5·717
19 45·5	5·692
	° ′ ′
20 46·5	67 30·5 31
21 39	67 30 30·5
	I.–II. **p** : 193 **s** : 194
	° ′ ′
20 24 42	63 11 11·5
25 23	63 11 11·5
	r
27 5·5	97 302 6·133
28 26	6·179
29 37·5	102 298 3·040
30 43	2·991
	° ′ ′
31 37	63 55·5 56
32 18	63 59 60

Chronometer Times.	Readings of Scales or Position-Circle.
September 10—*continued.*	
	I.–IV. **p** : 195 **s** : 196
h m s	° ′ ′
20 34 19	71 3 3·5
35 37	71 2·5 3
	r
36 59·5	98 303 3·929
39 26	3·800
42 18	102 298 5·560
44 36	5·628
	° ′ ′
45 32	71 51·5 52
46 40	72 6 7
	Standards. **p** : 197 **s** : 198, 199
	Victoria Standards. **s** : 200
	B *t* T
h m	° ′ ′
19 59	30·46 48·2 47·5
21 21	30·48 48·5 48·0
September 13.	GILL.
	II.–III. **p** : 201 **s** : 202
h m s	° ′ ′
19 56 24	70 10 10·5
57 23	70 11·5 12
	r
58 35	99 301 3·400
20 0 27	3·411
3 6	100 299 5·845
4 20	5·844
	° ′ ′
5 15	74 32 32·5
6 14	74 26 27

Chronometer Times.	Readings of Scales or Position-Circle.
September 13—*continued.*	
	I.–II. **p** : 203 **s** : 204
h m s	° ′ ′
20 10 14	66 41·5 42·5
11 25	66 43 44
	r
12 33	96 303 5·490
15 2	5·370
17 43	103 297 3·961
21 49	4·110
	° ′ ′
23 0	67 28 29
25 17	67 28 29
	I–III. **p** : 205 **s** : 206
	° ′ ′
20 26 38	68 14·5 15·5
28 40	68 13 14
	r
29 31	104 297 3·630
31 23	3·690
33 7	96 304 5·488
33 57	5·431
	° ′ ′
35 39	67 43 43·5
36 32	67 40·5 41
	Standards. **p** : 207 **s** : 208
	III.–IV. **p** : 209 **s** : 210
	° ′ ′
20 57 20	65 33 34
58 36	65 36·5 37
	r
59 31	92 307 5·480
21 1 49	5·481
3 23	107 293 3·768
4 43	3·762
	° ′ ′
6 0	65 52 53
7 36	65 50 50·5

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.
September 13—*continued.*	
	II.–IV. p : 211 s : 212
h m s	
21 8 52	65° 15′·5 16′·5
10 0	65 20·5 21
	r
10 41	106 293 4·770
12 0	4·810
13 10	93 307 4·422
14 29	4·425
	65° 0′ 0′·5
15 9	65 0·5 1
16 31	
	I.–IV. p : 213 s : 214
21 18 44	63° 2′·5 3′
19 35	63 1·5 2·5
	r
21 47	96 304 4·568
22 8	4·649
23 18	103 296 4·560
24 48	4·542
25 47	63° 34′·5 35′
26 48	63 34 35
h m	B in. t ° T °
20 8	30·34 49·8 47·8
21 28	30·35 47·5 45·3
September 14.	GILL.
	Victoria Standards. p : 215
	II.–III. p : 216 s : 217
h m s	
19 27 59	68° 39′·5 40′·0
29 24	68 55 56
	r
31 40	99 302 3·904 (67° 22′·5)
32 59	3·882 (71 10)
34 16	101 299 5·390 (69 10)
35 52·5	5·430 (72 49)
37 45	70° 55′ 56′
38 32·5	70 40 40·5

Chronometer Times.	Readings of Scales or Position-Circle.
September 14—*continued.*	
	I.–III. p : 218 s : 219
h m s	
19 41 2·5	63° 37′ 38′
42 10	63 32·5 33
	r
43 0·5	97 302 6·308
45 0	6·272
46 37	102 298 3·072
47 41	3·060
48 45	64° 16′·5 17′·5
49 54	64 17·5 18·5
	I.–II. p : 220 s : 221
h m s	
19 51 25	65° 38′ 38′·5
52 46·5	65 38 38·5
	r
56 9·5	96 304 3·260
59 6	3·220
20 0 48	103 296 6·117
1 52·5	6·103
2 44	66° 8′ 9
3 27	66 3·5 4·5
	II.–IV. p : 222 s : 223
h m s	
20 6 22	65° 21·5 22′·5
7 35	65 21·5 22·5
	r
9 26	95 304 5·970
12 59	6·010
14 10	104 296 3·321
16 20	3·330
17 27	65° 41′·5 42′·5
18 53·5	65 41·5 42·0

Chronometer Times.	Readings of Scales or Position-Circle.
September 14—*continued.*	
	III.–IV. p : 224 s : 225
h m s	
20 21 5	66° 37′ 38′
22 0	66 42·5 43
	r
23 57	105 295 2·510
24 49·5	2·548
25 53	94 306 2·816
26 52	2·801
27 33	66° 20′ 20′·5
30 9	66 18 19
	I.–IV. p : 226 s : 227
20 31 57	65° 35′ 35′·5
32 51	65 35·5 36
	r
33 59·5	92 308 5·870
35 4	5·820
38 25·5	108 293 3·495
39 16	3·490
39 53·5	65° 50′ 50′·5
40 49	65 50 50·5
	Standards. s : 228
	III.–IV. s : 229
	r
20 52 54	94 305 6·390
53 44	6·403
55 27	105 295 2·892
57 11	2·922
	II.–IV. s : 230
	r
20 59 51	104 296 3·488
21 0 27	3·461
3 51	95 304 5·760
5 10·5	5·772
	Standards. p : 231 s : 232
	Victoria Standards. s : 233
h m	B in. t ° T °
18 28	30·20 54·1 53·0
20 4	30·20 53·5 52·8
21 35	30·20 52·5 51·0

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.				Chronometer Times.	Readings of Scales or Position-Circle.				Chronometer Times.	Readings of Scales or Position-Circle.			
September 18.			FINLAY.		September 18—*continued.*					September 19.			GILL.	
	I.–II.		*p* : 234	*s* : 235		II.–III.		*p* : 240	*s* : 241		I.–II.		*p* : 248	*s* : 249
h m s	°	′	′		h m s	°	′	′		h m s	°	′	′	
19 46 20	68	29	29·5		20 24 50	63	59·5	60		19 25 31	68	46	46·5	
47 54	68	14	14		25 53	64	0·5	1		26 39	68	50·5	51	
			r					r					r	
49 24	102*	299	4·230		27 14	95	305	2·990		28 48	99	301	3·602	
51 3			4·330		29 24			3·007		29 44·5			3·561	
52 18	98	301	4·958		31 14	104	295	6·272		34 6	100	299	5·810	
53 53			4·964		32 46			6·240		35 59			5·853	
54 55	66	16	17		33 55	64	39	40		37 10	73	38	39	
56 22	66	6	7		34 55	64	40·5	41		38 46·5	73	35	36	
	II.–IV.†		*p* : 236	*s* : 237		I.–III.		*p* : 242	*s* : 243		I.–III.		*p* : 250	*s* : 251
20 0 19	70	2	1		20 36 32	65	12·5	13·5						
1 36	70	10	10·5		37 48	65	12	13		19 43 16	65	28	29	
			r					r		44 31	65	27	27·5	
2 39	98	303	2·600		39 1	105	294	4·894					r	
3 31			2·550		41 12			4·922		46 47·5	97	303	5·162	
5 21	103	297	4·869		50 20	94	306	4·549		48 20			5·153	
6 22			4·849		51 6			4·559		55 46	103	298	4·244	
7 29	69	7	8		52 23	64	43	44		56 35			4·259	
8 42	69	75	8·5		54 21	64	44	45		59 37	66	22	22·5	
	I.–IV.†		*p* : 238	*s* : 239		III.–IV.		*p* : 244	*s* : 245	20 0 23	66	20·5	21	
20 11 31	70	10·5	11		20 57 35	66	13·5	14·5						
12 54	70	7	8		59 0	66	16	17			II.–III.		*p* : 252	*s* : 253
			r					r						
13 49	98	303	2·499		21 0 34	92	308	4·148						
15 32			2·500		1 39			4·127		20 1 51	67	18	18·5	
17 37	103	297	5·206		3 11	107	292	5·188		2 33·5	67	19·5	20	
18 26			5·260		4 57			5·179					r	
										4 48	103	297	4·058	
19 45	69	56	57		6 16	66	35·5	36		6 9·5			4·094	
21 3	70	0	0·5		7 11	66	39	40		7 12	96	303	5·221	
										8 51			5·170	
						Standards.		*p* : 246	*s* : 247					
										9 47	66	32	33	
						B		*t*	T	11 3	66	30	30·5	
					h m	in.		°	°					
					19 59	30·09		52·0	51·0					
					21 25	30·09		52·0	51·1		Standards.		*s* : 254	*p* : 255

* Altered to 101. (The combination 102 299, 98 301 would not give the same result for the two positions of the semi-lenses). *Rejected.*

† In both these observations the first half is an observation of II.–IV., and the second half of I.–IV. *Rejected.*

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.
September 19—*continued.*	
	III.–IV. p : 256 s : 257
h m s	° ′ ′
20 25 1·5	67 22 22·5
25 56	67 22·5 23
	r
28 13·5	93 307 3·130
31 1	3·118
32 12	106 293 6·230
33 32	6·277
	° ′ ′
34 55·5	67 46·5 47·5
35 44	67 43·5 44·5
	I.–IV. p : 258 s : 259
	° ′ ′
20 39 5	69 14 14·5
40 16	69 11·5 12
	r
43 8	103 296 4·673
45 24	4·702
46 12	96 304 4·623
47 1	4·589
	° ′ ′
48 30	68 32 33
49 50	68 32 32·5
	II.–IV. p : 260 s : 261
	° ′ ′
20 51 12	67 47 48
52 43	67 48 49
	r
55 35	96 303 5·307
21 0 1	5·340
1 11	103 297 3·957
2 30	3·936
	° ′ ′
4 36	68 30·5 31
5 30	68 31 31·5
	Standards. p : 262 s : 263,264
	Victoria Standards. s : 265

h m	B in.	t °	T °
19 41	30·18	55·0	53·1
21 23	30·20	54·2	52·2

Chronometer Times.	Readings of Scales or Position-Circle.
September 20.	GILL.
	I.–IV. p : 266 s : 267
h m s	° ′ ′
19 35 46·5	61 56 57
36 50	61 53 54
	r
38 6	97 303 4·599
39 44	4·631
40 47	102 297 4·680
42 20	4·699
	° ′ ′
43 51·5	62 51 51·5
44 53	62 48 48·5
	I.–II. p : 268 s : 269
	° ′ ′
19 48 47	66 49 50
50 4	66 50·5 51
	r
50 57·5	104 296 6·218
53 5	6·261
54 11	96 305 3·030
54 55	2·990
	° ′ ′
55 46	66 14 15
56 36	66 17 18
	I.–III. p : 270 s 271
	° ′ ′
19 58 30	67 2·5 3
59 31	67 0 0·5
	r
20 1 5	95 305 4·496
2 53	4·450
3 55	104 295 4·911
4 56	4·941
	° ′ ′
6 11·5	67 30 30·5
7 15	67 29 30·25
	Standards p : 272 s : 273

Chronometer Times.	Readings of Scales or Position-Circle.
September 20—*continued.*	
	III.–IV. p : 274 s : 275
h m s	° ′ ′
20 18 49	73 44 45
20 0	73 50 51
	r
21 28	98 302 5·820
22 52	5·789
24 40	102 299 3·590
25 35	3·630
	° ′ ′
26 20	75 1 1·5
27 1	74 56 57
	II.–IV. p : 276 s : 277
	° ′ ′
20 28 42	75 37 38
30 6	75 46 47
	r
31 31	99 302 3·820
32 12	3·780
33 12	101 299 5·540
34 8	5·572
	° ′ ′
34 51	77 37 37·5
35 40	77 42 42·5
	II.–III. p : 278 s : 279
	° ′ ′
20 37 52	69 35·5 36
38 29	69 31·5 32
	r
39 28	99 301 4·330 (72° 22′)
41 0	4·330 (67 14)
43 14	100 299 4·942 (69 52)
44 20	5·002 (75 23)
	° ′ ′
45 23	72 39·25 40
46 40	72 43 43·5
	Standards. p : 280 s : 281
	Victoria Standards. s : 282

h m	B in.	t °	T °
19 34	30·19	56·2	55·2
21 25	30·17	54·9	53·2

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
September 21.			GILL.	
	II.–III.		*p* : 283	*s* : 284
h m s	°	′	′	
19 52 56	68	24	25	
54 42	68	22	23	
			r	
55 42	98	301	5·941	
56 42			5·969	
57 33	101	299	3·396	
58 42			3·400	
59 27	68	42·5	43·5	
20 0 23·5	68	46	47	
	Standards.		*p* : 285	*s* : 286
	II.–IV.		*p* : 287	*s* : 288
20 12 12	63	58·5	59	
13 3·5	63	58·5	59	
			r	
15 4·5	94	307	3·860	
17 16			3·937	
18 38	106	294	5·372	
19 43			5·320	
20 19	64	19	19·5	
20 55	64	18	18·5	
	III.–IV.		*p* : 289	*s* : 290
20 22 52	62	32	32·5	
23 39	62	31	32	
			r	
26 24	95	305	5·094	
27 34			5·142	
28 31·5	105	296	4·190	
29 23			4·146	
31 6	62	55	55·5	
31 42	62	55	56	

Chronometer Times.	Readings of Scales or Position-Circle.			
September 21—*continued.*				
	I.–III.		*p* : 291	*s* : 292
h m s	°	′	′	
20 33 47	63	11	11·5	
34 56	63	17	18	
			r	
36 42	97	302	4·514	
37 50			4·524	
39 6	102	298	4·822	
40 0			4·820	
40 49	64	3·5	4·5	
41 33	64	2·5	3	
	I.–II.		*p* : 293	*s* : 294
20 48 3	65	31	31·5	
48 56	65	28·5	29	
			r	
50 15·5	96	304	3·551	
51 23			3·560	
53 11·5	103	296	5·810	
54 7			5·839	
54 58	66	5	5·5	
55 55	66	4	4·5	
	I.–IV.		*p* : 295	*s* : 296
20 58 2·5	61	48	49	
59 4	61	48	48·5	
			r	
21 0 10	97	303	4·524	
0 55·5			4·580	
2 8	102	297	4·714	
3 11			4·690	
4 14	62	35	35·5	
4 51	62	33	33·5	
	Standards.		*p* : 297	*s* : 298

h. m	B in.	*t* °	T °
20 2	29·98	57·2	56·0
21 6	29·99	56·5	56·0

Chronometer Times.	Readings of Scales or Position-Circle.			
September 22.			GILL.	
	I.–II.		*p* : 299	*s* : 300
h m s	°	′	′	
19 54 24	63	34·5	35·5	
59 46	63	44	45	
			r	
20 3 41	99	301	3·040	(61 2)
5 13			3·093	(66 43)
6 6	100	299	6·229	(66 43)
7 34·5			6·236	(71 27)
12 14	68	47	48	
12 57	68	51	51·5	
	I.–III.		*p* : 301	*s* : 302
20 16 57	63	32	32·5	
17 55	63	25	25·5	
			r	
18 45	96	303	4·053	
20 4·5			4·100	
21 35	103	297	5·130	
22 47			5·036	
23 47	64	19	19·5	
24 58	64	25·5	26·5	
	II.–III.		*p* : 303	*s* : 304
20 25 52	64	2	2·5	
26 32	64	2	2·5	
			r	
27 28	102	297	5·446	
29 33			5·382	
30 56	97	303	3·980	
31 50			4·040	
32 49	62	56·5	57	
33 20	62	55	56	

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.
September 22—*continued.*	
	III.–IV. p : 305 s : 306
h m s	° ′ ′
20 35 56	65 0 0·5
36 36	64 58·5 59
	r
39 0	89 310 4·950
40 22	4·968
41 44	110 290 4·309
42 43	4·280
	° ′ ′
44 0	65 17 17·5
45 17	65 19 19·5
	Standards. p : 307 s : 308
	I.–IV. p : 309 s : 310
	° ′ ′
20 58 25	65 52 53
59 52	65 51·5 52
	r
21 2 22	107 294 3·149
3 25	3·170
4 29	93 307 6·116
5 25	6·091
	° ′ ′
6 12	65 30 30·5
7 18	65 28·5 29
	II.–IV. p : 311 s : 312
	° ′ ′
21 8 36	65 31·5 32
9 16	65 31·5 32·5
	r
10 7	92 308 2·901
11 58	2·850
13 36	107 293 2·479
15 10	2·483
	° ′ ′
15 59	65 54 55
16 55	65 57 57·5
	Standards. p : 313 s : 314
	Victoria Standards. s : 315

h m	B in.	t °	T °
19 31	30·25	52·0	50·1
21 38	30·29	50·0	47·1

Chronometer Times.	Readings of Scales or Position-Circle.
September 23. GILL.	
	Standards. p : 316 s : 317
	III.–IV. p : 318 s : 319
h m s	° ′ ′
20 3 32	65 58·5 59
4 18	65 58 59
	r
6 5	88 312 6·037
7 22	6·080
8 50	112 289 3·286
9 47	3·302
	° ′ ′
10 43	66 12 12·5
12 16	66 11·5 12
	II.–III. p : 320 s : 321
	° ′ ′
20 16 53	66 15 16
17 55	66 16 17·5
	r
18 38	107 294 2·730
22 4	2·730
27 59	93 308 2·617
29 31	2·621
	° ′ ′
30 22	65 53 53·5
31 44	65 54 54·5
	II.–IV. p : 322 s : 323
	° ′ ′
20 34 37	65 53 53·5
35 50	65 50·5 51
	r
36 48	95 305 6·014
37 32	6·009
38 58	105 296 3·317
40 10	3·350
	° ′ ′
41 28	66 25 25·5
43 8	66 28 28·5
	III.–IV. s : 324
	r
20 46 53	88 312 5·908
49 16	5·927
50 24	112 289 3·400
51 43	3·396

Chronometer Times.	Readings of Scales or Position-Circle.
September 23—*continued.*	
	Standards. p : 325 s : 326
	Victoria Standards s : 327

h m	B in.	t °	T °
19 47	30·34	54·3	50·2
21 12	30·32	51·0	48·1

September 24. GILL.

Half distance I.–II. p : 328 s : 329

h m s	r	° ′ ′
19 33 38	100 301 3·086	117 12 12·5
38 28	3·080	124 47 47·5
41 43	100 300 6·270	190 41 41·5
44 1	6·240	199 34·5 35
45 12·5	6·244	200 20 20·5
46 24·5	6·210	204 31 31·5
50 53·5	100 301 3·239	175 41 41·5
53 4	3·255	183 10 10·5

h m	B in.	t °	T °
20 6	30·16	52·0	51·2

September 28. GILL.

	Standards. p : 330 s : 331
	I.–II. p : 332 s : 333
h m s	° ′ ′
20 17 51	65 16·5 17
18 42	65 10 10·5
	r
22 24	96 303 6·561
23 14	6·547
26 35	103 297 2·688
28 39	2·691
	° ′ ′
31 7	65 42·5 43
32 13	65 42 42·5

h m	B in.	t °	T °
19 57	30·40	53·7	53·2
20 33	30·40	54·0	53·0

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
September 29.			GILL.	
	I.–II.		*p* : 334	*s* : 335
h m s	°	′	′	
19 38 16	64	52·5	53	
39 41	64	51	51·5	
			r	
40 44	99	301	3·058	(62° 44′)
42 29			3·069	(67 27)
43 55	100	299	6·238	(64 50)
45 45			6·260	(70 58)
48 2	68	10·5	11	
49 6	67	59·5	60	
	III.–IV.		*p* : 336	*s* : 337
19 50 52	72	42	42·5	
51 4	73	32·5	33	
			r	
53 47	99	300	6·110	(69° 0′)
54 50			6·111	(77 41)
55 58	100	300	3·170	(71 40)
57 44			3·161	(81 59)
20 1 13	77	10	10·5	
2 31	76	40	40·5	
	Standards.		*p* : 338	*s* : 339
	I.–III.		*p* : 340	*s* : 341
20 18 33	63	0·5	1	
21 46	62	57	57·5	
			r	
25 20	97	302	5·515	
27 4			5·610	
31 58	102	298	3·469	
33 43			3·432	
35 10	63	42	42·5	
35 57	63	40	40·5	

h m	B in.	*t* °	T °
20 4	30·28	55·8	55·0
20 39	30·28	55·5	55·1

Chronometer Times.	Readings of Scales or Position-Circle.			
October 4.			GILL.	
	II.–III.		*p* : 342	*s* : 343
h m s	°	′	′	
19 51 14	68	35·5	36·5	
52 12	68	50·5	51·5	
			r	
54 7	99	302	2·858	(66° 54′)
56 34			2·860	(71 40)
58 6	101	300	2·462	(72 44)
59 26			2·487	(67 38)
20 1 18	70	1	1·5	
2 5	69	58	58·5	
	I.–IV.		*p* : 344	*s* : 345
	68	4·5	5	
20 4 3	68	3	4	
4 53			r	
6 7	95	304	6·140	
9 15			6·117	
10 38	104	296	3·224	
12 47			3·301	
13 56			3·312	
15 32			3·343	
16 52	68	16·5	17	
17 47	68	18·5	19·5	
	I.–II.		*p* : 346	*s* : 347
20 19 58	65	50	50·5	
21 11	65	50·5	51	
			r	
22 37	95	304	5·434	
23 49			5·430	
25 28	104	296	3·878	
26 25			3·881	
27 49	66	4·5	5	
29 31	66	3	4	
	Standards.		*p* : 348	*s* : 349

Chronometer Times.	Readings of Scales or Position-Circle.			
October 4—*continued.*				
	III.–IV.		*p* : 350	*s* : 351
h m s	°	′	′	
20 45 0	67	24	25	
46 58	67	22·5	23·5	
			r	
48 12	109	290	5·240	
49 11			5·309	
50 37	90	310	3·979	
52 10			3·990	
52 40	67	18	18·5	
53 34	67	20	20·5	
	II.–IV.		*p* : 352	*s* : 353
20 56 29	67	2·5	3·5	
57 37	67	2·5	3	
			r	
59 9	91	309	3·832	
21 0 1			3·819	
1 5	108	291	5·491	
2 9			5·500	
3 11	67	10	10·5	
3 59	67	10·5	11	
	I.–III.		*p* : 354	*s* : 355
21 5 55	66	38	39	
6 46	66	40	40·5	
			r	
8 12	94	305	5·420	
9 6			5·410	
10 7	105	295	3·901	
11 7			3·910	
11 52	66	48·5	49	
12 53	66	48	49	

h m	B in.	*t* °	T °
20 34	29·98	67·0	67·0
21 14	29·98	67·0	64·3

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
October 6.			FINLAY.	
	III.–IV.		p : 356	s : 357
h m s	°	′	′	
20 5 26	59	34	35	
6 39	59	40	40·5	
			r	
8 13	102	298	4·932	
9 6			4·980	
11 14	97	302	4·466	
13 42			4·585	
	°	′	′	
16 29	59	7	8	
17 27	58	54	55	
	II.–IV.		p : 358	s : 359
	°	′	′	
20 19 7	60	19·5	20	
20 59	60	28·5	30	
			r	
22 23	101	299	4·249	
24 42			4·240	
27 51	98	301	5·230	
29 31			5·200	
	°	′	′	
30 57	59	7·5	9	
32 4	59	16	17	
	I.–IV.		p : 360	s : 361
	°	′	′	
20 35 24	55	0	0	
36 44	54	52	53	
			r	
38 56	100	299	5·124	(60° 43′)
40 55			5·119	(49 26)
44 0	99	301	4·313	(47 46)
46 13			4·279	(57 11)
	°	′	′	
47 36	52	53	54	
49 17	52	31	32	
	Standards.		p : 362	s : 363

Chronometer Times.	Readings of Scales or Position-Circle.			
October 6—*continued.*				
	I.–III.		p : 364	s : 365
h m s	°	′	′	
21 15 37	62	2·5	3	
16 53	62	0·5	1	
			r	
17 51	98	302	5·074	(65° 53′)
19 31			5·110	(59 38)
22 20	102	299	4·190	(65 44)
23 54			4·127	(60 15)
	°	′	′	
25 41	63	3·5	4·5	
26 48	63	7	8	

h m	B in.	t °	T °
20 33	30·07	60·2	59·4
21 27	30·07	60·7	59·0

October 9.			GILL.	
	I.–III.		p : 366	s : 367
h m s	°	′	′	
23 53 1	62	52·5	53	
55 42	62	58	58·5	
			r	
57 34	97	303	4·506	
58 51·5			4·606	
0 1 6	102	297	4·624	
4 14			4·508	
	°	′	′	
4 54	63	19·5	20	
5 54	63	22·5	23	
	I.–II.		p : 368	s : 369
	°	′	′	
0 8 25	66	36	37	
9 30	66	36	37	
			r	
10 33	98	302	4·783	
11 47			4·785	
12 43·5	101	298	4·567	
13 25			4·608	
	°	′	′	
14 27	67	11	11·5	
15 34·5	67	9	9·5	

Chronometer Times.	Readings of Scales or Position-Circle.			
October 9—*continued.*				
	III.–IV.		p : 370	s : 371
h m s	°	′	′	
0 17 29	66	51·5	52	
19 19	66	47	47·5	
			r	
20 32·5	94	305	3·306	
22 39			3·236	
23 46	105	295	6·110	
24 38			6·128	
	°	′	′	
26 18	67	1·5	2	
27 49	67	1	1·5	
	Standards.		p : 372	s : 373
	II.–IV.		p : 374	s : 375
	°	′	′	
0 43 32	65	49	49·5	
45 37	65	49	49·5	
			r	
47 40	90	310	4·271	
49 44			4·255	
50 34	109	290	5·085	
51 46			5·079	
	°	′	′	
52 43	65	53	54	
53 59	65	54·5	55	
	I.–IV.		p : 376	s : 377
	°	′	′	
0 56 28	65	44·5	45	
57 16	65	44·5	45	
			r	
58 17	108	292	5·852	
59 6			5·851	
1 0 28	92	309	3·540	
2 2			3·591	
	°	′	′	
2 57	65	35	36	
3 49	65	35	35·5	

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.		
October 9—*continued.*			
	II.–III.	***p*** : 378	***s*** : 379
h m s	°	′	′
1 6 1	64	37	37·5
6 49	64	39	39·5
			r
8 13	95	305	4·373
9 38			*
10 29	104	295	4·900
11 17			4·910
12 7	64°	51·5′	52′
13 6	64	53	53·5
15 27	64	54·5	55
16 19	64	52·5	53

h m	B in.	*t* °	T °
0 7	30·10	66·0	66·0
0 40	30·10	65·5	65·5
1 18	30·09	64·9	64·6

October 10. GILL.

h m s	I.–III.	***p*** : 380	***s*** : 381
23 57 14	65° 36·5′	37·5′	
58 59	65 45·5	~~47·5~~	
		r	
0 0 8	97 303	4·632	
2 4		4·640	
3 13	102 297	4·657	
4 36		4·690	
5 18	66° 7·5′	8′	
6 29·5	66 2·5	3	
	III.–IV.	***p*** : 382	***s*** : 383
0 11 8	66° 59·5′	60′	
12 27	66 53	53·5	
		r	
14 19	96 303	6·230	
16 6		6·181	
17 20	103 297	3·090	
22 5		3·090	
23 17	67° 19′	19·5′	
24 41	67 24·5	25·0	

* One pointing not printed; *rejected.*

Chronometer Times.	Readings of Scales or Position-Circle.		
October 10—*continued.*			

h m	B in.	*t* °	T °
23 55	29·98	67·1	66·1

October 13. GILL.

h m s	I–II.	***p*** : 384	***s*** : 385
0 0 55	65° 5′	5′	
1 56	64 59·5	59·5	
		r	
2 48	99 301	4·201	(63° 35′)
5 33		4·220	(67 4)
6 50	100 299	5·057	(69 47)
8 26		5·043	(64 45)
9 58	67° 29·5′	30′	
10 48	67 22·5	23	
	II.–III.	***p*** : 386	***s*** : 387
0 12 14	59° 20·5′	21′	
12 57	59 13·5	14	
13 49	98 301	3·188	
14 44		3·201	
15 36	101 299	6·056	
17 5		6·021	
18 44	60° 47·5′	48′	
19 27	61 10	10·5	
	I.–III.	***p*** : 388	***s*** : 389
0 21 20	62° 6·5′	7′	
23 12	62 12	12	
		r	
23 57	98 302	5·120	
25 1		5·137	
27 40	101 298	4·030	
28 33		3·974	
29 53	63° 5′	5·5′	
30 41	63 8	8·5	
	Standards	***p*** : 390	***s*** : 391

Chronometer Times.	Readings of Scales or Position-Circle.		
October 13—*continued.*			
	III.–IV.	***p*** : 393	***s*** : 394
h m s	°	′	′
0 45 14	66°	58·5′	59′
46 16	66	57	57·5
			r
47 13	107	293	6·278
48 25			6·320
50 36	93	308	2·955
53 8			2·970
54 24	66°	43′	43·5′
55 48	66	45	45·5
	II.–IV.	***p*** : 395	***s*** : 396
	°	′	′
0 57 55	67°	59′	59·5′
58 44·5	68	0·25	0·75
			r
1 0 34	94	306	4·950
3 56			4·836
4 50			4·789
5 55			4·772
7 7	105	294	4·540
8 16			4·603
9 28	68°	15′	15·5′
10 58	68	14·5	15
	I.–IV.	***p*** : 397	***s*** : 398
	°	′	′
1 12 56	68°	34′	35′
14 26	68	34·5	35·5
			r
20 40	105	296	2·929
23 18			3·080
24 5			3·069
24 55			3·086
27 11	95	305	6·056
29 21			6·015
30 23	68°	21′	21·5′
31 27	68	19	20

h m	B in.	*t* °	T °
0 0	30·40	54·6	53·5
0 43	30·40	54·2	53·1
1 32	30·40	53·6	52·5

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
October 14.			FINLAY.	
	I.–II.		p : 399	s : 400
h m s	°	′	′	
0 4 14	62	3·5	4	
5 55	61	53·5	54	
			r	
7 13	98	301	5·331	
8 27			5·420	
10 22	101	299	3·778	
11 23			3·680	
	°	′	′	
12 32	63	30	30·5	
14 47	63	13	13·5	
	II.–IV.		p : 401	s : 402
	°	′	′	
0 15 59	53	46	47	
16 56	54	9	9·5	
			r	
18 55	99	302	2·881	(58° 36′)
20 52			2·960	(49 44)
23 10	101	299	6·312	(51 8)
25 5			6·200	(59 19)
	°	′	′	
26 31	55	36	37	
28 0	55	50	50	
	I.–IV.		p : 403	s : 404
	°	′	′	
0 31 43	80	30·5	31	
32 52	80	40	40·5	
			r	
34 55	99	300	6·137	(89° 46′)
36 44			6·116	(69 20)
38 7	100	300	3·123	(69 20)
39 52			3·130	(95 0)
	°	′	′	
41 41	83	13·5	14	
42 40	84	0	0·5	

Chronometer Times.	Readings of Scales or Position-Circle.			
October 14—*continued.*				
	I.–III.		p : 405	s : 406
h m s	°	′	′	
0 46 32	66	38	39	
48 1	66	36	37	
			r	
49 27	95	306	3·010	
50 45			2·990	
52 47	105	296	2·405	
54 6			2·430	
	°	′	′	
55 20	66	58	59·5	
56 35	66	56	57·5	
	III.–IV.		p : 407	s : 408
	°	′	′	
0 59 46	68	10	10·5	
1 0 41	68	9	10	
			r	
1 36	105	295	2·899	
3 23			2·860	
5 35	94	306	2·378	
8 0			2·351	
	°	′	′	
9 0	67	55	56	
10 57	67	55·5	57	
	II.–III.		p : 409	s : 410
	°	′	′	
1 12 40	65	50	50	
13 34	65	52·5	53·5	
			r	
14 37	93	307	4·019	
15 56			4·020	
17 55	106	293	5·181	
19 20			5·152	
	°	′	′	
20 41	66	4	4·5	
21 30	66	6	7	
	Standards.		p : 411	s : 412

h m	B in.	t °	T °
0 0	30·38	55·6	55·0
1 35	30·38	55·2	54·9

Chronometer Times.	Readings of Scales or Position-Circle.			
October 15.			FINLAY.	
	III.–IV.		p : 413	s : 414
h m s	°	′	′	
23 58 54	81	31	31·5	
0 0 18	81	43	43·5	
			r	
2 12	99	301	5·832	(86° 40′)
4 5			5·783	(75 18)
5 45	101	300	3·578	(75 18)
7 24			3·609	(87 59)
	°	′	′	
8 39	82	10	10	
9 35	82	13	13·5	
	II.–IV.		p : 415	s : 416
	°	′	′	
0 12 52	64	2	3	
13 41	63	56	57	
			r	
20 58	97	303	5·039	
22 36			4·980	
25 19	102	297	4·279	
27 22			4·314	
	°	′	′	
28 58	64	21·5	22	
29 55	64	16	17	
	II.–III.		p : 417	s : 418
	°	′	′	
0 32 50	68	37·5	38·5	
33 48	68	29·5	30	
			r	
35 8	103	297	2·461	
36 31			2·536	
39 0	96	304	2·688	
42 47			2·520	
44 11			2·406	
	°	′	′	
45 45	68	23·5	24	
47 10	68	23	23·5	

h m	B in.	t °	T °
23 57	30·24	61·0	60·2
0 56	30·22	60·3	60·9

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.		
October 16.			FINLAY.
	II.–IV.	*p* : 419	*s* : 420
h m s	° ′	′	
0 10 50	62 45·5	46·5	
11 44·5	62 49	49·5	
		r	
18 56	98 302	3·757	
19 43		3·757	
21 29·5	101 298	5·536	
22 17		5·490	
23 5	63 39·5	40	
23 53	63 32·5	33	
	II.–III.	*p* : 421	*s* : 422
0 26 16	64 5·5	6·5	
27 0·5	64 8	9	
		r	
28 24	96 304	5·719	
30 55		5·769	
32 45	104 297	3·432	
34 11		3·410	
35 3	64 26	27	
35 54	64 31	32	
	I.–II.	*p* : 423	*s* : 424
0 38 23	67 0	0·5	
39 33·5	67 4·5	5	
		r	
40 37	102 298	6·065	
42 5		6·150	
43 20	98 303	3·146	
44 25		3·117	
45 35	66 21	22	
46 20	66 24	24·5	

Chronometer Times.	Readings of Scales or Position-Circle.		
October 16—*continued.*			
	I.–IV.	*p* : 425	*s* : 426
h m s	° ′	′	
0 49 13	64 57·5	58·5	
50 50	64 58	59	
		r	
51 32	96 304	4·306	
52 51		4·308	
55 6	103 296	5·060	
56 11		5·080	
57 20·5	65 22	22·5	
58 7	65 20	20	
	Standards.	*p* : 427	*s* : 428
	I.–III.	*p* : 429	*s* : 430
1 17 18	62 30·5	31	
18 20	62 25·5	26·5	
		r	
20 9	102 298	5·730	
21 17·5		5·682	
22 38·5	98 303	3·678	
23 42		3·717	
24 33	62 6	7	
25 23	62 2	3	
	III.–IV.	*p* : 431	*s* : 432
1 28 2·5	64 0	0	
28 46	64 1·5	2·5	
		r	
29 47·5	94 306	5·060	
31 4		5·012	
33 44	106 295	4·191	
35 16	105 294	4·121	
36 56	64 13	13·5	
38 36·5	64 17·5	18	

h m	B in.	*t* °	T °
0 9	30·06	63·5	62·8
0 59	30·06	63·0	61·0
1 42	30·05	62·1	60·1

Chronometer Times.	Readings of Scales or Position-Circle.		
October 17.			FINLAY.
	I.–III.	*p* : 433	*s* : 434
h m s	° ′	′	
0 0 51	65 43	44	
1 43	65 35·5	37	
		r	
3 13·5	97 303	5·411	
4 36·5		5·430	
6 32	103 298	3·900	
7 55	102 297	3·925	
9 27	66 10·5	11	
10 19	66 5·5	6·5	
	I.–IV.	*p* : 435	*s* : 436
0 13 51	65 27·5	28·5	
15 33·5	65 28	29	
		r	
16 38·5	92 308	2·590	
18 23		2·690	
21 2	107 293	2·530	
22 21·5		2·514	
24 9·5	65 37·5	38·5	
25 5	65 40	41	
	III.–IV.	*p* : 437	*s* : 438
0 27 49	65 43·5	44·5	
28 40	65 44·5	45	
		r	
29 32·5	110 290	3·882	
31 9·5		3·859	
32 52·5	89 310	5·451	
34 54		5·507	
36 47·5	65 38	39	
37 45	65 36·5	38	
	Standards.	*p* : 439	*s* : 440

h m	B in.	*t* °	T °
0 0	30·05	63·7	62·0
0 56	30·05	63·4	64·1

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
October 19.			Finlay.	
	I.–III.		p : 441	s : 442
h m s	°	′	′	
0 15 37	57	1·5	2	
17 8	57	13	13·5	
			r	
18 26	99	301	4·157	
19 49			4·195	
21 26	100	299	5·019	
23 10			4·980	
25 4	56*	28	29	
26 7	56*	10·5	11	
	II.–III.		p : 443	s : 444
	°	′	′	
0 30 4	68	0·5	2	
31 58	67	54	55·5	
			r	
33 26	102	299	2·670	
35 31			2·621	
36 39	98	303	2·696	
37 40			2·677	
38 52	67	30·5	31	
40 20	67	33·5	34·5	
	I.–II.		p : 445	s : 446
	°	′	′	
0 43 40	64	59·5	60	
44 40	65	8	9	
			r	
46 0	97	303	4·830	
47 12			4·817	
48 39	102	297	4·436	
50 18			4·390	
51 24	65	25	25·5	
52 36	65	24·5	25	
	Standards.		p : 447	s : 448

* Altered to 58°. (The motion of the Satellites *increases* the position-angle.) *Rejected.*

Chronometer Times.	Readings of Scales or Position-Circle.			
October 19—*continued.*				
	I.–IV.		p : 449	s : 450
h m s	°	′	′	
1 12 24	66	43	44	
13 12	66	46	47	
			r	
14 33	109	291	4·390	
16 27			4·420	
18 27	90	309	4·816	
19 51			4·811	
20 50	66	35	36	
22 26	66	36	37·5	
	III.–IV.		p : 451	s : 452
	°	′	′	
1 24 27	67	20	20·5	
26 1	67	21	22	
			r	
27 32	91	308	5·959	
28 41			5·906	
32 3	108	292	3·450	
33 22			3·469	
34 24	67	30·5	30·5	
35 36	67	30	30	
	II.–IV.		p : 453	s : 454
	°	′	′	
1 37 41	67	21·5	22	
38 47	67	23	24	
			r	
39 59	106	294	3·880	
41 1			3·876	
43 46	93	306	5·281	
45 15			5·223	
47 0	67	10	10	
48 5	67	13	13·5	

h m	B in.	t °	T °
0 10	29·95	61·1	56·9
1 5	29·94	60·9	56·9
1 49	29·93	60·0	56·0

Chronometer Times.	Readings of Scales or Position-Circle.			
October 23.			Finlay.	
	II.–IV.		p : 455	s : 456
h m s	°	′	′	
23 57 59	62	19	20	
59 56	62	16·5	17·5	
			r	
0 0 55	97	303	5·744	
2 12			5·799	
4 46·5	103	298	3·457	
6 1			3·439	
7 30·5	63	2·5	3	
8 51	63	3	4	
	II.–III.		p : 457	s : 458
	°	′	′	
0 10 46	63	54·5	55·5	
11 33	63	55	56	
			r	
13 44·5	103	298	2·957	
15 14			2·862	
16 43	97	304	2·475	
17 43			2·510	
19 0	63	14·5	15·5	
19 52	63	15·5	17	
	III.–IV.		p : 459	s : 460
	°	′	′	
0 28 27	89	13	13·5	
29 30	87	51	51·5	
			r	
32 43	100	301	3·132	(126° 18′)
34 57			3·056	(66 4)
36 17	100	300	6·197	(66 4)
38 7			6·380	(128 1)
41 11	120	40·5	41	
42 2	117	13	13·5	
	Standards.		p : 461	s : 462

h m	B in.	t °	T °
23 55	30·19	53·8	53·0
0 56	30·18	53·3	52·4

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
November 5.			GILL.	
h m s	I.–III.		p : 463	s : 464
23 49 23	66°	47′·5	48′	
50 57	66	46·5	47	
			r	
53 29	98	303	3·756	
54 49	97	302	3·735	
56 25	102	298	5·540	
58 9			5·530	
0 1 12	67°	15′·5	16′	
2 52	67	11·5	12	
	I.–II.		p : 465	s : 466
0 6 14	66°	42′	42′·5	
8 1	66	46	47	
			r	
9 55	96	304	4·220	
10 47			4·220	
13 18	103	296	5·150	
15 44			5·210	
16 47	66°	58′·5	59′·5	
18 2	66	57	57·5	
	II.–III.*		p : 467	s : 468
0 25 12	66°	57′	57′·5	
28 26	67	0	0·5	
			r	
30 57	105	294	4·338	
32 53			4·400	
37 9	96	304	3·475	
38 12			3·430	
39 40	66°	45′	46′	
40 46	66	42·5	43	
	Standards.		p : 469	s : 470

* The first half is an observation of II.–III., the second half of III.–IV. *Rejected.*

Chronometer Times.	Readings of Scales or Position-Circle.			
November 5—*continued.*				
h m s	II.–IV.		p : 471	s : 472
1 8 3	66°	55′	55′·5	
9 38	66	54	54·5	
			r	
12 30	109	291	5·116	
15 5			5·101	
16 20	91	310	4·147	
17 12	90	309	4·079	
20 20	66°	49′	49′·5	
21 14	66	48	49	
	I.–IV.		p : 473	s : 474
1 25 55	66°	44′	44′·5	
29 9	66	41·5	42	
			r	
31 52	94	306	4·661	
32 44			4·749	
34 6	105	294	4·527	
36 15			4·531	
37 3	66°	51′	51′·5	
39 30	66	50	51	
	III.–IV.		p : 475	s : 476
1 41 45	66°	30′	30′·5	
42 38	66	28	28·5	
			r	
43 48	96	304	3·065	
44 40			3·102	
46 24	103	296	6·162	
47 50			6·210	
48 56	66°	48′	48′·5	
49 49	66	52	53	

h m	B	t	T
	°		
23 47	30·09	61·0	60′·5
1 3	30·09	61·0	60·0
1 50	30·09	61·0	60·0

Chronometer Times.	Readings of Scales or Position-Circle.			
November 6.			GILL.	
h m s	II.–III.		p : 477	s : 478
0 6 34	*58°	15′·5	16′	
7 52	56	4	4·5	
			r	
8 52	99	301	6·077	
10 1			6·103	
11 41	101	300	3·148	
12 45			3·120	
14 17	59°	40′·5	41′	
15 16	59	7	8	
	I.–III.		p : 479	s : 480
0 17 25	69°	48′	48′·5	
18 30	69	40	40·5	
			r	
19 31	98	302	4·726	
21 25			4·690	
22 36	101	298	4·701	
23 27			4·740	
24 17	71°	6′	6′·5	
25 22	71	24·5	25	
	I.–II.		p : 481	s : 482
0 27 3	65°	35′·5	36′·0	
28 14	65	38·5	39·0	
			r	
31 8	97	303	4·410	
32 43			4·411	
34 1	102	297	4·943	
35 22			4·952	
36 5	66°	36′	36′·5	
37 14	66	39·5	40·5	
	Standards.		p : 483	s : 484

* Altered to 56°. *Rejected.*

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
November 6—*continued.*				
	I.–IV.		p : 485	s : 486
h m s	°	′	′	
0 56 17	67	43	43·5	
57 7	67	45·5	46·0	
			r	
59 50	106	293	4·459	
1 4 23			4·621	
5 17			4·630	
5 57			4·696	
7 30	93	307	4·611	
8 46			4·593	
10 30	67	22·0	22·5	
11 18	67	20·5	21·0	
	III.–IV.		p : 487	s : 488
1 14 30	66	43	44	
15 44	66	42·5	43	
			r	
17 45	105	295	4·520	
19 7			4·485	
21 6	94	305	4·824	
22 45			4·825	
26 21	66	15	15·5	
27 18	66	15·5	16	

h m	B in.	t °	T °
0 5	30·09	62·9	62·0
0 48	30·07	62·8	62·2
1 30	30·09	62·5	62·1

November 8.			GILL.	
	II.–III.		p : 489	s : 490
h m s	°	′	′	
0 15 36	66	22	22·5	
16 19·5	66	23	23·5	
			r	
17 24	98	302	5·415	
18 29			5·430	
20 59	102	299	3·966	
22 29			3·960	
23 12	67	54	54·5	
24 2·5	67	52	52·5	

Chronometer Times.	Readings of Scales or Position-Circle.			
November 8—*continued.*				
	II.–IV.		p : 491	s : 492
h m s	°	′	′	
0 26 11·5	67	40·5	41	
28 10	67	39	39·5	
			r	
29 30	96	304	4·410	
31 22			4·375	
32 48	103	296	4·905	
34 42			4·923	
36 5	68	25	26	
38 7	68	31	31·5	
	Standards.		p : 493	s : 494
	III.–IV.		p : 495	s : 496
0 54 49·5	67	29	29·5	
56 22	67	32	32·5	
			r	
58 29	94	306	2·587	
1 2 21			2·533	
3 26	105	295	2·821	
4 45			2·869	
5 58·5	68	5	5·5	
6 45	68	5	5·5	

h m	B in.	t °	T °
0 14	30·07	67·0	65·8
1 8	30·06	67·0	68·1

November 9.				GILL.	
	Double	distance.	I.–II.	p : 497	s : 498
h m s	°	′			r
0 8 45	356	55	100	301	3·883
10 45	3	57			3·993
12 48	20	35	100	300	5·130
14 51	24	27			5·053
	III.–IV.				s : 499
					r
0 23 39	98	301			5·908
26 34					5·844
28 9	101	299			3·463

h m	B in.	t °	T °
0 15	29·97	69·9	64·1

Chronometer Times.	Readings of Scales or Position-Circle.			
November 19.			GILL.	
	III.–IV.		p : 500	s : 501
h m s	°	′	′	
0 37 34	62	0	0·5	
38 35	61	53·5	54	
			r	
40 8	98	301	3·818	
41 5			3·820	
42 20	101	299	5·580	
43 39			5·510	
44 20	62	41·5	42	
45 44	62	32	32·5	

h m	B in.	t °	T °
0 48	30·24	57·5	54·6

November 22.			GILL.	
	I.–II.		p : 502	s : 503
h m s	°	′	′	
0 39 50	59	19·5	20	
41 3	59	28·5	29	
			r	
42 39	99	301	4·313	
43 59			4·368	
44 46	100	299	4·950	
45 35			4·920	
46 36	61	1	1·5	
47 41	60	56·5	57	
	II.–III.		p : 504	s : 505
0 51 5	66	13	14	
52 45	66	22	22·5	
			r	
54 6	97	302	4·460	
55 44			4·430	
58 51·5	102	298	4·975	
59 58·5			5·020	
1 1 36	66	46·5	47	
2 46·5	66	47	47·5	

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.		
November 22—*continued.*			
h m s	III–IV.	*p* : 506	*s* : 507
1 6 10·5	66° 34′	34′·5	
7 5	66 32	32	
		r	
9 30	110 290	4·950	
12 2		4·889	
14 13	89 310	4·510	
14 50		4·480	
15 24	66° 25′	25′·5	
17 16	66 27	27·5	
	Stan lards	*p* : 508	*s* : 509
	II.–IV.	*p* : 510	*s* : 511
1 37 22	66° 31′·5	32′	
39 31	66 32	32·5	
		r	
41 1	108 292	6·056	
41 46		6·066	
43 38	92 309	3·383	
45 46		3·411	
47 22	66° 24′	24′·5	
48 11	66 27	27·5	
	I.–IV.	*s* : 512	
		r	
1 50 20	93 308	2·760	
51 46		2·770	
53 1	107 294	2·720	
53 47		2·749	

	B	*t*	T
h m	in.		
0 49	30·17	63°·0	62°·3
1 54	30·18	63·0	63·0

November 27. GILL.

h m s	I.–IV.	*p* : 513	*s* : 514
0 46 16	64° 9′·5	10′	
47 55	64 8	8·5	
		r	
49 28	97 303	2·446	
51 29		2·489	
52 42	102 298	2·958	
53 46		2·930	
54 45	64° 38′	38′·5	
56 16	64 39·5	40	

Chronometer Times.	Readings of Scales or Position-Circle.		
November 27—*continued.*			
	I.–III.	*p* : 515	*s* : 516
0 58 1	67° 4′	4′·5	
59 3	67 7·5	8	
		r	
1 2 0	96 304	4·850	
2 45		4·880	
4 46	103 296	4·603	
5 23		4·575	
6 12	67° 29′	29′·5	
7 9	67 27	27·5	
	III.–IV.	*p* : 517	*s* : 518
1 9 8	66° 12′·5	13′	
10 22	66 12·5	13	
		r	
11 11	106 294	4·744	
13 21		4·740	
14 14	93 306	4·663	
15 3		4·648	
15 47	65° 59′·5	60′	
16 51	66 0	0·5	
	Standards	*p* : 519	*s* : 520

	B	*t*	T
h m	in.		
0 45	29·89	79°·9	82°·5
1 31	29·89	79·2	82·0

November 30. FINLAY.

h m s	I.–II.	*p* : 521	*s* : 522
0 55 27	66° 56′	57′	
56 16	66 49·5	50	
		r	
57 19	97 303	2·630	
58 35		2·559	
1 0 0	102 298	2·950	
0 58		3·012	
2 47	67° 38′	39′	
4 59	67 49·5	50	

Chronometer Times.	Readings of Scales or Position-Circle.		
November 30—*continued.*			
	I.–III.	*p* : 523	*s* : 524
1 7 47	66° 53′	54′	
8 57	66 50	50	
		r	
10 39	96 305	3·006	
13 17		2·888	
17 26	104 297	2·690	
20 24		2·780	
22 5	67° 27′·5	28′·5	
23 54	67 25·5	26	

	B	*t*	T
h m	in.		
1 5	30·18	62°·0	61°·0
1 40	30·19	62·0	60·9

December 8. FINLAY.

h m s	I.–III.	*p* : 525	*s* : 526
1 6 5	67° 34′·5	35′·5	
7 10	67 44·5	45·5	
		r	
9 5	99 302	2·840	
10 17		2·819	
11 31	101 300	2·627	
14 17		2·610	
15 12	69° 17′	18′	
16 23	69 27·5	29	
	I.–II.	*p* : 527	*s* : 528
1 18 18	65° 29′·5	30′	
19 25	65 24	25	
		r	
20 46	98 302	6·150	
21 43		6·140	
23 7	102 299	3·310	
24 1		3·320	
25 41	66° 17′·5	18′·5	
26 42	66 14	15	

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TABLE II.—*continued.*

Chronometer Times.	Readings of Scales or Position-Circle.			
December 8—*continued.*				
	I.–IV.		p : 529	s : 530
1 28 53	66°	47′	48′	
30 15	66	47	48	
			r	
31 19	107	294	2·709	
33 1			2·777	
34 0	66°	46′	47′	
35 26	66	45	46	
			r	
38 10	93	308	2·539	
39 14			2·501	
40 6	66°	29′	30′	
41 26	66	30	30·5	
	Standards.		p : 531	s : 532
	III.–IV.		p : 533	s : 534
h m s				
2 6 35	66°	45′·5	46′	
7 48	66	46·5	47·5	
			r	
9 17	92	309	2·423	
10 30			2·417	
12 22	108	293	3·092	
13 33			3·104	
15 22	67°	0′	0′·5	
16 31	67	2	2·5	
	II.–IV.		p : 535	s : 536
2 19 29	67°	5′·5	6′·5	
21 23	67	7	8	
			r	
22 17	105	296	3·070	
23 18			3·040	
24 28	95	306	2·440	
25 36			2·415	
27 3	66°	54′	55′	
27 48	66	43	44	

Chronometer Times.	Readings of Scales or Position-Circle.			
December 8—*continued.*				
	II.–III.		p : 537	s : 538
2 30 43	67°	11′·5	12′	
31 35	67	9	9·5	
			r	
32 49	103	298	3·000	
34 27			2·984	
35 30	97	304	2·469	
37 49			2·491	
38 44	66°	35′	35′·5	
40 35	66	29	29·5	
	B		t	T
h m	in.		°	°
1 43	30·08		65·0	60·7
2 43	30·08		63·3	60·1
December 14.			GILL.	
	III.–IV.		p : 539	s : 540
h m s				
1 17 45	55°	58′	58′·5	
19 15·5	56	13	13·5	
			r	
23 23	99	301	3·093	(61° 20′)
25 15			3·054	(50 55)
27 23	100	299	6·281	(53 31)
28 31			6·280	(65 42)
29 53	59°	40′	40′·5	
31 4	60	13	13·5	
	II.–III.*		p : 541	s : 542
1 36 56	65°	33′	33′·5	
38 37	65	24·5	25	
			r	
40 5	98	301	6·080	
41 30			6·096	
43 12	102	299	2·766	
44 6			2·730	
45 56	64°	27′	27′·5	
50 3·5	64	32	32·5	

* The first half of this observation is an observation of II.–III.; the second half of II.–IV. *Rejected.*

Chronometer Times.	Readings of Scales or Position-Circle.			
December 14—*continued.*				
	II.–IV.		p : 543	s : 544
1 54 36·5	64°	38′	38′·5	
56 32	64	34·5	35	
			r	
58 9	102	299	2·531	
59 22			2·513	
2 2 45	98	303	3·020	
4 12·5			3·028	
5 55	63°	29′·5	30′	
7 17	63	22·5	23	
	Standards.		p : 545	s : 546
	I.–IV.		p : 547	s : 548
h m s				
2 24 40	65°	41′·5	42′	
26 43	65	40	40·5	
			r	
28 15	105	295	3·750	
29 16			3·800	
31 5	94	305	5·620	
32 50			5·630	
33 40	65°	14′	14′	
35 13	65	14	14	
	I.–III.		p : 549	s : 550
2 39 31	66°	5′	5′·5	
41 21	66	6	6·5	
			r	
42 27	95	305	4·299	
46 1			4·193	
47 29	104	295	5·256	
48 15			5·272	
49 28	66°	42′	42′·25	
50 28	66	39	39·5	
	B		t	T
h m	in.		°	°
1 51	30·02		65·0	60·1
2 53	30·04		64·0	60·8

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Instrumental Constants.—The instrumental distances have been derived from the recorded scale readings by means of the tables given in *Cape Annals*, vol. vii. pages 46–54. The run, which is required as argument of Table C, was determined on every night by measuring the space 300–301 on the scale. It is given below in Table VIII. The periodic errors of the screw (vol. vii., Table D, page 54) have been neglected for the observations of the satellites, but were applied to the observations of the standard stars.

The position-angles must be corrected for equatorial adjustment. The formula used for this purpose is given in *Cape Annals*, vol. vi. page 162. As there are some errors in the signs of the formula as there given, it is here transcribed with the correct signs. The correction to be applied to any instrumental position-angle is :—

$$\begin{aligned}\Delta p = -35' \; 43'' &\mp 21''\cdot 2 \, [\sin\phi \cos\delta_0 - \cos\phi \sin\delta_0 \cos\tau_0] \\ &+ 27\cdot 3 \sin\tau_0 \sec\delta_0 \\ &- 26\cdot 1 \cos\tau_0 \sec\delta_0 \\ &\pm 25\cdot 4 \sec\delta_0 \\ &\mp 6\cdot 5 \tan\delta_0 \\ &- 3\cdot 7 \cos\phi \tan\delta_0 \sin\tau_0,\end{aligned}$$

where ϕ is the latitude and δ_0 and τ_0 are the declination and hour-angle of the middle point between the two stars. The upper signs must be taken for *east* hour-angles and the lower sign for *west* hour-angles. The corrections for equatorial adjustment are given in Table III.

TABLE III.

Corrections for Equatorial Adjustment.

Hour-Angle East.	Correction.		Hour-Angle West.	Correction.	
	Decl. −7°.	Decl. −10°.		Decl. −7°.	Decl. −10°.
h m	′ ″	′ ″	h m	′ ″	′ ″
−1 0	−35 38	−35 37	+1 0	−36 40	−36 43
1 30	39	38	1 30	35	38
2 0	40	39	2 0	30	33
2 30	42	40	2 30	25	27
3 0	43	41	3 0	20	21
3 30	43	42	3 30	15	16
4 0	−35 43	−35 42	4 0	−36 10	−36 11

For those observations of position-angle where the microscopes were read, we require the reading for coincidence of the fixed and movable wires. These readings are given in Table IV.

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TABLE IV.

COINCIDENCE OF POSITION-CIRCLE MICROSCOPE WIRES.

Date.	Microscope.		Date.	Microscope.	
	A.	B.		A.	B.
August 19	5′ 0″	5′ 20″	September 5	5′ 6″	5′ 20″
20	5	18	8	6	21
21	5	15	13	3	20
24	5	20	14	7	19

The run of the position-circle microscopes was determined on August 24 and found to be negligible. No corrections for run were accordingly applied.

Table V. contains the details of the chronometer comparisons.

Aberration, Refraction, Parallax.—The observed sidereal times after the application of the chronometer corrections are given in Tables XI. and XII. in the fourth column. These were then converted into Greenwich mean times.

The adopted longitude and latitude of the Cape Observatory are

$$\lambda = 1^h\ 13^m\ 54^s{\cdot}8 \text{ east of Greenwich.}$$
$$\phi = -33^\circ\ 56'\ 3''{\cdot}5.$$

The Greenwich mean times include the correction for aberration which was applied, using the distance of Jupiter from the earth as given in the *Nautical Almanac* (the correction $\delta \log \Delta$, to reduce from true distance to distance from earth at time of observation to Jupiter at time when the observed light left it, having been first applied, as explained in Chapter II). The adopted light time is $498^s{\cdot}46$. These corrected times are given in the fifth column of the Tables XI. and XII. The observed distances and position-angles were not corrected for aberration of the fixed stars (except, of course, those of the standard stars). The refractions were computed by means of the tables given in *Cape Annals*, vol. vii., Part 3. For computing the hour-angles and zenith-distances for this purpose, the R.A. and declination of Jupiter, as given in the *Nautical Almanac*, were used instead of the exact places for the satellites.

The observations were not reduced to the centre of the earth.

The corrections applicable on this acount are

$$\frac{\delta s}{s} = -\pi\rho \cos \zeta_0$$
$$\delta p = \pi\rho \cos \phi' \tan \delta_0 \sin \tau_0$$

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where

π = equatorial horizontal parallax
ρ = radius-vector } of the observer
ϕ' = reduced latitude }
δ_0 = declination }
ζ_0 = zenith-distance } of the middle point
τ_0 = hour-angle }

TABLE V.

CHRONOMETER COMPARISONS.

Date.	Transit Clock.	Chrono-meter.	Chrono-meter. slow on T.C.	T.C. slow on S.T.	Chrono-meter slow on S.T.	Adopted at 20^h.	Adopted Hourly Rate.	Date.	Transit Clock.	Chrono-meter.	Chrono-meter slow on T.C.	T.C. slow on S.T.	Chrono-meter slow on S.T.	Adopted at 20^h. / at 0^h.	Adopted Hourly Rate.
1891.	h m s	m s	+ m s	− s	s	s	s	1891.	h m s	m s	+ m s	− s	s	s	s
Aug. 17	22 28 0	26 52·8	1 7·2	46·9	+20·7	+20·5	+0·06	Sept. 22	19 23 0	22 5·0	55·0	42·7	+12·3	+12·3	+0·02
19	22 26 0	24 51·5	1 8·5	46·2	22·3	22·0	0·06		21 47 0	46 4·8	55·2	42·7	12·5	...	...
20	19 23 0	21 51·0	1 9·0	45·7	23·3	23·5	0·14	23	19 34 0	33 2·8	57·2	42·7	14·5	14·5	0·08
	21 55 0	53 50·5	1 9·5	45·6	23·9	...	...		21 22 0	21 2·6	57·4	42·7	14·7	...	...
21	19 25 0	23 48·2	1 11·8	45·3	26·5	26·7	0·14	24	19 24 0	22 2·0	58·0	42·6	15·4	15·4	0·04
	21 10 0	8 47·8	1 12·2	45·2	27·0	...	...	28	19 52 0	50 57·5	1 2·5	42·4	20·1	20·0	+0 01
24	19 37 0	35 41·8	1 18·2	44·8	33·4	33·5	0·13	29	20 51 0	49 57·7	1 2·3	42·3	20·0	20·0	0·00
	21 37 0	35 41·5	1 18·5	44·7	33·8	...	...	30	19 35 0	33 57·7	1 2·3	42·1	20·2	20·0	−0·01
25	19 23 0	21 39·9	1 20·1	44·3	35·8	36·1	0·12	Oct. 2	19 43 0	42 1·6	58·4	42·2	16·2	16·3	0·06
	21 32 0	30 39·4	1 20·6	44·2	36·4	...	...	4	19 42 0	41 2·5	57·5	42·4	15·1	15·0	0·04
27	21 19 0	17 37·2	1 22·8	43·7	39·1	38·9	0·12		21 25 0	24 2·8	57·2	42·4	14·8	...	...
29	19 26 0	24 35·1	1 24·9	43·5	41·4	41·5	0·12	6	21 41 0	40 5·8	54·2	42·5	11·7	11·9	0·06
	22 4 0	2 34·8	1 25·2	43·4	41·8	...	...	9	1 28 0	27 8·1	52·9	42·7	10·2	10·4	0·02
30	19 50 0	48 32·3	1 27·7	43·3	44·4	44·4	0·12	10	23 49 0	48 8·1	52·9	42·9	10·0	9·7	−0·02
31	19 27 0	25 30·2	1 29·8	43·1	46·7	46·8	0·10		1 3 0	2 8·0	52·0	43·0	9·0	...	...
	21 58 0	56 30·0	1 30·0	43·1	46·9	...	...	13	23 55 0	54 6·3	53·7	43·5	10·2	10·1	0·00
Sept. 1	19 35 0	33 28·7	1 31·3	43·0	48·3	48·4	0·07		1 44 0	43 6·6	53·4	43·6	9·8	...	...
	21 41 0	39 28·5	1 31·5	43·0	48·5	...	...	14	23 57 0	56 6·3	53·7	43·6	10·1	10 1	0·00
3	19 48 0	46 26·3	1 33·7	43·0	50·7	50·7	0·05		1 46 0	45 6·1	53·9	43·6	10·3	...	...
5	17 47 0	45 25·5	1 34·5	42·8	51·7	52·0	0·03	15	1 9 25	8 30·6	54·4	43 6	10·8	10·1	0·00
	22 14 0	12 25·1	1 34·9	42·7	52·2	...	...	16	0 0 0	59 6·5	53·5	43·6	10·1	10·1	0·00
6	19 43 0	41 23·6	1 36·4	42·7	53·7	53·7	0·08		1 56 0	55 6·5	53·5	43·7	9 8	...	...
7	19 43 0	41 18·6	1 41·4	42·7	58·7	58·7	0·20	17	1 11 0	10 6·3	53·7	43·9	9·8	9·8	+0·02
	21 18 0	16 18·4	1 41·6	42·7	58·9	...	...	19	2 2 0	1 3·7	56·3	44·6	11·7	11·6	0·04
8	19 27 0	25 15·9	1 44·1	42·6	61·5	61·5	0·12	23	23 39 0	38 2·4	57·6	45·6	12·0	12·0	0·00
	21 25 0	23 15·8	1 44·2	42·6	61·6	...	...		1 11 0	10 2·4	57·6	45·7	11·9	...	...
10	19 27 0	25 13·4	1 46·6	42·7	63·9	63·9	0·05	Nov. 5	23 41 0	38 58·5	1 1·5	46·8	14·7	14·7	0·03
	21 31 0	29 13·2	1 46·8	42·7	64·1	...	...		2 1 0	59 58·5	1 1·5	46·9	14·6	...	...
11	18 33 0	32 9	51	42·7	8·3	...	...	6	23 4 0	2 57·8	1 2·2	47·0	15·2	15·3	0·05
13	19 37 0	36 10·1	49·9	42·7	7·2	7·2	0·06		1 40 0	38 57·3	1 2·7	47·0	15·7	...	...
	21 41 0	40 10·0	50·0	42·7	7·3	...	...	8	0 11 0	9 53·6	1 6·4	47·4	19·0	19·0	0·03
14	17 57 0	56 9·0	51·0	42·7	8·3	8·6	0·04		1 19 0	17 53·6	1 6·4	47·4	19·0	...	...
	21 48 0	47 8·5	51·5	42·7	8·8	...	...	9	0 1 0	59 53·0	1 7·0	47·6	+19·4	+19·4	0·02
18	19 34 0	33 5·0	55·0	42·7	12·3	12·3	0·05	22	2 5 0	4 47·3	12·7	52·7	−40·0	−40·0	0·04
	21 41 0	40 4·9	55·1	42·7	12·4	...	...	23	17 16 14	16 0·5	13·5	52·8	39·3	39·0	0·04
19	19 19 0	18 3·8	56·2	42·7	13·5	13·5	+0·01	27	0 40 0	39 42·0	18·0	54·6	36·6	36·6	+0·03
	21 53 0	52 3·8	56·2	42·7	13·5	...	...		1 38 0	37 42·0	18·0	54·7	36·7	...	...
20	19 29 0	28 3·6	56·4	42·7	13·7	13·6	0·00	Dec. 8	2 56 30	55 42·2	47·8	59·8	12·0	11·9	−0·05
	21 35 0	34 3·7	56·3	42·7	13·6	...	...	14	1 8 0	7 16·9	43·1	62·6	−19·5	−19·5	−0·05
21	21 31 0	30 4·4	55·6	42·7	+12·9	+13·0	−0·01								

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If we take

$$\pi = 2''\cdot 1 \qquad \phi' = -34^\circ$$
$$\delta = -9^\circ \qquad \rho = 1$$

we get the following corrections :—

hour-angle	$\pm 1^h$	$\pm 2^h$	$\pm 3^h$	$\pm 4^h$
$\frac{\delta s}{s}$	$-0\cdot00000088$	$-0\cdot00000081$	$-0\cdot00000068$	$-0\cdot00000051$
δp	$\mp 0''\cdot 1$	$\mp 0''\cdot 1$	$\mp 0''\cdot 2$	$\mp 0''\cdot 2$

which can safely be neglected. In the case of the distances a very small systematic error can thus be introduced. We will return to this point later on (Chapter III.).

Standard Stars.—The heliometer cannot give *absolute* determinations of distance and position-angle, because the scale-value, the index-error of the position-circle, and the equatorial adjustment, alter with the temperature of the different parts of the instrument. A pair of stars was therefore selected near the place of Jupiter and approximately in the position-angle of the plane of the satellites, and this standard pair was observed on every night when observations of the satellites were made.

The standard stars are

S.D. $-8^\circ\cdot 5905$ mag. 7·0 and S.D. $-7^\circ\cdot 5838$ mag. 6·2.

They were compared by the heliometer with the standard stars of the triangulation of the Victoria Comparison stars, of which the distance and position-angle are probably better known than of any other pair of stars in the heavens.

The final R.A. and declination of the Victoria Standards, as derived from the discussion of the triangulation, and from the meridian observations, are, for the equinox 1889·0 and the epoch 1889·55 :

			Proper Motions.	
p :	$19^h\ 35^m\ 56^s\cdot 005$	$-4^\circ\ 32'\ 49''\cdot 33$	$-0''\cdot 021$	$+0''\cdot 004$
t :	19 43 4·463	$-4\ 46\ 18\cdot 32$	$-0\cdot 010$	$-0\cdot 016$.

From these we derive, for the epoch 1891·69, and referred to the equinox of 1891·0, the following values of the true distance and position-angle of these stars (the position-angle being taken at the middle point) :

$$s_0 = 6456''\cdot 404 \qquad \pm 0''\cdot 025$$
$$p_0 = 97^\circ\ 11'\ 22''\cdot 2 \qquad \pm 0\cdot 80.$$

The approximate distance of the Jupiter standard stars is $6705''$. Consequently the correction for scale-value to be applied to the observed distances of the Jupiter standard stars is

(1) $\qquad 1\cdot 0467\{6456''\cdot 404 - \text{Obs. Dist. of Vict. Standards}\}$.

The comparisons are given in Table VI.

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TABLE VI.

Observations of Victoria Standards, and corresponding Observations of Jupiter Standards.

Distances.

Date.	Victoria Standards.										Jupiter Standards.				
	Rotation No.	Sidereal Time.	Scale Reading. 19 381	Scale Reading. 180 220	Instrumental Distance.	Corrections for Refraction.	Corrections for Aberration.	Observed Distance.	True—Observed.	Definition. I S	Rotation No.	Observed Distance.	Correction for Scale Value.	True Distance.	Weight.
1891.		h m	r	r		+	−								
Aug. 29	107	21 48·2	2·588	2·756	6454″·407	2″·675	0″·397	6456″·685	−0″·281	2–3 2–3	106	6705″·42	−0″·29	6705″·13	1
31	119	21 9·3	*2·644	2·710	·923	2·200	0·414	6·709	−0·305	2–3 2–3	118	5·65	−0·32	5·33	½
Sept. 1	136	20 50·7	2·683	2·749	·930	2·047	0·421	6·556	−0·152		135	5·46	−0·16	5·30	1
5	160	21 56·3	2·665	2·717	·994	2·730	0·450	7·274	[−0·870]	2 2–3	159	5·39	−0·91	[4·48]	1
8	184	21 2·5	*2·598	*2·659	·951	2·140	0·471	6·620	−0·216		183	5·75	−0·23	5·52	1
10	200	21 16·7	*2·638	*2·713	·882	2·314	0·484	6·712	−0·308	2 2–3	{198 199}	5·65	−0·32	5·33	1½
14	233	21 31·6	2·549	†2·739	·298	2·425	0·508	6·215	+0·189	1–2 1–2	232	5·05	+0·20	5·25	1
19	265	21 32·7	*2·622	2·740	·660	2·428	0·536	6·552	−0·148	1–2 2	{263 264}	5·52	−0·15	5·37	1½
20	282	21 18·9	2·578	*2·713	·584	2·281	0·540	6·352	+0·079	1–2 2	281	[4·28]	+0·08	[4·36]	1
22	315	21 33·9	2·575	2·732	·462	2·470	0·550	6·382	+0·022	1–2 2	314	5·34	+0·02	5·36	1
23	327	21 8·2	2·591	2·726	·579	2·219	0·555	6·243	+0·161	1–2 3	326	5·52	+0·17	5·69	1

Position-Angles.

Date.	Victoria Standards.									Jupiter Standards.				
	Rotation No.	Sidereal Time.	Instrumental Position Angle.	Corrections for Equatorial Adjustment.	Corrections for Refraction.	Corrections for Prec. & Aberr.	Observed Position Angle.	True—Observed.	Definition. I S	Rotation No.	Observed Position Angle.	Correction for Index Error.	True Position Angle.	Weight.
1891.		h m												
Sept. 5	143	18 1·0	*97° 46′ 59″	−35′ 42″	−1″·5	+10″	97° 11′ 12″	+10″	3 3–4	148	68° 4′ 23″	+10″	68° 4′ 33″	1
										152	18	+10	28	4
14	215	18 17·4	51	−35 41	−1·2	+11	9	+13	3 3–4	231	19	+13	32	4

Taking the weighted mean of the true distances given in the last column, we find

$$s = 6705''\cdot20 \pm 0''\cdot08 .$$

If, however, we reject the determinations on September 5 and September 20, the weighted mean becomes

$$s = 6705''\cdot36 \pm 0''\cdot05 .$$

On September 5 the distance of the Victoria standards is evidently affected by

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some large error or mistake. The same thing must be assumed for the measure No. 281 of the Jupiter standards on September 20. The other observation (No. 273) on the same date gives the observed distance 6705″·23.

For the position-angles we find similarly, as the weighted mean of the three determinations,

$$p = 68^\circ\ 4'\ 30'' \pm 1''\cdot7 .$$

The Jupiter standards were also observed on the meridian on 15 nights each. The resulting R.A. and declinations are given in the *Cape Catalogue* for 1890. They are (equinox 1890·0, epoch 1891·8) :—

			Proper Motions.	
S.D.−8°·5905	$22^h\ 30^m\ 30^s\cdot930$	$-8^\circ\ 14'\ 2''\cdot67$	...	...
S.D.−7 ·5838	22 37 29·590	−7 32 19·36	$-0^s\cdot0029$	$+0''\cdot023$.

The distance and position-angle derived from these co-ordinates are

$$s = 6705''\cdot29 \qquad p = 68^\circ\ 4'\ 32''.$$

The agreement is excellent. The values derived from the heliometer comparisons have been finally adopted, viz. :—

$$s = 6705''\cdot36 \pm 0''\cdot065,$$
$$p = 68^\circ\ 4'\ 30'' \pm 2''\cdot1 .$$

The probable errors have been increased to take account of the probable error of the distance and position-angle of the Victoria standards.

The observations of the Jupiter standard stars are given in Table VII.

TABLE VII.

Observations of Jupiter Standard Stars.

Date.	Observer.	Distances.									Position-Angles.							
		Rot. No.	Sidereal Time.	Scale Readings. 16 384	Scale Readings. 183 216	Instrumental Distance.	Refraction.	Aberration.	Observed Distance.	True − Observed.	Rot. No.	Sidereal Time.	Instrumental Position-Angle.	Equatorial Adjustment.	Refraction.	Prec. Aberr. Nut.	Observed Position-Angle.	True − Observed.
							+						68°	−		+	68°	
1891.			h m	r	r							h m						
Aug. 19	G	14	20 14·4	†3·271	†6·059	6701″·81	3″·14	+0″·12	6705″·07	+0″·29	13	20 13·9	*40′ 24″	35′ 44″	−21″	10″	4′ 29″	+ 1″
20	G	32	20 24·7	†3·391	†6·036	2·53	3·01	·11	5·65	−0·29	31	20 24·6	*40 16		43	−20	10	23	+ 7
		42	21 35·8	†3·417	†5·988	2·90	2·25	·11	5·26	+0·10	41	21 34·8	*39 49		42	−14	10	3	+27
21	G	48	20 20·5	†3·399	†6·097	2·25	3·03	·10	5·38	−0·02	47	20 19·5	*40 18		44	−20	10	24	+ 6
24	G	56	20 29·1	†3·400	†6·030	2·60	2·94	·07	5·61	−0·25	55	20 29·0	40 8		43	−19	10	16	+14
		66	21 22·1	†3·425	†5·950	3·13	2·35	·07	5·55	−0·19	65	21 21·6	40 0		42	−15	10	13	+17
25	G	68	19 34·8	†3·225	†6·191	0·92	4·17	·05	5·14	+0·22	67	19 34·7	40 15		45	−27	10	13	+17
		76	20 29·0	†3·389	†6·009	2·66	2·94	·05	5·65	−0·29	75	20 29·3	40 15		44	−19	10	22	+ 8
27	F	91	20 52·9	†3·370	†6·059	2·31	2·55	·03	4·89	+0·47	90	20 52·7	39 52		42	−16	10	4	+26
29	G	99	20 46·8	†3·351	†5·972	2·63	2·70	·01	5·34	+0·02	98	20 46·3	40 15		43	−17	10	25	+ 5
		106	21 37·6	3·402	5·914	3·18	2·23	+0·01	5·42	−0·06								

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TABLE VII.—*continued.*

Date.	Observer.	Distances. Rot. No.	Sidereal Time.	Scale Readings. 16 384	Scale Readings. 183 216	Instrumental Distance.	Refraction.	Aberration.	Observed Distance.	True − Observed	Position-Angles. Rot. No.	Sidereal Time.	Instrumental Position Angle.	Equatorial Adjustment.	Refraction.	Prec. Aberr. Nut.	Observed Position-Angle.	True − Observed.
							+						68°	−		+	68°	
1891.			h m	r	r		″	″	″	″		h m	′ ″	′ ″	″	″	′ ″	″
Aug. 31	G	111	19 59·7	†3·353	†6·037	6702·34	3·40	−0·01	6705·73	−0·37	110	19 59·9	40 11	35 44	−22	10	4 15	+15
		117	20 47·9	†3·429	†5·944	3·18	2·63	·01	5·80	−0·44	116	20 48·0	40 0	43	−17	10	10	+20
		118	20 57·7	3·414	5·935	3·15	2·51	·01	5·65	−0·29								
Sept. 1	G	129	19 57·7	†3·343	†6·070	2·11	3·43	·03	5·51	−0·15	128	19 57·8	40 8	44	−22	10	12	+18
		135	20 33·5	3·382	5·990	2·71	2·78	·03	5·46	−0·10	134	20 33·4	40 8	43	−18	10	17	+13
5	G	149	20 17·6	†3·315	†5·980	2·43	3·05	·08	5·40	−0·04	148	20 16·5	*40 17	44	−20	10	23	+ 7
		159	21 43·1	3·423	5·909	3·33	2·14	·08	5·39	−0·03	152	20 52·9	*40 6	42	−16	10	18	+12
7	G	173	21 2·2	3·310	5·860	3·00	2·46	·10	5·36	0·00								
8	G	180	20 16·4	3·342	5·935	2·79	3·07	·11	5·75	−0·39	179	20 15·5	*40 17	44	−20	10	23	+ 7
		183	20 47·8	3·373	5·874	3·25	2·61	·11	5·75	−0·39								
10	G	190	20 7·7	3·349	5 980	2·58	3·33	·13	5·78	−0·42	197	20 55·2	40 0	42	−17	10	11	+19
		198	21 0·1	3·383	5·866	3·23	2·55	·13	5·65	−0·29								
		199		3·361	5·883													
13	G	208	20 48·4	3·364	5·905	3·06	2·67	·16	5·56	−0·20	207	20 47·8	*40 13	43	−17	10	23	+ 7
14	G	228	20 46·5	†3·389	†5·810	3·65	2·65	·17	6·13	[−0·77]	231	21 14·1	*40 6	42	−15	10	19	+11
		232	21 23·1	3·346	5·910	2·93	2·29	·17	5·05	+0·31								
18	F	247	21 18·4	3·375	5·922	3·04	2·33	·21	5·16	+0·20	246	21 18·1	40 0	42	−14	11	15	+15
19	G	254	20 18·2	3·378	5·945	2·92	3·06	·22	5·76	−0·40	255	20 17·9	40 30	44	−20	11	37	− 7
		263	21 16·0	3·414	5·888	3·40	2·34	·22	5·52	−0·16	262	21 12·5	40 0	42	−15	11	14	+16
		264		3·410	5·881													
20	G	273	20 13·2	†3·336	†5·965	2·59	2·87	·23	5·23	+0·13	272	20 13·0	40 4	44	−20	11	11	+19
		281	21 5·0	3·373	5·905	2·08	2·43	·23	4·28	[+1·08]	280	21 5·3	40 0	42	−15	11	14	+16
21	G	286	20 7·5	†3·320	†6·031	2·19	3·24	·24	5·19	+0·17	285	20 7·1	40 8	44	−21	11	14	+16
		298	21 15·8	†3·416	†5·890	3·38	2·32	·24	5·46	−0·10								
22	G	308	20 51·8	3·388	5·893	3·24	2·61	·25	5·60	−0·24	307	20 51·5	40 15	42	−17	11	27	+ 3
		314	21 24·2	3·414	5·909	3·28	2·31	·25	5·34	+0·02	313	21 23·4	40 11	42	−14	11	26	+ 4
23	G	317	19 52·8	†3·290	†5·998	2·22	3·60	·26	5·56	−0·20	316	19 52·8	40 11	44	−23	11	25	+ 5
		326	20 59·2	3·418	5·920	3·25	2·53	·26	5·52	−0·16	325	20 58·6	40 26	42	−16	11	39	− 9
28	G	331	20 9·6	3·343	5·935	2·79	3·24	·31	5·72	−0·36	330	20 8·9	40 26	44	−21	11	32	− 2
29	G	339	20 12·3	3·316	5·947	2·59	3·17	·32	5·44	−0·08	338	20 11·3	40 19	44	−21	11	25	+ 5
Oct. 4	G	349	20 38·8	3·377	5·860	3·34	2·65	·37	5·62	−0·26	348	20 38·8	40 0	43	−17	10	10	+20
6	F	363	21 1·4	3·453	5·903	3·53	2·43	·39	5·57	−0·21	362	21 0·7	40 15	35 42	−16	10	27	+ 3
9	G	373	0 35·4	3·460	5·873	3·69	1·97	·42	5·24	+0·12	372	0 34·9	40 19	36 28	+16	10	17	+13
13	G	391	0 38·4	†3·380	†5·820	3·55	2·05	·45	5·15	+0·21	390	0 38·3	40 11	27	+17	10	11	+19
14	F	412	1 29·3	†3·409	†5·911	3·23	2·41	·46	5·18	+0·18	411	1 29·3	40 0	19	+35	10	26	+ 4
16	F	428	1 7·5	3·418	5·820	3·76	2·16	·48	5·44	−0·08	427	1 7·8	40 8	23	+26	10	21	+ 9
17	F	440	0 49·3	3·466	5·839	3·92	2·04	·49	5·47	−0·11	439	0 49·6	40 15	16	+20	10	29	+ 1
19	F	448	1 0·9	†3·410	†5·800	3·82	2·11	·50	5·43	−0·07	447	1 0·8	40 0	24	+23	10	9	+21
23	F	462	0 51·0	3·429	5·797	3·93	2·10	·53	5·50	−0·14	461	0 50·9	40 0	26	+21	10	5	+25
Nov. 5	G	470	0 54·9	3·423	5·787	3·94	2·09	·60	5·43	−0·07	469	0 54·9	22	25	+21	10	28	+ 2
6	G	484	0 45·0	3·315	6·028	2·17	2·03	·61	3·59	+1·77	483	0 44·5	0	27	+18	10	1	+29
8	G	494	0 46·9	3·286	6·018	2·08	2·03	·62	3·49	+1·87	493	0 47·0	8	27	+19	10	10	+20
22	G	509	1 27·8	3·495	5·871	3·86	2·35	·66	5·55	−0·19	508	1 26·7	11	21	+34	9	33	− 3
27	G	520	1 23·8	3·569	5·919	4·01	2·22	·66	5·57	−0·21	519	1 24·0	40 0	20	+31	9	20	+10
Dec. 8	F	532	1 56·7	3·478	5·961	3·36	2·70	·65	5·40	−0·04	531	1 56·8	39 45	13	+51	9	32	− 2
14	G	546	2 16·8	†3·367	†5·990	6702·63	3·10	−0·62	6705·10	+0·26	545	2 16·1	39 26	36 12	+68	9	4 31	− 1

228 and 281 *rejected* for discordance.

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The correction for scale-value to be applied to any observed distance s, is now

$$\Delta s = s. \frac{\text{True} - \text{Observed}}{6705''{\cdot}36} \quad (2)$$

When more than one observation of the standard stars is taken on the same night, the mean was adopted. For those nights on which observations of the Victoria standards are available, these were also used for the determination of the correction for scale value.

The index correction to be applied to any observed position-angle is simply the value "*True — Observed*" given in the last column of Table VII.

The instrumental corrections thus derived are contained in Table VIII.

TABLE VIII.

Instrumental Corrections.

Date	Observer.	Run.	Scale Value per 1000″.	Index Correction.	Date.	Observer.	Run.	Scale Value per 1000″.	Index Correction.
1891.		+ r	″	″	1891.		+ r	″	″
Aug. 17	G	0·026	...	...	Sept. 28	G	(0·024)	−0·051	− 2
19	G	·023	+0·041	+ 1	29	G	·025	− ·011	+ 5
20	G	·024	− ·013	+ 17	Oct. 4	G	·023	− ·037	+ 20
21	G	·025	− ·003	+ 6	6	F	·018	− ·030	+ 3
24	G	·024	− ·031	+ 16	9	G	·026	+ ·017	+ 13
25	G	·021	− ·005	+ 12	10	G	·029	(+ ·020)	(+ 14)
27	F	·021	+ ·067	+ 26	13	G	·026	+ ·030	+ 19
29	G	·027	− ·003	+ 5	14	F	·029	+ ·026	+ 4
31	G	·022	− ·052	+ 18	15	F	·015	...	(+ 6)
Sept. 1	G	·025	− ·018	+ 16	16	F	·020	− ·011	+ 9
5	G	·021	− ·005	+ 11	17	F	·014	− ·016	+ 1
7	G	·023	·000	(+ 8)	19	F	·020	− ·010	+ 21
8	G	·023	− ·056	+ 7	23	F	·020	− ·020	+ 25
10	G	·027	− ·051	+ 19	Nov. 5	G	·022	− ·010	+ 2
13	G	·021	− ·028	+ 7	* 6	G	·025	+ ·253	+ 29
14	G	·025	+ ·044	+ 11	* 8	G	·025	+ ·267	+ 20
18	F	·016	+ ·028	+ 15	* 9	G	(·025)	(+ ·260)	(+ 20)
19	G	·022	− ·040	+ 4	22	G	·029	− ·027	− 3
20	G	·027	+ ·018	+ 18	27	G	·025	− ·030	+ 10
21	G	·024	+ ·005	+ 16	30	G	(·025)	...	...
22	G	·022	− ·016	+ 4	Dec. 8	F	·019	− ·006	− 2
23	G	·021	− ·026	− 2	14	G	·027	+ ·037	− 1
24	G	(·022)	(− ·030)	(− 2)					

* On November 6 an alteration of ·09 in the focal reading of the eyepiece was made.
( ) Values, which are not derived directly from observations, but were determined by interpolation, are inclosed in parentheses.

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Systematic errors in distance.—Having regard to the agreement of the different determinations of the distance of the standard stars, we can confidently assert that this distance is known to within $\frac{1}{100,000}$th of its amount. Consequently the mass of Jupiter can be determined from these observations within $\frac{1}{33,000}$th of its amount, apart from the purely accidental errors of observation, if the observations are *free from systematic errors depending on the distance.*

Systematic errors in the distances depending on the position-angles have been shown not to exist in the triangulation of the Victoria comparison stars, executed by the same observers and the same instrument in the previous year 1890 [see *Cape Annals*, vol. vi., part 1, p. 146]. Further, all errors which are proportional to the distance, whether depending on the position-angle or not (such as, *e.g.*, a possible small error in the adopted refraction constant), are eliminated by the use of the instantaneous scale-value derived from the standard stars, which are in approximately the same position-angle and the same hour-angle as the satellites. The only systematic errors which are of importance for our purpose are those depending on the distance by a law differing from simple proportionality.

Now in the polar triangulation, Mr Hough found evidence of a systematic error depending on the square of the distance [see *Cape Annals*, vol. xi., part 1, p. 112]. If we take the mean of the two observers g and L (the observations of S are too few in number, but if they were included, the result would be practically the same), the following differences *True – Observed* are derived for different distances :—

s	1000″	2000″	3000″	4000″	5000″	6000″	7000″
error	+0″·137	+ ″·040	− ″·018	− ″·036	− ″·016	+ ″·044	+ ″·142.

No weight whatever is to be attached to the large positive values for short distances, which must be considered not as derived from observations, but as forced on by the parabolic nature of the formula used. In fact there are only four distances below 1500″ in the whole triangulation, and none below 900″. We need thus only consider distances of the size of the distance of the standard stars. Whether the short distances between the satellites are affected by systematic errors or not must be decided by a discussion of the residuals of these observations themselves.

Supposing now for a moment that the error depending on s^2 had also existed in 1891, its amount would have been :

for the Victoria standards . . .	+0″·084
for the Jupiter standards . . .	+0″·108.

The formula (1) should then have been replaced by

(1*) $$1\cdot0467\{6456''\cdot404 - \text{Obs. dist. of Vict. standards } +0''\cdot084\}.$$

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The true distances of the Jupiter standard stars as given in the last column of Table VI. would thus require the correction

$$-0''\cdot108 + 1\cdot0467 \times 0''\cdot084 = -0''\cdot020,$$

and the same correction must be added to the final mean, which would thus become $6705''\cdot34$. In Table VII. the observed distances would require a correction of $-0''\cdot108$, the differences *True − Observed* given in that table thus would require a correction of $-0''\cdot108 + 0''\cdot020 = -0''\cdot088$. Consequently the correction for scale-value as actually applied to the observations of the satellites, and computed by the formula (2), would require the further correction

$$(2^*) \qquad \delta s = -s \cdot \frac{0''\cdot088}{6705''} = -0''\cdot0013 \cdot \frac{s}{100''}.$$

In other words, if the error depending on s^2 were assumed to exist in the observations of the standard stars, the measured distances of the satellites after correction for scale-value by the formula (2) would all be too large by $\frac{1}{70,000}$th of their amount, and the resulting reciprocal of the mass would be too small by three times this fraction, *i.e.* it would require a correction of $+0\cdot045$. Though smaller than the probable error $\pm0\cdot058$ due to accidental errors of observation (which will be derived in Chapter III.), this is a quantity which would be too large to be neglected. It is therefore important to discuss the probability of its reality.

In 1890 an error of this kind did certainly not exist in the observations by Gill and Finlay of the Victoria comparison stars, ranging over distances between $600''$ and $7000''$. This is shown conclusively by the comparison with the meridian observations [see *Cape Annals*, vol. vi., part 1, p. 231, and *History of the Cape Observatory*, page xci]. Mr Cookson in 1905 made a special investigation to decide whether it existed in his observations, again with a negative result [see *Cape Annals*, vol. xii., part 2, pp. 182–188, and *History*, p. xc]. It thus becomes probable that it is peculiar to the polar triangulation. Now in 1897, before this triangulation was begun, the instrument was partly dismounted and cleaned. At that time no skilled labour for such work was available at the Cape. In the summer of 1899 a new overhauling of the instrument had become an urgent necessity,* and it was executed by the optical fitter who had been appointed in the meantime. Now the bulk of the observations for the polar triangulation were made between these two dates, and the possibility is not excluded that the error is instrumental in origin and did not exist before 1897 nor after 1899.

However this may be, it is almost certain that it cannot have existed in the

* The writer remembers these circumstances well, as he was at that time staying at the Cape, and observing with the heliometer himself.

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observations by Gill and Finlay in 1891, when the instrument was in the same state as in the previous year when the same observers used it for the Victoria triangulation. The correction (2 *) has accordingly not been applied.

There are, however, other sources of systematic errors depending on the distance, which are undoubtedly real.

Personal error depending on amount of "crossing."—In the Cape heliometer, as in all modern heliometers, there is in the focal plane of the eyepiece a square formed by flat wires, adjusted parallel and perpendicular to the direction of motion of the semilenses. This system of wires rotates with the instrument, and is the means by which directions are estimated in the illuminated field. Before observations are commenced the reversing prism is turned so as to make the apparent direction of these wires vertical and horizontal.

Since the superposition of two images is not an accurate method of measuring, every heliometer observer tries to make the images of the two stars pass and repass through each other by a small motion in position-angle $\pm\psi$. (The equality of the two angles $+\psi$ and $-\psi$ is easily estimated in terms of the diameter of the star discs). In causing the images to "swing through" he instinctively uses the pair of lines at right angles to the distance-motion as a guide. He adjusts the distance till the stars appear as represented in fig. 1, and then immediately moves the instrument in position-angle, when the stars will appear as in fig. 2. When he has satisfied himself that the line joining the images is parallel to the wires p in both positions, he gives the signal "stop" and proceeds to read the scales.

The most refined judgment of this parallelism with the wires is inseparable from the plan of rapidly "crossing" (that is exchanging the position of) the images. The

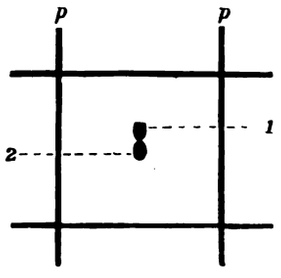

FIG. 1.

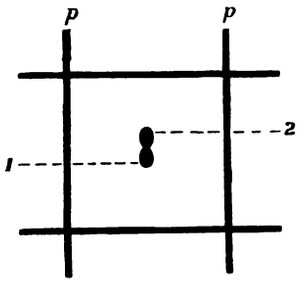

FIG. 2.

eye in this operation is extremely sensitive to a minute change in the apparent position-angle, and the method is probably the most refined that we possess as a practical means of measuring the angular distance between optical discs with high precision. The

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optical discs formed by the half lenses are elongated in the direction perpendicular to the diameter along which the lens has been cut, that is parallel to the wires p. This circumstance greatly facilitates the judgment of parallelism.

Figs. 3 and 4 represent the positions of the images in the focal plane. The angle ψ is of course greatly exaggerated, as is also the distance between the two wires p.

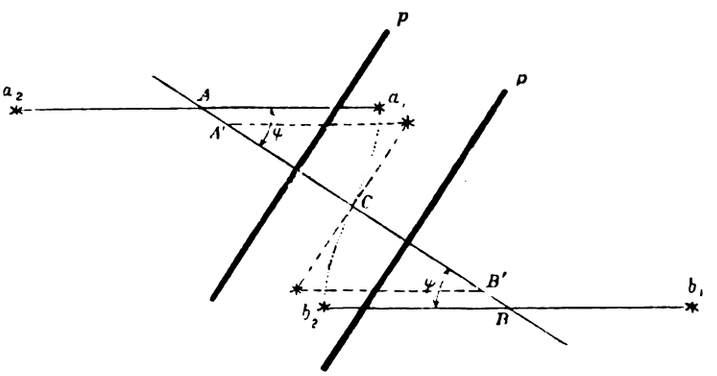

FIG 3

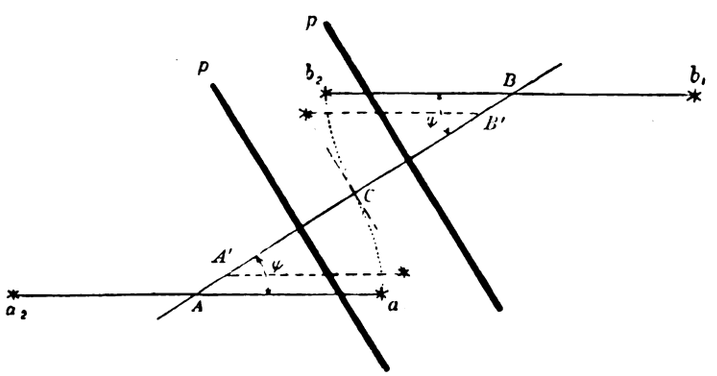

FIG. 4.

A and B are the projections on the focal plane of the optical centres of the two semilenses. The line AB shows the direction of motion of the slides. C is the intersection of the central axis of the instrument with the focal plane. The optical axis of lens A is supposed to be pointing to star 1 and that of lens B to star 2. Lens A gives images of the two stars at a_1 and a_2, and lens B at b_1 and b_2. The direction of the lines $a_2\,a_1$ and $b_2\,b_1$ represents thus the true position-angle of the stars in the heavens. In the figures the separation of the lenses is supposed to correspond to the exact distance between the stars, so that by a rotation in position-angle the two images a_1 and b_2 can be brought to coincidence in C. The observer does not, however, try to effect that coincidence, but, while the line of motion makes an angle $\pm\psi$ with the true

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position-angle, he tries to make the line joining the images a_1 and b_2 parallel to the wires p. To do this he must diminish the separation of the optical centres A and B till they occupy the positions A′ and B′. The measured distance is thus not s, but $A'B' = s \cos \psi$, and the correction to be applied to the measured distance is

$$\Delta s = s\,(1 - \cos \psi) = + \frac{\kappa^2}{2s},$$

where κ is the distance of the artificial double star formed by the images a_1 and b_2, or $\kappa = s \sin \psi$.*

Before and after crossing through, that is in the two positions represented by figs. 3 and 4, there is a small difference in the direction of the line of motion of the slides. The observer is not aware of this, as the wires p, by which he can only be guided in estimating directions, *rotate with the instrument* and remain always perpendicular to the line AB. If the distance is so short that the change in direction of the wires p, caused by the small motion in position-angle needed to cross the images, becomes noticeable, a better method of observing is *not* to cross through, but to make one pointing as represented in fig. 3, but with the images just touching each other (as in fig. 1), so that the major axes of the optical ellipses, which are always parallel to the wires p, are in one line. Then, after rotating the reversing prism through 90°, the next pointing is made as in fig. 4 (or 2). The distance actually measured is again $s \cos \psi$, and needs the correction $+\frac{\kappa^2}{2s}$. This plan was adopted by Gill in all the observations of the satellites.

For all distances under 100″, with a few exceptions, and for one distance of 145″, the angle ψ actually used has been recorded; that is to say the images were placed as in fig. 1 and distance *and position-angle* read off; then for the next pointing the images were placed as in fig. 2, and the distance and position-angle were again read.

The difference between the two position-angles, corrected for the motion of the satellites in the interval, is 2ψ. These distances have of course been corrected accordingly, and are thus free from personal error.

From these observed values of ψ we find the following values of κ for the two observers.

* This explanation of the error depending on κ was already given in *Cape Annals*, vol. vi. p. 234 *sqq*. It is repeated here for the sake of clearness, and to facilitate comparison with the somewhat similar error depending on the separation of optical centres, which is treated below.

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TABLE IX.

VALUES OF κ FROM OBSERVED ψ.

GILL.											FINLAY.			
Rot. No.	s	κ	Definition.	Mean s	Mean κ	Rot. No.	s	κ	Definition.	Mean s	Mean κ	Rot. No.	s	κ	Definition.
	″	″		″	″		″	″		″	″		″	″	
1	22·0	3·71	2			540	44·0	4·36	2–3			404	35·2	7·18	2–3
28	22·4	2·40	2–3	22·3	2·89	335	44·1	2·09	2			361	56·0	5·01	2–3
26	22·5	2·56	2–3			300	44·2	2·01	2			414	71·3	7·49	3
						142	49·9	2·80		50·1	2·72	402	83·5	6·23	2–3
109	30·4	3·65	3–4			385	55·9	2·09	1–2			365	145·1	7·44	2–3
83	31·2	3·08	2–3			172	56·0	3·04	1–2						
186	33·0	2·69	2			279	56·9	2·65	1–2						
337	34·9	2·90	2	34·9	3·23	78	80·5	2·67	2–3			Mean $\kappa = 6''·67$			
81	38·6	3·16	2–3			343	82·2	3·54	3						
80	39·0	3·30	2–3			147	83·2	3·61	3	85·5	3·08				
145	39·4	3·81	3			127	88·7	3·77							
						217	92·7	3·02	3						
Mean of all observations: $\kappa = 3''·04$															

It appears that Gill's κ is very small indeed and very constant. Finlay's observations are too few to permit very definite conclusions to be drawn.

If we arrange Gill's observations according to the estimated quality of images, we get the following means :—

Definition 1 and 1–2	$\kappa = 2''·59$	3 observations
,, 2 ,, 2–3	2 ·91	12 ,,
,, 3 ,, 3–4	3 ·55	5 ,,

The influence of the sharpness of the images is very marked. It seems to be impossible with images 3 or 3–4 to bring the discs closer together than 3″.

It is a very important result that κ shows no sign of variation with the distance. If this is the case with distances up to 1000″ we may be tolerably certain that after correction for κ all distances are free from systematic error.

As the observer was constantly conscious of the necessity of keeping the amount of "crossing" as nearly constant as possible, it is very probable that in the ordinary observations, by crossing through without measuring ψ, κ is even more constant, and perhaps also smaller, than in these exceptional cases, where this necessity does not

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exist, since the value of ψ is recorded and the correction due to the amount of crossing can afterwards be applied.

The value of κ for average definition must be determined for each observer from the observations themselves, by introducing a corresponding unknown quantity into the equations of condition. It must be assumed that the quality of definition is in the mean the same for all distances. That this assumption is not very far from the truth, will be seen from the following little table containing the number of observations of different distances, made with different qualities of images.

Gill.					Finlay.				
Distances.	No. of Observations in Definition.			Mean Definition.	Distances.	No. of Observations in Definition.			Mean Definition.
	1 & 1-2	2 & 2-3	3 & 3-4			1 & 1-2	2 & 2-3	3 & 3-4	
100″— 200″	5	19	2	2·1	100″— 200″	0	8	2	2·3
200— 400	9	30	9	2·2	200— 400	0	7	4	2·6
400—1000	6	33	11	2·4	400—1000	0	8	7	2·6

Observations in which ψ was recorded, rejected observations, and observations in which there is no note about the quality of the images, are excluded from this table. Whether the observations corrected for κ are really free from systematic error will appear from the final residual errors.

For the different values of κ we have the following corrections to different distances:—

κ \ s	50″	100″	200″	500″	1000″
2″·5	0″·062	0″·031	0″·016	0″·006	0″·003
3·0	0·090	0·045	0·022	0·009	0·004
3·5	0·122	0·061	0·031	0·012	0·006
6·7	0·449	0·224	0·212	0·045	0·022

It will be seen that with Gill's κ for average definition the correction for 100″ (the smallest distance observed without recording ψ) is only $\frac{1}{2000}$ of the distance, and for the average distance of 300″ it is only $\frac{1}{20000}$ of the distance. It will therefore be sufficient if the correction due to κ can be determined within $\frac{1}{5}$ of its amount in order that the distances may in the mean, apart from errors of observation, be reliable within $\frac{1}{100000}$ and consequently the mass of Jupiter within $\frac{1}{330000}$ of its amount.

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Distance of optical centres.—There is still another source of error, which, though generally of small importance, does considerably affect the distances measured by placing the images at half or double distance. The optical centres of the two semilenses have so far been supposed to be in the plane of motion, of which the intersection with the focal plane is represented by the line AB in fig. 3. This will however generally not be the case. Let the distances of the optical centres of the semilenses A and B from that plane be δ_A and δ_B. Then, when the lenses are crossed, the optical centres never come into exact coincidence, but pass clear of each other at a minimum distance $\delta = \delta_A + \delta_B$. (It may be remarked that δ can be both positive or negative. The two semilenses are made by cutting one complete lens in two pieces, and these may be afterwards brought closer together than the thickness of the sawcut, which has been taken out).

Fig. 5 represents a measure of double distance.

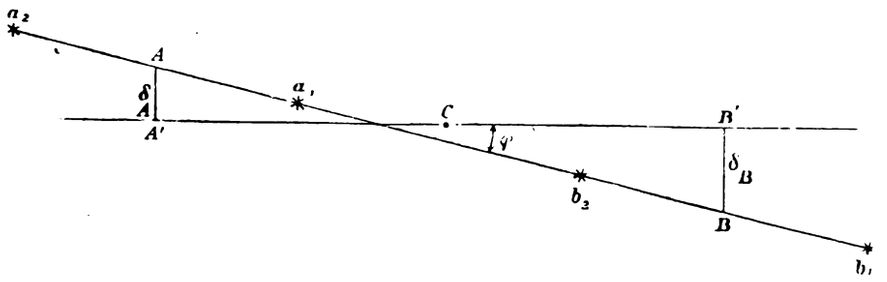

FIG. 5.

The four images are visible in the field at the same time, and are placed so that they lie in a straight line, and the distances $a_2 a_1$, $a_1 b_2$ and $b_2 b_1$ are equal. It is then evident from the diagram that the separation of the lenses A′ B′ is not $2s$, but $2s \cos \phi$, while ϕ is determined by the relation

$$\sin \phi = \frac{\delta_A + \delta_B}{2s}.$$

The measured position-angle will be $p + \phi$ instead of p.

On reversing the lenses, A′ and B′ change places (A and B moving parallel to the line A′ B′), and the measured distance will again be $2s \cos \phi$, but now the measured position-angle is $p - \phi$.

The position-angle is therefore correct in the mean, and the difference between the two measured values affords a means of determining ϕ and hence $\delta = \delta_A + \delta_B$.

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The same reasoning applies, of course, to measures of half distance, when the images are placed so that b_2 comes midway between a_2 and a_1, and a_1 midway between b_2 and b_1.

The distances so measured thus require a correction

$$\Delta s = s\,(1 - \cos\phi),$$

where

$$\sin\phi = \frac{\delta}{2s} \text{ for observations of double distance,}$$

$$\sin\phi = \frac{2\delta}{s} \text{ ,, ,, ,, half ,, .}$$

The value of δ can be determined from the comparison of the readings of the position-circle in the two positions of the semilenses. If observations of satellites are used for this purpose, the motion of the satellites in the interval between the two pointings must be taken into account. This, of course, is not necessary for the standard stars. From the observations of the standard stars we find the following values of δ. Where two decimals of a second are given, the position-angles were read by microscope.

TABLE X.

SEPARATION OF OPTICAL CENTRES FROM OBSERVATIONS OF JUPITER STANDARDS.

Date.	δ.	Date.	δ.	Date.	δ.	Date.	δ.
Aug. 19	+0".94	Sept. 8	+0".81	Sept. 28	+1".1	Oct. 19	+0".5
20	1·10	10	1·5	29	1·1	23	0·5
21	0·41	13	1·46	Oct. 4	0·5	Nov. 5	1·2
24	0·6	14	1·87	6	1·0	6	2·4
25	1·0	18	1·5	9	0·6	8	2·7
27	0·7	19	1·7	13	1·1	22	0·3
29	1·0	20	1·3	14	0·5	27	1·0
31	0·5	21	1·2	16	0·7	Dec. 8	0·5
Sept. 1	1·2	22	2·1	17	1·0	14	1·2
5	0·64	23	1·2				

It appears, thus, that the value of δ has been fairly constant throughout the period of observations. The mean value is practically $+1''{\cdot}0$.

It is not impossible that, owing to slight irregularities in the slides, δ has different values for different distances. For this reason the measures of half and double distance have as a rule been corrected by means of the value of δ derived from these same measures.

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In ordinary observations made by the method of crossing through, the influence of this distance of optical centres is similar to the effect of κ, though much smaller.

Figs. 6 and 7 represent the positions of the images in such an observation as they appear in the field when the apparent direction of the wires p has been made vertical by the use of the reversing prism. The meaning of the different symbols is the same as in the preceding figures. One pointing is represented by fig. 6; the measured distance is $A'B' = s \cos(\psi - \phi)$. Then, after crossing through, a second pointing is made as in fig. 7, and now the measured distance is $s \cos(\psi + \phi)$. When ψ and ϕ are both small the mean will be $s \cos\phi \cos\psi$, and therefore depends on ϕ, that is on the distance of optical centres.*

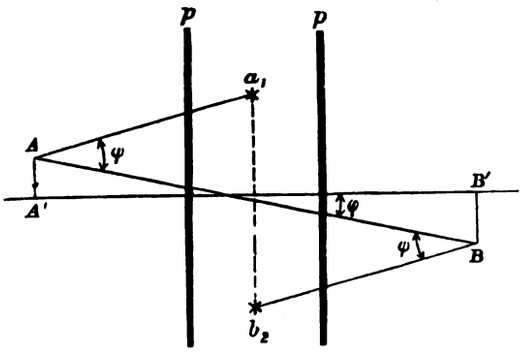

Fig. 6.

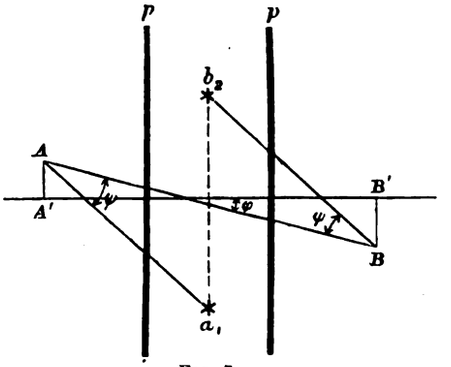

Fig. 7.

We have now $\sin\phi = \frac{\delta}{s}$, and the total correction to be applied to the measured distance is

$$\Delta s = + \frac{\kappa^2 + \delta^2}{2s}.$$

* It may be remarked that the angles ϕ in figs. 6 and 7 are not exactly equal, but the difference is a small quantity of the third order, if ϕ itself is of the first order. The same thing is true of ψ. If we suppose that the distance $a_1 b_2$ of the artificial star is the same in the two positions represented by the figures 6 and 7, and equal to κ in both cases, and if we denote the angles ψ, ϕ and the measured distance $A'B'$ in the case of fig. 6 by the suffix 1, and by 2 in the case of fig. 7, then the rigorous formulas are:

$$\kappa + \delta = s \sin(\psi_1 - \phi_1), \qquad \kappa - \delta = s \sin(\psi_2 + \phi_2),$$
$$\delta = (A'B')_1 \tan\phi_1, \qquad \delta = (A'B')_2 \tan\phi_2,$$
$$(A'B')_1 = s \cos(\psi_1 - \phi_1), \qquad (A'B')_2 = s \cos(\psi_2 + \phi_2).$$

From these formulas it is easily verified that the approximations

$$\psi_1 = \psi_2 = \frac{\delta}{s} \qquad \phi_1 = \phi_2 = \frac{\kappa}{s}$$

are correct to the second order inclusive.

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The part of this correction which is due to δ is practically negligible, except for very small distances. Assuming $\delta = 1''$, then

for	$s =$	$20''$	$50''$	$100''$	$200''$	$500''$	$1000''$,
we have	$\frac{\delta^2}{2s} =$	$+0''{\cdot}025$	$+0''{\cdot}010$	$+0''{\cdot}005$	$+0''{\cdot}002$	$+0''{\cdot}001$	$+0''{\cdot}0004$.

Not only is the correction negligible for large distances, but it would be wrong to apply it, since an unknown corresponding to κ is to be introduced into the equations of condition. The only effect of the separation of optical centres is thus that the value derived from the solution of these equations will not be κ itself, but $\sqrt{\kappa^2 + \delta^2}$.

However, for this same reason it is necessary to apply the correction for δ to those observations in whose equations this unknown is not introduced, *i.e.* (besides the measures of half and double distances) those in which ψ was recorded. To these observations a correction was applied, assuming $\delta = +1''$. It might have been better, perhaps, to derive a value of δ for each date from observations of short distances when available. This determination would, however, always remain to a large extent uncertain, and, as the whole correction is very small, it was thought better to adopt a constant value throughout.

For long distances (over $1000''$) the most accurate method of observing is by crossing through. If the distance of optical centres is not zero, the line of the images, if parallel to the wires p, will be no longer so after crossing through. The difference will, however, be extremely small and scarcely, if at all, noticeable. The observer while crossing through finds himself unable to satisfy the condition of exact parallelism in both positions (figs. 1 and 2) by the same adjustment of the separation of the lenses, and will instinctively make a compromise between the two adjustments which would produce exact parallelism in the two cases. Thus his measure practically represents the mean of the two measures illustrated by figs. 6 and 7, and the only effect of the distance of optical centres will be to make the observations perhaps slightly more difficult.

As is already explained, in observing the satellites the two pointings represented by figs. 6 and 7, corresponding to figs. 1 and 2, were always made separately without crossing through (at least certainly so by Gill, and probably also by Finlay). In observing distances below $100''$ the angle ψ was moreover recorded, as is explained above.

For distances under $20''$ the most reliable method is to put the images at half distance, and for distances under $5''$ at double distance, provided ϕ or δ be determined by simultaneous observations of position-angle. This method was always employed for very short distances, with one exception, when Mr Finlay measured a distance of

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5″ by crossing through. This observation (the rotation number 459–460) has been rejected.

In how far the distances, after application of the corrections for κ and δ, are systematically correct, must be decided from the discussion of the residuals remaining after the solution.

Position-angles.—The position-angles can be supposed to be not affected by systematic errors. It has already been shown that the distance of optical centres does not affect the mean of the measures in the two positions of the lenses. The crossing through, which is here done by rapid and alternately reversed motion in distance, does not introduce any error at all. We must further assume that there is no personal error. The personal difference of 6″ between Gill and Finlay which was found from the Victoria triangulation work (*Cape Annals*, vol. vi., part 1, p. 173) is absolutely negligible for the present purpose. Mr Finlay's observations are too few to allow a reliable determination of personal errors to be made from the satellite observations themselves. If there were a systematic error depending on the position-angle, this would be eliminated by the determination of the instantaneous index error from the observations of the standard stars, which are in approximately the same position-angle as the satellites. A systematic error depending on the distance is not very probable *a priori*. Whether it exists or not must appear from a discussion of the residuals.

The corrections for equatorial adjustment have been applied using the same constants which were used in the discussion of the Victoria triangulation. The instantaneous index error was determined every night by observations of the standard stars, either at the beginning and end, or in the middle of the night's work. As these stars were very near Jupiter, the index error thus determined includes the effect of deviations from the adopted equatorial adjustment at the mean hour-angle of the night.

In the comparison of the standard stars with the Victoria standards the assumption is made that the adopted adjustment is so nearly correct that the index correction is the same for the two pairs. It has already been pointed out that the position-angle of the standard stars derived from this comparison can be considered correct to a few seconds. If there are any systematic errors in the adopted refraction constants, such errors will be eliminated by the use of the index error derived from the standard stars owing to the circumstance that these stars are in approximately the same position-angle as the satellites. The measured position-angles of the satellites, when corrected for equatorial adjustment and instantaneous index error, therefore represent, within the errors of observation, absolute position-angles referred to the true pole of the heavens at the epoch of observation.

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Observed distances and position-angles.—The Tables XI. and XII. contain the principal details of the reduction of the observations. In Table XI. are given the observed distances, and in Table XII. the observed position-angles. An asterisk in the column "Rotation No." in both tables refers to a footnote. The sidereal times given are the means of the recorded times for the separate pointings; these have been reduced to Greenwich mean time and the correction for aberration has been applied, using the distance of Jupiter from the earth. The so corrected times are also given in the tables. In the distance tables an asterisk in the column of "Scale Readings" means that four pointings were made instead of two, a dagger that only one pointing was made. In the column of "Instrumental Distance" the observations in which the amount of crossing was measured are marked with the letter ψ, and the observations of half and double distances with h and d respectively. In the columns of "Scale Value" (for the distances) and "Index" (for the position-angles), those values which are not based on observations of the standard stars on the same day but on interpolation, are inclosed in brackets.

It has been a strict rule to reject every observation, in the course of the reduction of which any alteration was made in the original records. Such cases are referred to in the footnotes.

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TABLE XI.

Distances of I.–II.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Scale Divisions. A (+) B	Scale Divisions. A (−) B	Scale Readings. (+)	Scale Readings. (−)	Instrumental Distance.	Refraction.	Scale Value.	Observed Distance.	Definition. I	Definition. S
	1891		h m s	h m			r	r	″	+ ″	″	″		
3	Aug. 17	G	20 56 24·8	9 25·13	98 301	101 299	5·677	3·841	109·517	0·050	...	109·567	2	2
20	19	G	21 11 31·7	9 32·42	96 304	104 297	5·936	3·770	311·665	·132	+ ·012	311·809	2–3	3
34	20	G	20 44 51·0	9 1·93	96 304	104 297	6·236	3·166	316·222	·159	− ·004	316·377	2	3
*44	21	G	19 46 1·1	7 59·36	100 301	100 300	*3·686	*5·838	d 9·195	·002	− 0	4·601	3	3
*50			20 45 16·8	8 58·46	100 301	100 300	*3·660	*5·821	d 9·150	·001	− 0	4·577	3	3
64	24	G	21 10 35·2	9 12·00	98 302	102 299	5·955	3·261	153·928	·066	− 5	153·989	2–3	2–3
*80	25	G	20 53 12·2	8 50·76	99 301	100 300	2·609	2·824	ψ 39·039	·017	− 0	39·208	2–3	2–3
*81			21 3 42·7	9 1·24	99 301	100 300	2·575	2·879	ψ 38·589	·016	− 0	38·749	2–3	2–3
103	29	G	21 8 4·8	8 49·95	98 302	101 298	3·489	5·761	128·848	·056	− 0	128·904	2–3	2–3
121	31	G	21 24 44·4	8 58·74	97 303	102 298	2·408	3·071	197·105	·076	− 10	197·171	2–3	2–3
*142	Sept. 1	G	21 25 2·6	8 55·12	99 301	100 299	3·679	†5·709	ψ 49·945	·019	− 1	50·053	...	
*145	5	G	19 42 48·1	6 57·44	99 301	100 300	2·482	2·628	ψ 39·386	·029	− 0	39·612	3	3–4
164	7	G	20 4 32·9	7 11·26	96 303	103 297	4·724	4·530	261·600	·155	0	261·755	1–2	2–3
194	10	G	20 30 1·9	7 24·85	97 302	102 298	6·156	3·016	196·265	·099	− 10	196·354	2	2–3
204	13	G	20 16 54·0	6 59·91	96 303	103 297	5·430	4·036	267·647	·143	− 8	267·782	2	3
221	14	G	19 59 37·6	6 38·73	96 304	103 296	3·240	6·110	286·263	·172	+ 13	286·448	3	3, 3–4
*235	18	F	19 51 51·8	6 15·15	98 301	101 299	4·961	4·280	103·680	·064	+ 3	103·747	2	2
249	19	G	19 32 22·9	5 51·76	99 301	100 299	3·582	5·832	48·843	·034	− 2	48·875	2	3
269	20	G	19 53 30·7	6 8·86	96 305	104 296	3·010	6·240	324·547	·197	+ 6	324·750	1–2	2
294	21	G	20 52 27·2	7 3·67	96 304	103 296	3·555	5·824	289·296	·120	+ 1	289·417	1	1–2
*300	22	G	20 5 50·9	6 13·22	99 301	100 299	3·066	6·232	ψ 44·227	·024	− 1	44·305	2	2–3
*329	24	G	19 44 26·0	5 43·91	100 301	100 300	*3·165	*6·241	h 4·547	·003	[0]	10·015	2–3	...
333	28	G	20 25 33·0	6 9·00	96 303	103 297	6·554	2·690	280·104	·135	− 14	280·225	3	3–4
*335	29	G	19 43 33·2	5 23·12	99 301	100 299	3·063	6·249	ψ 44·116	·028	− 0	44·208	2	3
347	Oct. 4	G	20 24 49·8	5 44·30	95 304	104 296	5·432	3·880	348·636	·159	− 13	348·782	2–3	2–3
369	9	G	0 12 17·5	9 11·10	98 302	101 298	4·784	4·588	141·306	·040	+ 2	141·348	2–3	2–3
*385	13	G	0 6 4·3	8 48·84	99 301	100 299	4·210	5·050	ψ 55·947	·016	+ 2	56·008	1–2	1–2
400	14	F	0 9 31·3	8 48·26	98 301	101 299	5·375	3·729	108·566	·032	+ 3	108·601	2–3	3
424	16	F	0 42 46·9	9 13·39	98 303	102 298	3·132	6·108	165·383	·046	− 2	165·427	2–3	3
446	19	F	0 48 14·1	9 6·75	97 303	102 297	4·823	4·413	222·594	·065	− 2	222·657	2	3
466	Nov. 5	G	0 12 40·7	7 22·61	96 304	103 296	4·220	5·180	295·898	·084	− 3	295·979	2–3	3–4
482	6	G	0 33 33·8	7 39·38	97 303	102 297	4·410	4·947	217·816	·062	+ 54	217·932	2	2–3
*498	9	G	0 12 6·6	7 5·83	100 301	100 300	3·938	5·092	d 14·230	·002	[+ 2]	7·141	...	
503	22	G	0 43 34·8	6 44·47	99 301	100 299	4·341	4·935	57·180	·015	− 1	57·194	2	2–3
*522	30	F	0 58 38·5	6 26·99	97 303	102 298	2·595	2·981	198·496	·061	...	198·557	2–3	3–4
528	Dec. 8	F	1 22 12·2	6 18·00	98 302	102 299	6·145	3·315	154·599	·049	− 1	154·647	2	3

Aug. 21 — Adopted distance of optical centres $\delta = +0''\cdot15$. A correction of $+0''\cdot001$ has been applied to distances 44 and 50.

25 80 — Measured $\psi = 4^\circ\ 50'$. A correction of $+0''\cdot140$ has been applied. Correction for δ $+0''\cdot012$.

81 — Measured $\psi = 4^\circ\ 45'$. A correction of $+0''\cdot131$ has been applied. Correction for δ $+0''\cdot013$.

Sept. 1 142 — Measured $\psi = 3^\circ\ 13'$. A correction of $+0''\cdot080$ has been applied. Correction for δ $+0''\cdot010$. One pointing not printed; *rejected* on account of consequent uncertainty in time.

5 145 — Measured $\psi = 5^\circ\ 31'$. A correction of $+0''\cdot185$ has been applied. Correction for δ $+0''\cdot012$.

18 235 — Recorded scale divisions were 102 299. This is evidently a mistake, the combination 98 301 and 102 299 does not give the same distance for the two halves of the observation reduced separately; *rejected*.

22 300 — Measured $\psi = 2^\circ\ 36'$. A correction of $+0''\cdot044$ has been applied. Correction for δ $+0''\cdot011$.

24 329 — Distance of optical centres $\delta = +2''\cdot1$. A correction of $+0''\cdot918$ has been applied. Impossible to measure double distance on account of interference of planet.

29 335 — Measured $\psi = 2^\circ\ 43'$. A correction of $+0''\cdot053$ has been applied. Correction for δ $+0''\cdot011$.

Oct. 13 385 — Measured $\psi = 2^\circ\ 8'$. A correction of $+0''\cdot034$ has been applied. Correction for δ $+0''\cdot009$.

Nov. 9 498 — Distance of optical centres $\delta = +1''\cdot2$. A correction of $+0''\cdot022$ has been applied.

30 — All observations on this date were *rejected* on account of bad definition.

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TABLE XI.—*continued.*

DISTANCES OF I–III.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Scale Divisions. (+) A	(+) B	(−) A	(−) B	Scale Readings. (+)	(−)	Instrumental Distance.	Refraction.	Scale Value.	Observed Distance.	Definition. I	S
	1891		h m s	h m					r	r	"	+ "	"	"		
10	Aug. 17	G	21 47 18·0	10 15·84	97	302	102	298	4·106	5·231	174·745	0·067	...	174·812	2	2
*26	19	G	22 6 34·3	10 27·32	99	300	100	300	4·918	4·441	ψ 22·480	·006	+ ·001	22·655	2–3	3
*28	20	G	19 39 39·9	7 56·92	99	300	100	300	4·884	4·418	ψ 22·422	·012	·000	22·588	2–3	3
62	24	G	21 0 40·1	9 2·11	97	302	102	298	4·876	4·557	182·024	·082	− ·006	182·100	2–3	2–3
72	25	G	20 8 11·1	8 5·86	94	306	106	295	5·997	3·365	474·314	·295	− ·002	474·607	3	3
101	29	G	20 57 5·7	8 39·00	96	303	103	297	3·741	5·510	251·696	·113	− ·001	251·808	2–3	3
123	31	G	21 32 17·5	9 6·27	97	302	102	298	4·889	4·455	182·603	·204	− ·010	182·797	2–3	2–3
133	Sept. 1	G	20 20 20·9	7 50·60	94	306	105	295	2·580	2·741	440·220	·241	− ·008	440·453	...	
156	5	G	21 21 35·4	8 35·96	97	304	103	298	2·643	2·665	240·438	·097	− ·001	240·534	2	2–3
166	7	G	20 20 4·0	7 26·74	97	302	102	298	6·327	2·887	197·756	·106	·000	197·862	1–2	2–3
188	10	G	19 48 55·4	6 43·86	98	302	101	298	4·799	4·501	141·821	·093	− ·007	141·907	2	2–3
206	13	G	20 32 6·7	7 15·08	96	304	104	297	5·460	3·660	309·808	·150	− ·009	309·949	2	3
219	14	G	19 45 43·2	6 24·85	97	302	102	298	6·290	3·066	196·678	·133	+ ·009	196·820	3	3–4
241	18	F	20 30 21·8	6 53·54	95	305	104	295	2·999	6·256	364·509	·167	+ ·010	364·686	2	3
251	19	G	19 52 5·6	6 11·41	97	303	103	298	5·158	4·252	225·046	·139	− ·009	225·176	2	2–3
271	20	G	20 3 25·8	6 18·75	95	305	104	295	4·473	4·926	378·613	·214	+ ·007	378·834	1–2	2
292	21	G	20 38 37·5	6 49·88	97	302	102	298	4·519	4·821	178·896	·080	+ ·001	178·977	1	1–2
302	22	G	20 21 0·2	6 28·34	96	303	103	297	4·077	5·083	255·550	·129	− ·004	255·675	2	2–3
341	29	G	20 29 51·2	6 9·29	97	302	102	298	5·563	3·451	191·070	·090	− ·002	191·158	3–4	4
355	Oct. 4	G	21 9 53·0	6 29·23	94	305	105	295	5·415	3·906	428·643	·156	− ·016	428·783	3	3
*365	6	F	21 21 5·8	6 32·41	98	302	102	299	5·092	4·159	ψ 145·052	·051	− ·004	145·287	2–3	3
367	9	G	0 0 36·8	8 59·46	97	303	102	297	4·556	4·566	220·475	·060	+ ·004	220·539	3	3
381	10	G	0 2 39·9	8 57·49	97	303	102	297	4·636	4·674	220·336	·060	[+ ·004]	220·400	2–3	3
389	13	G	0 26 27·9	9 9·18	98	302	101	298	5·129	4·002	145·997	·043	+ ·004	146·044	1–2	1–2
406	14	F	0 51 56·3	9 30·56	95	306	105	296	3·000	2·418	403·884	·122	+ ·011	404·017	3–4	3–4
430	16	F	1 22 6·9	9 52·61	98	303	102	298	3·698	5·706	170·258	·054	− ·002	170·310	3	3–4
434	17	F	0 5 44·0	8 32·42	97	303	{103 / 102}	{298 / 297}	5·421	3·913	228·088	·070	− ·004	228·154	2–3	3–4
442	19	F	0 20 54·4	8 39·50	99	301	100	299	4·176	5·000	56·033	·015	− ·001	56·047	2	3
464	Nov. 5	G	23 55 37·7	7 5·94	{98 / 97}	{303 / 302}	102	298	3·746	5·535	171·378	·048	− ·002	171·424	2–3	3–4
480	6	G	0 22 0·1	7 27·86	98	302	101	298	4·708	4·721	140·253	·048	+ ·035	140·336	2	2–3
516	27	G	1 3 6·9	6 43·64	96	304	103	296	4·865	4·589	302·128	·089	− ·009	302·208	2–3	2–3
*524	30	F	1 14 52·0	6 43·17	96	305	104	297	2·947	2·735	321·864	·102	...	321·966	...	
526	Dec. 8	F	1 11 5·5	6 6·92	99	302	101	300	2·830	2·619	81·326	·026	·000	81·352	2	3
550	14	G	2 45 43·4	7 16·92	95	305	104	295	4·246	5·264	375·763	·179	+ ·014	375·956	3	3–4

Aug. 19 26 Measured ψ 6° 30′. A correction of +0″·146 has been applied. Correction for δ +0″·022.
20 28 Measured ψ 6° 6′. A correction of +0″·132 has been applied. Correction for δ +0″·022.
Oct. 6 365 Measured ψ 2° 56′. A correction of +0″·188 has been applied.
Nov. 30 All observations on this date are *rejected* on account of bad definition.

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TABLE XI.—*continued.*

DISTANCES OF I.–IV.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Scale Divisions. A (+) B	Scale Divisions. A (−) B	Scale Readings. (+)	Scale Readings. (−)	Instrumental Distance.	Refraction. +	Scale Value.	Observed Distance.	Definition. I	Definition. S
	1891.		h m s	h m			r	r	"	"	"	"		
*1	Aug. 17	G	20 36 2·6	9 4·81	99 300	100 300	4·923	4·544	ψ 21·985	0·008	...	22·344	2	2
18	19	G	20 45 40·8	9 6·64	93 307	106 294	2·490	2·902	519·099	·257	+ ·021	519·377	2–3	3
38	20	G	21 10 32·6	9 27·55	94 305	105 295	4·326	4·995	417·661	·181	− 0·06	417·836	2–3	3
60	24	G	20 49 49·2	8 51·29	94 306	105 295	2·642	2·830	440·087	·208	− ·014	440·281	2–3	2–3
*83	25	G	21 15 16·1	9 12·76	99 300	100 300	5·764	3·558	ψ 31·187	·010	·000	31·369	2–3	2–3
84	27	F	19 50 10·5	7 40·09	94 306	105 294	4·798	4·422	462·955	·325	+ ·031	463·311	3–4	3
93	29	G	20 3 52·5	7 45·92	91 310	109 292	2·946	2·525	723·836	·456	− ·002	724·290	2–3	3
125	31	G	21 41 18·4	9 15·26	93 307	106 293	5·323	4·180	546·996	·194	− ·029	547·161	2–3	2–3
*131	Sept. 1	G	20 8 27·5	7 38·74	93 307	106 294	2·627	2·850	520·049	·305	− ·009	520·345	...	
151	5	G	20 33 35·9	7 48·10	96 305	104 297	2·816	2·423	322·775	·120	− ·002	322·893	3	3–4
196	10	G	20 41 53·8	7 36·68	98 303	102 298	3·865	5·594	171·647	·078	− ·009	171·716	2	2–3
214	13	G	21 23 7·5	8 5·95	96 304	103 296	4·609	4·551	301·026	·117	− ·009	301·134	2–3	3
227	14	G	20 36 49·8	7 15·83	92 308	108 293	5·845	3·493	633·310	·296	+ ·028	633·634	1–2	1–2
259	19	G	20 45 39·7	7 4·83	96 304	103 296	4·606	4·688	300·326	·128	− ·012	300·442	1–2	2
267	20	G	19 40 27·8	5 55·85	97 303	102 297	4·615	4·690	220·148	·151	+ ·004	220·303	1–2	2
296	21	G	21 1 49·1	7 13·00	97 303	102 297	4·552	4·702	219·769	·089	+ ·001	219·859	1	1–2
310	22	G	21 4 7·5	7 11·34	93 307	107 294	6·104	3·160	556·051	·233	− ·009	556·275	2	2–3
345	Oct. 4	G	20 10 42·1	5 30·21	95 304	104 296	6·129	3·295	355·092	·176	− ·013	355·255	3	3
*361	6	F	20 42 42·9	5 54·13	99 301	100 299	4·296	5·122	ψ 56·025	·025	− ·002	56·289	2–3	3
377	9	G	1 0 8·6	9 58·83	92 309	108 292	3·566	5·852	650·028	·194	+ ·011	650·233	2	2–3
398	13	G	1 25 55·3	10 8·14	95 305	105 296	6·036	3·041	396·046	·135	+ ·012	396·193	3	3
*404	14	F	0 37 34·6	9 16·24	99 300	100 300	6·127	3·127	ψ 35·220	·011	+ ·001	35·981	3	3
426	16	F	0 54 5·1	9 24·66	96 304	103 296	4·307	5·070	296·894	·088	− ·003	296·979	2–3	3
436	17	F	0 19 46·0	8 46·41	92 308	107 293	2·640	2·522	601·989	·169	− ·010	602·148	2–3	3–4
450	19	F	1 17 31·2	9 35·95	90 309	109 291	4·814	4·405	743·849	·237	− ·007	744·079	2	3
474	Nov. 5	G	1 33 58·9	8 43·68	94 306	105 294	4·705	4·529	461·947	·158	− ·005	462·100	2–3	3–4
486	6	G	1 6 15·3	8 11·97	93 307	106 293	4·602	4·602	541·240	·169	+ ·137	541·546	3	3–4
512	22	G	1 51 33·6	7 52·26	93 308	107 294	2·765	2·735	561·428	·206	− ·015	561·619	3	3–4
514	27	G	0 51 14·7	6 31·80	97 303	102 298	2·468	2·944	198·044	·056	− ·006	198·094	2–3	2–3
530	Dec. 8	F	1 35 14·0	6 30·99	93 308	107 294	2·520	2·743	560·152	·184	− ·003	560·333	2	3
548	14	G	2 30 1·9	7 1·27	94 305	105 295	5·625	3·775	430·369	·179	+ ·016	430·564	2	3

Aug. 17 — 1 — Measured ψ 9° 34′. A correction of +0″·328 has been applied. Correction for δ +0″·023.

25 — 83 — Measured ψ 5° 38′. A correction of +0″·156 has been applied. Correction for δ +0″·016.

Sept. 1 — 131 — Recorded scale division was 306. This is evidently a mistake, as the combination 93 306, 106 294 would not give the same result for the two positions of the semilenses. *Rejected.*

Oct. 6 — 361 — Measured ψ 5° 5′. A correction of +0″·232 has been applied. Correction for δ +0″·009.

14 — 404 — Measured ψ 11° 32′. A correction of +0″·735 has been applied. Correction for δ +0″·014.

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TABLE XI.—*continued.*

DISTANCES OF II.–III.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Scale Divisions. A (+) B	Scale Divisions. A (−) B	Scale Readings. (+)	Scale Readings. (−)	Instrumental Distance.	Refraction.	Scale Value.	Observed Distance.	Definition. I	Definition. S
	1891.		h m s	h m			r	r		+				
12	Aug. 17	G	22 10 1·4	10 38·54	99 302	101 299	3·416	5·791	88″·344	0″·030	...	88″·374	2	2
22	19	G	21 34 12·2	9 55·04	96 305	104 296	3·514	5·298	331·845	·126	+ ·014	331·985	2–3	3
36	20	G	20 59 34·4	9 16·61	96 305	104 296	3·215	6·318	325·199	·151	− ·004	325·346	2	3
*52	24	G	19 55 18·0	7 56·92	99 300	100 300	5·441	4·021	d 27·233	·009	·000	13·628	2–3	2–3
74	25	G	20 18 12·7	8 15·86	94 305	105 295	6·294	3·244	436·398	·253	− ·002	436·649	2–3	2–3
105	29	G	21 23 42·7	9 5·54	98 302	101 299	2·912	2·560	122·092	·047	·000	122·139	2–3	2–3
*109	31	G	19 46 15·4	7 20·52	99 300	100 300	5·667	3·624	ψ 30·371	·019	− ·002	30·630	3–4	3–4
140	Sept. 1	G	21 15 1·0	8 45·12	95 306	105 295	3·445	6·013	408·029	·162	− ·007	408·184	...	
154	5	G	21 10 40·2	8 25·07	98 303	102 298	3·406	6·045	167·075	·067	− ·001	167·141	2	2–3
*172	7	G	20 52 44·4	7 59·32	99 301	100 299	4·194	5·021	ψ 56·013	·024	·000	56·130	...	
175	8	G	19 36 28·0	6 39·32	96 305	104 297	2·723	2·415	322·351	·240	− ·018	322·573	...	
*186	10	G	19 37 9·9	6 32·13	99 300	100 300	5·879	3·326	ψ 32·958	·025	− ·002	33·108	2	2–3
202	13	G	20 1 44·2	6 44·79	99 301	100 299	3·406	5·845	47·894	·027	− ·001	47·920	2–3	3
*217	14	G	19 33 50·5	6 13·02	99 302	101 299	3·893	5·410	ψ 92·673	·067	+ ·004	92·806	3	3–4
243	18	F	20 45 37·1	7 8·76	94 306	105 294	4·554	4·908	459·278	·203	+ ·013	459·494	2	3
253	19	G	20 6 58·6	6 26·26	96 303	103 297	5·196	4·076	266·263	·146	− ·011	266·398	2	3
*279	20	G	20 42 14·1	6 57·45	99 301	100 299	4·330	4·972	ψ 56·944	·024	+ ·001	57·041	1–2	2
284	21	G	19 57 22·8	6 8·78	98 301	101 299	5·955	3·398	113·149	·065	+ ·001	113·215	1	1–2
304	22	G	20 30 9·1	6 37·46	97 303	102 297	4·010	5·414	213·449	·101	− ·003	213·457	2	2–3
321	23	G	20 24 47·5	6 28·14	93 308	107 294	2·619	2·730	560·716	·273	− ·015	560·974	2	2–3
*343	Oct. 4	G	19 57 18·2	5 16·85	99 302	101 300	2·859	2·475	ψ 82·204	·044	− ·003	82·333	3	3
*379	9	G	1 9 43·4	10 8·38	95 305	104 295	†4·373	4·905	378·216	·116	+ ·006	378·338	2	2–3
387	13	G	0 15 28·6	8 58·22	98 301	101 299	3·195	6·039	85·907	·024	+ ·003	85·934	1–2	1–2
410	14	F	1 17 7·1	9 55·66	93 307	106 293	4·020	5·167	535·448	·173	+ ·014	535·635	3	3–4
418	15	F	0 38 42·1	9 13·34	96 304	103 297	2·576	2·499	281·080	·083	...	281·163	3–4	3–4
422	16	F	0 31 43·9	9 2·37	96 304	104 297	5·744	3·421	312·443	·088	− ·003	312·528	2–3	3
444	19	F	0 36 0·6	8 54·56	98 303	102 299	2·687	2·646	160·581	·046	− ·002	160·625	2	3
458	23	F	0 16 3·1	8 18·53	97 304	103 298	2·493	2·910	238·448	·068	− ·005	238·511	2	3
478	Nov. 6	G	0 11 5·1	7 16·97	99 301	101 300	6·090	3·134	75·112	·021	+ ·019	75·152	2	2–3
490	8	G	0 20 9·2	7 17·91	98 302	102 299	5·423	3·963	147·709	·041	+ ·040	147·790	2	2–3
505	22	G	0 56 30·0	6 57·35	97 302	102 298	4·445	4·998	177·629	·054	− ·005	177·678	2	2–3
538	Dec. 8	F	2 34 56·8	7 30·53	97 304	103 298	2·480	2·992	237·971	·108	− ·001	238·078	3	3–4

Aug. 24 52 Distance of optical centres $\delta = -0''{\cdot}63$. A correction of $+0''{\cdot}003$ has been applied.

31 109 Measured ψ 6° 50′. A correction of $+0''{\cdot}225$ has been applied. Correction for δ $+0''{\cdot}017$.

Sept. 7 172 Measured ψ 3° 6′. A correction of $+0''{\cdot}084$ has been applied. Correction for δ $+0''{\cdot}009$.

10 186 Measured ψ 4° 40′. A correction of $+0''{\cdot}112$ has been applied. Correction for δ $+0''{\cdot}015$.

14 217 Measured ψ 1° 52′. A correction of $+0''{\cdot}056$ has been applied. Correction for δ $+0''{\cdot}006$.

20 279 Measured ψ 2° 40′. A correction of $+0''{\cdot}063$ has been applied. Correction for δ $+0''{\cdot}009$.

Oct. 4 343 Measured ψ 2° 30′. A correction of $+0''{\cdot}082$ has been applied. Correction for δ $+0''{\cdot}006$.

9 379 One pointing not printed. *Rejected* on account of consequent uncertainty of time.

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TABLE XI.—*continued.*

DISTANCES OF II.–IV.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Scale Divisions. A (+) B	Scale Divisions. A (−) B	Scale Readings. (+)	Scale Readings. (−)	Instrumental Distance.	Refraction. +	Scale Value.	Observed Distance.	Definition. I	Definition. S
	1891.		h m s	h m			r	r	″	″	″	″		
6	Aug. 17	G	21 17 4·5	9 45·73	98 302	101 298	3·713	5·563	130·982	0·056	...	131·038	2	2
24	19	G	21 54 54·7	10 15·69	97 303	102 297	5·048	4·115	225·226	·082	+ ·009	225·317	2–3	3
30	20	G	20 1 41·5	8 18·85	91 309	109 292	6·024	3·320	715·327	·485	− ·010	715·802	2–3	3
58	24	G	20 40 18·2	8 41·80	92 308	107 293	2·857	2·534	603·025	·305	− ·019	603·311	2–3	2–3
*78	25	G	20 41 51·2	8 39·44	99 302	101 300	2·796	2·750	ψ 80·497	·036	·000	80·588	2–3	2–3
88	27	F	20 31 0·0	8 20·80	98 303	102 298	3·765	5·706	170·595	·088	+ ·011	170·694	3–4	3
95	29	G	20 17 35·7	7 59·60	89 310	110 290	5·380	4·045	828·671	·471	− ·002	829·140	2–3	3
115	31	G	20 33 38·1	8 7·77	95 305	104 296	2·775	2·656	361·463	·178	− ·019	361·622	3–4	3–4
138	Sept. 1	G	21 3 54·7	8 34·04	94 307	106 295	2·624	2·720	480·623	·199	− ·009	480·813	...	
158	5	G	21 32 3·5	8 46·40	96 303	103 297	4·823	4·489	262·307	·097	− ·001	262·403	2	2–3
168	7	G	20 29 51·9	7 36·51	89 311	111 290	5·681	3·481	873·067	·434	·000	873·501	1–2	2–3
177	8	G	19 50 21·0	6 53·16	92 309	108 292	3·425	5·960	648·782	·422	− ·036	649·168	...	
192	10	G	20 18 58·0	7 13·82	95 305	104 295	3·541	5·705	369·983	·197	− ·019	370·161	2	2–3
212	13	G	21 12 42·3	7 55·56	93 307	106 293	4·424	4·790	539·390	·216	− ·015	539·591	2	3
223	14	G	20 13 22·4	6 52·44	95 304	104 296	5·990	3·326	354·257	·193	+ ·015	354·465	3–4	3–4
230		G	21 2 28·5	7 41·41	95 304	104 296	5·766	3·475	352·366	·144	+ ·015	352·525	1–2	1–2
261	19	G	21 0 2·7	7 19·18	96 303	103 297	5·324	3·947	267·562	·108	− ·011	267·659	2–3	3
277	20	G	20 32 59·4	6 48·23	99 302	101 299	3·800	5·556	91·463	·040	+ ·002	91·505	1–2	2
288	21	G	20 17 53·4	6 29·20	94 307	106 294	3·899	5·346	493·832	·252	+ ·002	494·086	1	1–2
312	22	G	21 12 55·1	7 20·10	92 308	107 293	2·876	2·481	603·385	·234	− ·010	603·609	2	2–3
323	23	G	20 38 36·6	6 41·92	95 305	105 296	6·012	3·334	394·432	·178	− ·010	394·600	2	2–3
353	Oct. 4	G	21 0 51·0	6 20·22	91 309	108 291	3·826	5·496	693·269	·261	− ·026	693·504	3	3
359	6	F	20 26 18·7	5 37·77	98 301	101 299	5·215	4·245	105·139	·049	− ·003	105·185	2–3	3
375	9	G	0 50 6·4	9 48·81	90 310	109 290	4·263	5·082	777·726	·228	+ ·013	777·967	2–3	3
396	13	G	1 5 55·2	9 48·19	94 306	105 294	*4·837	4·572	462·399	·148	+ ·014	462·561	2–3	2–3
*402	14	F	0 22 10·6	9 0·88	99 302	101 299	2·921	6·256	ψ 83·484	·023	+ ·002	83·749	2–3	3
416	15	F	0 24 13·9	8 58·92	97 303	102 297	5·010	4·297	224·111	·064	...	224·175	3–4	3–4
420	16	F	0 20 46·5	8 51·44	98 302	101 298	3·757	5·513	131·470	·036	− ·001	131·505	2–3	3
454	19	F	1 42 41·9	10 1·06	93 306	106 294	5·252	3·878	508·044	·180	− ·005	508·219	2	3
456	23	F	0 3 40·6	8 6·19	97 303	103 298	5·772	3·448	232·184	·066	− ·005	232·245	2	3
472	Nov. 5	G	1 15 31·5	8 25·28	{91 310 / 90 309}	109 291	4·113	5·109	736·768	·236	− ·007	736·997	2–3	3–4
492	8	G	0 32 24·5	7 30·13	96 304	103 296	4·393	4·914	298·106	·086	+ ·080	298·272	2	2–3
511	22	G	1 42 22·9	7 43·10	92 309	108 292	3·397	6·061	648·109	·225	− ·018	648·316	3	3–4
536	Dec. 8	F	2 23 42·8	7 19·33	95 306	105 296	2·428	3·055	397·783	·169	− ·002	397·950	3	3–4
544	14	G	2 0 47·5	6 32·11	98 303	102 299	3·024	2·522	162·908	·058	+ ·006	162·972	2	3

Aug. 25 78 Measured $\psi = 1^\circ\ 54'$. A correction of +0″·048 has been applied. Correction for δ +0″·007.
Oct. 14 402 Measured $\psi = 4^\circ\ 16'$. A correction of +0″·234 has been applied. Correction for δ +0″·006.

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TABLE XI.—*continued.*

DISTANCES III.–IV.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Scale Divisions. A (+) B	Scale Divisions. A (−) B	Scale Readings. (+)	Scale Readings. (−)	Instrumental Distance.	Refraction.	Scale Value.	Observed Distance.	Definition. I	Definition. S
	1891.		h m s	h m			r	r	"	+	"	"		
8	Aug. 17	G	21 33 40·8	10 2·29	97 303	102 297	3·797	5·642	211·219	0·083	...	211·302	2	2
16	19	G	20 32 40·8	8 53·68	93 307	107 294	5·823	3·512	552·865	·296	+ ·023	553·184	2–3	3
40	20	G	21 23 39·6	9 40·66	95 306	105 296	2·971	2·478	403·428	·162	− ·005	403·585	2	3
46	21	G	20 6 49·1	8 20·08	96 304	103 296	3·153	6·374	284·493	·181	− ·001	284·673	2–3	3
54	24	G	20 10 31·5	8 12·10	92 308	107 292	4·780	4·625	622·210	·384	− ·019	622·575	2–3	2–3
70	25	G	19 56 20·6	7 54·05	93 307	106 293	3·372	6·024	527·876	·359	− ·003	528·232	3	3
86	27	F	20 12 21·3	8 2·20	95 306	105 296	2·954	3·150	399·953	·238	+ ·027	400·218	3–4	3
*97	29	G	20 33 53·1	8 15·85	88 313	112 289	2·925	2·455	964·634	·495	− ·003	965·126	2–3	3
113	31	G	20 18 46·9	7 52·96	95 305	105 296	5·310	3·967	387·709	·213	− ·020	387·902	3–4	3–4
*127	Sept. 1	G	19 43 58·3	7 14·32	99 302	101 299	3·565	5·875	ψ 88·673	·059	− ·002	88·825	...	
*147	5	G	19 57 38·6	7 12·24	99 302	101 300	2·943	2·370	ψ 83·153	·054	·000	83·296	3	3–4
170	7	G	20 41 0·7	7 47·62	90 311	110 290	4·142	5·141	816·853	·381	·000	817·234	1–2	2–3
178	8	G	20 3 53·3	7 6·66	88 313	112 288	3·998	5·276	975·841	·571	− ·055	976·357	...	
181		G	20 28 51·1	7 31·56	{87 / 88} {312 / 313}	112 288	* 3·980	*5·265	975·796	·480	− ·055	976·221	...	
189	10	G	19 56 20·9	6 51·26	95 304	104 296	3·600	5·809	329·668	·205	− ·017	329·856	2	2–3
210	13	G	21 2 28·8	7 45·36	92 307	107 293	5·481	3·766	589·973	·247	− ·017	590·203	2	3
225	14	G	20 25 31·5	7 4·56	94 306	105 295	2·809	2·529	442·444	·221	+ ·019	442·684	3–4, 4	3–4, 4
229		G	20 54 57·6	7 33·91	94 305	105 295	6·397	2·907	438·631	·184	+ ·019	438·834	1–2	1–2
245	18	F	21 2 47·6	7 25·88	92 308	107 292	4·138	5·184	616·169	·249	+ ·017	616·435	2	3
257	19	G	20 31 28·1	6 50·68	93 307	106 293	3·124	6·254	525·468	·244	− ·021	525·691	1–2	2
275	20	G	20 23 52·4	6 39·14	98 302	102 299	5·805	3·610	151·419	·070	+ ·003	151·492	1–2	2
290	21	G	20 28 11·1	6 39·47	95 305	105 296	5·118	4·168	385·731	·185	+ ·002	385·918	1	1–2
306	22	G	20 41 9·5	6 48·43	89 310	110 290	4·959	4·295	825·293	·373	− ·013	825·653	2	2–3
319	23	G	20 8 15·5	6 11·65	88 312	112 289	6·059	3·294	956·129	·516	− ·025	956·620	2	2–3
324		G	20 49 48·6	6 53·09	88 312	112 289	5·918	3·398	954·896	·407	− ·025	955·278	2	2–3
*337	29	G	19 55 54·8	5 35·44	99 300	100 300	6·111	3·166	ψ 34·925	·019	·000	35·080	2	3
351	Oct. 4	G	20 50 17·5	6 9·69	90 310	109 290	3·985	5·275	775·356	·307	− ·029	775·634	3	3
357	6	F	20 10 45·7	5 22·27	97 302	102 298	4·526	4·956	178·257	·090	− ·005	178·342	2–3	3
371	9	G	0 23 4·3	9 21·85	94 305	105 295	3·271	6·119	406·667	·114	+ ·007	406·788	2–3	3
383	10	G	0 17 37·2	9 12·40	96 303	103 297	6·206	3·090	276·349	·078	[+ ·006]	276·433	2–3	3
394	13	G	0 50 0·6	9 32·32	93 308	107 293	2·963	6·299	564·505	·172	+ ·017	564·694	2–3	2–3
408	14	F	1 4 48·6	9 43·39	94 306	105 295	2·365	2·880	438·434	·139	+ ·011	438·584	3	3–4
*414	15	F	0 5 1·6	8 39·76	99 301	101 300	5·808	3·594	ψ 71·338	·022	...	71·766	3	3
432	16	F	1 32 38·0	10 3·10	94 306	{106 / 105} {295 / 294}	5·036	4·156	465·490	·152	− ·005	465·637	3	3–4
438	17	F	0 32 16·9	8 58·89	89 310	110 290	5·479	3·871	830·034	·240	− ·013	830·261	3	3
452	19	F	1 30 36·2	9 49·00	91 308	108 292	5·933	3·460	673·991	·227	− ·007	674·211	2	3
*460	23	F	0 35 43·0	8 38·14	100 301	100 300	3·094	6·289	3·957	·002	·000	4·588	2	3
476	Nov. 5	G	1 45 55·2	8 55·59	96 304	103 296	3·084	6·186	285·101	·102	− ·003	285·200	2–3	3–4
488	6	G	1 20 26·2	8 26·12	94 305	105 295	4·825	4·503	422·658	·138	+ ·107	422·903	3, 3–4	3, 3–4
496	8	G	1 2 34·2	8 0·20	94 306	105 295	2·560	2·845	439·598	·136	+ ·118	439·852	2–3, 3	2–3, 3
*499	9	G	0 26 57·2	7 20·64	98 301	101 299	5·876	†3·463	112·493	·032	[+ ·029]	112·554	...	
507	22	G	1 11 58·8	7 12·79	89 310	110 290	4·495	4·920	819·807	·255	− ·022	820·040	2	2–3
518	27	G	1 12 50·6	6 53·34	93 306	106 294	4·656	4·742	500·687	·149	− ·015	500·821	2	2–3
534	Dec. 8	F	2 11 13·5	7 7·04	92 309	108 293	2·420	3·098	638·110	·250	− ·004	638·356	3	3–4
*540	14	G	1 25 48·4	5 57·22	99 301	100 299	3·074	6·281	ψ 44·001	·020	+ ·002	44·250	2–3	3

Sept. 1 127 Measured $\psi = 2^\circ\ 26'$. A correction of +0″·089 has been applied. Correction for δ +0″·006.

5 147 Measured $\psi = 2^\circ\ 29'$. A correction of +0″·083 has been applied. Correction for δ +0″·006.

Oct. 29 337 Measured $\psi = 4^\circ\ 45'$. A correction of +0″·122 has been applied. Correction for δ +0″·014.

15 414 Measured $\psi = 6^\circ\ 0'$. A correction of +0″·399 has been applied. Correction for δ +0″·007.

23 460 Measured $\psi = 30^\circ\ 12'$. A correction of +0″·519 has been applied. Correction for δ +0″·110. *Rejected.*

Nov. 9 499 Last pointing missing through cloud. Half weight on account of assymmetry.

Dec. 14 540 Measured $\psi = 5^\circ\ 39'$. A correction of +0″·216 has been applied. Correction for δ +0″·011.

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TABLE XII.

Position-Angles of I.–II.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Instrumental Position-Angle.	Refraction.	Equatorial Adjustment.	Index.	Observed Position-Angle.	Definition. I	Definition. S
	1891.		h m s	h m	° ′ ″	″	′ ″	″	° ′ ″		
4	Aug. 17	G	21 5 54·9	9 34·60	67 33 8	− 18	− 35 40	...	66 57 10	2	2
19	19	G	21 9 44·9	9 30·65	246 11 4	− 17	35 40	+ 1	245 35 8	2–3	3
33	20	G	20 43 37·1	9 0·70	65 7 56	− 20	35 41	+ 17	64 32 12	2	3
*43	21	G	19 46 1·1	7 59·36	*84 59 15	− 56	35 43	+ 6	84 22 42	3	3–4
*49			20 45 28·3	8 58·65	*82 18 0	− 28	35 41	+ 6	81 41 57	2	
63	24	G	21 11 6·0	9 12·51	66 55 19	− 18	35 40	+ 16	66 19 37	2–3	2–3
79	25	G	20 54 26·8	8 52·00	70 22 52	− 21	35 41	+ 12	69 47 2	2–3	2–3
102	29	G	21 9 2·0	8 50·90	242 27 4	− 15	35 40	+ 5	241 51 14	2–3	2–3
120	31	G	21 24 6·8	8 58·11	66 35 54	− 15	35 38	+ 18	66 0 19	2–3	2–3
141	Sept. 1	G	21 25 31·6	8 55·60	68 24 19	− 14	35 38	+ 16	67 48 43		
144	5	G	19 42 39·2	6 57·30	235 55 11	− 12	35 43	+ 11	235 19 27	3	3–4
163	7	G	20 5 36·9	7 12·32	66 6 52	− 24	35 43	[+ 8]	65 30 53	1–2	2–3
193	10	G	20 29 33·9	7 24·38	63 34 41	− 18	35 42	+ 19	62 59 0	2	2–3
203	13	G	20 17 36·2	7 0·61	247 5 38	− 23	35 42	+ 7	246 29 40	2	3
220	14	G	19 57 44·2	6 36·84	65 52 15	− 24	35 43	+ 11	65 16 19	3	3, 3–4
234	18	F	19 51 35·1	6 14·87	67 16 34	− 26	35 42	+ 15	66 40 41	2	2
248	19	G	19 32 15·1	5 51·62	71 12 45	− 37	35 42	+ 4	70 36 30	2	3
268	20	G	19 53 1·8	6 8·38	246 33 4	− 25	35 42	+ 18	245 57 15	1–2	2
293	21	G	20 52 11·0	7 3·40	65 47 22	− 16	35 39	+ 16	65 11 43	1	1–2
299	22	G	20 5 2·5	6 12·42	66 14 34	− 24	35 40	+ 4	65 38 34	2	2–3
*328	24	G	19 44 26·0	5 43·91	*354 29 49	+ 15	35 41	[− 2]	353 54 21	2–3	
332	28	G	20 25 18·2	6 8·75	65 28 0	− 19	35 40	− 2	64 51 59	3	3–4
334	29	G	19 44 6·2	5 23·66	66 28 38	− 27	35 41	+ 5	65 52 35	2	3
346	Oct. 4	G	20 24 52·2	5 44·34	245 57 19	− 18	35 40	+ 20	245 21 41	2–3	2–3
368	9	G	0 12 9·5	9 10·97	66 53 22	+ 8	36 38	+ 13	66 17 5	2–3	2–3
384	13	G	0 6 4·3	8 48·84	66 14 15	+ 7	36 37	+ 19	65 38 4	1–2	1–2
399	14	F	0 9 32·1	8 48·28	242 40 15	+ 6	36 38	+ 4	242 3 47	2–3	3
423	16	F	0 42 38·0	9 13·24	66 42 26	+ 15	36 33	+ 9	66 6 17	2–3	3
445	19	F	0 48 16·6	9 6·79	65 14 34	+ 16	36 33	+ 21	64 38 38	2	3
465	Nov. 5	G	0 12 30·7	7 22·44	246 51 15	+ 9	36 38	+ 2	246 14 48	2–3	3–4
481	6	G	0 32 24·3	7 38·23	66 7 41	+ 13	36 34	+ 29	65 31 49	2	2–3
*497	9	G	0 12 6·6	7 5·83	11 28 30	− 7	36 38	[+ 20]	10 52 5		...
502	22	G	0 43 7·5	6 44·02	240 11 38	+ 12	36 33	− 3	239 35 14	2	2–3
*521	30	F	0 59 17·7	6 27·64	247 18 38	+ 17	36 29	...	246 42 26	2–3	3–4
527	Dec. 8	F	1 22 19·5	6 18·12	65 51 41	+ 24	36 26	− 2	65 15 37	2	3

Aug. 21 43 and 49 Images at double distance.
Sept. 24 328 Images at half distance.
Nov. 9 497 Images at double distance.
30 All observations on this date were *rejected* on account of bad definition.

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TABLE XII.—*continued.*

POSITION-ANGLES OF I.–III.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Instrumental Position-Angle.	Refraction.	Equatorial Adjustment.	Index.	Observed Position-Angle.	Definition. I	Definition. S
	1891.		h m s	h m	° ′ ″	″	′ ″	″	° ′ ″		
9	Aug. 17	G	21 47 47·2	10 16·36	64 3 0	− 13	− 35 39	...	63 27 8	2	2
25	19	G	22 6 14·6	10 26·99	88 4 30	− 10	35 38	+ 1	87 28 43	2–3	3
27	20	G	19 38 32·0	7 55·79	97 25 38	− 67	35 43	+ 17	96 49 5	2–3	3
61	24	G	21 0 29·2	9 1·93	64 4 19	− 17	35 40	+ 16	63 28 38	2–3	2–3
*71	25	G	20 7 31·1	8 5·20	65 15 8	− 24	35 43	+ 12	64 39 13	3	3
100	29	G	20 57 23·6	8 39·30	245 44 24	− 17	35 40	+ 5	245 8 32	2–3	2–3
122	31	G	21 32 21·5	9 6·33	64 6 11	− 24	35 39	+ 18	63 30 26	2–3	2–3
132	Sept. 1	G	20 19 2·9	7 49·30	65 0 34	− 21	35 42	+ 16	64 24 47	...	
155	5	G	21 21 13·4	8 35·60	245 43 30	− 15	35 39	+ 11	245 7 47	2	2–3
165	7	G	20 19 31·3	7 26·19	64 7 56	− 20	35 42	[+ 8]	63 32 2	1–2	2–3
187	10	G	19 48 56·0	6 43·86	69 9 22	− 30	35 43	+ 19	68 33 28	2	2–3
205	13	G	20 31 59·4	7 14·96	247 58 8	− 21	35 42	+ 7	247 22 12	2	3
218	14	G	19 45 36·5	6 24·75	63 56 19	− 24	35 42	+ 11	63 20 24	3	3, 3–4
240	18	F	20 30 5·5	6 53·27	244 20 11	− 17	35 40	+ 15	243 44 29	2	3
250	19	G	19 52 10·3	6 11·49	245 54 41	− 25	35 42	+ 4	245 18 38	2	3
270	20	G	20 3 5·5	6 18·42	247 15 43	− 24	35 42	+ 18	246 39 55	1–2	2
291	21	G	20 37 59·2	6 49·24	63 38 52	− 17	35 40	+ 16	63 3 11	1	1–2
301	22	G	20 21 6·5	6 28·44	63 55 41	− 19	35 41	+ 4	63 19 45	2	2–3
340	29	G	20 28 11·5	6 7·63	63 20 8	− 17	35 40	+ 5	62 44 16	3–4	4
354	Oct. 4	G	21 9 36·5	6 28·96	246 44 0	− 14	35 38	+ 20	246 8 28	3	3
364	6	F	21 21 26·6	6 32·75	62 33 45	− 12	35 38	+ 3	61 57 58	2–3	3
366	9	G	0 0 3·2	8 58·90	243 7 22	+ 5	36 40	+ 13	242 31 0	3	3
380	10	G	0 2 9·8	8 56·99	245 53 19	+ 6	36 40	[+ 14]	245 16 59	2–3	3
388	13	G	0 26 26·6	9 9·16	62 38 4	+ 9	36 35	+ 19	62 1 57	1–2	1–2
405	14	F	0 51 47·1	9 30·41	66 47 38	+ 18	36 31	+ 4	66 11 29	2–3	3–4
429	16	F	1 21 33·6	9 52·06	242 16 26	+ 25	36 25	+ 9	241 40 35	3	3–4
433	17	F	0 5 44·8	8 32·43	245 54 8	+ 7	36 39	+ 1	245 17 37	2–3	3–4
*441	19	F	0 21 10·6	8 39·76	57 43 34	+ 6	36 36	+ 21	57 7 25	2	3
463	Nov. 5	G	23 56 20·7	7 6·32	67 0 30	+ 6	36 40	+ 2	66 23 58	2–3	3–4
479	6	G	0 21 38·8	7 27·50	70 29 49	+ 30	36 25	+ 29	69 54 23	2	2–3
515	27	G	1 1 59·6	6 42·52	67 17 8	+ 19	36 29	+ 10	66 41 8	2–3	2–3
*523	30	F	1 15 6·3	6 43·41	247 9 19	+ 23	36 32	...	246 33 10	Very bad	
525	Dec. 8	F	1 11 0·5	6 6·83	248 31 26	+ 22	36 28	− 2	247 55 18	2	3
549	14	G	2 44 52·4	7 16·07	246 23 13	+ 81	36 11	− 1	245 48 22	3	3–4

Aug. 25 71 Evident mistake of 1° in the recorded position-angle, which was corrected. The observation should have been rejected, but was included by mistake.

Oct. 19 441 Evident mistake of 2° in the recorded position-angle, which was corrected. *Rejected.*

Nov. 30 All observations on this date are *rejected* on account of bad definition.

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TABLE XII.—*continued.*

POSITION-ANGLES OF I.–IV.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Instrumental Position-Angle.	Refraction.	Equatorial Adjustment.	Index.	Observed Position-Angle.	Definition. I	Definition. S
	1891.		h m s	h m	° ′ ″	″	′ ″	″	° ′ ″		
2	Aug. 17	G	20 43 57·4	9 12·70	191 24 15	+ 18	− 35 42	...	190 48 51	2	2
17	19	G	20 45 16·5	9 6·24	244 18 15	− 19	35 42	+ 1	243 42 15	2–3	3
37	20	G	21 11 25·0	9 28·42	244 22 11	− 16	35 40	+ 17	243 46 32	2	3
59	24	G	20 49 50·7	8 51·32	246 58 52	− 20	35 41	+ 16	246 23 7	2–3	2–3
82	25	G	21 15 30·3	9 13·00	282 7 52	− 17	35 40	+ 12	281 32 7	2–3	2–3
85	27	F	19 50 27·1	7 40·36	64 3 22	− 25	35 43	+ 26	63 27 40	3–4	3
92	29	G	20 3 44·0	7 45·78	65 45 22	− 26	35 43	+ 5	65 9 18	2–3	3
124	31	G	21 41 14·8	9 15·20	67 7 49	− 13	35 39	+ 18	66 32 15	2–3	2–3
130	Sept. 1	G	20 8 7·5	7 38·41	66 45 28	− 25	35 43	+ 16	66 9 36	...	
150	5	G	20 33 29·6	7 48·00	243 35 41	− 16	35 39	+ 11	242 59 57	3	3–4
161	7	G	19 54 19·3	7 1·06	245 34 34	− 25	35 43	[+ 8]	244 58 34	1–2	2–3
195	10	G	20 41 35·9	7 36·38	251 31 4	− 22	35 41	+ 19	250 55 20	2	2–3
213	13	G	21 22 50·8	8 5·68	63 18 30	− 13	35 39	+ 7	62 42 45	2	3
226	14	G	20 36 31·2	7 15·52	65 42 52	− 18	35 41	+ 11	65 7 4	1–2, 2	3–4, 4
258	19	G	20 44 38·7	7 3·82	68 52 41	− 18	35 40	+ 4	68 16 47	1–2	2
266	20	G	19 40 33·8	5 55·95	242 22 22	− 22	35 42	+ 18	241 46 36	1–2	2
295	21	G	21 1 45·9	7 12·95	242 11 19	− 14	35 39	+ 16	241 35 42	1	1–2
309	22	G	21 3 9·1	7 10·36	245 40 49	− 11	35 39	+ 4	245 5 3	2	2–3
344	Oct. 4	G	20 11 8·8	5 30·66	68 11 0	− 22	35 41	+ 20	67 35 17	3	3
360	6	G	20 42 27·2	5 53·87	233 49 22	− 10	35 39	+ 3	233 13 36	2–3	3
376	9	G	1 0 17·9	9 58·98	245 40 4	+ 18	36 30	+ 13	245 4 5	2	2–3
397	13	G	1 22 28·1	10 4·69	248 27 34	+ 30	36 25	+ 19	247 51 58	3	3
403	14	F	0 37 24·1	9 16·06	262 6 15	+ 18	36 34	+ 4	261 30 3	2–3	3
425	16	F	0 54 2·7	9 24·62	65 9 41	+ 17	36 31	+ 9	64 33 36	2–3	3
435	17	F	0 19 49·6	8 46·47	65 33 45	+ 9	36 37	+ 1	64 57 18	2–3	3–4
449	19	F	1 17 24·7	9 35·84	66 40 34	+ 26	36 26	+ 21	66 4 55	2	3
473	Nov. 5	G	1 33 8·9	8 42·85	66 46 56	+ 34	36 22	+ 2	66 11 10	2–3	3–4
485	6	G	1 4 3·4	8 9·79	67 33 0	+ 22	36 28	+ 29	66 57 23	3, 3–4	3, 3–4
513	27	G	0 50 41·4	6 31·25	244 24 0	+ 15	36 32	+ 10	243 47 53	2–3	2–3
529	Dec. 8	F	1 36 15·2	6 32·00	66 38 19	+ 30	36 23	− 2	66 2 24	2	3
547	14	G	2 29 44·4	7 0·98	245 27 30	+ 63	36 14	− 1	244 52 18	2	3

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TABLE XII.—*continued.*

POSITION-ANGLES OF II.–III.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Instrumental Position-Angle.	Refraction.	Equatorial Adjustment.	Index.	Observed Position-Angle.	Definition. I	Definition. S
	1891.		h m s	h m	° ′ ″	″	′ ″	″	° ′ ″		
11	Aug. 17	G	22 7 17·2	10 35·41	59 53 0	− 11	− 35 38	...	59 17 11	2	2
21	19	G	21 34 25·2	9 55·25	67 50 30	− 15	35 39	+ 1	67 14 37	2–3	3
35	20	G	20 57 40·4	9 14·71	242 56 38	− 17	35 40	+ 17	242 20 58	2	3
*51	24	G	19 55 18·0	7 56·92	29 31 49	+ 23	35 43	+ 16	28 56 45	2–3	2–3
73	25	G	20 18 6·0	8 15·75	64 49 30	− 22	35 42	+ 12	64 13 38	3	3
104	29	G	21 23 18·9	9 5·15	249 21 38	− 16	35 39	+ 5	248 45 48	2–3	2–3
108	31	G	19 46 21·0	7 20·61	259 20 41	− 45	35 43	+ 18	258 44 31	3–4	3–4
139	Sept. 1	G	21 14 26·0	8 44·54	64 44 4	− 15	35 39	+ 16	64 8 26		...
153	5	G	21 10 27·6	8 24·86	248 7 34	− 16	35 40	+ 11	247 31 49	2	2–3
171	7	G	20 52 53·9	7 59·48	253 20 45	− 20	35 40	[+ 8]	252 44 53	1–2	2–3
174	8	G	19 36 19·7	6 39·18	64 14 11	− 26	35 43	+ 7	63 38 9		...
185	10	G	19 37 51·9	6 32·86	216 35 34	+ 20	35 42	+ 19	216 0 31	2	2–3
201	13	G	20 1 26·2	6 44·49	252 20 11	− 30	35 42	+ 7	251 44 6	2	3
216	14	G	19 33 33·7	6 12·74	249 46 30	− 34	35 42	+ 11	249 10 25	3	3, 3–4
242	18	F	20 45 28·3	7 8·63	244 58 22	− 17	35 40	+ 15	244 22 40	2	3
252	19	G	20 6 32·1	6 25·82	246 55 11	− 22	35 41	+ 4	246 19 12	2	3
278	20	G	20 42 19·6	6 57·54	251 7 36	− 20	35 40	+ 18	250 31 54	1–2	2
*283	21	G	19 57 5·1	6 8·45	248 34 8	− 26	35 41	+ 16	247 58 17	1	1–2
303	22	G	20 29 50·5	6 37·15	63 29 11	− 17	35 40	+ 4	62 53 18	2	2–3
320	23	G	20 24 28·0	6 27·81	66 4 56	− 19	35 40	− 2	65 28 55	2	2–3
342	Oct. 4	G	19 56 57·2	5 16·50	249 21 38	− 25	35 41	+ 20	248 45 52	3	3
378	9	G	1 10 30·3	10 9·16	*244 45 42	+ 21	36 27	+ 13	244 9 49	2	2–3
386	13	G	0 16 0·6	8 58·75	60 8 8	+ 6	36 38	+ 19	59 31 55	1–2	1–2
409	14	F	1 17 16·3	9 55·82	65 58 26	+ 25	36 26	+ 4	65 22 29	3	3–4
417	15	F	0 40 3·3	9 14·69	68 28 41	+ 14	36 33	[+ 6]	67 52 28	3–4	3–4
421	16	F	0 31 13·5	9 1·86	244 18 8	+ 11	36 33	+ 9	243 41 55	2–3	3
443	19	F	0 35 30·1	8 54·05	247 45 11	+ 14	36 33	+ 21	247 9 13	2	3
457	23	F	0 15 29·8	8 17·98	243 35 26	+ 8	36 37	+ 25	242 59 22	2	3
*467	Nov. 5	G	0 33 45·7	7 43·63	66 51 26	+ 13	36 33	+ 2	66 15 8	2–3	3–4
*477	6	G	0 11 15·1	7 17·13	237 47 4	+ 5	36 38	+ 29	237 11 0	2	2–3
489	8	G	0 20 6·5	7 17·86	247 8 0	+ 11	36 36	+ 20	246 31 55	2	2–3
504	22	G	0 56 23·1	6 57·24	246 32 26	+ 18	36 31	− 3	245 56 10	2	2–3
537	Dec. 8	F	2 35 12·2	7 30·79	246 51 22	+ 72	− 36 13	− 2	246 16 19	3	3–4

Aug. 24	51	Images at double distance.
Sept. 21	283	Large discordance; perhaps mistake of 30′. *Rejected in final solution.*
Nov. 5	467	One half of this observation belongs to the pair III.–IV. *Rejected in final solution.*
6	477	Evident mistake of 1° in record, which was corrected. *Rejected.*

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TABLE XII.—*continued.*

POSITION-ANGLES OF II.-IV.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Instrumental Position-Angle.	Refraction.	Equatorial Adjustment.	Index.	Observed Position-Angle.	Definition. I	Definition. S
	1891.		h m s	h m	° ′ ″	″	′ ″	″	° ′ ″		
5	Aug. 17	G	21 16 26·8	9 45·11	238 47 30	− 13	− 35 40	...	238 11 37	2	2
23	19	G	21 53 54·9	10 14·70	241 55 52	− 12	35 38	+ 1	241 20 3	2–3	3
29	20	G	20 1 20·5	8 18·54	244 39 40	− 25	35 43	+ 17	244 3 49	2–3	3
57	24	G	20 39 32·5	8 41·04	246 58 11	− 21	35 42	+ 16	246 22 24	2–3	2–3
77	25	G	20 41 22·2	8 38·96	261 28 30	− 28	35 41	+ 12	260 52 33	2–3	2–3
89	27	F	20 34 18·9	8 24·10	63 5 26	− 19	35 42	+ 26	62 29 51	3–4	3
94	29	G	20 17 49·6	7 59·84	65 15 0	− 22	35 42	+ 5	64 39 1	2–3	3
114	31	G	20 33 14·1	8 7·37	67 23 41	− 21	35 42	+ 18	66 47 56	3–4	3–4
137	Sept. 1	G	21 4 0·0	8 34·13	66 37 41	− 17	35 40	+ 16	66 2 0	...	
157	5	G	21 31 33·6	8 45·90	244 36 15	− 13	35 39	+ 11	244 0 34	2	2–3
167	7	G	20 30 11·4	7 36·83	245 45 30	− 20	35 42	[+ 8]	245 9 36	1–2	2–3
176	8	G	19 49 9·5	6 51·97	246 55 30	− 27	35 42	+ 7	246 19 28	thro' cloud	
191	10	G	20 19 1·0	7 13·87	247 19 0	− 23	35 41	+ 19	246 43 15	2	2–3
211	13	G	21 12 45·3	7 55·61	65 9 26	− 15	35 39	+ 7	64 33 39	2	3
222	14	G	20 12 43·0	6 51·78	65 31 56	− 21	35 42	+ 11	64 56 4	3–4, 4	3–4, 4
260	19	G	20 58 43·7	7 17·86	68 9 30	− 16	35 39	+ 4	67 33 39	2–3	3
276	20	G	20 32 33·4	6 47·80	76 40 52	− 24	35 41	+ 18	76 5 5	1–2	2
287	21	G	20 16 50·4	6 28·15	244 8 45	− 19	35 41	+ 16	243 33 1	1	1–2
311	22	G	21 12 53·8	7 20·08	245 43 52	− 14	35 39	+ 4	245 8 3	2	2–3
322	23	G	20 39 0·4	6 42·31	246 9 22	− 18	35 40	− 2	245 33 22	2	2–3
352	Oct. 4	G	21 0 34·0	6 19·94	67 6 41	− 15	35 39	+ 20	66 31 7	3	3
358	6	F	20 25 58·7	5 37·44	239 48 26	− 14	35 40	+ 3	239 12 35	2–3	3
374	9	G	0 49 8·2	9 47·84	245 51 41	+ 16	36 32	+ 13	245 15 38	2–3	3
395	13	G	1 4 26·5	9 46·72	248 7 26	+ 22	36 29	+ 19	247 31 38	2–3	2–3
401	14	F	0 22 1·6	9 0·73	54 50 34	+ 5	36 36	+ 4	54 14 7	2–3	3
415	15	F	0 21 31·6	8 56·21	64 9 19	+ 9	36 36	[+ 6]	63 32 58	3	3
419	16	F	0 17 33·2	8 48·23	63 10 41	+ 8	36 37	+ 9	62 34 21	2–3	3
453	19	F	1 43 4·9	10 1·44	67 17 8	+ 37	36 21	+ 21	66 41 45	2	3
455	23	F	0 3 46·1	8 6·28	242 40 41	+ 6	36 39	+ 25	242 4 33	2	3
471	Nov. 5	G	1 15 5·9	8 24·86	66 52 19	+ 26	36 26	+ 2	66 16 21	2–3	3–4
491	8	G	0 32 27·4	7 30·18	68 4 11	+ 13	36 34	+ 20	67 28 10	2	2–3
510	22	G	1 42 26·6	7 43·16	66 28 52	+ 36	36 21	− 3	65 53 4	3	3–4
535	Dec. 8	F	2 23 43·8	7 19·35	66 57 52	+ 62	36 15	− 2	66 22 37	3	3–4
543	14	G	2 0 45·5	6 32·08	244 1 22	+ 40	36 19	− 1	243 25 42	2	3

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TABLE XII.—*continued.*

POSITION-ANGLES OF III.–IV.

Rotation No.	Date.	Observer.	Sidereal Time.	Greenwich Mean Time.	Instrumental Position-Angle.	Refraction.	Equatorial Adjustment.	Index.	Observed Position Angle.	Definition. I	Definition. S
	1891.		h m s	h m	° ′ ″	″	′ ″	″	° ′ ″		
7	Aug. 17	G	21 33 27·7	10 2·08	239 9 0	− 13	− 35 39	...	238 33 8	2	2
15	19	G	20 32 7·1	8 53·12	245 20 30	− 21	35 42	+ 1	244 44 28	2–3	3
39	20	G	21 23 43·5	9 40·72	246 10 11	− 16	35 39	+ 17	245 34 33	2	3
45	21	G	20 6 15·8	8 19·55	246 17 45	− 26	35 43	+ 6	245 41 42	2–3	3
53	24	G	20 10 47·7	8 12·37	246 7 27	− 25	35 43	+ 16	245 31 35	2–3	2–3
69	25	G	19 55 52·3	7 53·58	247 21 11	− 30	35 43	+ 12	246 45 10	3	3
87	27	F	20 12 14·7	8 2·10	62 19 26	− 20	35 43	+ 26	61 43 49	3–4	3
96	29	G	20 34 1·0	8 15·98	65 46 52	− 21	35 42	+ 5	65 10 54	2–3	3
112	31	G	20 18 11·0	7 52·36	68 27 52	− 25	35 42	+ 18	67 52 3	3–4	3–4
126	Sept. 1	G	19 45 7·4	7 15·47	75 38 37	− 41	35 43	+ 16	75 2 29	...	
146	5	G	19 58 40·6	7 13·28	237 23 49	− 14	35 43	+ 11	236 48 3	3	3–4
169	7	G	20 41 0·4	7 47·62	245 14 45	− 18	35 41	[+ 8]	244 38 54	1–2	2–3
182	8	G	20 37 25·1	7 40·10	246 5 16	− 19	35 41	+ 7	245 29 23	thro' cloud	
209	13	G	21 2 30·3	7 45·39	65 43 15	− 16	35 40	+ 7	65 7 26	2	3
224	14	G	20 25 20·4	7 4·37	66 29 45	− 20	35 41	+ 11	65 53 55	3–4, 4	3–4, 4
244	18	F	21 2 42·9	7 25·81	66 26 26	− 16	35 39	+ 15	65 50 46	2	3
256	19	G	20 30 37·7	6 49·84	67 34 0	− 20	35 40	+ 4	66 58 4	1–2	2
274	20	G	20 23 16·1	6 38·53	74 23 11	− 25	35 41	+ 18	73 47 23	1–2	2
289	21	G	20 27 32·8	6 38·83	242 43 38	− 17	35 40	+ 16	242 7 57	1	1–2
305	22	G	20 40 39·5	6 47·93	245 8 52	− 17	35 39	+ 4	244 33 0	2	2–3
318	23	G	20 7 56·7	6 11·34	246 5 19	− 22	35 41	− 2	245 29 14	2	2–3
336	29	G	19 56 45·0	5 36·28	255 1 22	− 32	35 42	+ 5	254 25 13	2	3
350	Oct. 4	G	20 49 48·0	6 9·20	67 21 30	− 16	35 39	+ 20	66 45 55	3	3
356	6	F	20 11 42·1	5 23·20	239 19 11	− 14	35 40	+ 3	238 43 20	2–3	3
370	9	G	0 22 54·2	9 21·68	246 55 30	+ 10	36 36	+ 13	246 19 17	2–3	3
382	10	G	0 18 2·9	9 12·83	247 9 15	+ 9	36 37	[+ 14]	246 33 1	2–3	3
393	13	G	0 50 35·6	9 32·90	246 51 8	+ 17	36 32	+ 19	246 15 12	2–3	2–3
407	14	F	1 5 16·1	9 43·85	248 2 52	+ 22	36 29	+ 4	247 26 49	3	3–4
413	15	F	0 4 31·6	8 39·26	261 54 26	+ 11	36 40	[+ 6]	261 18 3	3	3
431	16	F	1 33 15·3	10 3·72	64 8 15	+ 30	36 23	+ 9	63 32 31	3	3–4
437	17	F	0 32 55·2	8 59·53	65 41 8	+ 12	36 35	+ 1	65 4 46	3	3
451	19	F	1 30 18·7	9 48·70	67 25 34	+ 31	36 24	+ 21	66 50 2	2	3
*459	23	F	0 35 29·5	8 37·92	103 44 38	+ 17	36 34	+ 25	103 8 46	2	3
475	Nov. 5	G	1 46 1·7	8 55·70	66 39 49	+ 40	36 20	+ 2	66 4 11	2–3	3–4
487	6	G	1 21 13·6	8 26·91	66 29 19	+ 28	36 25	+ 29	65 53 51	3, 3–4	3, 3–4
495	8	G	1 1 17·8	7 58·94	67 48 0	+ 22	36 29	+ 20	67 12 13	2–3, 3	2–3, 3
506	22	G	1 10 48·9	7 11·62	66 29 41	+ 23	36 27	− 3	65 53 34	2	2–3
517	27	G	1 12 25·4	6 52·92	246 6 22	+ 22	36 27	+ 10	245 30 27	2	2–3
533	Dec. 8	F	2 11 22·0	7 7·02	66 53 49	+ 51	36 18	− 2	66 18 20	3	3–4
539	14	G	1 24 9·8	5 55·58	238 1 15	+ 19	36 26	− 1	237 25 7	2–3	3

Oct. 23 459 *Rejected*—See corresponding distance (460).

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CHAPTER II.

Tabular Places and Equations of Condition.

Choice of the Theory.—For the determination of the elements of the orbits of the satellites it is necessary to compare the observed distances and position-angles with their values as computed from some theory, in order to derive from the differences *observed minus computed* corrections to the constants on which the theory is based.

A theory which is to be used for this purpose must first of all be self-consistent. It must represent, so far as our mathematical resources allow, and within a predetermined limit of accuracy, the motions of the bodies in question under the action of gravitation, assuming certain elements and masses. This system of fundamental constants should be consistently carried through the whole theory from beginning to end, and therefore also through the tables based on it.

Damoiseau's tables do not satisfy this condition. Very little is known with certainty concerning the manner in which they were constructed, but it can hardly be doubted that they are largely of an empirical nature. Probably most of the important inequalities, as they occur in the tables, were derived by Damoiseau directly from the observations, but some were also adopted unaltered from his predecessor Delambre, and the smaller inequalities were either taken from Delambre or computed theoretically from a set of masses and elements, which was not consistent with the new values of the chief inequalities.

For the purpose of computing positions of the satellites at any other time than the epochs of eclipse, Damoiseau's tables are moreover entirely unsuitable, but even if this were not so, their use as a basis for the computation of tabular places would be excluded by the considerations just given.

Souillart's theory, published in 1880 in the *Memoirs of the Royal Astronomical Society*, vol. xlv., is mathematically entirely consistent. It is undoubtedly a very thorough and admirable piece of work, representing a great improvement on previously existing theories. Naturally it does stop at a certain order of approximation, which may perhaps not in all cases be sufficient. So, to take one instance, the differential equations for the latitude are integrated by two different methods. The differences between the results of these two methods were entirely negligible so long as only

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eclipse observations were available, but may become of importance when series of refined heliometer observations are compared with the theory. These differences, however, only affect the slow changes of the inclinations and nodes, and are unimportant for the purposes of the present discussion. Similarly, other imperfections of Souillart's theory can be disregarded for the moment.

Souillart's theory was reduced to numbers in 1887 (*Mémoires présentés par divers savants à l'académie des sciences de l'Institut de France*, tome xxx.). It cannot be denied that Souillart's choice of his fundamental constants was a rather unfortunate one. Instead of deriving the masses and elements from those inequalities, which had been determined anew by Damoiseau, he took the masses stated in Damoiseau's introduction, which are not consistent with his values of the inequalities. The inequalities which result from Souillart's numerical theory thus, of course, do not agree with Damoiseau's tables. If we assume, as seems natural, that Damoiseau's values of the inequalities are nearer the truth than Delambre's, then Souillart's theory will require larger corrections than would be the case if he had made a new determination of the masses from Damoiseau's inequalities, as was done by Adams. But this is not of much importance. The all-important point is that all Souillart's numerical work is consistent with one and the same system of elements and masses. It was therefore decided to use Souillart's theory for the computation of tabular places.

Mean Distances.—The position of any satellite is determined by its longitude, radius vector, and latitude. Instead of the latter we can use the inclination and node of the oscillating orbital plane. The radius vector is expressed in the mean distance as unit. The term "mean distance" requires some explanation, as different writers have used it in slightly different meanings. The non-periodic part of the radius vector consists of:

I. The mean distance as computed from the mean motion by Kepler's third law pure and simple.

II. The corrections of the first order due to the compression of Jupiter.

III. The corrections due to perturbations by the sun and the other satellites, together with terms of higher orders depending on the compression.

I adopt as "mean distance" the sum of I. and II., and include III. under the "perturbations." This is Laplace's definition of the mean distance. It is thus given by the formula:

$$n_i^2 a_i^3 = f \frac{1+m_i}{M(1+\Sigma m)}\left[1+\frac{Jb^2}{a_i^2}\right] \quad (1)$$

Here f is the Gaussian constant of attraction; $\log f = 6{\cdot}4711629 - 10$.

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The coefficient J depends on the compression of Jupiter. It is independent of the particular units used. But it always occurs multiplied with b^2, the square of the equatorial radius of the planet. The effect produced by a compression corresponding to a given coefficient J therefore depends on this diameter, and inversely the value of J corresponding to a given effect depends on the value which is adopted for b. Which values are actually adopted in the computation of the tabular places is of little importance, so long as they are not too far outside the limits of possibility, but it is important to realise the fact that not J, but Jb^2, is the quantity which enters into the theory.

Souillart expresses the values of a_i in b as unit. Therefore, from the value of Jb^2 used in his theory, we can derive the value of b which is implied by it, when a_i has been computed from (1). Souillart's value of Jb^2, taking $b=1$, is

$$J = 0{\cdot}0247187.$$

From the values of a_i stated below we find then

$$\log b = 6{\cdot}668410 - 10.$$

This corresponds to the following value of the apparent equatorial diameter of Jupiter at its mean distance from the earth ($\Delta = 5{\cdot}202800$):

$$d = 39''{\cdot}9512.$$

The value of Jb^2 expressed in astronomical units is

$$Jb^2 = 0{\cdot}000000005368 41.$$

Souillart includes in his "mean distance" a part of the perturbations mentioned above under III., viz.: those of the first order. If these are removed, the remaining values, when expressed in the semi-major axis of the earth's orbit as unit, are in agreement with the formula (1).

The sidereal mean daily motions are:

$$\begin{aligned} n_1 &= 203^\circ{\cdot}488955 \\ n_2 &= 101{\cdot}374724 \\ n_3 &= 50{\cdot}317608 \\ n_4 &= 21{\cdot}571071 \end{aligned}$$

For the reciprocal of the mass of the system to which a correction must be determined from the observations, we choose the value determined by Dr Kempf* from previous observations of satellites, made by Bessel, Airy, Schur, and Vogel, viz.—

$$M = 1047{\cdot}700.$$

* *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, No. 10 (Bd. III. Stück 2), 1882.

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We then find from the formula (1) the following values of the logarithms of the semi-major axes:

$$\log a_1 = 7{\cdot}4501051 - 10$$
$$\log a_2 = 7{\cdot}6517881 - 10$$
$$\log a_3 = 7{\cdot}8545797 - 10$$
$$\log a_4 = 8{\cdot}0997955 - 10.$$

The radii vectores are then given by the formula:

$$r_i = a_i \rho_i.$$

Souillart gives the radii vectores in the form $r_i = a_i + \textit{perturbations}$, but his a_i contains, as already remarked, perturbations of the first order. Souillart's mean distances are thus:

$$(a_i)_{\text{Souillart}} = a_i + \Delta a_i,$$

where Δa_i are the perturbations of the first order. These are for the four satellites, expressed in Souillart's units,

$$\Delta a_1 = -0{\cdot}00001744 \quad \text{or} \quad \frac{\Delta a_1}{a_1} = -0{\cdot}0000029$$
$$\Delta a_2 = +0{\cdot}00001426 \qquad \frac{\Delta a_2}{a_2} = +0{\cdot}0000015$$
$$\Delta a_3 = +0{\cdot}00024213 \qquad \frac{\Delta a_3}{a_3} = +0{\cdot}0000158$$
$$\Delta a_4 = +0{\cdot}00139728 \qquad \frac{\Delta a_4}{a_4} = +0{\cdot}0000518.$$

We have, therefore, for the computation of ρ_i,

$$\rho_i = 1 + \left(\frac{r_i - a_i}{a_i}\right)_{\text{Souillart}} + \frac{\Delta a_i}{a_i}. \tag{2}$$

The ephemeris; fundamental plane.—In *Monthly Notices of the Royal Astronomical Society*, vol. li. pp. 505–539, Mr A. Marth has published tables based on Souillart's theory. Souillart refers the satellites to the plane of Jupiter's orbit, but Mr Marth has adopted a fundamental plane which nearly coincides with Jupiter's equator. On p. 510 the node and inclination of this plane on Jupiter's orbit are given; and also with reference to the ecliptic and the mean equinox (*i.e.* the longitudes are reckoned from the mean position of the first point of Aries, including precession, but not nutation). Then on p. 511 the quantities defining the position of this same plane with reference to the earth's true equator and true equinox (*i.e.* including nutation) are given. These latter quantities have been correctly derived from those immediately preceding them, but the quantities referring to the ecliptic are not consistent with those referring to Jupiter's orbit, nor have these latter been entirely correctly derived from Souillart's data.

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In the present investigation, however, this plane must only be considered as an arbitrarily adopted plane, so chosen as to facilitate the computations. The hypothesis that it coincides with Jupiter's equator is not made use of, nor is it possible to verify this hypothesis from observations made at one epoch only. The fundamental plane must thus be considered as being defined by its inclination and node on the ecliptic. These are :

Ascending node . . .	$N_0 = 336^\circ\ 21'\ 28''{\cdot}39 + 0''{\cdot}126783\ t$
Inclination	$J_0 = 2\ 9\ 3{\cdot}94 + 0{\cdot}000950\ t$

where t is the time counted in days from 1900 January, 0·0 Greenwich mean time.

The node and inclination of Jupiter's orbit are, for 1900·0 :

$$\Im = 99^\circ\ 26'\ 35''{\cdot}60$$
$$\phi = 1\ 30\ 33{\cdot}11.$$

From these we find for the relative position of these two planes :

Longitude of node counted in fundamental plane	$\theta_0' = 135^\circ\ 24'\ 34''{\cdot}34$
„ „ „ Jupiter's orbit	$\theta_0 = 135\ 25\ 48{\cdot}51$
Inclination	$\omega_0 = 3\ 4\ 4{\cdot}75.$

The value of ω_0 is exactly identical to the value stated by Marth ; the difference of 0″·08 in θ_0 is unimportant.

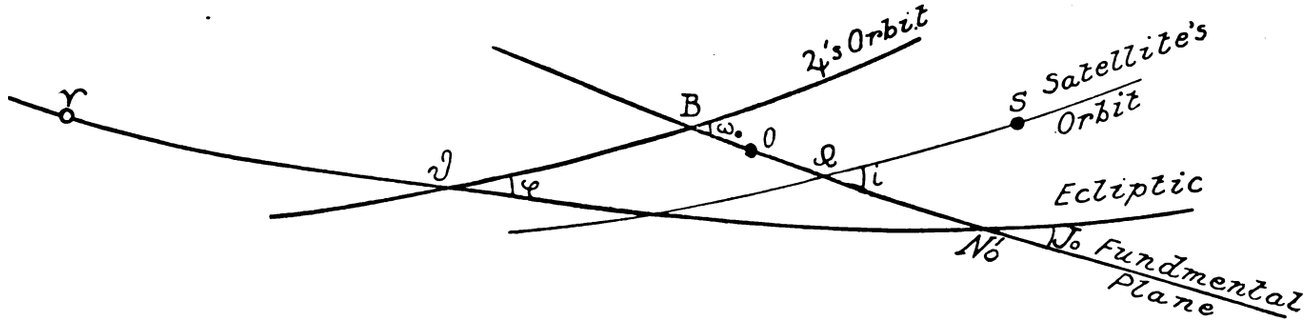

FIG. 8.

Marth evidently intended to adopt the descending node of the fundamental plane on the orbit (*i.e.* Jupiter's adopted vernal equinox) as his origin of longitudes, *i.e.* to adopt for "longitude of 0" the value of θ_0'. Probably he applied the correction needed to derive θ_0' from θ_0 with the wrong sign. Anyhow, the longitudes are actually counted not from the node, but from a point 0 whose longitude, counted in the orbit, is

$$\text{Longitude of 0} \ . \ . \ . \quad O_0 = 135^\circ\ 27'\ 2''{\cdot}55 + 0''{\cdot}129894\ t.$$

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It should be noted that the motions here given imply Newcomb's precession, though Marth uses Leverrier's.

In fig. 8 we have, ♈ being the first point of Aries:

$$N_0 = \Upsilon N_0' + 180^\circ$$
$$\theta_0 = \Upsilon\mathfrak{S} + \mathfrak{S}B$$
$$\theta_0' = \Upsilon N_0' - N_0'B$$
$$O_0 = \Upsilon N_0' - N_0'O.$$

The longitudes of the satellites are counted "in the orbit." Thus, if the satellite is at S, we have

$$\Omega = O\Omega$$
$$v = O\Omega + \Omega S.$$

Errors in Ephemeris.—Marth's data and tables were very carefully examined and compared with Souillart's theory, on which they profess to be based, and in addition to a few unimportant errors in the last decimal places, the following discrepancies were found.

Of Marth's perijoves, given on p. 511, ϖ_2 and ϖ_4 agree respectively with Souillart's $g_1t + \beta_1 - b_4t - \gamma_4 + 180^\circ$, and $g_3t + \beta_3 - b_4t - \gamma_4 + 180^\circ$, but ϖ_1 and ϖ_3 differ from Souillart's values by $+20^\circ{\cdot}0022$ and $+21^\circ{\cdot}0000$ (Marth—Souillart). The motions of all the perijoves agree with Souillart. As the coefficients of the inequalities do not depend on the adopted longitudes of the perijoves, it is not of the least importance which values are adopted. The values printed on p. 511 have actually been used in the formation of the arguments, and in the computation of the inequalities S on p. 512.

The values of G_2 given on p. 512 are in error by a constant quantity $-1^\circ{\cdot}9755$ (Marth—Souillart). The *correct* values (*i.e.* those given by Marth $+1^\circ{\cdot}9755$) have been used in the inequalities S and in the formulas on p. 513.

The coefficients of the inequalities are all in agreement with those given by Souillart. A curious mistake has been made by Marth in the tables of the second satellite. On p. 514 he gives the inequalities:

Radius Vector.		Longitude.	
$-\cdot0000672\cos\beta_2$		$+48''{\cdot}578\sin\beta_2$	
$+\quad 634\cos 2\beta_2$	β_2	$-20{\cdot}498\sin 2\beta_2$	β_2
$+\quad 061\cos 3\beta_2$		$-\ 1{\cdot}505\sin 3\beta_2$	
$+\quad 770$			

In these formulas $2\beta_2$ must be replaced by $3\beta_2$, and $3\beta_2$ by $5\beta_2$. In the table for the longitudes, p. 531, the *correct* arguments ($3\beta_2$ and $5\beta_2$) have been used, but the table for the radius vector (pp. 530–531) is computed with the *erroneous*

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arguments ($2\beta_2$ and $3\beta_2$). Thus the radii vectores, computed from Marth's tables, require a correction:

$$\delta\rho_2 = +0{\cdot}0000634\ (\cos 3\beta_2 - \cos 2\beta_2) + 0{\cdot}0000061\ (\cos 5\beta_2 - \cos 3\beta_2).$$

This error of Marth was only discovered after the computation of the tabular places. Appropriate corrections to the tabular distances and position-angles were then applied by means of differential formulas:

$$(3) \qquad \delta s = \frac{\partial s}{\partial \rho_2}\,\delta\rho_2, \qquad \delta p = \frac{\partial p}{\partial \rho_2}\,\delta\rho_2.$$

In the ephemeris on p. 521 all values of ϵ_3 must be increased by $+\cdot 070$. The whole of the perturbation depending on this argument is, however, negligible.

The most important error is, however, the following. Marth adopts the same definition of the mean distance as has been explained above, *i.e.* not including any "perturbations" except those of the first order depending on the compression. He should thus have added the quantities $\frac{\Delta a_i}{a_i}$ stated above in formula (2) to his perturbations in radius vector. This he has not done, but his perturbations are identical to Souillart's. At the time when Marth's "mean distances" were examined, the Cape Observatory did not possess a copy of Souillart's second paper. When this afterwards arrived, I verified that Marth's perturbations agreed with Souillart's, but I failed to notice that they corresponded to different definitions of the "mean distances."

Afterwards my attention was again drawn to the mean distances on the occasion of a discussion about the influence of the compression, and thus the error was detected and corrected. We have, by formula (2):

$$\Delta\rho_i = \frac{\Delta a_i}{a_i}.$$

The distances and position-angles were then corrected differentially by the formulas

$$(4) \qquad \begin{aligned} \Delta s &= \frac{\partial s}{\partial \rho_1}\,\frac{\Delta a_1}{a_1} + \frac{\partial s}{\partial \rho_2}\,\frac{\Delta a_2}{a_2} \\ \Delta p &= \frac{\partial p}{\partial \rho_1}\,\frac{\Delta a_1}{a_1} + \frac{\partial p}{\partial \rho_2}\,\frac{\Delta a_2}{a_2}. \end{aligned}$$

In addition to these errors the following errata were noticed:

page 510	line 6	for	$65^\circ{\cdot}073$	read	$65^s{\cdot}073$
513	line 8	,,	sin V	,,	cos V
	line 31	,,	$a_2 = l_1 - l_2$	,,	$a_2 = l_1 - l_2 + 180^\circ$
514	First Satellite, Radius Vector . .	,,	$-\cdot 0000607$	,,	$-\cdot 0000703$
515	Fourth Satellite, Longitude . .	,,	$2{\cdot}208 \sin \epsilon_4$	,,	$4{\cdot}208 \sin \epsilon_4$
516	column: log a_2 argument 7·8 . .	,,	794	,,	774

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page 517	line 23	for	14	read 11
526	column a_1, argument ·190 . .	,,	8458	,, 8558
	·310 . .	,,	1328	,, 1338
529	column a_2, argument 8^m . . .	,,	·00148	,, ·00158
530	Longitude, column a_2, argument ·310 .	,,	·0493	,, ·9493
531	Longitude (continued), column γ_2, argument ·05	,,	$\mp$·0039	,, $\pm$·0039
532	column $l_3 - O$, argument 1^d 0^h . .	,,	50·3196	,, 50·3176
	footnote	,,	$\gamma_3, \zeta, \gamma, \beta$	,, $\eta_3, \zeta_1, \eta_1, \theta_1$
538	Radius Vector, column a_4, argument ·120	,,	·9412	,, ·4412
539	Longitude (continued), Heading . .	,,	$\eta_4\beta_4$	,, $\eta_4\theta_4$

Tabular places.—The longitudes and radii vectores were interpolated from Marth's tables for the epochs of observation in distance; the tabular distances and position-angles were then computed by the following formulas:

$$\Delta' = \Delta + a\rho \cos (v - \mathrm{L}) \cos \mathrm{B}.$$

(5)
$$x'' = \frac{a''\rho}{\Delta'} [\sin (v - \mathrm{L}) - \sin (v - \Omega) \sin i \tan \tfrac{1}{2} i \cos (\mathrm{L} - \Omega)]$$

$$y'' = \frac{a''\rho}{\Delta'} [\cos (v - \mathrm{L}) \sin \mathrm{B} + \sin (v - \Omega) \sin i \cos \mathrm{B}]$$

where

a = mean distance
ρ = radius vector
v = true longitude
i = inclination
Ω = ascending node
} of the satellite

$\mathrm{L} + 180^\circ$ = jovicentric longitude
B = ,, latitude
Δ = ,, distance
} of the earth

$$a'' = \frac{a}{\sin 1''}.$$

The quantities belonging to the satellites I., II., III., IV. will be distinguished by the suffixes 1, 2, 3, 4.

x'' and y'' are then the co-ordinates, expressed in seconds of arc, of the projections of the satellites on a sphere through Jupiter's centre, the axes of x'' and y'' being perpendicular and parallel to the projection on that sphere of a line perpendicular to the fundamental plane.

The unit of Δ and of a is the mean distance of the earth from the sun.

The second term in the expression for x'' has been neglected for satellites I and III.

The co-ordinates so computed are given in Table XIII. They still require some corrections, which will be explained below.

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TABLE XIII.

SATELLITES I.–II. CO-ORDINATES.

Date.	Rot. No. p.	Rot. No. s.	v_1.	v_2.	ρ_1.	ρ_2.	$\log \frac{a_1''\rho_1}{\Delta_1'}$.	$\log \frac{a_2''\rho_2}{\Delta_2'}$.	x_1''.	y_1''.	x_2''.	y_2''.
			°	°								
Aug. 17		3	257·295	320·861	1·002147	1·004454	2·160681	2·363741	+ 106″·308	+ 3″·045	+ 215″·979	− 0″·581
	4		258·628	321·522	1·002078	1·004541	2·160662	2·363788	+ 108·563	+ 2·969	+ 215·038	− 0·655
19	19	20	305·237	162·588	0·999041	0·992674	2·160692	2·359243	+ 144·126	− 0·406	− 167·850	+ 2·892
20	33	34	145·182	263·390	1·002151	0·995874	2·162387	2·361187	− 131·196	+ 1·912	+ 185·118	+ 5·010
21	43	44	339·207	359·782	0·997144	1·008605	2·161003	2·367879	+ 111·585	− 2·832	+ 115·917	− 4·525
	49	50	347·610	3·870	0·996821	1·008795	2·160913	2·367998	+ 96·855	− 3·304	+ 101·199	− 4·843
24	63	64	239·922	309·615	1·002744	1·003570	2·164282	2·366674	+ 74·571	+ 3·856	+ 228·795	+ 0·588
25	79	80	81·082	47·963	0·998421	1·007675	2·163217	2·369182	− 114·755	− 2·757	− 75·735	− 6·416
		81	82·568	48·690	0·998508	1·007609	2·163251	2·369154	− 117·052	− 2·667	− 78·536	− 6·410
29	102	103	174·670	92·962	1·003561	1·001544	2·166149	2·367424	− 81·753	+ 3·696	− 210·153	− 4·195
31	120	121	222·543	298·350	1·003220	1·002649	2·166339	2·368073	+ 36·083	+ 4·316	+ 233·378	+ 1·719
Sept. 1	141	142	66·085	37·974	0·997871	1·008082	2·164697	2·371045	− 89·789	− 3·499	− 40·020	− 6·317
		145	143·360	74·736	1·002681	1·003559	2·166647	2·369176	− 132·243	+ 1·899	− 171·359	− 5·354
5	144								− 133·328	+ 1·829	− 169·971	− 5·398
									− 131·167	+ 1·965	− 172·677	− 5·311
7	163	164	191·957	280·455	1·003734	1·000636	2·166864	2·367358	− 39·063	+ 4·228	+ 222·899	+ 3·367
10	193	194	84·804	225·182	0·999378	0·99299	2·165071	2·363351	− 123·773	− 2·323	+ 72·093	+ 6·068
13	203	204	331·103	166·590	0·996753	0·991235	2·163287	2·362018	+ 120·017	− 2·415	− 147·951	+ 3·327
14	220	221	171·894	267·793	1·003697	0·999415	2·165599	2·365446	− 83·093	+ 3·528	+ 203·519	+ 4·252
18	234	235	262·020	311·378	1·000400	1·006170	2·162814	2·367294	+ 120·688	+ 2·367	+ 224·581	+ 0·109
19		249	103·037	49·506	1·001021	1·005651	2·162889	2·366953	− 141·889	− 0·946	− 93·430	− 5·881
	248								− 141·391	− 1·008	− 91·833	− 5·900
									− 142·336	− 0·887	− 94·953	− 5·861
20	268	269	308·209	152·467	0·997489	0·991717	2·160877	2·359687	+ 141·409	− 0·888	− 183·377	+ 2·000
21	293	294	159·890	259·170	1·003602	0·998853	2·162732	2·362281	− 104·109	+ 2·907	+ 185·301	+ 4·647
22	299	300	356·131	355·929	0·996270	1·009144	2·159510	2·366930	+ 70·763	− 3·595	+ 114·793	− 4·051
24	328	329	39·268	197·051	0·997363	0·991499	2·158877	2·357234	− 32·126	− 4·000	− 34·700	+ 5·138
									− 34·475	− 3·985	− 32·814	+ 5·146
									− 35·489	− 3·977	− 31·997	+ 5·150
									− 37·604	− 3·962	− 30·287	+ 5·156
28	332	333	136·709	244·914	1·003101	0·997421	2·158302	2·357267	− 133·581	+ 1·495	+ 146·676	+ 5·139
29	334	335	333·345	342·188	0·996424	1·009047	2·154990	2·362308	+ 111·593	− 2·486	+ 155·612	− 2·853
Oct. 4	346	347	273·494	129·792	0·998865	0·992715	2·151684	2·350664	+ 132·645	+ 1·392	− 215·954	− 0·166
9	368	369	240·159	292·682	1·000798	1·005759	2·147557	2·351617	+ 83·366	+ 3·062	+ 224·661	+ 1·693
13	384	385	331·322	335·849	0·996297	1·009127	2·141867	2·349243	+ 109·448	− 2·256	+ 165·050	− 2·133
14	399	400	174·673	76·076	1·003602	0·998806	2·143514	2·343653	− 66·868	+ 3·236	− 175·461	− 4·168
16	423	424	224·927	282·523	1·001421	1·005099	2·140297	2·343754	+ 50·982	+ 3·389	+ 216·800	+ 2·430
19	445	446	115·132	226·284	1·002875	0·996912	2·137697	2·336383	− 137·223	+ 0·113	+ 85·390	+ 4·873
Nov. 5	465	466	319·424	141·299	0·996229	0·990803	2·112902	2·311886	+ 115·889	− 1·438	− 180·125	+ 0·699
6	481	482	165·138	244·928	1·003450	1·001803	2·114296	2·315160	− 79·426	+ 2·552	+ 138·695	+ 4·072
9	497	498	51·468	186·405	1·000100	0·993639	2·109124	2·307350	− 61·090	− 2·797	− 57·799	+ 3·482
									− 62·220	− 2·783	− 56·825	+ 3·490
22	502	503	173·071	61·044	1·002748	0·996879	2·091295	2·290976	− 62·259	+ 2·575	− 119·427	− 3·904
30	521	522	359·171	152·259	0·997806	0·991919	2·078235	2·276863	+ 50·255	− 2·618	− 148·510	+ 1·229
Dec. 8	527	528	185·060	243·011	1·001504	1·005008	2·068265	2·271389	− 39·512	+ 2·660	+ 115·565	+ 3·767

144 $\Delta t = -7^m·03$ and $+6^m·74$ from 145.
248 $\Delta t = -6^m·06$ and $+5^m·80$ from 249.
328, 329 $\Delta t = -8^m·10$, $-1^m·31$, $+1^m·63$, and $+7^m·78$ from the mean.
497, 498 $\Delta t = -2^m·03$ and $+2^m·03$ from the mean.

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TABLE XIII.—*continued.*

SATELLITES I.–III. CO-ORDINATES.

Date.	Rot. No.		v_1.	v_3.	ρ_1.	ρ_3.	$\log \frac{a_1'' \rho_1}{\Delta_1'}$.	$\log \frac{a_3'' \rho_3}{\Delta_3'}$.	x_1''.	y_1''.	x_3''.	y_3''.
	p.	*s.*										
Aug. 17	9	10	264°.433	262°.968	1.001748	0.999982	2.160557	2.563981	+ 117".668	+ 2".615	+ 292".362	+ 7".191
19		26	313.014	3.857	0.998561	1.001072	2.160543	2.566914	+ 140.897	− 1.004	+ 161.424	− 9.536
	25								+ 141.223	− 0.960	+ 162.249	− 9.523
									+ 140.613	− 1.041	+ 160.733	− 9.546
20		28	136.030	48.839	1.001634	1.000596	2.162186	2.567027	− 139.413	+ 1.243	− 120.980	− 10.611
	27								− 140.177	+ 1.160		
									− 138.836	+ 1.302		
24	61	62	238.532	252.593	1.002807	0.999857	2.164303	2.567202	+ 71.506	+ 3.909	+ 253.596	+ 8.455
25	71	72	74.715	300.924	0.998056	1.000572	2.163072	2.568452	− 104.065	− 3.129	+ 370.021	+ 0.408
29	100	101	173.134	143.216	1.003527	0.999225	2.166137	2.568672	− 84.988	+ 3.629	− 336.642	+ 3.841
31	122	123	223.600	244.953	1.003183	0.999696	2.166326	2.568961	+ 38.701	+ 4.295	+ 221.265	+ 9.137
Sept. 1	132	133	56.924	292.624	0.997407	1.000364	2.164515	2.569933	− 70.246	− 3.889	+ 369.735	+ 1.799
5	155	156	157.200	135.310	1.003231	0.999490	2.166821	2.569634	− 113.195	+ 2.795	− 353.710	+ 2.537
7	165	166	194.128	233.667	1.003716	0.999431	2.166852	2.569070	− 33.669	+ 4.270	+ 164.171	+ 9.884
10	187	188	79.002	23.037	0.999009	1.000806	2.164940	2.570785	− 115.224	− 2.681	+ 25.663	− 10.530
13	205	206	333.260	175.071	0.996684	0.999219	2.163263	2.567999	+ 116.819	− 2.547	− 193.365	+ 8.517
14	218	219	169.946	223.731	1.003677	0.999241	2.165599	2.567656	− 87.145	+ 3.441	+ 109.644	+ 10.153
18	240	241	267.443	65.849	1.000045	1.000648	2.162672	2.568149	+ 127.789	+ 2.025	− 237.226	− 8.358
19	250	251	105.808	114.664	1.001195	0.999877	2.162944	2.566809	− 143.301	− 0.747	− 368.737	− 0.941
20	270	271	309.614	165.292	0.997421	0.999160	2.160851	2.565421	+ 140.590	− 0.989	− 238.403	+ 7.252
21	291	292	157.955	216.798	1.003564	0.999028	2.162728	2.564685	− 107.480	+ 2.803	+ 71.303	+ 10.044
22	301	302	358.284	266.439	0.996273	0.999588	2.159510	2.564786	+ 65.983	− 3.669	+ 320.953	+ 5.518
29	340	341	339.917	257.997	0.996325	0.999496	2.154948	2.560136	+ 100.632	− 2.827	+ 291.212	+ 6.375
Oct. 4	354	355	279.860	150.085	0.998471	0.999204	2.151506	2.555940	+ 137.331	+ 0.979	− 291.414	+ 4.965
6	364	365	327.555	251.023	0.996433	0.999037	2.149057	2.553991	+ 117.478	− 2.114	+ 261.930	+ 6.866
9	366	367	238.516	46.938	1.000900	1.000875	2.147605	2.552998	+ 80.098	+ 3.124	− 140.658	− 8.724
10	380	381	82.561	97.136	1.000721	1.000098	2.146913	2.551178	− 120.077	− 1.962	− 341.058	− 3.256
13	388	389	334.218	248.709	0.996276	0.999430	2.141854	2.546990	+ 105.005	− 2.399	+ 250.250	+ 6.772
14	405	406	180.608	299.808	1.003477	1.000256	2.143415	2.546845	− 53.874	+ 3.400	+ 350.038	− 0.319
16	429	430	230.456	41.036	1.001083	1.000843	2.140131	2.545460	+ 63.092	+ 3.244	− 107.159	− 8.595
17	433	434	63.449	88.497	0.999811	1.000269	2.138972	2.543733	− 88.953	− 2.771	− 317.691	− 4.266
19		442	111.302	189.473	1.002705	0.999059	2.137665	2.539842	− 137.230	− 0.128	− 81.708	+ 8.204
	441								− 137.173	− 0.166	− 82.598	+ 8.198
									− 137.271	− 0.085	− 80.717	+ 8.210
Nov. 5	463	464	317.051	321.744	0.996241	1.000407	2.112913	2.519425	+ 118.203	− 1.318	+ 289.252	− 3.233
6	479	480	163.521	12.741	1.003491	1.001148	2.114328	2.518680	− 82.309	+ 2.496	+ 57.446	− 7.544
27	515	516	110.816	347.920	1.003634	1.001115	2.084781	2.488675	− 121.400	− 0.161	+ 180.281	− 5.370
30	523	524	1.467	138.625	0.997932	0.999457	2.078278	2.482918	+ 45.867	− 2.665	− 276.354	+ 2.365
Dec. 8	525	526	183.499	179.977	1.001596	0.998828	2.068319	2.471256	− 42.500	+ 2.634	− 124.221	+ 5.927
14	549	550	335.004	124.234	0.997121	0.999957	2.058616	2.464080	+ 88.333	− 1.761	− 287.764	+ 0.453

25 $\Delta t = -4^m.08$ and $+3^m.42$ from 26.
27 $\Delta t = -7^m.85$ and $+5^m.59$ from 28. Co-ordinates of III. taken from 28.
441 $\Delta t = -4^m.33$ and $+4^m.86$ from 442.

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TABLE XIII.—*continued.*

SATELLITES I.–IV. CO-ORDINATES.

Date.	Rot. No.		v_1.	v_4.	ρ_1.	ρ_4.	$\log \frac{a_1''\rho_1}{\Delta_1'}$.	$\log \frac{a_4''\rho_4}{\Delta_4'}$.	x_1''.	y_1''.	x_4''.	y_4''.
	p.	*s.*										
Aug. 17		1	254·436	22·097	1·002299	1·003889	2·160728	2·812668	+ 101″·282	+ 3″·204	+ 89″·850	− 16″·857
	2		255·547	22·214	1·002238	1·003880	2·160709	2·812669	+ 102·990	+ 3·153	+ 88·524	− 16·856
									+ 103·542	+ 3·134		
19	17	18	301·588	65·143	0·999272	0·998849	2·160763	2·811366	+ 144·729	− 0·124	− 374·546	− 12·432
20	37	38	148·786	87·135	1·002338	0·996369	2·162460	2·810433	− 127·024	+ 2·164	− 544·735	− 7·244
24	59	60	237·011	173·951	1·002870	0·993656	2·164325	2·809171	+ 68·106	+ 3·967	− 371·942	+ 14·657
25	82	83	84·199	196·090	0·998609	0·995304	2·163290	2·810045	− 119·483	+ 2·564	− 145·099	+ 16·557
27	85	84	118·144	238·044	1·000859	1·000100	2·164714	2·812879	− 146·111	− 0·068	+ 317·236	+ 13·457
29	92	93	165·684	281·033	1·003331	1·004956	2·166069	2·816274	− 99·755	+ 3·259	+ 624·601	+ 3·001
31	124	125	224·863	324·949	1·003139	1·007168	2·166309	2·818653	+ 41·810	+ 4·270	+ 588·680	− 9·396
Sept. 1	130	131	55·240	344·794	0·997334	1·006833	2·164488	2·819023	− 66·486	− 3·945	+ 452·794	− 13·588
5	150	151	150·479	70·685	1·002985	0·998199	2·166743	2·815523	− 123·353	+ 2·377	− 446·303	− 10·499
	161		190·526	113·476	1·003740	0·993910	2·166868	2·812672	− 42·588	+ 4·197	− 648·124	+ 1·054
10	195	196	86·477	179·613	0·999488	0·993931	2·165111	2·811016	− 126·007	− 2·215	− 297·544	+ 15·218
13	213	214	340·498	245·129	0·996483	1·001091	2·163196	2·813610	+ 104·905	− 2·961	+ 405·615	+ 11·101
14	226	227	177·097	265·853	1·003740	1·003608	2·165595	2·814798	− 71·810	+ 3·735	+ 561·827	+ 6·207
19	258	259	113·335	12·230	1·001650	1·004883	2·163084	2·815489	− 145·435	− 0·198	+ 153·834	− 15·813
20	266	267	306·361	32·609	0·997580	1·002721	2·160911	2·814130	+ 142·357	− 0·756	− 77·986	− 15·232
21	295	296	161·199	55·270	1·003624	0·999948	2·162733	2·812227	− 101·760	+ 2·974	− 321·311	− 12·338
22	309	310	4·406	76·871	0·996314	0·997332	2·159526	2·810222	+ 51·905	− 3·852	− 504·827	− 7·821
Oct. 4	344	345	271·498	334·847	0·998991	1·006971	2·151737	2·805869	+ 130·833	+ 1·517	+ 485·241	− 11·219
6	360	361	322·107	17·828	0·996554	1·004324	2·149111	2·803357	+ 124·359	− 1·800	+ 68·539	− 14·503
9	376	377	246·896	86·148	1·000359	0·996476	2·147356	2·796179	+ 96·011	+ 2·776	− 554·251	− 4·434
13	397	398	342·613	173·646	0·996265	0·993641	2·141835	2·789039	+ 90·656	− 2·783	− 305·809	+ 12·824
14		404	178·599	194·679	1·003523	0·995181	2·143450	2·788504	− 58·345	+ 3·347	− 93·076	+ 13·606
	403								− 59·928	+ 3·327		
									− 56·861	+ 3·365		
16	425	426	226·515	238·158	1·001325	1·000102	2·140249	2·788616	+ 54·514	+ 3·351	+ 351·567	+ 9·628
17	435	436	65·426	259·094	0·999941	1·002647	2·139011	2·788938	− 92·536	− 2·690	+ 509·208	+ 5·567
19	449	450	119·234	302·549	1·003042	1·006507	2·137724	2·789154	− 136·532	+ 0·369	+ 607·049	− 4·410
Nov. 5	473	474	330·967	308·394	0·996263	1·006931	2·112885	2·767365	+ 101·892	− 1·987	+ 563·608	− 5·295
6	485	486	169·712	329·192	1·003322	1·007220	2·114196	2·766525	− 70·939	+ 2·702	+ 470·003	− 8·751
22		512	182·602	314·251	1·002282	1·006918	2·091009	2·743359	− 43·737	+ 2·789	+ 517·457	− 5·757
27	513	514	109·156	59·919	1·003601	0·999483	2·084787	2·733585	− 121·171	− 0·246	− 319·692	− 7·318
Dec. 8	529	530	186·890	298·386	1·001391	1·006270	2·068202	2·720306	− 35·970	+ 2·689	+ 524·144	− 2·505
14	547	548	332·779	67·073	0·997020	0·998652	2·058580	2·709638	+ 91·081	− 1·674	− 339·935	− 6·274

2 $\Delta t = -1^m·10$ and $+1^m·10$ from the mean. Co-ordinates of I. computed for separate epochs, those of IV. for mean epoch.
84, 85 Co-ordinates computed for epoch of observation in position-angle (85).
403 $\Delta t = -5^m·10$ and $+4^m·75$ from 404. Co-ordinates of IV. computed for epoch of 404.

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TABLE XIII.—*continued.*

SATELLITES II.–III. CO-ORDINATES.

Date.	Rot. No. p.	Rot. No. s.	v_2.	v_3.	ρ_2.	ρ_3.	$\log \frac{a_2''\rho_2}{\Delta_2'}$.	$\log \frac{a_3''\rho_3}{\Delta_3'}$.	x_2''.	y_2''.	x_3''.	y_3''.
Aug. 17	11	12	325°·979	263°·761	1·005141	0·999996	2·364108	2·564004	+ 207″·957	− 1″·152	+ 295″·411	+ 7″·077
19	21	22	164·205	2·732	0·992520	1·001074	2·359174	2·566910	− 163·368	+ 3·051	+ 167·919	− 9·426
20	35	36	264·431	51·620	0·996023	1·000550	2·361265	2·567022	+ 187·596	+ 4·937	− 137·796	− 10·459
24	51	52	304·364	250·314	1·002779	0·999820	2·366268	2·567149	+ 231·468	+ 1·170	+ 242·671	+ 8·725
25	73	74	45·544	301·273	1·007878	1·000578	2·369267	2·568463	− 66·328	− 6·427	+ 369·952	+ 0·341
29	104	105	94·055	144·145	1·001369	0·999219	2·367342	2·568662	− 211·997	− 4·101	− 334·076	+ 4·008
31		109	291·465	241·255	1·001576	0·999650	2·367539	2·568900	+ 231·430	+ 2·434	+ 201·542	+ 9·495
	108								+ 231·244	+ 2·480	+ 200·540	+ 9·515
									+ 231·613	+ 2·386	+ 202·578	+ 9·475
Sept. 1	139	140	37·281	294·528	1·008131	1·000390	2·371065	2·569975	− 37·215	− 6·309	+ 370·763	+ 1·443
5	153	154	80·866	134·929	1·002638	0·999494	2·368735	2·569641	− 187·219	− 4·971	− 354·458	+ 2·468
7	171	172	283·832	234·807	1·001177	0·999439	2·367617	2·569078	+ 226·647	+ 3·054	+ 170·776	+ 9·802
8	174	175	18·292	282·364	1·008900	1·000381	2·371473	2·569895	+ 36·641	− 5·643	+ 358·972	+ 3·459
10		186	221·417	22·628	0·992624	1·000808	2·363185	2·570787	+ 57·495	+ 6·070	+ 28·320	− 10·520
	185								+ 56·516	+ 6·078	+ 29·124	− 10·518
									+ 58·856	+ 6·059	+ 27·202	− 10·523
13		202	165·506	174·011	0·991282	0·999219	2·362047	2·568012	− 151·275	+ 3·233	− 199·187	+ 8·402
	201								− 152·280	+ 3·207	− 200·081	+ 8·385
									− 150·393	+ 3·255	− 198·407	+ 8·417
14		217	265·981	223·316	0·999129	0·999242	2·365313	2·567657	+ 199·832	+ 4·384	+ 107·082	+ 10·170
	216								+ 199·103	+ 4·420	+ 106·005	+ 10·180
									+ 200·473	+ 4·351	+ 108·039	+ 10·162
18	242	243	315·104	66·377	1·006601	1·000641	2·367494	2·568136	+ 220·178	− 0·276	− 239·847	− 8·303
19	252	253	51·908	115·183	1·005338	0·999869	2·366799	2·566794	− 102·259	− 5·840	− 368·779	− 0·849
20	278	279	155·946	166·646	0·991507	0·999148	2·359555	2·565392	− 174·711	+ 2·327	− 231·690	+ 7·422
21	283	284	255·295	215·359	0·998240	0·999021	2·362010	2·564694	+ 175·516	+ 4·870	+ 62·218	+ 10·071
22	303	304	357·605	266·758	1·009136	0·999592	2·366922	2·564788	+ 108·811	− 4·172	+ 321·943	+ 5·472
23	320	321	97·356	316·745	0·997871	1·000478	2·361208	2·565267	− 218·472	− 3·281	+ 342·206	− 2·986
Oct. 4	342	343	127·831	147·551	0·992904	0·999238	2·350778	2·556027	− 217·949	− 0·349	− 300·525	+ 4·605
9	378	379	296·666	49·342	1·006245	1·000850	2·351820	2·552927	+ 224·514	+ 1·344	− 154·308	− 8·590
13	386	387	336·498	248·326	1·009129	0·999423	2·349241	2·546993	+ 163·329	− 2·185	+ 248·586	+ 6·811
14	409	410	80·836	300·684	0·998057	1·000269	2·343249	2·546843	− 185·784	− 3·911	+ 349·391	− 0·458
15	417	418	180·653	349·444	0·991434	1·000875	2·338604	2·546544	− 84·062	+ 3·780	+ 196·247	− 6·980
16	421	422	281·755	39·284	1·004995	1·000854	2·343713	2·545511	+ 216·208	+ 2·489	− 96·882	− 8·655
19	443	444	225·421	190·000	0·996784	0·999057	2·336332	2·539827	+ 82·364	+ 4·877	− 78·601	+ 8·227
23	457	458	268·365	30·008	1·003819	1·000937	2·334766	2·537213	+ 196·576	+ 3·287	− 42·412	− 8·457
Nov. 5	467	468	142·899	323·105	0·990799	1·000436	2·311853	2·519414	− 177·305	+ 0·820	+ 285·358	− 3·397
6	477	478	243·356	12·362	1·001551	1·001144	2·315065	2·518689	+ 134·412	+ 4·117	+ 59·595	− 7·530
8	489	490	84·256	112·843	0·995027	1·000201	2·309961	2·514800	− 179·351	− 3·231	− 327·190	− 0·618
22	504	505	61·956	96·566	0·996745	1·000425	2·290902	2·495163	− 121·851	− 3·882	− 299·367	− 2·651
Dec. 8	537	538	248·062	182·905	1·005658	0·998801	2·271623	2·471153	+ 128·095	+ 3·647	− 110·338	+ 6·091

108 $\Delta t = -5^m\cdot28$ and $+5^m\cdot47$ from 109.
185 $\Delta t = -3^m\cdot55$ and $+4^m\cdot94$ from 186.
201 $\Delta t = -4^m\cdot71$ and $+4^m\cdot11$ from 202.
216 $\Delta t = -4^m\cdot99$ and $+4^m\cdot44$ from 217.

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TABLE XIII.—*continued.*

SATELLITES II.–IV. CO-ORDINATES.

Date.	Rot. No. p.	Rot. No. s.	v_2.	v_4.	ρ_2.	ρ_4.	$\log \frac{a_2''\rho_2}{\Delta_2'}$.	$\log \frac{a_4''\rho_4}{\Delta_4'}$.	x_2''.	y_2''.	x_4''.	y_4''.
Aug. 17	5	6	322°·298	22°·705	1·004647	1·003827	2·363844	2·812661	+ 213″·898	− 0″·742	+ 82″·984	− 16″·862
19	23	24	165·680	66·180	0·992390	0·998722	2·359118	2·811322	− 159·173	+ 3·192	− 384·059	− 12·224
20	29	30	260·330	86·099	0·995444	0·996475	2·360965	2·810475	+ 177·495	+ 5·208	− 538·373	− 7·517
24	57	58	307·504	173·807	1·003254	0·993647	2·366511	2·809166	+ 230·103	+ 0·823	− 373·268	+ 14·636
25	77	78	47·178	195·586	1·007743	0·995256	2·369210	2·810019	− 72·699	− 6·421	− 150·651	+ 16·539
27	89		250·280	238·699	0·994770	1·000182	2·363389	2·812931	+ 152·852	+ 5·689	+ 323·774	+ 13·343
		88							+ 152·145	+ 5·709	+ 323·288	+ 13·351
29	94	95	89·429	281·236	1·002098	1·004970	2·367682	2·816287	− 203·661	− 4·483	+ 625·319	+ 2·944
31	114	115	294·779	323·951	1·002095	1·007163	2·367798	2·818623	+ 232·789	+ 2·094	+ 593·722	− 9·150
Sept. 1	137	138	36·514	345·611	1·008184	1·006799	2·371088	2·819029	− 34·103	− 6·299	+ 445·877	− 13·729
5	157	158	82·360	71·562	1·002407	0·998087	2·368626	2·815460	− 190·761	− 4·869	− 453·541	− 10·299
7	167	168	282·229	114·013	1·000920	0·993873	2·367493	2·812641	+ 224·964	+ 3·204	− 648·467	+ 1·208
8	176	177	19·248	135·214	1·008861	0·992900	2·371457	2·811633	+ 32·750	− 5·684	− 616·496	+ 7·030
10	191	192	224·395	179·267	0·992920	0·993910	2·363315	2·811013	+ 69·060	+ 6·072	− 301·027	+ 15·184
13	211	212	170·578	244·973	0·991092	1·001073	2·361924	2·813601	− 135·286	+ 3·665	+ 404·213	+ 11·134
14	222	223	268·759	265·505	0·999569	1·003571	2·365516	2·814780	+ 205·402	+ 4·180	+ 559·760	+ 6·298
		230	272·207	266·233	1·000127	1·003650	2·365774	2·814819	+ 211·652	+ 3·914	+ 564·063	+ 6·107
19	260	261	55·595	12·443	1·004841	1·004867	2·366553	2·815479	− 115·436	− 5·763	+ 151·451	− 15·817
20	276	277	155·286	33·389	0·991545	1·002632	2·359580	2·814073	− 176·412	+ 2·266	− 87·015	− 15·171
21	287	288	256·737	54·614	0·998468	1·000031	2·362110	2·812286	+ 179·252	+ 4·791	− 314·843	− 12·451
22	311	312	0·552	77·003	1·009106	0·997319	2·366905	2·810210	+ 98·073	− 4·377	− 505·744	− 7·789
23	322	323	98·330	98·160	0·997716	0·995126	2·361128	2·808264	− 219·601	− 3·198	− 614·321	− 2·351
Oct. 4	352	353	132·360	335·586	0·992474	1·006955	2·350515	2·805845	− 212·961	+ 0·074	+ 479·780	− 11·342
6	358	359	332·923	17·585	1·008921	1·004348	2·356321	2·803376	+ 176·774	− 1·969	+ 71·232	− 14·509
9	374	375	295·305	85·979	1·006086	0·996494	2·351753	2·796198	+ 224·688	+ 1·464	− 553·507	− 4·469
13	395	396	339·952	173·346	1·009149	0·993625	2·349213	2·789049	+ 153·834	− 2·462	− 308·639	+ 12·800
14		402	76·967	194·446	0·998664	0·995157	2·343576	2·788506	− 177·487	− 4·123	− 95·546	+ 13·608
	401								− 176·588	− 4·155		
									− 178·330	− 4·093		
15	415	416	179·620	216·149	0·991375	0·997426	2·338592	2·788403	− 87·677	+ 3·720	+ 136·307	+ 12·527
16	419	420	280·993	237·661	1·004893	1·000044	2·343671	2·788610	+ 215·582	+ 2·545	+ 347·157	+ 9·710
19	453	454	230·129	302·922	0·997499	1·006527	2·336605	2·789151	+ 98·630	+ 4·836	+ 606·379	− 4·492
23	455	456	267·503	26·445	1·003692	1·003620	2·334715	2·784156	+ 195·179	+ 3·338	− 37·240	− 12·678
Nov. 5	471	472	145·794	308·122	0·990808	1·006921	2·311800	2·767372	− 171·856	+ 1·038	+ 564·360	− 5·244
8	491	492	85·125	11·172	0·994911	1·004981	2·309892	2·763264	− 180·781	− 3·187	+ 116·780	− 11·818
22	510	511	65·192	314·116	0·996164	1·006914	2·290588	2·743364	− 130·179	− 3·801	+ 517·925	− 5·735
Dec. 8	535	536	247·283	299·091	1·005561	1·006315	2·271589	2·720293	+ 126·224	+ 3·667	+ 523·685	− 2·628
14	543	544	131·686	66·638	0·991350	0·998706	2·257561	2·709695	− 173·786	− 0·276	− 337 057	− 6·335

88 $\Delta t = -3^m·30$ from 89.

401 $\Delta t = -5^m·54$ and $+5^m·24$ from 402. Co-ordinates of IV. taken from 402.

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TABLE XIII.—*continued.*

SATELLITES III.–IV. CO-ORDINATES.

Date.	Rot. No. p.	Rot. No. s.	v_3.	v_4.	ρ_3.	ρ_4.	$\log \frac{a_3''\rho_3}{\Delta_3'}$.	$\log \frac{a_4''\rho_4}{\Delta_4}$.	x_3''.	y_3''.	x_4''.	y_4''.
Aug. 17	7	8	262°.495	22°.951	0·999972	1·003793	2·563965	2·812653	+ 290″.508	+ 7″.258	+ 80″.199	− 16″.860
19	15	16	0·592	64·949	1·001080	0·998875	2·566903	2·811376	+ 180·092	− 9·209	− 372·749	− 12·473
20	39	40	52·458	87·333	1·000535	0·996347	2·567019	2·810424	− 142·798	− 10·407	− 545·930	− 7·190
21	45	46	99·950	107·888	0·999665	0·994505	2·566647	2·809631	− 347·258	− 4·395	− 631·726	− 1·378
24	53	54	250·845	173·356	0·999828	0·993623	2·567161	2·809156	+ 245·251	+ 8·662	− 377·402	+ 14·573
25	69	70	300·512	194·899	1·000566	0·995193	2·568441	2·809983	+ 370·089	+ 0·487	− 158·195	+ 16·515
27	87	86	41·263	238·370	1·000597	1·000140	2·569835	2·812905	− 79·985	− 11·386	+ 320·492	+ 13·398
29	96	97	142·406	281·477	0·999233	1·004993	2·568683	2·816305	− 338·807	+ 3·697	+ 626·166	+ 2·876
31	112	113	242·389	323·733	0·999662	1·007159	2·568918	2·818615	+ 207·685	+ 9·390	+ 594·802	− 9·095
Sept. 1	126	127	291·357	344·433	1·000348	1·006846	2·569907	2·819023	+ 368·829	+ 2·037	+ 455·820	− 13·526
5	146	147	132·381	70·146	0·999521	0·998264	2·569685	2·815561	− 359·061	+ 1·999	− 441·788	− 10·620
7	169	170	234·397	114·182	0·999435	0·993864	2·569075	2·812632	+ 168·412	+ 9·831	− 648·564	+ 1·256
8		178	283·319	135·419	1·000394	0·992898	2·569913	2·811626	+ 360·531	+ 3·290	− 615·764	+ 7·083
		181	284·189	135·797	1·000406	0·992887	2·569927	2·811611	+ 361·864	+ 3·135	− 614·387	+ 7·180
	182		284·487	135·927	1·000407	0·992882	2·569931	2·811605	+ 362·301	+ 3·082	− 613·917	+ 7·213
10		189	23·295	178·925	1·000806	0·993885	2·570784	2·811009	+ 23·980	− 10·537	− 304·463	+ 15·151
13	209	210	176·130	244·821	0·999218	1·001052	2·567985	2·813592	− 187·481	+ 8·628	+ 402·841	+ 11·166
14	224	225	225·120	265·685	0·999252	1·003592	2·567660	2·814790	+ 118·192	+ 10·092	+ 560·834	+ 6·253
		229	226·149	266·122	0·999259	1·003638	2·567660	2·814814	+ 124·474	+ 10·043	+ 563·410	+ 6·135
18	244	245	66·974	351·216	1·000633	1·006472	2·568124	2·816408	− 242·769	− 8·237	+ 373·438	− 14·274
19	256	257	116·037	12·020	0·999854	1·004903	2·566768	2·815500	− 368·778	− 0·696	+ 156·185	− 15·806
20	274	275	166·006	33·254	0·999155	1·002644	2·565408	2·814081	− 234·883	+ 7·342	− 85·488	− 15·183
21	289	290	216·433	54·768	0·999027	1·000013	2·564687	2·812273	+ 69·008	+ 10·052	− 316·362	− 12·425
22	305	306	267·141	76·526	0·999597	0·997373	2·564790	2·810255	+ 323·121	+ 5·414	− 502·416	− 7·903
23	318	319	316·169	97·703	1·000469	0·995165	2·565262	2·808303	+ 343·535	− 2·889	− 612·831	− 2·474
		324	317·616	98·329	1·000490	0·995108	2·565273	2·808248	+ 340·129	− 3·133	− 614·860	− 2·307
29	336	337	256·813	228·005	0·999479	0·999039	2·560134	2·804396	+ 286·653	+ 6·523	+ 252·211	− 12·670
Oct. 4	350	351	149·400	335·430	0·999213	1·006959	2·555963	2·805831	− 293·931	+ 4·869	+ 480·939	− 11·316
6	356	357	248·567	17·355	0·999001	1·004371	2·553996	2·803396	+ 251·211	+ 7·133	+ 73·784	− 14·511
9	370	371	47·719	85·591	1·000867	0·996535	2·552976	2·796243	− 145·123	− 8·683	− 551·474	− 4·565
10	382	383	97·657	107·216	1·000085	0·994567	2·551156	2·793910	− 341·949	− 3·178	− 618·220	+ 0·706
13	393	394	249·519	173·105	0·999441	0·993612	2·546986	2·789059	+ 253·733	+ 6·688	− 310·888	+ 12·778
14	407	408	300·256	195·090	1·000263	0·995218	2·546843	2·788498	+ 349·717	− 0·389	− 88·705	+ 13·605
15	413	414	348·272	215·860	1·000868	0·997392	2·546559	2·788400	+ 202·196	− 6·866	+ 133·278	+ 12·553
16	431	432	41·402	238·733	1·000840	1·000174	2·545449	2·788624	− 109·293	− 8·581	+ 356·628	+ 9·532
17	437	438	89·421	259·280	1·000253	1·002668	2·543694	2·788940	− 319·982	− 4·142	+ 510·333	+ 5·529
19	451	452	191·906	302·741	0·999050	1·006518	2·539773	2·789154	− 67·313	+ 8·309	+ 606·707	− 4·452
23	459	460	30·692	26·921	1·000932	1·003570	2·537194	2·784107	− 46·489	− 8·451	− 42·269	− 12·651
Nov. 5	475	476	325·572	308·570	1·000486	1·006936	2·519393	2·767359	+ 277·888	− 3·700	+ 563·112	− 5·329
6	487	488	14·772	329·401	1·001163	1·007219	2·518635	2·766515	+ 45·883	− 7·609	+ 468·727	− 8·780
8	495	496	114·321	11·618	1·000169	1·004939	2·514728	2·763218	− 327·020	− 0·420	+ 112·354	− 11·819
9		499	163·291	32·433	0·999157	1·002745	2·512412	2·760901	− 207·065	+ 5·476	− 96·374	− 11·014
22	506	507	97·106	313·668	1·000408	1·006902	2·495135	2·743380	− 300·184	− 2·588	+ 519·457	− 5·662
27	517	518	348·258	60·242	1·001118	0·999443	2·488668	2·733543	+ 178·804	− 5·397	− 322·107	− 7·274
Dec. 8	533	534	182·083	298·919	0·998806	1·006303	2·471180	2·720296	− 114·265	+ 6·046	+ 523·805	− 2·598
14	539	540	121·451	66·114	1·000023	0·998773	2·464212	2·709762	− 289·651	+ 0·126	− 333·582	− 6·407

86, 87 Quantities computed for epoch of 87.

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From the co-ordinates the distances and position-angles were computed by the formulas:

$$(6) \qquad \begin{aligned} s_0 \sin (p-\mathrm{P})_0 &= x_2'' - x_1'' \\ s_0 \cos (p-\mathrm{P})_0 &= y_2'' - y_1'' \end{aligned}$$

where $(p-\mathrm{P})_0$ is the position-angle reckoned in the usual direction from the direction of the y''-axis, that is from the adopted position-angle of Jupiter's axis of rotation.

The position-angles so computed are those for the epochs of the observations in distance. They were reduced to the instants of the corresponding observations in position-angle by means of the differential coefficients $\frac{dp}{dt}$. In consequence of the symmetrical arrangement of the observations, the difference between these two epochs is seldom more than a minute, and it is sufficiently exact to take

$$\frac{dp}{dt} = n_1 \frac{dp}{dv_1} + n_2 \frac{dp}{dv_2},$$

where n_1 and n_2 are the mean motions in longitude.

The column "Δt" in Table XIV. contains the interval of time between the observations of distance and position-angle, in the sense $p - s$. The only exception is formed by the observations 84, 85 and 86, 87, which are mentioned in the footnotes. The differential coefficient $\frac{dp}{dt}$ is given in the next column.

In Table XIV. the column "computed s_0" contains the distance computed as here explained. In pairs in which satellite II. enters the corrections given in the column headed $\Delta s_0(\zeta_2, \eta_2)$ must be added to s_0. This correction has become necessary because the inequalities in the longitude of the second satellite, depending on Marth's arguments ζ_2 and η_2, had inadvertently been applied with the wrong sign. The longitudes v_2 as contained in Table XIII. have *not* been corrected: they require the corrections Δv_2 given below.

Date.	Δv_2.	Date.	Δv_2.	Date.	Δv_2.	Date.	Δv_2.
Aug. 17	−0°·023	Sept. 5	−0°·051	Sept. 22	−0°·073	Oct. 15	−0°·098
19	·025	7	·054	23	·074	16	·099
20	·027	8	·055	24	·075	19	·101
21	·029	10	·058	28	·082	23	·104
24	·033	13	·062	29	·084	Nov. 5	·112
25	·035	14	·063	Oct. 4	·088	6	·113
27	·038	18	·068	6	·089	8	·113
29	·041	19	·069	19	·092	9	·114
31	·044	20	·071	13	·096	22	·117
Sept. 1	−0°·045	21	−0°·072	14	−0°·097	Dec. 8	·117
						14	−0°·115

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The co-ordinates x_2'' and y_2'', as given in Table XIII., belong to the erroneous value of the longitude v_2. The correction Δs_0 to the distances s_0 was computed differentially by the formula

$$\Delta s_0 = \frac{\partial s}{\partial v_2} \Delta v_2.$$

The corrections to the position-angles were similarly computed and applied. As they are very small, they are not given separately in Table XIV., but are combined with the other corrections, which will be explained below.

When two satellites are close together and crossing each other at a short distance, so that the change in distance and position-angle is rapid and cannot be assumed proportional to the time, the co-ordinates x'' and y'' were computed separately for the epochs of each pointing.

In this computation, however, the value of the factor $\frac{\alpha'' \rho}{\Delta'}$, which changes very slowly, was always taken for the mean epoch. The longitudes, taken from Marth's tables for the mean epoch, were reduced to the epochs of the separate pointings by means of corrections $\Delta v = n \Delta t$. The co-ordinates x'', y'', and the position-angles $p - P$ computed from them are thus not rigorously the true values for the separate epochs, but the mean of the separate position-angles will represent the true position-angle at the mean epoch with all necessary accuracy.

For the observations treated in this way the values of the co-ordinates for the separate epochs are given in Table XIII., and the difference in time of the separate epochs from the mean is given in the footnotes to this Table. In Table XIV. these differences of time are given in the column Δt, *printed in italics*, and the separate values of s_0 and $(p - P)_0$ are also given.

Throughout the computation of tabular distances and position-angles 6-figure logarithms were used. All the computations were carefully examined by Mr. A. COCHRANE, one of the most experienced computers of the Cape Observatory. Moreover, the longitudes, radii vectores, and rectangular co-ordinates were checked by an independent computation of the differential coefficients $\frac{dv}{dt}$, $\frac{ds}{dt}$, $\frac{dx''}{dt}$, $\frac{dy''}{dt}$, and of the differences of these quantities in different observations on the same date by the formulas

$$\Delta v = \frac{dv}{dt} \Delta t \ldots . \text{ etc.}$$

The tabular distances and position-angles were checked by recomputing from them the differences of the rectangular co-ordinates ($x_2'' - x_1''$ and $y_2'' - y_1''$).

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TABLE XIV.

SATELLITES I.–II. COMPUTED DISTANCES AND POSITION-ANGLES.

Rot. No. p.	Rot. No. s.	Δt.	$\frac{s}{1000}\cdot\frac{dp}{dt}$.	Computed s_0.	Δs_0 (ζ_2, η_2).	Corrections s.	Computed $(p-P)_0$.	Computed p_0.	Corrections p.	$\rho_1 \frac{\partial s}{\partial \rho_1}$.	$\rho_2 \frac{\partial s}{\partial \rho_2}$.	$s \sin 1'' \rho_1 \frac{\partial p}{\partial \rho_1}$.
		m		″	″	″	° ′ ″	° ′ ″	′ ″			
	3			109·731	+ 0·032	0·000	91 53	66 56		− 106·2	+ 215·9	
4				106·5			91 56 58	66 59 5	− 27			+ 6·7
19	20	− 1·77	+ 4·2	311·993	+ ·068	− ·024	270 36 20	245 39 11	0	+ 144·1	+ 167·8	− 1·1
33	34	− 1·23	+ 3·0	316·329	− ·064	− ·012	89 26 20	64 29 58	− 4	+ 131·2	+ 185·2	+ 3·2
43	44	·00		4·651	+ ·094	− ·018	111 20 46	86 25 11	− 25 49	− 104·9	+ 109·6	+ 38·0
49	50	+ 0·19	− 0·9	4·609	+ ·098	− ·017	109 30 30	84 34 17	− 26 2	− 92·4	+ 97·0	+ 29·2
63	64	+ 0·51	+ 2·0	154·259	+ ·024	+ ·005	91 12 50	66 19 18	− 20	− 74·5	+ 228·8	+ 5·4
79	80	+ 1·24	+ 2·7	39·191	+ ·136	− ·022	95 21 26	70 29 50	− 37	+ 113·9	− 74·8	− 13·5
	81			38·697	+ ·134	− ·022	95 33	70 40		+ 116·2	− 77·6	
102	103	+ 0·95	+ 5·4	128·642	− ·072	+ ·030	266 29 0	241 39 30	+ 11	− 81·4	+ 210·0	− 8·7
120	121	− 0·63	+ 1·9	197·312	·000	+ ·007	90 45 15	65 55 32	− 14	− 36·0	+ 233·4	+ 4·8
141	142	+ 0·48	+ 1·6	49·849	+ ·182	− ·018	93 14 27	68 26 45	− 21	+ 89·4	− 39·6	− 8·6
	145			39·782	− ·140	+ ·039	259 30	234 45		− 129·7	+ 169·5	
144		− *7·03*					258 50 34	234 42 10	− 1 33			− 26·1
		+ *6·74*					260 3 29					
163	164	+ 1·06	+ 2·1	261·963	− ·064	+ ·011	90 11 18	65 28 10	− 14	+ 39·1	+ 222·9	+ 4·1
193	194	− 0·47	+ 6·0	196·046	− ·222	+ ·021	87 32 49	62 51 43	− 44	+ 123·6	+ 72·3	+ 3·0
203	204	+ 0·70	+ 4·9	268·030	+ ·190	− ·037	271 13 39	246 35 26	+ 2	+ 120·0	+ 148·0	− 0·2
220	221	− 1·89	+ 2·2	286·612	− ·122	+ ·014	89 51 19	65 13 28	− 6	+ 83·1	+ 203·5	+ 3·7
234	235	− 0·28	+ 1·0	103·915	+ ·074	− ·011	91 14 42	66 40 24	− 34	− 120·5	+ 224·6	+ 5·0
	249			48·710	+ ·256	− ·034	95 49	71 16		+ 141·1	− 92·4	
248		− *6·06*					95 38 15	71 15 31	− 1 16			− 15·3
		+ *5·80*					95 59 34					
268	269	− 0·48	+ 3·9	324·798	+ ·170	− ·022	270 30 34	245 57 55	+ 1	+ 141·4	+ 183·4	− 0·4
293	294	− 0·27	+ 2·3	289·416	− ·172	+ ·017	89 39 20	65 7 35	− 3	+ 104·1	+ 185·3	+ 3·5
299	300	− 0·80	− 0·1	44·032	+ ·256	− ·001	90 35 36	66 4 45	− 52	− 70·8	+ 114·8	− 2·9
328	329	− *8·10*		9·493	− ·068	+ ·010	344 16 5	349 4 41	− 79 20	+ 12·1	− 2·0	+ 30·9
		− *1·31*		9·281			10 18 35					
		+ *1·63*		9·772			20 56 13					
		+ *7·78*		11·691			38 44 46					
332	333	− 0·25	+ 2·7	280·280	− ·244	− ·003	89 15 18	64 49 13	− 4	+ 133·5	+ 146·8	+ 3·2
334	335	+ 0·54	− 0·4	44·020	+ ·242	+ ·003	90 28 40	66 3 19	− 56	− 111·6	+ 155·6	− 1·6
346	347	+ 0·04	+ 3·3	348·601	+ ·092	− ·001	269 44 38	245 23 2	+ 4	+ 132·6	+ 216·0	− 0·8
368	369	− 0·13	+ 0·7	141·300	− ·006	+ ·004	90 33 18	66 15 6	− 31	− 83·4	+ 224·7	+ 3·9
384	385	·00		55·602	+ ·252	+ ·009	89 52 24	65 36 32	− 52	− 109·4	+ 165·0	− 2·5
399	400	+ 0·02	+ 6·3	108·845	− ·224	+ ·011	266 5 58	241 50 39	− 17	− 66·5	+ 175·4	− 7·8
423	424	− 0·15	+ 0·7	165·822	− ·072	+ ·010	90 19 53	66 5 32	− 29	− 51·0	+ 216·8	+ 3·7
445	446	+ 0·04	+ 3·0	222·665	− ·352	+ ·016	88 46 30	64 33 30	− 7	+ 137·2	+ 85·4	+ 3·0
465	466	− 0·17	+ 3·0	296·021	+ ·192	+ ·028	270 24 42	246 14 49	− 1	+ 115·9	+ 180·1	+ 0·6
481	482	− 1·15	+ 1·4	218·126	− ·300	+ ·014	89 36 3	65 25 53	− 8	+ 79·4	+ 138·7	+ 3·1
497	498	− *2·03*		7·089	− ·220	− ·012	27 39 37	10 0 21	− 138 54	+ 36·8	− 29·1	+ 49·1
		+ *2·03*		8·274			40 41 48					
502	503	− 0·45	+ 10·1	57·534	− ·314	− ·008	263 32 3	239 16 4	− 2 5	− 61·6	+ 119·1	− 9·6
521	522	+ 0·65	+ 3·2	198·802	+ ·240	+ ·021	271 6 32	246 47 29	− 10	+ 50·3	+ 148·5	+ 1·6
527	528	+ 0·12	+ 0·7	155·081	− ·300	+ ·022	89 35 28	65 10 17	− 12	+ 39·5	+ 115·6	+ 2·9

329 Mean s_0: 10″·059.
498 Mean s_0: 7″·682.

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TABLE XIV.—*continued.*

SATELLITES I.–III. COMPUTED DISTANCES AND POSITION-ANGLES.

Rot. No.		Δt.	$\frac{s}{1000}\frac{dp}{dt}$.	Computed s_0.	Corrections s.	Computed $(p-P)_0$.	Computed p_0.	Corrections p.	$\rho_1\frac{\partial s}{\partial \rho_1}$.	$\rho_3\frac{\partial s}{\partial \rho_3}$.	$s \sin 1'' \rho_1 \frac{\partial p}{\partial \rho_1}$.
p.	*s.*										
		m									
9	10	+0·52	−1·2	174″·754	+0″·010	88° 29′ 58″	63° 32′ 1″	− ′ 22″	−118·1	+292·4	− 0·5
	26			22·230	− ·035	112 34	87 37		−130·5	+152·7	
25		*−4·08*				112 9 32	87 35 27	+ 1 51			+ 53·1
		+3·42				112 54 52					
	28			21·916	− ·038	122 45	97 48		+117·9	− 96·0	
27		*−7·85*				121 30 55	97 40 36	+ 4 34			− 74·4
		+5·59				123 42 36					
61	62	−0·18	−1·0	182·147	+ ·017	88 34 12	63 40 34	− 16	− 71·6	+253·7	+ 2·1
71	72	−0·66	+3·4	474·098	+ ·009	89 34 21	64 41 16	− 15	+104·1	+370·0	− 2·3
100	101	+0·30	0·0	251·654	− ·006	270 2 54	245 12 44	+ 28	− 85·0	+336·6	− 3·5
122	123	+0·06	−0·8	182·628	+ ·008	88 28 51	63 40 9	− 11	− 38·8	+221·4	+ 3·3
132	133	−1·30	+3·4	440·018	+ ·026	89 15 34	64 27 28	− 16	+ 70·0	+369·7	− 3·0
155	156	−0·36	−0·4	240·515	+ ·002	269 56 19	245 11 28	+ 33	−113·2	+353·7	− 2·9
165	166	−0·55	+0·2	197·920	+ ·021	88 22 28	63 39 12	− 6	+ 33·5	+164·4	+ 5·3
187	188	+0·01	+2·0	141·105	− ·039	93 11 22	68 30 30	+ 10	+114·9	+ 26·2	− 9·1
205	206	−0·12	+5·5	310·381	− ·026	272 2 34	247 24 6	+ 12	+116·8	+193·5	− 1·7
218	219	−0·10	+1·0	196·903	+ ·023	88 2 48	63 25 11	+ 5	+ 87·0	+109·8	+ 6·4
240	241	−0·27	+4·7	365·163	+ ·054	268 22 14	243 47 56	− 47	+127·8	+237·3	+ 1·6
250	251	+0·08	−0·8	225·436	+ ·010	269 57 3	245 23 40	+ 39	−143·3	+368·7	+ 0·6
270	271	−0·33	+4·1	379·083	− ·008	271 14 44	246 42 8	+ 11	+140·6	+238·5	− 2·1
291	292	−0·64	+1·5	178·932	+ ·022	87 40 45	63 8 57	+ 9	+107·3	+ 71·6	+ 7·1
301	302	+0·10	+3·2	255·136	+ ·045	87 56 11	63 25 19	− 24	− 65·8	+320·9	− 6·1
340	341	−1·66	+3·3	190·802	+ ·038	87 14 8	62 48 23	− 27	−100·4	+291·2	− 7·7
354	355	−0·27	+3·0	428·764	+ ·017	270 31 58	246 10 20	+ 15	+136·4	+292·7	− 2·3
364	365	+0·34	+3·5	144·731	+ ·032	86 26 34	62 6 28	+ 1 44	−117·2	+261·8	− 9·4
366	367	−0·56	+6·8	221·074	+ ·050	266 55 40	242 37 10	+ 7	+ 80·2	+141·0	+ 1·2
380	381	−0·50	−0·7	220·985	+ ·020	269 39 52	245 22 20	+ 35	−120·1	+341·1	+ 1·3
388	389	−0·02	+4·2	145·534	+ ·031	86 23 13	62 7 20	− 25	−104·6	+250·2	− 9·0
405	406	−0·15	+2·5	403·929	+ ·029	90 31 39	66 16 18	+ 33	+ 53·9	+350·0	+ 2·9
429	430	−0·55	+8·4	170·662	+ ·043	266 1 20	241 46 33	− 8	+ 63·2	+107·5	+ 1·2
433	434	+0·01	−0·4	228·743	+ ·020	269 37 32	245 23 40	+ 10	− 89·0	+317·7	+ 2·2
	442			56·144	+ ·005	81 28	57 15		+135·7	− 79·6	
441		*−4·33*				81 17 14	57 15 17	+ 37			+ 20·3
		+4·86				81 39 22					
463	464	+0·38	−0·7	171·060	+ ·019	90 38 29	66 28 29	− 31	−118·2	+289·3	0·0
479	480	−0·36	+7·9	140·115	+ ·012	94 6 33	69 56 10	+ 4	+ 82·3	+ 57·8	− 3·4
515	516	−1·12	+2·6	301·726	+ ·045	90 59 21	66 41 49	− 8	+121·3	+180·3	− 2·3
523	524	+0·24	+2·5	322·260	+ ·052	270 53 40	246 34 29	− 1 50	+ 45·8	+276·4	+ 2·0
525	526	−0·09	−1·0	81·787	+ ·031	272 18 27	247 53 17	+ 2 31	− 42·6	+124·3	− 0·9
549	550	−0·85	+2·2	376·104	+ ·052	270 20 14	245 49 59	+ 6	+ 88·3	+287·8	+ 1·3

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TABLE XIV.—*continued.*

SATELLITES I.–IV. COMPUTED DISTANCES AND POSITION-ANGLES.

Rot. No.		Δt.	$\frac{s}{1000}\frac{dp}{dt}$.	Computed s_0.	Corrections s.	Computed $(p-P)_0$.	Computed p_0.	Corrections p.	$\rho_1 \frac{\partial s}{\partial \rho_1}$.	$\rho_4 \frac{\partial s}{\partial \rho_4}$.	$s \sin 1'' \rho_1 \frac{\partial p}{\partial \rho_1}$.
p.	s.										
		m		″	″	° ′ ″	° ′ ″	′ ″			
	1			23·090	+0·037	209 41	184 43		+ 52·9	− 29·8	
2		*−1·10*				215 52 3	191 25 39	+ 9 14			+ 81·3
		+1·10				216 55 0					
17	18	−0·40	+ 3·5	519·420	+ ·064	268 38 32	243 41 44	+ 15	+144·7	+374·7	+ 3·5
37	38	+0·87	0·0	417·816	+ ·057	268 42 35	243 46 25	+ 41	−127·0	+544·8	− 5·1
59	60	+0·03	+ 0·6	440·178	− ·025	271 23 30	246 29 51	+ 20	+ 67·9	+372·2	− 5·7
82	83	+0·24	+44·2	31·965	− ·040	306 44 22	281 56 54	+ 3 26	− 94·2	+126·1	+ 73·6
85	84	−0·27		463·503	+ ·052	88 19 41	63 28 5	− 9	+146·0	+317·5	+ 4·2
92	93	−0·14	+ 2·4	724·356	+ ·035	90 1 13	65 11 3	− 31	+ 99·8	+624·6	+ 3·3
124	125	−0·06	+ 2·3	547·041	− ·015	91 25 53	66 37 11	− 34	− 41·7	+588·8	+ 5·3
130	131	−0·33	+ 0·8	519·370	− ·019	91 3 50	66 15 52	− 23	+ 66·5	+453·0	− 5·1
150	151	−0·10	+ 1·4	323·206	+ ·088	267 43 1	242 58 9	+ 41	−123·1	+446·1	− 7·3
161				605·5		269 42 9	244 58 53	+ 46			− 4·4
195	196	−0·30	+ 5·3	172·420	− ·037	275 48 11	251 7 10	+ 37	−125·2	+297·6	+ 15·0
213	214	−0·27	+ 2·7	301·038	+ ·082	87 19 22	62 40 54	− 24	−104·7	+405·7	− 7·8
226	227	−0·31	+ 1·7	633·641	+ ·062	89 46 35	65 8 58	− 26	+ 71·8	+561·8	+ 4·0
258	259	−1·01	+ 3·8	299·676	− ·063	92 59 13	68 25 37	+ 7	+145·2	+154·4	− 7·8
266	267	+0·10	+ 3·7	220·818	+ ·088	266 14 28	241 41 57	+ 2	+142·1	+ 78·8	+ 10·1
295	296	−0·05	+ 4·8	220·084	+ ·098	266 0 38	241 28 54	+ 37	−101·3	+321·4	− 10·1
309	310	−0·98	+ 1·1	556·746	+ ·069	269 35 29	245 4 34	+ 34	+ 51·6	+505·3	+ 4·3
344	345	+0·45	+ 0·5	354·637	+ ·002	92 3 29	67 41 54	− 44	−130·6	+485·4	+ 6·2
360	361	−0·26	+ 1·8	57·247	+ ·055	257 10 46	232 50 24	+ 29	+120·9	− 63·6	+ 29·4
376	377	+0·15	+ 2·8	650·301	+ ·094	269 21 53	245 3 41	+ 40	+ 96·0	+554·3	− 1·7
397	398	−3·45	+ 4·7	396·771	+ ·013	272 15 16	247 58 43	+ 20	+ 90·5	+306·1	− 0·8
	404			36·214	− ·010	286 27	262 12		− 57·0	+ 93·1	
403		*−5·10*				287 13 42	262 15 13	+ 28			+ 13·3
		+4·75				285 47 24					
425	426	−0·04	− 1·0	297·119	+ ·054	88 47 22	64 33 2	− 31	− 54·6	+351·7	+ 2·2
435	436	+0·06	+ 2·9	601·801	+ ·089	89 12 50	64 58 58	− 34	+ 92·5	+509·3	− 1·4
449	450	−0·11	+ 2·6	743·596	+ ·079	90 22 6	66 9 5	− 40	+136·5	+607·1	− 0·5
473	474	−0·83	− 1·0	461·728	+ ·073	90 24 38	66 14 42	− 51	−101·9	+563·6	− 1·3
485	486	−2·18	+ 2·9	541·063	+ ·077	91 12 46	67 2 31	− 31	+ 71·0	+470·1	+ 1·2
	512			561·259	+ ·105	90 52	66 38		+ 43·7	+517·5	
513	514	−0·55	− 0·4	198·647	+ ·014	267 57 35	243 40 14	+ 27	−121·1	+319·8	− 4·1
529	530	+1·01	+ 1·5	560·136	+ ·104	90 31 52	66 6 44	− 35	+ 36·0	+524·3	+ 2·4
547	548	−0·29	+ 1·4	431·040	+ ·027	269 23 19	244 53 9	+ 10	+ 91·1	+340·0	+ 2·7

84, 85 Quantities computed for epoch of position-angle 85. $\frac{ds}{dt} = +0·152$. Correction $\Delta s = \Delta t \frac{ds}{dt} = -0''·041$ is included in computed s_0.

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TABLE XIV.—*continued.*

SATELLITES II.–III. COMPUTED DISTANCES AND POSITION-ANGLES.

Rot. No. p.	Rot. No. s.	Δt.	$\frac{s}{1000}\cdot\frac{dp}{dt}$.	Computed s_0.	Δs_0. (ζ_2, η_2).	Corrections s.	Computed $(p-P)_0$.	Computed p_0.	Corrections p.	$\rho_2 \frac{\partial s}{\partial \rho_2}$.	$\rho_3 \frac{\partial s}{\partial \rho_3}$.	$s \sin 1'' \rho_2 \frac{\partial p}{\partial \rho_2}$.
		m		″	″	″	° ′ ″	° ′ ″	′ ″			
11	12	− 3·13	+ 4·4	87·840	− 0·040	+ 0·011	84 37 28	59 36 59	− 33	− 206·9	+ 294·8	− 20·6
21	22	+ 0·21	+ 5·2	331·521	+ ·070	− ·063	92 9 25	67 12 43	0	+ 163·4	+ 168·2	− 3·1
35	36	− 1·90	+ 5·1	325·756	− ·062	+ ·038	267 17 28	242 20 49	− 4	+ 187·6	+ 138·1	+ 4·0
51	52	0·00		13·512	− ·012	+ ·007	56 0 19	31 6 40	− 15	− 192·6	+ 206·1	− 128·4
73	74	− 0·11	+ 2·2	436·332	− ·138	+ ·029	89 6 40	64 13 39	− 18	+ 66·4	+ 369·8	− 5·4
104	105	− 0·39	+ 3·0	122·348	+ ·068	− ·035	273 48 1	248 57 41	+ 37	− 211·6	+ 333·5	+ 18·2
	109			30·712	− ·022	− ·008	283 18	258 29		+ 224·6	− 194·0	
108		*− 5·28*					282 54 17	258 30 2	− 13			− 55·6
		+ 5·47					283 43 12					
139	140	− 0·58	+ 2·3	408·051	− ·184	+ ·040	88 54 41	64 6 41	− 19	+ 37·3	+ 370·7	− 5·6
153	154	− 0·21	+ 2·6	167·404	+ ·124	− ·036	272 32 49	247 47 53	+ 38	− 186·9	+ 354·3	+ 13·3
171	172	+ 0·16	+ 4·1	56·276	− ·058	− ·010	276 53 13	252 10 8	− 11	+ 225·7	− 168·3	− 30·2
174	175	− 0·14	+ 2·6	322·459	− ·224	+ ·049	88 22 57	63 40 28	− 23	− 36·4	+ 358·9	− 6·6
	186			33·562	− ·196	+ ·056	240 23	215 42		+ 53·0	− 19·4	
185		*+ 3·55*					238 47 23	215 53 24	− 7 15			+ 23·1
		+ 4·94					242 21 8					
	202			48·190	− ·186	+ ·011	276 9	251 31		− 150·7	+ 199·0	
201		*− 4·71*					276 10 57	251 31 8	+ 2 21			+ 13·0
		+ 4·11					276 8 11					
	217			92·930	− ·128	− ·015	273 34	248 57		+ 199·2	− 106·3	
216		*− 4·99*					273 32 25	248 56 32	− 7			− 16·8
		+ 4·44					273 35 50					
242	243	− 0·13	+ 2·6	460·095	+ ·092	+ ·053	269 0 1	244 25 45	+ 5	+ 220·2	+ 240·0	+ 4·1
252	253	− 0·44	+ 2·0	266·567	+ ·252	− ·025	271 4 22	246 30 55	+ 35	− 102·1	+ 368·7	+ 7·7
278	279	+ 0·09	− 0·6	57·206	− ·182	+ ·016	275 6 35	250 34 2	+ 1 52	− 174·2	+ 231·5	+ 13·2
283	284	− 0·33	+ 1·0	113·417	− ·186	− ·006	272 37 42	248 5 56	− 5	+ 174·9	− 61·6	− 13·0
303	304	− 0·31	+ 3·4	213·350	− ·262	+ ·047	87 24 33	62 53 35	− 33	− 108·5	+ 321·9	− 9·1
320	321	− 0·33	+ 2·4	560·678	− ·092	+ ·006	89 58 11	65 28 7	− 10	+ 218·5	+ 342·2	− 3·2
342	343	− 0·35	+ 0·4	82·724	− ·080	+ ·018	273 26 0	249 4 23	+ 1 15	− 217·4	+ 300·3	+ 13·4
378	379	+ 0·78	+ 2·7	378·952	+ ·018	+ ·056	268 29 52	244 11 44	− 5	+ 224·5	+ 154·5	+ 4·5
386	387	+ 0·53	+ 6·9	85·730	− ·254	+ ·021	83 58 36	59 43 27	− 1 10	− 162·3	+ 248·0	− 19·3
409	410	+ 0·16	+ 2·1	535·187	− ·200	+ ·041	89 37 49	65 22 30	− 13	+ 185·8	+ 349·4	− 2·7
417	418	+ 1·35	+ 4·9	280·516	+ ·344	+ ·047	92 11 54	67 57 29	− 21	+ 84·1	+ 196·4	+ 0·6
421	422	− 0·51	+ 3·4	313·288	− ·076	+ ·051	267 57 41	243 43 16	− 16	+ 216·2	+ 97·1	+ 5·0
443	444	− 0·51	+ 0·2	161·000	− ·354	+ ·011	271 11 32	246 58 30	+ 4	+ 82·2	+ 78·8	− 6·6
457	458	− 0·55	+ 4·2	239·276	− ·162	+ ·028	267 11 12	242 59 23	− 22	+ 196·5	+ 42·8	+ 6·3
467	468	− 1·28	+ 2·4	462·7			90 31 20	66 21 15	+ 4	+ 177·3	+ 285·4	− 0·8
477	478	+ 0·16	+ 12·5	75·718	− ·304	+ ·003	261 9 6	236 59 29	− 2 33	+ 133·4	− 57·7	+ 16·6
489	490	− 0·05	+ 0·7	147·862	+ ·192	+ ·025	271 0 45	246 50 31	+ 43	− 179·2	+ 327·1	+ 6·4
504	505	− 0·11	+ 0·7	177·520	+ ·312	+ ·023	270 23 50	246 9 10	+ 31	− 121·9	+ 299·4	+ 4·8
537	538	+ 0·26	+ 0·8	238·445	− ·278	+ ·056	270 35 14	246 10 4	− 3	+ 128·1	+ 110·4	− 5·1

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TABLE XIV.—*continued.*

SATELLITES II.–IV. COMPUTED DISTANCES AND POSITION-ANGLES.

Rot. No.		Δt.	$\frac{s}{1000}\frac{dp}{dt}$.	Computed s_0.	Δs_0. (ζ_2, η_2).	Corrections s.	Computed $(p-P)_0$.	Computed p_0.	Corrections p.	$\rho_2 \frac{\partial s}{\partial \rho_2}$.	$\rho_4 \frac{\partial s}{\partial \rho_4}$.	$s \sin 1'' \rho_2 \frac{\partial p}{\partial \rho_2}$.
p.	*s.*											
		m		"	"	"	° ′ ″	° ′ ″	′ ″			
5	6	− 0·62	+ 3·1	131·903	+ 0·034	+ 0·075	262 58 49	238 0 41	− 12	+ 212·2	− 80·3	+ 26·8
23	24	− 0·99	+ 4·0	225·413	− ·072	+ ·087	266 4 43	241 7 40	+ 37	− 158·7	+ 384·0	− 14·1
29	30	− 0·31	+ 2·8	715·982	− ·068	+ ·049	268 58 54	244 2 43	+ 21	+ 177·5	+ 538·4	− 2·1
57	58	− 0·76	+ 2·9	603·529	+ ·020	− ·020	271 18 41	246 24 58	+ 10	+ 230·0	+ 373·5	− 6·1
77	78	− 0·48	+ 25·5	81·263	+ ·130	− ·069	286 24 43	261 29 12	− 20	− 67·9	+ 149·2	+ 26·7
89				171·1			87 26 10	62 34 34	− 16			− 1·1
	88			171·314	+ ·114	+ ·050	87 26	62 35		− 152·4	+ 323·8	
94	95	+ 0·24	+ 2·4	829·012	− ·080	+ ·067	89 29 12	64 39 3	− 27	+ 203·7	+ 625·3	− 2·7
114	115	− 0·40	− 0·1	361·106	+ ·010	− ·021	91 47 4	66 58 22	− 45	− 232·6	+ 593·7	+ 9·3
137	138	+ 0·09	− 0·2	480·037	− ·184	− ·005	90 53 13	66 5 16	− 22	+ 34·0	+ 446·0	− 6·8
157	158	− 0·50	− 0·5	262·836	+ ·120	+ ·051	268 48 58	244 4 7	+ 50	− 190·8	+ 453·6	+ 1·0
167	168	+ 0·32	+ 2·3	873·434	− ·058	+ ·058	269 52 9	245 8 54	+ 28	+ 225·0	+ 648·5	− 2·7
176	177	− 1·19	+ 2·8	649·416	+ ·224	− ·062	271 7 18	246 24 45	+ 36	+ 33·1	+ 616·6	+ 5·1
191	192	+ 0·05	− 0·3	370·200	− ·224	− ·013	271 24 38	246 43 46	+ 23	+ 68·9	+ 301·3	− 7·8
211	212	+ 0·05	+ 1·6	539·552	+ ·202	+ ·042	89 12 25	64 33 59	− 11	+ 135·2	+ 404·4	+ 5·6
222	223	− 0·66	− 0·3	354·364	+ ·118	+ ·047	89 39 27	65 1 52	− 42	− 205·4	+ 559·8	+ 3·0
	230			352·417	+ ·104	+ ·047	89 39	65 1		− 211·7	+ 564·1	
260	261	− 1·32	− 0·2	267·076	− ·244	− ·026	92 9 27	67 36 5	+ 9	+ 115·1	+ 152·0	− 10·1
276	277	− 0·43	+ 14·8	91·082	+ ·176	− ·109	101 2 13	76 28 30	− 25	+ 173·6	− 82·5	− 31·6
287	288	− 1·05	+ 3·7	494·395	− ·180	+ ·113	268 0 5	243 28 14	+ 12	+ 179·4	+ 315·1	+ 1·5
311	312	− 0·02	+ 1·6	603·825	+ ·268	+ ·067	269 40 34	245 9 41	+ 30	+ 98·1	+ 505·7	+ 4·9
322	323	+ 0·39	− 0·4	394·714	+ ·086	+ ·068	270 7 23	245 37 20	+ 1 4	− 219·6	+ 614·3	+ 3·7
352	353	− 0·28	+ 2·5	692·835	+ ·106	− ·001	90 56 39	66 35 2	− 19	+ 212·9	+ 479·8	− 3·4
358	359	− 0·33	+ 1·0	106·286	+ ·222	+ ·065	263 13 27	238 53 10	+ 29	+ 175·4	− 69·0	+ 22·8
374	375	− 0·97	+ 2·1	778·218	+ ·010	+ ·097	269 33 47	245 15 31	+ 29	+ 224·7	+ 553·5	+ 0·2
395	396	− 1·47	+ 3·7	462·724	+ ·272	+ ·025	271 53 24	247 37 20	+ 8	+ 153·9	+ 308·9	− 2·6
	402			83·837	− ·216	+ ·021	77 47	53 32		+ 174·4	− 90·5	
401		− *5·54*					77 38 14	53 31 42	− 1 31			+ 33·6
		+ *5·24*					77 55 50					
415	416	− 2·71	+ 0·4	224·157	+ ·342	+ ·034	87 44 54	63 30 0	+ 4	+ 87·5	+ 136·7	+ 7·1
419	420	− 3·21	+ 0·2	131·770	+ ·080	+ ·045	86 52 59	62 38 34	− 24	− 215·4	+ 346·6	− 9·3
453	454	+ 0·38	+ 1·5	507·833	+ ·342	+ ·062	91 3 9	66 50 9	− 1 0	− 98·5	+ 606·2	+ 6·6
455	456	+ 0·09	+ 4·9	232·971	− ·168	+ ·051	266 3 29	241 51 51	− 27	+ 194·9	+ 38·0	+ 10·1
471	472	− 0·42	+ 2·0	736·243	+ ·218	+ ·098	90 29 20	66 19 21	− 33	+ 171·9	+ 564·4	− 0·5
491	492	+ 0·05	+ 1·0	297·686	− ·188	+ ·020	91 39 41	67 29 27	+ 5	+ 180·6	+ 117·9	− 8·5
510	511	+ 0·06	+ 0·8	648·107	− ·298	+ ·095	90 10 16	65 55 36	− 33	+ 130·2	+ 517·9	− 4·2
535	536	+ 0·02	+ 0·7	397·505	+ ·282	+ ·079	90 54 28	66 29 17	− 50	− 126·1	+ 523·6	+ 5·7
543	544	− 0·03	+ 0·7	163·384	− ·102	+ ·005	267 52 29	243 22 19	+ 32	− 173·5	+ 337·0	− 6·1

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TABLE XIV.—*continued.*

SATELLITES III.–IV. COMPUTED DISTANCES AND POSITION-ANGLES.

Rot. No.		Δt.	$\frac{s}{1000}\frac{dp}{dt}$.	Computed s_0.	Corrections s.	Computed $(p-P)_0$.	Computed p_0.	Corrections p.	$\rho_3\frac{\partial s}{\partial \rho_3}$.	$\rho_4\frac{\partial s}{\partial \rho_4}$.	$s \sin 1'' \rho_3 \frac{\partial p}{\partial \rho_3}$.
p.	s.	m									
7	8	−0·21	+ 8·2	211″·687	+0″·086	263° 27′ 29″	238° 29′ 28″	− ′ 13″	+289·5	− 77·8	+ 25·9
15	16	−0·56	+ 1·3	552·851	+ ·027	269 39 42	244 42 56	+ 9	+180·0	+372·8	+ 10·3
39	40	+0·06	+ 0·5	403·145	+ ·009	270 27 26	245 31 16	+ 41	−142·7	+545·8	+ 11·5
45	46	−0·53	− 0·3	284·483	+ ·021	270 36 28	245 40 54	+ 58	−348·1	+631·7	+ 8·1
53	54	+0·27	+ 1·2	622·682	+ ·039	270 32 38	245 39 0	+ 9	+245·2	+377·5	− 11·0
69	70	−0·47	+ 2·5	528·527	− ·044	271 44 16	246 51 14	− 10	+369·9	+158·6	− 11·7
87	86	+0·10		401·281	+ ·109	86 27 31	61 35 55	− 9	+ 80·6	+320·7	− 6·4
96	97	+0·13	+ 2·2	964·973	+ ·029	90 2 55	65 12 45	− 17	+338·8	+626·2	+ 3·4
112	113	−0·60	+ 2·6	387·558	− ·030	92 44 2	67 55 16	− 46	−207·1	+594·6	+ 19·3
126	127	+1·15	+ 4·7	88·372	− ·046	100 8 35	75 21 39	− 30	−362·7	+451·1	+ 67·0
146	147	+1·04	+ 5·1	83·684	+ ·096	261 19 37	236 35 48	+ 1 4	−354·7	+438·3	− 56·5
169	170	−0·00(5)	+ 1·9	817·021	+ ·068	269 23 55	244 40 39	+ 34	+168·5	+648·6	− 8·0
	178			976·301	+ ·034	270 13	245 31		+360·5	+615·8	
	181			976·259	+ ·033	270 14	245 32		+361·9	+614·4	
182				976·2		270 14 33	245 32 5	+ 17			− 4·6
	189			329·446	− ·077	274 28	249 47		+ 24·7	+304·7	
209	210	+0·03	+ 1·3	590·328	+ ·052	89 45 13	65 6 47	− 6	+187·5	+402·8	+ 9·4
224	225	−0·19	+ 0·7	442·659	+ ·038	90 29 49	65 52 13	− 40	−118·1	+560·7	+ 11·1
	229			438·953	+ ·039	90 31	65 53		−124·4	+563·3	
244	245	−0·07	+ 1·1	616·237	+ ·008	90 33 41	65 59 26	− 2	+243·3	+373·5	− 10·7
256	257	−0·84	+ 2·4	525·181	− ·051	91 38 55	67 5 28	+ 20	+368·6	+156·6	− 11·3
274	275	−0·61	+11·1	151·083	− ·097	98 34 27	74 1 9	+ 48	+233·4	− 82·2	− 27·7
289	290	−0·64	+ 5·1	386·025	+ ·119	266 39 43	242 7 52	+ 25	+ 69·5	+316·6	− 6·0
305	306	−0·50	+ 2·5	825·644	+ ·116	269 4 33	244 33 38	+ 16	+323·1	+502·4	− 0·2
318	319	−0·31	+ 2·0	956·366	+ ·077	270 1 29	245 31 25	+ 19	+343·5	+612·8	+ 2·8
	324			954·993	+ ·074	270 3	245 33		+340·1	+614·9	
336	337	+0·84	+ 1·1	34·986	− ·029	280 7 9	255 42 19		+281·1	−246·0	− 56·8
350	351	−0·49	+ 2·5	775·040	+ ·020	91 11 48	66 50 10	− 10	+294·0	+481·0	− 1·2
356	357	+0·93	+ 8·8	178·743	+ ·084	263 2 42	238 43 14	− 12	+250·3	− 71·4	+ 23·3
370	371	−0·17	+ 0·6	406·371	+ ·042	270 34 50	246 16 37	+ 57	−145·0	+551·5	+ 10·2
382	383	+0·43	− 0·2	276·298	+ ·041	270 48 20	246 30 46	+ 1 14	−341·9	+618·2	+ 8·0
393	394	+0·58	+ 1·0	564·654	+ ·048	270 37 5	246 21 14	+ 7	+253·6	+311·0	− 9·4
407	408	+0·46	+ 2·3	438·645	+ ·020	271 49 42	247 34 24	− 18	+349·5	+ 89·1	− 10·8
413	414	−0·50	+19·0	71·602	− ·036	285 44 10	261 27 8	+ 6	+196·5	−124·9	− 48·3
431	432	+0·62	+ 3·5	466·273	+ ·096	87 46 25	63 32 9	− 18	+109·5	+356·8	− 4·4
437	438	+0·64	+ 2·1	830·371	+ ·111	89 19 58	65 6 8	− 17	+320·0	+510·4	− 0·4
451	452	−0·30	+ 1·9	674·142	+ ·082	91 5 4	66 52 2	− 45	+ 67·5	+606·7	+ 7·0
459	460	−0·22	− 7·3	5·954	− ·010	134 51 50	110 44 40	+ 5 11	+ 27·0	− 21·1	− 38·8
475	476	+0·11	− 0·4	285·229	+ ·053	90 19 38	66 9 40	− 1 5	−277·9	+563·1	− 2·1
487	488	+0·79	+ 0·2	422·846	+ ·068	90 9 31	65 59 28	− 40	− 45·9	+468·7	− 7·5
495	496	−1·26	+ 1·8	439·522	+ ·044	91 29 10	67 18 51	+ 16	+327·0	+112·7	− 9·9
	499			111·913	+ ·014	98 28	74 18		+205·5	− 93·7	
506	507	−1·17	+ 1·4	819·647	+ ·117	90 12 54	65 58 12	− 18	+300·2	+519·5	− 3·7
517	518	−0·42	+ 1·0	500·915	+ ·059	269 47 7	245 29 44	+ 7	+178·8	+322·1	+ 6·1
533	534	−0·02	+ 1·4	638·128	+ ·135	90 46 34	66 21 23	− 31	+114·4	+523·8	+ 4·4
539	540	−1·64	+ 3·2	44·414	− ·028	261 32 29	237 0 20	− 14	−286·5	+330·8	− 42·7

86, 87 Quantities computed for epoch of position-angle 87; $\frac{ds}{dt} = +0·368$. Correction $\Delta s = \frac{ds}{dt}\Delta t = +0''·037$ is included in computed s_0.

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Corrections to tabular places.—I now come to the explanation of the corrections which must further be applied to the computed distances $s_0 + \Delta s_0$ and p_0. These are of two kinds.

Firstly, there are the corrections necessary on account of errors in Marth's ephemeris, which were detected too late to be included in $s_0 + \Delta s_0$ and p_0. These are the error in the inequality of the radius vector of II. depending on β_2,* and the error in the mean distances. These were corrected by the formulas (3) and (4) already given above on page 84.

The second group of corrections are of the nature of refinements in the reduction, which were at first assumed to be unnecessary, but which were afterwards applied, having regard to the extraordinary refined character of the observations, which was shown by the solution of the equations. These corrections naturally require a somewhat lengthy explanation.

Imagine a system of co-ordinate axes fixed on the celestial sphere, and parallel to the axes to which the co-ordinates x'', y'' are referred. The axis of Y thus is parallel to the projection on the sphere of Jupiter's adopted axis of rotation, *i.e.* of a line perpendicular to the adopted fundamental plane.

Let the co-ordinates of the satellites with reference to these fixed axes be X_i, Y_i, and those of Jupiter ξ, η. We have then

$$X_i = \xi + x_i'', \qquad Y_i = \eta + y_i'',$$

where x_i'' and y_i'' are given by the formulas (5).

The observed light left the two satellites, which are observed together (say I. and II.), at the times

$$t_1 = t_0 - \tau \Delta_1' \text{ and } t_2 = t_0 - \tau \Delta_2'$$

respectively, where t_0 is the observed time, Δ_1' and Δ_2' are computed by (5), and $\tau = 498^s{\cdot}46$.

For the computation of X_1 and Y_1 we must therefore take out the longitude and radius vector of satellite I., which serve to compute x_1'' and y_1'', for the time t_1, and also ξ and η must be taken for that same time t_1. Similarly for X_2 and Y_2 we must use the time t_2.

The co-ordinates of the satellites as given in Tables XIII. and XIV. were, however, *not* computed for the time t_i, but for the time

$$t = t_0 - \tau \Delta,$$

i.e. the time at which the observed light left *Jupiter*. This is the time given in the

* The correction (3) was already applied in the original work. It is, however, *not* contained in the co-ordinates of Table XIII., and the computed quantities s_0, p_0 of Table XIV.

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columns "Greenwich Mean Time" of Tables XI. and XII. They therefore require a correction, which is, with sufficient accuracy,

$$\delta v_i = n_i(\Delta - \Delta_i')\tau, \qquad \delta\rho_i = 0,$$

from which we find

$$(7)\quad \begin{aligned} \delta \boldsymbol{s} &= \tau\left[n_1(\Delta-\Delta_1')\frac{\partial \boldsymbol{s}}{\partial v_1} + n_2(\Delta-\Delta_2')\frac{\partial \boldsymbol{s}}{\partial v_2}\right], \\ \delta \boldsymbol{p} &= \tau\left[n_1(\Delta-\Delta_1')\frac{\partial \boldsymbol{p}}{\partial v_1} + n_2(\Delta-\Delta_2')\frac{\partial \boldsymbol{p}}{\partial v_2}\right]. \end{aligned}$$

These corrections are exceedingly small.*

But also the co-ordinates ξ, η of Jupiter are different at the two epochs t_1 and t_2. We have thus

$$\begin{aligned} \mathrm{x}_2 - \mathrm{x}_1 &= (\xi)_{t_2} - (\xi)_{t_1} + x_2'' - x_1'' = x_2'' - x_1'' + (t_2 - t_1)\frac{d\xi}{dt}, \\ \mathrm{y}_2 - \mathrm{y}_1 &= (\eta)_{t_2} - (\eta)_{t_1} + y_2'' - y_1'' = y_2'' - y_1'' + (t_2 - t_1)\frac{d\eta}{dt}. \end{aligned}$$

We have with sufficient accuracy

$$\frac{d\xi}{dt} = \cos \mathrm{B}\left(\frac{d\mathrm{L}}{dt}\right)_{\mathfrak{U}}, \qquad \frac{d\eta}{dt} = 0,$$

where $\left(\frac{d\mathrm{L}}{dt}\right)_{\mathfrak{U}}$ denotes the variation of L in the unit of time *by the motion of Jupiter alone*, the earth being assumed not to move. In the notation introduced below we have thus

$$(t_2 - t_1)\left(\frac{d\mathrm{L}}{dt}\right)_{\mathfrak{U}} = \mathrm{L}(t_2, t_0) - \mathrm{L}(t_1, t_0).$$

We find thus

$$\begin{aligned} \mathrm{x}_2 - \mathrm{x}_1 &= x_2'' - x_1'' + \tau(\Delta_1' - \Delta_2')\cos \mathrm{B}\left(\frac{d\mathrm{L}}{dt}\right)_{\mathfrak{U}} \\ \mathrm{y}_2 - \mathrm{y}_1 &= y_2'' - y_1''. \end{aligned}$$

These must be substituted for $x_2'' - x_1''$ and $y_2'' - y_1''$ in the formula (6) on page 92. Therefore to the values of $\boldsymbol{s}_0$ and $(\boldsymbol{p} - \mathrm{P})_0$ computed by (6) we must apply the corrections

$$(8)\quad \begin{aligned} \delta \boldsymbol{s} &= \tau(\Delta_1' - \Delta_2')\cos \mathrm{B}\left(\frac{d\mathrm{L}}{dt}\right)_{\mathfrak{U}} \sin(p - \mathrm{P}) \\ \boldsymbol{s}\,\delta(\boldsymbol{p} - \mathrm{P}) &= \tau(\Delta_1' - \Delta_2')\cos \mathrm{B}\left(\frac{d\mathrm{L}}{dt}\right)_{\mathfrak{U}} \cos(p - \mathrm{P}). \end{aligned}$$

These can also be written:

$$(8')\quad \begin{aligned} \delta \boldsymbol{s} &= \tau\Delta\frac{\partial \boldsymbol{s}}{\partial \mathrm{L}}\left(\frac{d\mathrm{L}}{dt}\right)_{\mathfrak{U}} \\ \delta(\boldsymbol{p} - \mathrm{P}) &= \tau\Delta\frac{\partial(\boldsymbol{p} - \mathrm{P})}{\partial \mathrm{L}}\left(\frac{d\mathrm{L}}{dt}\right)_{\mathfrak{U}}. \end{aligned}$$

* This correction was already applied in the original work, and is there called "differential aberration." It is *not* contained in the values of $\boldsymbol{s}_0$ and $\boldsymbol{p}_0$ of Table XIV.

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To prove this, let ϕ and ψ be the longitude and latitude, referred to the fundamental plane, of satellite II. as seen from I., and let D be the distance of II. from I. Then, in a system of co-ordinate axes having its origin in I., and its plane of $\mathbf{x}$, $\mathbf{y}$ parallel to the fundamental plane, the negative axis of $\mathbf{y}$ being directed towards the projection of the earth on this plane, the co-ordinates of II. are

$$\mathbf{x} = \mathrm{D} \cos\psi \sin(\phi - \mathrm{L})$$
$$\mathbf{y} = \mathrm{D} \cos\psi \cos(\phi - \mathrm{L})$$
$$\mathbf{z} = \mathrm{D} \sin\psi.$$

Further, we have

$$s\Delta \sin(p - \mathrm{P}) = \mathbf{x}$$
$$s\Delta \cos(p - \mathrm{P}) = \mathbf{z} \cos \mathrm{B} + \mathbf{y} \sin \mathrm{B}$$

and

$$\Delta_2' - \Delta_1' = \mathbf{y} \cos \mathrm{B} - \mathbf{z} \sin \mathrm{B}.$$

From these formulas we find easily

$$\Delta \frac{\partial s}{\partial \mathrm{L}} = (\Delta_1' - \Delta_2') \cos \mathrm{B} \sin(p - \mathrm{P})$$
$$s\Delta \frac{\partial (p - \mathrm{P})}{\partial \mathrm{L}} = (\Delta_1' - \Delta_2') \cos \mathrm{B} \cos(p - \mathrm{P}) - s\Delta \sin \mathrm{B}.$$

The last term of the second equation is always entirely negligible. By means of these last two equations (8′) follows immediately from (8).

We must now consider more closely the values of L, B, Δ to be used in the formulas (5). I will denote by $\mathrm{L}(t, t')$, $\mathrm{B}(t, t')$; $\Delta(t, t')$ the values of these co-ordinates reckoned from the position of Jupiter at time t to that of the earth at time t'.

For the computation of the co-ordinates of the satellite I., we must use the values $\mathrm{L}(t_1, t_0)$, $\mathrm{B}(t_1, t_0)$, $\Delta(t_1, t_0)$, and for II. the same, with t_2 instead of t_1. It is, however, amply exact to use $\mathrm{L}(t, t_0)$, $\mathrm{B}(t, t_0)$, $\Delta(t, t_0)$ for both satellites.

Now what was actually done is the following.

The logarithm of Jupiter's distance from the earth was interpolated from the *Nautical Almanac* for the time t_0. This is thus $\Delta(t_0, t_0)$. To reduce this to $\Delta(t, t_0)$ a correction * must be applied, which is

$$\delta \log \Delta = 0{\cdot}00000394 \, \mathrm{R}_{\mathfrak{Z}} \sin(\Lambda - \mathrm{L}),$$

where $\mathrm{R}_{\mathfrak{Z}}$ and $\Lambda + 180°$ are the jovicentric distance and longitude of the sun. The numerical factor is the heliocentric motion of Jupiter in $498^{s}{\cdot}5$, expressed in radians and multiplied by the modulus of Briggian logarithms (0·434). The values of $\Delta(t, t_0)$

* This correction, as given by Marth in *M.N.*, vol. li. p. 360, is erroneous, as was pointed out by Marth himself on p. 575 of the same volume.

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so derived were tabulated in a table with argument t, from which they could easily be taken out for each observation.

The values of L and B were interpolated from Marth's "Ephemeris for Physical Observations of Jupiter" (*Monthly Notices of the R. A. Society*, vol. li. pp. 360–367). This ephemeris was computed by means of the apparent right ascensions and declinations of Jupiter given in the *Nautical Almanac*. It contains, therefore, *apparent* values of L and B. These were interpolated for time t. But the apparent value at time t is equal to the true value at time $t' = t - \tau\Delta = t_0 - 2\tau\Delta$. The values actually used are thus $L(t', t')$ and $B(t', t')$. The angle B is so small and changes so slowly, that the difference between this value and the correct value $B(t, t_0)$ is entirely negligible. For L this was originally assumed to be also true, but it is not so.

The corrections to be applied to $\boldsymbol{s}_0$ and $(\boldsymbol{p} - P)_0$ are therefore

$$\delta\boldsymbol{s} = \frac{\partial \boldsymbol{s}}{\partial L}\left\{ L(t, t_0) - L(t', t') \right\},$$
$$\delta(\boldsymbol{p} - P) = \frac{\partial(\boldsymbol{p} - P)}{\partial L}\left\{ L(t, t_0) - L(t', t') \right\}.$$

Now

$$L(t, t_0) - L(t', t') = L(t, t_0) - L(t_0, t_0) + L(t_0, t_0) - L(t', t') =$$
$$= -(t_0 - t)\left(\frac{dL}{dt}\right)_0 + (t_0 - t')\frac{dL}{dt}.$$

Since $t_0 - t = \tau\Delta$, and $t_0 - t' = 2\tau\Delta$, we have:

$$\delta\boldsymbol{s} = \tau\Delta\frac{\partial \boldsymbol{s}}{\partial L}\left\{ 2\frac{dL}{dt} - \left(\frac{dL}{dt}\right)_0 \right\},$$
$$\delta(\boldsymbol{p} - P) = \tau\Delta\frac{\partial(\boldsymbol{p} - P)}{\partial L}\left\{ 2\frac{dL}{dt} - \left(\frac{dL}{dt}\right)_0 \right\}. \qquad (9)$$

The correction (9) can be combined with (8′). The sum of the two is

$$\delta\boldsymbol{s} = 2\tau\Delta\frac{\partial \boldsymbol{s}}{\partial L}\frac{dL}{dt},$$
$$\delta(\boldsymbol{p} - P) = 2\tau\Delta\frac{\partial(\boldsymbol{p} - P)}{\partial L}\frac{dL}{dt}. \qquad (10)$$

If all these corrections are applied to the values of $\boldsymbol{s}_0 + \Delta\boldsymbol{s}_0$ and $(\boldsymbol{p} - P)_0$ as given in Table XIV., we get true distances and position-angles at the time of observation. But in order to be comparable to the observed quantities these must be reduced to apparent by the application of the aberration of the fixed stars. The corrections for aberration are independent of the position-angle, and are therefore the same for $\boldsymbol{p}$ and P. Consequently $(\boldsymbol{p} - P)_{app.} = (\boldsymbol{p} - P)_{true}$. We need thus not apply any correction for aberration to the position-angles, provided we take for P its *apparent* value at time t_0.

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For the distances the correction for aberration is proportional to the distance. If we call it $A\boldsymbol{s}$, the total correction becomes

$$\delta\boldsymbol{s} = A\boldsymbol{s} + 2\tau\Delta\frac{\partial\boldsymbol{s}}{\partial\Delta}\frac{d\mathrm{L}}{dt},$$

$$\delta\boldsymbol{p} = 2\tau\Delta\frac{\partial(\boldsymbol{p}-\mathrm{P})}{\partial\mathrm{L}}\frac{d\mathrm{L}}{dt} + \Delta\mathrm{P}.$$

The factor A was computed by the usual formulas for the correction of micrometrical measures for aberration.

The term ΔP will be explained below.

Now L occurs practically only in the combination $(v - \mathrm{L})$. Therefore

$$\frac{\partial\boldsymbol{s}}{\partial\mathrm{L}} = -\frac{\partial\boldsymbol{s}}{\partial v_1} - \frac{\partial\boldsymbol{s}}{\partial v_2}, \qquad \frac{\partial(\boldsymbol{p}-\mathrm{P})}{\partial\mathrm{L}} = -\frac{\partial\boldsymbol{p}}{\partial v_1} - \frac{\partial\boldsymbol{p}}{\partial v_2}.$$

Putting now

$$2\tau\Delta\frac{d\mathrm{L}}{dt} = -B,$$

we have finally *

$$\delta\boldsymbol{s} = A\boldsymbol{s} + B\left(\frac{\partial\boldsymbol{s}}{\partial v_1} + \frac{\partial\boldsymbol{s}}{\partial v_2}\right)$$

(11)

$$\delta\boldsymbol{p} = B\left(\frac{\partial\boldsymbol{p}}{\partial v_1} + \frac{\partial\boldsymbol{p}}{\partial v_2}\right) + \Delta\mathrm{P}.$$

---

* The same formulas (11) can also be derived in a totally different way. If we do not wish to apply the correction for the aberration of the fixed stars, then we must not use $\mathrm{L}(t, t_0)$, $\Delta(t, t_0)$ but $\mathrm{L}(t, t)$ and $\Delta(t, t)$.

The correction $\delta(\log\Delta)$ which was applied should therefore be removed, *i.e.* to the computed $\boldsymbol{s}_0$ a correction must be added, which is

$$\delta\boldsymbol{s} = \frac{\partial\boldsymbol{s}}{\partial\Delta}\left\{\Delta(t, t) - \Delta(t, t_0)\right\},$$

and similarly for $\boldsymbol{p}$. Now we have

$$\frac{\partial\boldsymbol{s}}{\partial\Delta} = -\frac{\boldsymbol{s}}{\Delta}, \qquad \frac{\partial\boldsymbol{p}}{\partial\Delta} = 0.$$

Further, if θ be the angle between the direction of motion of the earth and the direction in which Jupiter is seen from the earth, we have

$$\Delta(t, t) - \Delta(t, t_0) = k\Delta\cos\theta,$$

where k is the aberration-constant. On the other hand, we have

$$A = -k\cos\theta.$$

Thus the first term of (11) is reproduced.

The second term is at once verified if we consider that in (8) we must now use $\frac{d\mathrm{L}}{dt}$ instead of $\left(\frac{d\mathrm{L}}{dt}\right)_2$, and in (9) we must take for the correction to be applied to L:

$$\mathrm{L}(t, t) - \mathrm{L}(t', t') = \tau\Delta\frac{d\mathrm{L}}{dt}.$$

I have, for three dates, verified by actual computation that the correction $A\boldsymbol{s}$, which has been applied, is equal to $\frac{\partial\boldsymbol{s}}{\partial\Delta}\left\{\Delta(t, t) - \Delta(t, t_0)\right\}$. In doing this I took $\Delta(t, t) - \Delta(t, t_0) = \Delta(t, t) - \Delta(t_0, t_0) + \Delta(t_0, t_0) - \Delta(t, t_0)$, and the second of these differences was derived from the corrections $\delta\log\Delta$ which had been actually applied. Thus both these corrections and the corrections $A\boldsymbol{s}$ are thoroughly checked.

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The values of A and B are

	A.	B.
August 17 . . .	−0·000035	+0°·0053
September 16 . . .	+ ·000021	+ ·0058
October 16 . . .	+ ·000070	+ ·0029
November 15 . . .	+ ·000096	− ·0021
December 15 . . .	+ ·000097	− ·0074

The term ΔP in (11) is the difference between the value which should have been used and the value which was actually used. The value that should have been used is the apparent position-angle P_0 of the projection on the celestial sphere of a line drawn at the middle point between the two satellites perpendicular to the fundamental plane.

If we call P_J the position-angle of a line perpendicular to the fundamental plane through the centre of Jupiter, and P_M the value of P which was actually used, and which was taken from Marth's ephemeris, we have $\Delta P = P_0 - P_M$, or

$$\Delta P = \Delta_1 P + \Delta_2 P, \quad \begin{cases} \Delta_1 P = P_J - P_M, \\ \Delta_2 P = P_0 - P_J. \end{cases}$$

Marth's ephemeris gives apparent values of P_J. The correct value might thus have been taken from this ephemeris, but since Marth gives P only to three decimal places of a degree, it was impossible to interpolate correctly, and a constant value was taken for each night. Afterwards a new ephemeris was computed (which agreed exactly with Marth's in the decimals given by him) from which the value of P could be interpolated for every observation. Table XV. contains the new values of P_J. The apparent values for Greenwich mean noon are given. The correction $\Delta_1 P$ is now the value interpolated from this table for the time t_0 *minus* the value previously used. These corrections, of course, never amount to more than a few seconds of arc. For practical use a table was constructed giving the value of $\Delta_1 P$ with argument t.

TABLE XV.

APPARENT POSITION-ANGLE P_J AT JUPITER.

Date.	P_J Greenwich Mean Noon.	Increase in one Hour.	Date.	P_J Greenwich Mean Noon.	Increase in one Hour.
	° ′ ″	″		° ′ ″	″
August 17	335 1 51	+ 1·4	October 20	335 47 12	+ 1·0
25	6 47	1·7	28	49 30	+ 0·4
September 2	12 33	1·9	November 5	50 4	− 0·1
10	18 55	2·1	13	48 51	− 0·6
18	25 35	2·1	21	45 58	− 1·2
26	32 10	2·0	29	41 33	− 1·6
October 4	38 15	1·8	December 7	35 51	− 1·9
12	43·20	+ 1·4	15	29 8	− 2·2

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The correction $\Delta_2 P$ has been derived by Mr Cookson in Part IV. of this volume, p. 110. It is

$$\Delta_2 P = [(x_1'' + x_2'') \cos P + (y_1'' + y_2'') \sin P] \cdot \tfrac{1}{2} \tan \delta,$$

where δ is the declination of Jupiter. It must be applied with its sign to the computed values of $\boldsymbol{p}$. These corrections are also small, owing to the factor $\frac{1}{2} \tan \delta$, which is smaller than 0·085. In one or two cases, however, the correction reaches about a minute of arc.

The sum of the corrections (3), (4), (7), and (11), as computed by these formulas, is given in Table XIV. under the headings "Corrections $\boldsymbol{s}$" and "Corrections $\boldsymbol{p}$."* The corrected distances and position-angles are contained in the Tables XVI. and XVII. in the columns "Computed Distance" and "Computed Position-angle."

For three dates, viz. : August 20, September 22, and December 8, all distances and position-angles, 18 pairs in all, were recomputed entirely *de novo*, the appropriate corrections being applied to the elements of the computation (a_i, L, Δ', etc.), and the co-ordinates x_i'', y_i'', etc., being computed from the corrected elements.† The agreement with the differentially corrected distances and position-angles was in all cases within the uncertainty of the computation with six-figure logarithms. The exactness of the differential formulas is thereby verified.

The corrections are all small quantities. Most of them are, however, more or less systematic, and in the discussion of observations of so refined character as these heliometer observations are, they cannot be neglected.

Differential coefficients.—Subtracting the tabular from the observed quantities we find the absolute terms (O − C) of the equations of condition. For the position-angles these were multiplied by $\boldsymbol{s} \sin 1''$ to reduce them to seconds of arc of a great circle. These are given in the Tables XVI. and XVII.

To form the left-hand side of the equations we must now compute the differential coefficients

$$\begin{array}{ccccc} \frac{\partial \boldsymbol{s}}{\partial a_i} & \frac{\partial \boldsymbol{s}}{\partial e_i} & \frac{\partial \boldsymbol{s}}{\partial \rho_i} & \frac{\partial \boldsymbol{s}}{\partial p_i} & \frac{\partial \boldsymbol{s}}{\partial q_i} \\ \frac{\partial \boldsymbol{p}}{\partial a_i} & \frac{\partial \boldsymbol{p}}{\partial v_i} & \frac{\partial \boldsymbol{p}}{\partial \rho_i} & \frac{\partial \boldsymbol{p}}{\partial p_i} & \frac{\partial \boldsymbol{p}}{\partial q_i} \end{array}$$

where

$$\begin{aligned} p_i &= i_i \sin(-\Omega_i) \\ q_i &= i_i \cos(-\Omega_i) \end{aligned} \qquad (i = 1, 2, 3, 4).$$

(The minus sign is introduced in conformity with Marth's notation.)

* The "corrections $\boldsymbol{p}$" contain also the correction to the position-angle depending on the arguments ζ_2, η_2, which is for the distances given separately in the column $\Delta s_0(\zeta_2, \eta_2)$.

† In this check computation I adopted the second mode of procedure (see footnote on p. 104), applying no correction for the aberration of the fixed stars, and using true instead of apparent values for L and Δ'.

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The four mean distances are not mutually independent, but are given by the formula already mentioned on p. 79 :—

$$(1) \qquad n_i^2 a_i^3 = f\frac{1+m_i}{M(1+\Sigma m)}\left[1+\frac{Jb^2}{a_i^2}\right].$$

Since the observations extend over only four months, the mean motions must be assumed known. Therefore, instead of the four mean distances, we must introduce the single unknown

$$M = \frac{\text{mass of sun}}{\text{mass of Jupiter's system}}.$$

The equation (1) also contains the compression Jb^2. This is, of course, one of the constants of the theory whose adopted value is subject to considerable doubt. But, on the one hand, it is impossible from the present series of observations to determine it with any accuracy ; and, on the other hand, the effect of the uncertainty of the adopted value on the mean distance is very small.

We have then from (1)

$$\frac{dM}{M} = -3\frac{da_i}{a_i},$$

from which we find

$$(12) \qquad \frac{\partial \boldsymbol{s}}{\partial M} = -\tfrac{1}{3}\frac{\boldsymbol{s}}{M}, \qquad \frac{\partial \boldsymbol{p}}{\partial M} = 0.$$

If v, p, and q are expressed in degrees, and if we put

$$X_i = \frac{a_i'\rho_i}{\Delta_i'}\sin 1^\circ \cos(v_i - L),$$

$$Y_i = \frac{a_i''\rho_i}{\Delta_i'}\sin 1^\circ\{-\sin(v_i - L)\sin B + \sin i_i \cos(v_i - \Omega_i)\cos B\},$$

$$(i = 1, 2, 3, 4),$$

we find :

$$(13) \qquad \begin{aligned} \rho_1\frac{\partial \boldsymbol{s}}{\partial \rho_1} &= -x_1''\sin(p-P) - y_1''\cos(p-P), & \frac{\partial \boldsymbol{s}}{\partial v_1} &= -X_1\sin(p-P) - Y_1\cos(p-P), \\ \rho_2\frac{\partial \boldsymbol{s}}{\partial \rho_2} &= x_2''\sin(p-P) + y_2''\cos(p-P), & \frac{\partial \boldsymbol{s}}{\partial v_2} &= X_2\sin(p-P) + Y_2\cos(p-P), \end{aligned}$$

with the check equation :

$$\rho_1\frac{\partial \boldsymbol{s}}{\partial \rho_1} + \rho_2\frac{\partial \boldsymbol{s}}{\partial \rho_2} = \boldsymbol{s}.$$

and :

$$(14) \qquad \begin{aligned} \boldsymbol{s}\sin 1''\,\rho_1\frac{\partial \boldsymbol{p}}{\partial \rho_1} &= -x_1''\cos(\boldsymbol{p}-P) + y_1''\sin(\boldsymbol{p}-P), & \boldsymbol{s}\sin 1''\frac{\partial \boldsymbol{p}}{\partial v_1} &= -X_1\cos(\boldsymbol{p}-P) + Y_1\sin(\boldsymbol{p}-P), \\ \boldsymbol{s}\sin 1''\,\rho_2\frac{\partial \boldsymbol{p}}{\partial \rho_2} &= x_2''\cos(\boldsymbol{p}-P) - y_2''\sin(\boldsymbol{p}-P), & \boldsymbol{s}\sin 1''\frac{\partial \boldsymbol{p}}{\partial v_2} &= X_2\cos(\boldsymbol{p}-P) - Y_2\sin(\boldsymbol{p}-P), \end{aligned}$$

with the check equation :

$$\rho_1\frac{\partial \boldsymbol{p}}{\partial \rho_1} + \rho_2\frac{\partial \boldsymbol{p}}{\partial \rho_2} = 0.$$

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Further, if we put

$$Z_i = \frac{a_i'' \rho_i}{\Delta_i''} \sin 1'' \cos B \qquad (i = 1, 2, 3, 4),$$

we find

$$(15)\quad \begin{aligned} \frac{\partial s}{\partial p_1} &= -Z_1 \cos v_1 \cos (p - P), & \frac{\partial s}{\partial q_1} &= -Z_1 \sin v_1 \cos (p - P), \\ \frac{\partial s}{\partial p_2} &= Z_2 \cos v_2 \cos (p - P), & \frac{\partial s}{\partial q_2} &= Z_2 \sin v_2 \cos (p - P), \end{aligned}$$

$$(16)\quad \begin{aligned} s \sin 1'' \frac{\partial p}{\partial p_1} &= Z_1 \cos v_1 \sin (p - P), & s \sin 1'' \frac{\partial p}{\partial q_1} &= Z_1 \sin v_1 \sin (p - P), \\ s \sin 1'' \frac{\partial p}{\partial p_2} &= -Z_2 \cos v_2 \sin (p - P), & s \sin 1'' \frac{\partial p}{\partial q_2} &= -Z_2 \sin v_2 \sin (p - P). \end{aligned}$$

If we denote by $\Delta\rho_1$, $\Delta\rho_2$, Δv_1, etc., corrections to the adopted radii vectores, longitudes, etc., and if we put

$$\mu = \frac{100}{3} \frac{\Delta M}{M},$$

and

$$z = \frac{\kappa^2 + \delta^2}{200} = \text{the sum of the corrections due to "crossing through" (supposed different for the two observers) and to the distance of optical centres,}$$

then the equations of condition become in distance:

$$(Ia.)\quad O - C = -\frac{s}{100}\mu - \frac{100}{s} z + \rho_1 \frac{\partial s}{\partial \rho_1} \frac{\Delta \rho_1}{\rho_1} + \rho_2 \frac{\partial s}{\partial \rho_2} \frac{\Delta \rho_2}{\rho_2} + \frac{\partial s}{\partial v_1} \Delta v_1 + \frac{\partial s}{\partial v_2} \Delta v_2 + \frac{\partial s}{\partial p_1} \Delta p_1 + \frac{\partial s}{\partial q_1} \Delta q_1 + \frac{\partial s}{\partial p_2} \Delta p_2 + \frac{\partial s}{\partial q_2} \Delta q_2,$$

and in position-angle:

$$(Ib.)\quad s \sin 1'' (O - C) = s \sin 1'' \left\{ \rho_1 \frac{\partial p}{\partial \rho_1} \frac{\Delta \rho_1}{\rho_1} + \rho_2 \frac{\partial p}{\rho_2} \frac{\Delta \rho_2}{\rho_2} + \frac{\partial p}{\partial v_1} \Delta v_1 + \frac{\partial p}{\partial v_2} \Delta v_2 + \frac{\partial p}{\partial p_1} \Delta p_1 + \frac{\partial p}{\partial q_1} \Delta q_1 + \frac{\partial p}{\partial p_2} \Delta p_2 + \frac{\partial p}{\partial q_2} \Delta q_2 \right\}$$

The values of the coefficients $\rho_1 \frac{\partial s}{\partial \rho_1}$, $\rho_2 \frac{\partial s}{\partial \rho_2}$, and $s \sin 1'' \rho_1 \frac{\partial p}{\partial \rho_1}$ are given for each observation in Table XIV. The other coefficients need not be given, as they appear unchanged in the final equations of condition (Tables XVI. and XVII.).

Choice of unknowns.—The corrections Δv and $\Delta \rho$ must now be expressed in terms of the corrections to the fundamental constants of the theory. The question thus presents itself, Which constants must be selected for correction?

The fundamental data on which a self-consistent theory like Souillart's rests are:—

M = reciprocal of the mass of the whole system,
$J b^2$ = coefficient depending on the compression of Jupiter,
i and Ω = inclination and ascending node
e and ϖ = excentricity and perijove
n = mean daily motion
ϵ = mean longitude at epoch
m = mass
} of each of the satellites.

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The mean motions n must be assumed to be known, as already mentioned. M appears already explicitly in the equations.

It has already been remarked that a correction to the quantity Jb^2 cannot be determined from these observations with any weight. The only means of determining it is from the secular motions of the nodes or perijoves. The influence of a correction to the adopted value of Jb^2, amounting to as much as a fifth or a quarter of the whole amount, on the values of the other unknowns would be exceedingly small—in fact, entirely negligible in comparison with the errors of observation. Accordingly no correction to Jb^2 was introduced as unknown.

i and Ω are represented by p and q, which have been introduced because the coefficients for these quantities are more easily computed, and are independent of the adopted values of i and Ω.

For ϵ we have

$$\frac{\partial s}{\partial \epsilon_i} = \frac{\partial s}{\partial r_i} \quad \text{and} \quad \frac{\partial p}{\partial \epsilon_i} = \frac{\partial p}{\partial r_i};$$

these quantities require no additional computation.

Between the longitudes and mean motions of the three inner satellites there exist the relations

$$n_1 - 3n_2 + 2n_3 = 0,$$
$$\theta = l_1 - 3l_2 + 2l_3 = 180^\circ.$$

These relations are rigorous if the *mean* values of the involved quantities are considered. Laplace found theoretically that the deviations of the actual values at a certain epoch from the mean values can be expressed as a periodic inequality, which he called libration, and which is distributed in a fixed proportion among the three satellites. Delambre was unable to find any trace of this libration from the observations. Nevertheless it is not allowed to treat the equations just given as rigorously true at any epoch, and by their means to eliminate one of the unknowns $\Delta\epsilon_i$. Moreover, no practical advantage would be gained by such an elimination, as will be shown below. The three unknowns $\Delta\epsilon_1$, $\Delta\epsilon_2$ and $\Delta\epsilon_3$ were accordingly introduced separately into the equations.

It remains to introduce the unknowns e, ϖ, and m for each of the satellites.

These appear in the longitudes and radii vectores as periodic inequalities, analytically represented by expressions of the form:

$$P \sin(\phi - Q) \quad \text{for the longitudes,}$$
$$\text{and} \quad P' \cos(\phi - Q) \quad \text{for the radii vectores,}$$

where ϕ is proportional to the time, Q depends on the perijoves and P and P′ on the excentricities and masses.

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Thus to introduce m, e, and ϖ into the equations we must first select those periodic inequalities which it is possible to determine with any weight from the observations—that is, those of which the periods are short in comparison with the period covered by the observations (119 days), and which are at the same time of importance for the determination of the masses and excentricities.

We have then

$$\Delta v = \Delta\epsilon + \Sigma\{\sin\phi\ \Delta(\mathrm{P}\cos\mathrm{Q}) - \cos\phi\ \Delta(\mathrm{P}\sin\mathrm{Q})\},$$
$$\Delta\rho = \Sigma\frac{\mathrm{P}'}{\mathrm{P}}\{\cos\phi\ \Delta(\mathrm{P}\cos\mathrm{Q}) + \sin\phi\ \Delta(\mathrm{P}\sin\mathrm{Q})\},$$

where the summation extends over all the selected terms.

I will now enumerate the periodic terms corrections to which have actually been introduced as unknowns.

1°. Each of the satellites has *four equations of the centre, i.e. four* periodic terms of the general form given above, for which $\phi = l =$ mean longitude. These are of the form:

$$\mathfrak{a}_1 \sin(l-\varpi_1), \quad \mathfrak{a}_2 \sin(l-\varpi_2), \quad \mathfrak{a}_3 \sin(l-\varpi_3), \quad \mathfrak{a}_4 \sin(l-\varpi_4),$$

for the longitudes; and for the radii vectores:

$$\mathfrak{a}_1' \cos(l-\varpi_1), \quad \mathfrak{a}_2' \cos(l-\varpi_2), \quad \mathfrak{a}_3' \cos(l-\varpi_3), \quad \mathfrak{a}_4' \cos(l-\varpi_4),$$

where $\mathfrak{a}_1' = -\frac{1}{2}\sin 1^\circ.\,\mathfrak{a}_1$, $\mathfrak{a}_2' = -\frac{1}{2}\sin 1^\circ.\,\mathfrak{a}_2$, etc., if the longitudes are expressed in degrees, and the radii vectores in the mean distances as unit.

It is not possible to separate these four terms by the observations.

They can be combined in *one* term of the form:

$$2E\sin(l-\Omega) \qquad \text{in longitude,}$$

and

$$-E\sin 1^\circ\cos(l-\Omega) \qquad \text{in radius vector,}$$

where

$$2E\sin\Omega = \mathfrak{a}_1\sin\varpi_1 + \mathfrak{a}_2\sin\varpi_2 + \mathfrak{a}_3\sin\varpi_3 + \mathfrak{a}_4\sin\varpi_4,$$
$$2E\cos\Omega = \mathfrak{a}_1\cos\varpi_1 + \mathfrak{a}_2\cos\varpi_2 + \mathfrak{a}_3\cos\varpi_3 + \mathfrak{a}_4\cos\varpi_4.$$

The $\mathfrak{a}$'s, and therefore E and Ω, involve not only the excentricities but also the masses of the satellites.

If we put

$$h = 2E\sin\Omega, \quad k = 2E\cos\Omega,$$

the periodic terms become in longitude

$$k\sin l - h\cos l,$$

and in radius vector

$$-\tfrac{1}{2}\sin 1^\circ(k\cos l + h\sin l).$$

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These terms include the inequalities depending on Marth's arguments γ_1, δ_1; γ_2, δ_2, ϵ_2; α_3, β_3, ι_3; α_4, γ_4. Their periods are respectively $1^d \cdot 77$, $3^d \cdot 55$, $7^d \cdot 15$ and $16^d \cdot 69$.

2°. The principal inequalities of the three inner satellites are very large owing to the circumstance that the mean motions are very approximately proportional to small whole numbers. Their expressions are

Satellite.	Marth's Argument.	In Longitude.	In Radius Vector.	Period.	Max. Geoc. Effect.
				d	″
I.	α_1	$+0^\circ\cdot432 \sin 2(l_1-l_2)$	$-0\cdot00378 \cos 2(l_1-l_2)$	1·76	1·08
II.	α_2	$-1\cdot203 \sin (l_1-l_2)$	$+0\cdot00901 \cos (l_1-l_2)$	3·52	3·66
III.	γ_3	$-0\cdot063 \sin (l_2-l_3)$	$+0\cdot00058 \cos (l_2-l_3)$	7·05	0·37

In the last column is given the maximum effect of the perturbation on the place of the satellite as seen from the earth, when Jupiter is in opposition to the sun ($\log \Delta = 0\cdot6$).

The coefficients in radius vector are again very approximately equal to those in longitude multiplied by $-\frac{1}{2} \sin 1°$. These coefficients involve the masses m_1, m_2, and m_3 only.

If we call the corrections to the coefficients in longitude x_1, x_2, x_3, the differential coefficient of x_1 is in longitude $\sin \alpha_1$ and in radius vector $-\frac{1}{2} \sin 1° \cos \alpha_1$, and similarly for x_2 and x_3. The angles α_1, α_2, γ_3 can be taken from the computations of longitudes and radii vectores for the time of each observation.

3°. In addition to these large inequalities, I have taken into account such terms of short period, of which the maximum effect on the position of the satellite as seen from the earth is at least equal to $0''\cdot05$. These are the following:—

Satellite.	Marth's Argument.	In Longitude.	In Radius Vector.	Period.	Max. Geoc. Effect.	
					Longit.	Radius Vect.
				d	″	″
II.	β_2	$-0^\circ\cdot013 \sin (l_2-l_3)$	$+0\cdot00007 \cos (l_2-l_3)$	7·05	0·05	0·01
		$+0\cdot006 \sin 3(l_2-l_3)$	$-0\cdot00006 \cos 3(l_2-l_3)$		0·02	0·01
III.	δ_3	$-0\cdot004 \sin (l_3-l_4)$	$+0\cdot00002 \cos (l_3-l_4)$	12·53	0·02	0·01
		$-0\cdot014 \sin 2(l_3-l_4)$	$+0\cdot00013 \cos 2(l_3-l_4)$		0·07	0·05
IV.	β_4	$-0\cdot003 \sin (l_3-l_4)$	$+0\cdot00011 \cos (l_3-l_4)$	12·53	0·02	0·07
		$+0\cdot001 \sin 2(l_3-l_4)$	$-0\cdot00002 \cos 2(l_3-l_4)$		0·01	0·01
	δ_4	$+0\cdot006 \sin (l_4-2l_\odot+\varpi_4)$	$-0\cdot00005 \cos (l_4-2l_\odot+\varpi_4)$	16·69	0·06	0·03

The last of these terms involves only the mass of the sun, and e_4 and ϖ_4. For the determination of these last quantities it is of no value compared with the term depending on α_4 (see under 1°) of which the maximum geocentric effect is $6''\cdot95$. It has therefore not been considered as liable to correction by the introduction of a corresponding unknown.

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The coefficients of all the inequalities depending on multiples of the same argument are proportional to the same mass, *viz.* in satellite II. to m_3, in III. to m_4, and in IV. to m_3.

According to the above-mentioned rule only one term for each of the satellites II., III., and IV. should be taken into account. The other terms are, however, of so little importance that it makes practically no difference in the differential coefficients whether we take as the quantity to be introduced into the equations the whole of the inequality depending on multiples of the same argument or only its most important term. I have therefore, in drawing up the form of the differential coefficients to be computed, been guided entirely by practical considerations regarding the shortness and easiness of computation.

If we call $\mathbf{D}_3$ the sum of the terms in the longitude of satellite III. depending on $l_3 - l_4$ and its multiples—that is, on Marth's argument δ_3; and $\mathbf{D}'$ the sum of the corresponding terms in the radius vector, then $\mathbf{D}_3$ and $\mathbf{D}_3' + 0{\cdot}000144$ can be taken at once from the computations of longitude and radius vector, for each epoch of observation, or can be interpolated from Marth's tables. $\mathbf{D}_3$ and $\mathbf{D}_3'$ being both proportional to m_4 we have

$$\frac{\Delta(\mathbf{D}_3)}{\mathbf{D}_3} = \frac{\Delta m_4}{m_4} = \frac{\Delta(\mathbf{D}_3')}{\mathbf{D}_3'} = 10 y_3;$$

and if y_3 is introduced as an unknown, the differential coefficients in longitude and radius vector become simply $10\mathbf{D}_3$ and $10\mathbf{D}_3'$.

Similarly if $\mathbf{B}_4$ and $\mathbf{B}_4'$ denote the sums of the terms in the longitude and radius vector of satellite IV., depending on multiples of $l_3 - l_4$ (Marth's β_4), the quantity

$$\frac{\Delta(\mathbf{B}_4)}{10\mathbf{B}_4} = \frac{\Delta(\mathbf{B}_4')}{10\mathbf{B}_4'} = y_4$$

has been introduced as an unknown.

A few words must be said about the introduction of the factor 10. If it had been my object to make these differential coefficients of approximately the same order of magnitude as those of the h's, k's, and x's, it would have been better to introduce a factor 100 instead of 10; the accuracy, however, with which the quantities $\mathbf{D}_3$, $\mathbf{D}_3'$, $\mathbf{B}_4$, $\mathbf{B}_4'$ are computed is not sufficient to allow multiplication by 100, and also by introducing the factor 10 the magnitude of the coefficients is in better agreement with the relative importance of the different unknowns for a determination of the masses.

In the case of satellite II., only the first of the terms depending on $l_2 - l_3$ has been

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taken into account. If ${}^{1}_{10}y_2$ represents the correction to the coefficient in longitude, the differential coefficients become

in longitude	$0{\cdot}1 \sin \beta_2$,
in radius vector	$-0{\cdot}0273 \sin 1^\circ \cos \beta_2$.

By the introduction of the factor ${}^{1}_{10}$ the coefficient of y_2 becomes of about the same order of magnitude as those of y_3 and y_4, which are of the same relative importance for the determination of the masses. It would have been better perhaps for the sake of uniformity to adopt the same method of computation for y_2 as for y_3 and y_4, but the matter is of no great importance, and by rejecting the two small terms depending on multiples of β_2 the coefficients become independent of the error in Marth's tables, which was mentioned in the beginning of this chapter.

We have thus finally :—

$$
\begin{aligned}
&\Delta v_1 = \Delta\epsilon_1 - \cos l_1\,\Delta h_1 + \sin l_1\,\Delta k_1 + \sin a_1\, x_1\\
&\Delta \rho_1 = -\tfrac{1}{2}\sin 1^\circ \sin l_1\,\Delta h_1 - \tfrac{1}{2}\sin 1^\circ \cos l_1\,\Delta k_1 - \tfrac{1}{2}\sin 1^\circ \cos a_1\, x_1\\
&\Delta v_2 = \Delta\epsilon_2 - \cos l_2\,\Delta h_2 + \sin l_2\,\Delta k_2 + \sin a_2\, x_2 + 0{\cdot}1 \sin \beta_2\, y_2\\
&\Delta \rho_2 = -\tfrac{1}{2}\sin 1^\circ \sin l_2\,\Delta h_2 - \tfrac{1}{2}\sin 1^\circ \cos l_2\,\Delta k_2 - \tfrac{1}{2}\sin 1^\circ \cos a_2\, x_2 - 0{\cdot}0273 \cos \beta_2\, y_2\\
&\Delta v_3 = \Delta\epsilon_3 - \cos l_3\,\Delta h_3 + \sin l_3\,\Delta k_3 + \sin \gamma_3\, x_3 + 10\mathrm{D}_2\, y_3\\
&\Delta \rho_3 = -\tfrac{1}{2}\sin 1^\circ \sin l_3\,\Delta h_3 - \tfrac{1}{2}\sin 1^\circ \cos l_3\,\Delta k_3 - \tfrac{1}{2}\sin 1^\circ \cos \gamma_3\, x_3 + 10\mathrm{D}_3'\, y_3\\
&\Delta v_4 = \Delta\epsilon_4 - \cos l_4\,\Delta h_4 + \sin l_4\,\Delta k_4 + 10\mathrm{B}_4\, y_4\\
&\Delta \rho_4 = -\tfrac{1}{2}\sin 1^\circ \sin l_4\,\Delta h_4 - \tfrac{1}{2}\sin 1^\circ \cos l_4\,\Delta k_4 + 10\mathrm{B}_4'\, y_4.
\end{aligned}
\qquad \text{(II.)}
$$

The angles a_1, a_2, β_2, γ_3 are Marth's arguments and represent respectively $2(l_1 - l_2)$, $l_1 - l_2 + 180^\circ$, $l_2 - l_3 + 180^\circ$, $l_2 - l_3 + 180^\circ$.

These values have been introduced into the equations (I.), and so the complete equations of condition are formed. In this part of the work 4-figure logarithms were used. All the computations have been done in duplicate by Mr S. S. Hough, then chief assistant at the Cape Observatory, and the equations were finally written out from my own computations and then read over with the duplicate. The coefficients in the final equations were rounded off to two decimal places. The complete equations are given in the Tables XVI. and XVII. at the end of this chapter.

The next step would now be to express the h's, k's, x's, and y's in terms of the e's, ϖ's, and m's.

The approximate theoretical expressions of the x's and y's are

$$
\begin{aligned}
{}^{1}_{1000}\, x_1 &= 20\Delta m_2 & {}^{1}_{1000}\, y_2 &= 1{\cdot}7\Delta m_3\\
{}^{1}_{1000}\, x_2 &= 11\Delta m_3 + 6\Delta m_1 & {}^{1}_{1000}\, y_3 &= 2{\cdot}4\Delta m_4\\
{}^{1}_{1000}\, x_3 &= 3\Delta m_2 & {}^{1}_{1000}\, y_4 &= 1{\cdot}1\Delta m_3
\end{aligned}
$$

The coefficients of the x's are about ten times as large as those of the y's; it appears

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therefore that only two combinations of the masses, *viz.* Δm_2 and $11\Delta m_3 + 6\Delta m_1$, can be determined with such weight that the resulting values can be expected to be real corrections to the adopted masses.

The x's contain in addition to the terms of the first order in the masses considerable terms of the second order, which amount to respectively $\frac{1}{18}$, $\frac{1}{21}$, and $\frac{1}{7}$ of the terms of the first order.

The y's contain no terms of higher orders than the first. The factors of the masses in these terms are pure functions of the ratios a_1/a_2, etc., between the mean distances, and therefore to be regarded as known quantities. It would thus be permitted to introduce one unknown instead of y_2 and y_4 which are proportional to the same mass. The advantage gained thereby would, however, be very small, and the unknowns y_2 and y_4 have been kept separate. I find from Souillart's data

$$y_2 : y_4 = 1{\cdot}468305.$$

The h's and k's, as already pointed out, involve excentricities, perijoves, *and masses*, and must be taken into account even in a first approximation to a determination of the masses. We have thus for the determination of corrections to the 12 elements e, ϖ. m only 10 quantities, which are determinable with considerable weight from the observations.

The masses of the satellites can therefore not be determined from the present series of observations without combination with other observations made at such a distance of time as to allow a good determination of secular perturbations to be made. It appears thus useless to derive from these observations values of the masses, which cannot be final, but the result of this investigation must be presented in the form most suitable for comparison with older or more recent observations. I believe that this condition is best fulfilled by a derivation of corrections to the coefficients of the more important periodic inequalities for the mean epoch of the observations. Accordingly the masses, excentricities, and perijoves have not been introduced into the equations of condition.

We have thus finally the following unknown quantities :—

z_0 and z_P	2
μ	1
$\Delta\epsilon_1, \Delta\epsilon_2, \Delta\epsilon_3, \Delta\epsilon_4$	4
$\Delta p_1, \Delta q_1, \Delta p_2, \Delta q_2, \Delta p_3, \Delta q_3, \Delta p_4, \Delta q_4$ . .	8
$\Delta h_1, \Delta k_1, \Delta h_2, \Delta k_2, \Delta h_3, \Delta k_3, \Delta h_4, \Delta k_4$ . .	8
x_1, x_2, x_3	3
y_2, y_3, y_4	3
Total number of unknowns . .	29

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The conditions $\Delta\epsilon_1 = 3\Delta\epsilon_2 - 2\Delta\epsilon_3$ and $y_2 = 1{\cdot}468 y_4$ have not been introduced into the equations of condition. The introduction of these conditions, while reducing the *total* number of unknowns by two, would not diminish the number of unknowns in the individual equations, as will be seen from the following little table, where in the column A are given the numbers of unknowns in the equations derived from the distance-observations of the different pairs of satellites as actually used, and in column B these same numbers as they would be if $\Delta\epsilon_1$ and y_2 were eliminated. The number of unknowns in the position-angle equations is always two less than in the corresponding distance equations.

Pair.	A.	B.	B—A.
I.—II.	15	15	0
I.—III.	15	15	0
I.—IV.	14	15	+ 1
II.—III.	16	16	0
II.—IV.	15	14	− 1
III.—IV.	15	15	0

It appears that the elimination of $\Delta\epsilon_1$ and y_2 does not reduce the number of multiplications needed for the formation of the normal equations, whilst it would complicate the computation of the coefficients and involve more liability to mistakes than an eventual subsequent alteration of the normal equations.

First approximation.—In the final revision, after the application of the corrections mentioned above, the right-hand members of the equations were reduced by subtracting from them the values which the left-hand members take by substitution of the following set of approximate values of the unknowns. These were derived by applying to the values originally found the corrections corresponding to the corrections (3), (4), and (7) to the computed distances and position-angles. These first approximations are :—

$\mu = -0{\cdot}0120$
$z_0 = +0{\cdot}0425$
$z_r = +0{\cdot}3080$

$\Delta h_1 = +0^{\circ}{\cdot}0225$
$\Delta k_1 = -0{\cdot}0667$
$x_1 = +0{\cdot}0747$

$\Delta p_1 = +0^{\circ}{\cdot}0415$
$\Delta q_1 = -0{\cdot}0265$

$\Delta\epsilon_1 = +0^{\circ}{\cdot}1024$
$\Delta\epsilon_2 = -0{\cdot}0002$
$\Delta\epsilon_3 = -0{\cdot}0256$
$\Delta\epsilon_4 = -0{\cdot}0208$

$\Delta h_2 = +0^{\circ}{\cdot}0095$
$\Delta k_2 = -0{\cdot}0534$
$x_2 = -0{\cdot}0171$

$\Delta p_2 = +0^{\circ}{\cdot}0727$
$\Delta q_2 = +0{\cdot}0845$

$\Delta h_3 = +0^{\circ}{\cdot}0589$
$\Delta k_3 = -0{\cdot}0490$
$x_3 = -0{\cdot}0060$

$\Delta p_3 = -0^{\circ}{\cdot}0028$
$\Delta q_3 = -0{\cdot}0693$

$y_2 = -0{\cdot}1177$
$y_3 = +0{\cdot}0454$
$y_4 = -0{\cdot}0190$

$\Delta h_4 = +0^{\circ}{\cdot}0619$
$\Delta k_4 = +0{\cdot}0282$

$\Delta p_4 = +0^{\circ}{\cdot}0638$
$\Delta q_4 = -0{\cdot}0132$

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The result of the substitution of these values into the equations of condition is given in the Tables XVI. and XVII. in the columns headed "First Approximation."

The right-hand members of the equations for the determination of the final corrections to the unknowns are thus formed by subtracting these numbers from those given in the preceding columns $O-C$ and $(O-C).s \sin 1''$ respectively.

Weights.—The weights of the individual equations must to a large extent depend on judgment. The observations are too few and the number of unknowns is too large to allow a determination to be made, from the residuals of a first approximate solution, of the relative weights of observations by the two observers, made in different definition, of different distances and by different methods of observing. In assigning weights I have been guided by the following principles:—

Observations by the two observers have the same weight. This is in accordance with previous experience in the Victoria-triangulation.

The position-angles (after reduction to the great circle) are entitled to the same weight as the distances. This was also found to be the case in the Victoria-triangulation work.

Distances measured with the images at half or double distance have received half weight. There can be no doubt that this method is less accurate than the method of crossing through used for long distances. All position-angle equations have the same weight, whether the images were placed at half or double distance or the measures were made in the ordinary way.

Distances measured by crossing through have received weight unity independent of the distance. It is very probable that short distances, say under 400″ or 500″, where the curvature of the path described by the images in crossing is noticeable, are less accurately measured than distances of some thousands of seconds, but in the absence of any definite experience about the measuring of such short distances it has been thought better to give a uniform weight to all distances observed by crossing through, whether ψ was recorded or not.

Double weight was given to observations consisting of 8 pointings instead of 4. Such observations are distinguished in the tables by the addition of an asterisk (*) before the observed value.

Half weight was given:—1°. To all observations made with estimated quality of images 3,

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2°. To observations 50 and 499, in both of which the last pointing was not printed, on account of the consequent asymmetry.

A weight ¼ was given to all observations made when the quality of images was 3–4.

Entirely rejected were*: 1°. All observations on November 19 and November 30. On both dates the definition was very bad, and there is no chronometer comparison.

2°. The observations 142 and 379, in which the record of one pointing is not printed, while it is uncertain which pointing is missing, so that the epoch becomes uncertain.

3°. All observations in which the original readings were altered (*e.g.* by 1° in position-angle, or one scale-division in distance), viz. Nos. 131, 235, 441, 477. Under this head come also the observations 87 and 283, which were rejected on account of a discordance of 30′ in position-angle, though this discordance could not be explained by an evident mistake in the recorded readings; and 467, in which before and after exchanging the segments a different pair was observed.

4°. The observation 459–60, where a distance of 5″ was measured by "crossing."

5°. The observations 328–29 and 497–98, on account of their unusual difficult nature. The distances were very short, and both distance and position-angle were changing very rapidly, the difficulty of the observation being considerably increased by the near neighbourhood of the planet.

Of these the observations 87, 283, and 467 were originally inadvertently included with full weight; 328–29 and 497–98 with half weight. In the final solutions, however, these seven observations were rejected.

In order to facilitate the multiplication of the equations by the square root of the weights, 0·49 was substituted for ½.

Summary of adopted elements.—The values of the principal constants adopted for the computation of the tabular places are collected below.

I. The mean epoch of the observations is:—

1891 *October* 2, *Greenwich mean noon* = 1891·75.

The values of the constants given below are all for this epoch.

* The observations 162, 236–37, 238–39, 468, 541–42 had already been rejected in an early stage of the work, for reasons stated in the footnotes to Table II. For these observations no tabular places and equations of condition were computed.

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II. The position of the adopted fundamental plane and of the point 0 from which the longitudes in that plane are reckoned are :—

Relatively to the ecliptic and mean equinox of date :

Ascending node	$N_0 = 336^\circ\ 15'\ 6''.54$
Inclination	$J_0 = 2\ 9\ 1''.09$
Longitude of 0	$O_0 = 135\ 20\ 31''.31$
Daily motion of 0	0°.121990

Relatively to the true equator and true equinox of date :

Ascending node	$N_0 = -2^\circ.0164$
Inclination	$I_0 = 25^\circ.4368^5$
Longitude of 0	$O_0 = 135^\circ.1574$

From these quantities we derive, or get by interpolation from Marth's tables :—

$$L = 204^\circ.414$$
$$B = +1^\circ.594$$
$$P = 335^\circ\ 36'\ 58''.$$

III. The elements and constants adopted by Marth, and used in the computation of his tables are :—

$$M = 1047.700$$

$n_1 = 203^\circ.4889560$	$\log a_1 = 7.450105 - 10$	$\epsilon_1 = 178^\circ.2725$	$\varpi_1 = 235^\circ.6034$
$n_2 = 101^\circ.3747256$	$\log a_2 = 7.651788 - 10$	$\epsilon_2 = 263^\circ.3747$	$\varpi_2 = 45^\circ.5259$
$n_3 = 50^\circ.3176104$	$\log a_3 = 7.854580 - 10$	$\epsilon_3 = 35^\circ.9258$	$\varpi_3 = 205^\circ.4592$
$n_4 = 21^\circ.5710740$	$\log a_4 = 8.099795 - 10$	$\epsilon_4 = 286^\circ.8757$	$\varpi_4 = 146^\circ.7810$

$i_1 = 0^\circ.0081$	$\Omega_1 = 250^\circ.96$	$p_1 = +0^\circ.0076$	$q_1 = -0^\circ.0026$
$i_2 = 0^\circ.4915$	$\Omega_2 = 237^\circ.87$	$p_2 = +0^\circ.4162$	$q_2 = -0^\circ.2614$
$i_3 = 0^\circ.1358$	$\Omega_3 = 239^\circ.13$	$p_3 = +0^\circ.1166$	$q_3 = -0^\circ.0697$
$i_4 = 0^\circ.3470$	$\Omega_4 = 330^\circ.77$	$p_4 = +0^\circ.1694$	$q_4 = +0^\circ.3028$

The values of p and q have been computed from the adopted values of i and Ω. The following values of h and k have been computed from the periodic inequalities adopted by Marth :—

$h_1 = -0^\circ.0030$	$k_1 = -0^\circ.0034$
$h_2 = +0^\circ.0059$	$k_2 = -0^\circ.0211$
$h_3 = -0^\circ.0188$	$k_3 = -0^\circ.1727$
$h_4 = +0^\circ.4629$	$k_4 = -0^\circ.6813$

IV. The coefficients of the inequalities are taken from Souillart, and rest on the following adopted masses and value of the constant depending on the compression of Jupiter :—

$m_1 = 0.0000\ 16877$	$Jb^2 = 0^\circ.0247187$ $(b = 1)$
$m_2 = 0.0000\ 23227$	$= 0^\circ.000000005368^41$ (astronomical units)
$m_3 = 0.0000\ 88437$	$\log b = 6.66841 - 10.$
$m_4 = 0.0000\ 42475$	

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It is unnecessary here to repeat the formulas by which Marth's ephemeris was computed from these elements. They are given in full by Mr Cookson in Part II. of this volume, pp. 71 and 73–75.

Tables of equations of condition.—In the following tables are given the observed and computed values and the equations of condition. The meaning of the different columns is sufficiently explained by their headings. A few particulars, which have been explained above, may be repeated here.

Computed distance and *computed position-angle.*—These contain all the corrections enumerated in this chapter.

Observed distance and *observed position-angle.*—An asterisk (*) in this column denotes that eight pointings were taken instead of four.

Method (in the tables of distances) c denotes the ordinary method of crossing.

ψ ,, that the amount of crossing was measured.

d ,, ,, the images were at double distance.

h ,, ,, the images were at half distance.

O – C.—This is the difference between the third and fourth columns.

First approximation.—This is the result of substituting the values of the unknowns stated above on p. 115. The right-hand members to be used in the final equations are for the distances (O – C) – *First approximation*; and for the position-angles (O – C). $s \sin 1''$ – *First approximation.*

Residuals.—The residuals belonging to rejected observations are enclosed in brackets.

[TABLES XVI. and XVII.

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TABLE XVI.

SATELLITES I. AND II.—DISTANCES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Distance	Observed Distance.	Method.	Weight.	Coefficients of μ.	z_0.	x_1.	Δh_1.	Δk_1.	x_2.	Δh_2.	Δk_2.	y_2.	$\Delta\epsilon_1$.	$\Delta\epsilon_2$.	Δp_1.	Δq_1.	Δp_2.	Δq_2.	O—C.	First Approximation.	Residuals.	Rot. No.
	1891.	"	"																		"	"	"	
3	Aug. 17	109·763	109·567	e	1·0	−1·10	−0·91	+0·89	−1·27	+1·48	−0·39	+2·30	−0·52	+0·18	−1·72	−1·43	−0·02	−0·08	−0·10	+0·08	−0·196	−0·226	+0·103	3
20	19	312·037	311·809	e	1·0	−3·12	−0·32	−0·09	+1·16	−0·54	+0·48	−3·02	+0·63	+0·01	−0·24	−2·71	−0·02	+0·02	−0·04	+0·01	−0·228	−0·021	−0·035	20
34	20	316·253	316·377	e	1·0	−3·16	−0·32	−0·24	−1·55	+0·30	+1·35	+1·92	−2·14	+0·06	−1·09	+2·37	+0·02	−0·01	0·00	−0·04	+0·124	−0·060	+0·023	34
44	21	4·727	*4·001	d	0·5	−0·05	0·00	−0·24	−1·70	+0·34	−0·22	+3·26	−0·91	−0·33	+1·48	−3·20	+0·86	−0·33	−1·51	0·00	−0·126	+0·130	+0·001	44
50		4·690	*4·577	d	0·5	−0·05	0·00	−0·22	−1·58	+0·42	−0·12	+3·37	−1·06	−0·35	+1·75	−3·43	+0·82	−0·18	−1·36	−0·09	−0·113	+0·164	−0·010	50
64	24	154·288	153·989	e	1·0	−1·54	−0·65	+1·03	−1·65	+1·58	+0·05	+2·01	−0·67	+0·12	−2·19	−0·73	−0·03	−0·05	−0·06	+0·07	−0·299	−0·258	−0·008	64
80	25	39·305	39·208	ψ	1·0	−0·39	0·00	+0·98	−1·24	+1·38	+1·50	+3·03	−2·45	+0·38	+1·56	−3·85	+0·04	+0·23	−0·26	−0·28	−0·097	+0·160	−0·050	80
81		38·809	38·749	ψ	1·0	−0·39	0·00	+0·96	−1·21	+1·36	+1·52	+3·01	−2·46	+0·38	+1·51	−3·83	+0·03	+0·24	−0·26	−0·30	−0·060	+0·154	−0·009	81
103	29	128·600	128·904	e	1·0	−1·29	−0·78	+0·01	+2·18	−0·50	−1·43	−1·71	+1·87	+0·20	+2·12	+1·75	−0·16	+0·02	+0·01	−0·25	+0·304	+0·138	+0·039	103
121	31	197·319	197·171	e	1·0	−1·97	−0·51	+1·01	−2·03	+1·46	+0·54	+1·81	−0·93	+0·07	−2·48	0·00	−0·02	−0·02	+0·02	+0·05	−0·148	−0·207	+0·111	121
142	Sept. 1	50·013	50·053	ψ	0·0	−0·50	0·00	+1·19	−1·54	+1·51	+1·54	+3·37	−2·24	+0·49	+2·01	−4·03	+0·06	+0·13	−0·18	−0·14	+0·040	+0·218	[+0·043]	142
145	5	39·681	39·612	ψ	0·5	−0·40	0·00	−0·01	+1·56	−0·24	−1·95	−2·11	+2·27	+0·25	+1·11	+2·72	−0·38	+0·28	−0·20	−0·72	−0·069	−0·069	−0·174	145
164	7	261·910	261·755	e	1·0	−2·62	−0·38	+0·50	−2·35	+0·84	+1·27	+1·72	−1·48	−0·01	−2·47	+1·18	−0·01	0·00	0·00	+0·01	−0·155	−0·229	−0·005	164
194	10	195·845	196·354	e	1·0	−1·96	−0·51	+1·14	−1·21	+1·24	+1·97	+3·17	−2·23	+0·10	+1·35	+3·82	−0·01	−0·11	−0·12	−0·12	+0·509	+0·201	+0·027	194
204	13	268·183	267·782	e	1·0	−2·68	−0·37	−0·16	+1·77	−0·23	−0·44	−3·29	+0·57	+0·03	−1·44	−3·07	−0·05	+0·03	−0·08	+0·02	−0·401	−0·151	−0·052	204
221	14	286·504	286·448	e	0·5	−2·87	−0·35	+0·36	−2·19	+0·42	+1·78	+1·88	−1·84	−0·06	−2·10	+1·94	+0·01	0·00	0·00	−0·01	−0·056	−0·151	−0·035	221
249	19	48·932	48·875	e	1·0	−0·49	−2·05	+0·86	−1·08	+0·82	+2·44	+2·98	−2·34	−0·33	+0·57	−3·70	−0·06	+0·25	−0·27	−0·31	−0·057	+0·060	+0·086	249
269	20	324·946	324·750	e	1·0	−3·25	−0·31	−0·41	+1·31	−0·34	−0·49	−2·86	+0·35	+0·04	−0·55	−2·39	−0·01	+0·02	−0·03	+0·02	−0·196	−0·053	+0·019	269
294	21	289·261	289·417	e	1·0	−2·89	−0·35	+0·36	−1·98	+0·24	+2·13	+2·08	−2·00	−0·09	−1·77	+2·39	+0·01	0·00	0·00	−0·02	+0·156	−0·096	+0·090	294
300	22	44·287	44·305	ψ	1·0	−0·44	0·00	+0·64	−2·23	+0·47	+1·00	+3·59	−0·74	+0·35	+2·20	−3·53	+0·03	0·00	−0·04	0·00	+0·018	+0·212	+0·018	300
329	24	10·001	*10·015	h	0·0	−0·10	0·00	+0·30	−0·48	+0·25	+0·36	+0·87	−0·28	+0·02	+0·53	+0·91	−1·78	−1·46	−3·47	−1·06	+0·014	−0·313	[+0·271]	329
333	28	280·033	280·225	e	0·5	−2·80	−0·36	+0·42	−1·48	+0·20	+2·52	+2·49	−2·17	−0·12	−0·94	+3·04	+0·02	−0·02	−0·02	−0·05	+0·192	+0·015	−0·032	333
335	29	44·205	44·208	ψ	1·0	−0·44	0·00	+0·48	−1·83	+0·18	+0·89	+3·23	−0·37	+0·30	+1·56	−2·96	+0·02	−0·01	−0·03	+0·01	−0·057	+0·149	−0·030	335
347	Oct. 4	348·692	348·782	e	1·0	−3·49	−0·29	−1·18	+1·09	−0·95	−0·90	−2·13	+0·43	+0·06	+0·87	−1·05	0·00	−0·01	+0·01	−0·01	+0·090	+0·084	+0·094	347
369	9	141·208	141·348	e	1·0	−1·41	−0·71	+1·77	−1·60	+1·36	+1·28	+1·78	−0·79	−0·06	−1·97	+0·07	−0·01	−0·02	−0·02	+0·04	+0·050	−0·162	+0·188	369
385	13	55·863	56·008	ψ	1·0	−0·56	0·00	+0·72	−1·76	+0·13	+1·22	+2·98	−0·22	+0·27	+1·49	−2·63	0·00	0·00	+0·01	0·00	+0·145	+0·153	+0·145	385
406	Nov. 5	296·241	295·979	e	1·0	−2·96	−0·34	−0·95	+1·43	−0·11	−1·53	−2·33	+0·17	−0·08	−1·02	−1·71	−0·01	+0·01	−0·02	+0·02	−0·262	−0·109	−0·019	406
482	6	217·840	217·932	e	1·0	−2·18	−0·46	+1·34	−1·92	+0·21	+2·86	+2·26	−1·86	+0·16	−1·80	+2·67	+0·02	0·00	−0·01	−0·02	+0·092	−0·086	−0·037	482
498	9	7·450	7·141	d	0·0	−0·08	0·00	+1·08	−0·93	+0·64	+1·56	+1·90	−0·44	−0·09	+1·68	+1·93	−1·15	−1·44	−2·89	−0·32	−0·309	−0·094	[−0·345]	498
503	22	57·212	57·194	e	1·0	−0·58	−1·75	−1·65	+1·91	−0·32	−2·88	−2·18	+1·88	+0·20	+1·85	+2·68	−0·24	+0·03	−0·19	−0·34	−0·018	−0·087	−0·114	503

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of μ.	z_F.	x_1.	Δh_1.	Δk_1.	x_2.	Δh_2.	Δk_2.	y_2.	$\Delta\epsilon_1$.	$\Delta\epsilon_2$.	Δp_1.	Δq_1.	Δp_2.	Δq_2.	O—C.	First Approximation.	Residuals.	Rot. No.
	1891.	"	"																		"	"	"	
235	Sept. 18	103·978	103·747	e	0·0	−1·04	−0·96	+1·30	−1·23	+1·27	+0·50	+2·18	−0·45	−0·14	−1·42	−1·08	−0·01	−0·06	−0·06	+0·07	−0·231	−0·388	[+0·226]	235
400	Oct. 14	108·632	108·601	e	1·0	−1·09	−0·92	−1·12	+2·07	−0·38	−2·51	−2·01	+1·92	−0·22	+2·13	+2·32	−0·16	+0·02	−0·06	−0·25	−0·031	−0·151	+0·008	400
424	16	165·760	165·427	e	1·0	−1·66	−0·60	+1·90	−1·89	+1·28	+1·64	+1·70	−1·08	+0·02	−2·24	+0·72	−0·01	−0·01	0·00	+0·02	−0·333	−0·334	−0·066	424
446	19	222·329	222·657	e	1·0	−2·23	−0·45	+0·85	−1·05	+0·42	+3·00	+2·98	−1·94	−0·17	−0·08	+3·48	+0·02	−0·05	−0·06	−0·06	+0·328	−0·016	+0·098	446
522	Nov. 30	199·063	198·557	e	0·0	−1·99	−0·50	−1·80	+1·90	−0·40	−2·09	−2·42	+0·18	+0·12	−1·90	−2·04	−0·04	0·00	−0·06	+0·03	−0·506	−0·402	[+0·038]	522
528	Dec. 8	154·803	154·647	e	1·0	−1·55	−0·65	+1·90	−1·88	+0·53	+2·69	+2·08	−1·79	−0·20	−1·92	+2·56	+0·02	0·00	−0·01	−0·02	−0·156	−0·222	−0·118	528

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TABLE XVI.—*continued.*

SATELLITES I. AND III.—DISTANCES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of														O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z_r.	x_1.	Δh_1.	Δk_1.	x_3.	Δh_3.	Δk_3.	y_3.	$\Delta\epsilon_1$.	$\Delta\epsilon_3$.	Δp_1.	Δq_1.	Δp_3.	Δq_3.				
	1891.																						
10	Aug. 17	174·764	174·812	c	1·0	−1·75	−0·57	+0·82	−1·15	+1·36	−2·11	+3·01	−3·50	+0·72	−1·46	+3·85	+0·01	+0·07	−0·02	−0·17	+0·048	+0·098	−0·049	10
26	19	22·195	22·655	ψ	1·0	−0·22	0·00	−0·03	−1·17	+0·42	+0·26	+5·21	−1·08	+0·52	+0·50	−5·31	+0·66	−0·71	−2·46	−0·17	+0·460	+0·610	−0·190	26
28	20	21·878	22·588	ψ	1·0	−0·22	0·00	−0·06	−1·12	+0·34	−1·75	+4·01	−3·33	+0·61	−0·56	−5·14	−0·99	+0·95	−2·29	−2·62	+0·710	+0·585	+0·085	28
62	24	182·164	182·100	c	1·0	−1·82	−0·55	+1·01	−1·68	+1·58	−2·60	+3·52	−3·79	−0·35	−2·22	+4·68	+0·03	+0·05	−0·05	−0·15	−0·064	−0·013	−0·039	62
72	25	474·107	474·607	c	0·5	−4·74	−0·21	+1·03	−1·36	+1·46	−0·61	+2·88	−1·47	+0·38	+1·78	−0·21	0·00	−0·02	+0·02	−0·04	+0·500	+0·451	+0·063	72
101	29	251·648	251·808	c	1·0	−2·52	−0·40	+0·03	+2·16	−0·48	−0·18	−3·92	+0·75	−0·54	+2·08	−2·70	0·00	0·00	0·00	0·00	+0·160	+0·086	+0·069	101
123	31	182·636	182·797	c	1·0	−1·83	−0·55	+1·02	−2·01	+1·46	−2·97	+3·95	−3·87	+0·29	−2·47	+5·18	+0·05	+0·05	−0·07	−0·16	+0·161	+0·013	+0·161	123
133	Sept. 1	440·044	440·453	c	1·0	−4·40	−0·23	+1·16	−1·75	+1·53	−1·25	+2·74	−1·81	+0·16	+2·24	+0·63	−0·02	−0·03	+0·03	−0·08	+0·409	+0·473	−0·044	133
156	5	240·517	240·534	c	1·0	−2·41	−0·41	0·00	+1·89	−0·27	+0·30	−3·57	+0·82	+0·22	+1·63	−1·97	0·00	0·00	0·00	0·00	+0·017	+0·048	−0·036	156
166	7	197·941	197·862	c	1·0	−1·98	−0·51	+0·61	−2·35	+0·89	−3·25	+4·60	−3·82	−0·79	−2·50	+5·80	+0·07	+0·02	−0·11	−0·15	−0·079	−0·014	−0·037	166
188	10	141·066	141·907	c	1·0	−1·41	−0·70	+1·24	−1·29	+1·34	−2·27	+5·87	−2·74	−0·76	+1·57	−6·47	+0·03	+0·14	−0·33	−0·14	+0·841	+0·756	+0·037	188
206	13	310·355	309·949	c	1·0	−3·10	−0·32	−0·17	+1·81	−0·24	+0·99	−5·62	+1·22	+0·50	−1·51	−5·49	−0·08	+0·04	−0·23	+0·02	−0·406	−0·325	−0·102	206
219	14	196·926	196·820	c	0·5	−1·97	−0·51	+0·32	−2·15	+0·38	−3·41	+5·11	−3·56	−0·72	−2·05	+6·15	+0·09	−0·02	−0·16	−0·15	−0·106	+0·063	−0·138	219
251	19	225·446	225·176	c	1·0	−2·25	−0·44	−0·80	+1·09	−0·76	+1·57	−2·87	+1·47	+0·38	−0·44	+0·13	0·00	0·00	0·00	0·00	−0·270	−0·257	−0·007	251
271	20	379·075	378·834	c	1·0	−3·79	−0·26	−0·39	+1·33	−0·32	+1·04	−5·25	+0·78	+0·77	−0·61	−4·88	−0·04	+0·04	−0·14	+0·04	−0·241	−0·205	−0·048	271
292	21	178·954	178·977	c	1·0	−1·79	−0·56	+0·34	−1·94	+0·22	−3·58	+5·40	−3·25	−0·66	−1·71	+6·28	+0·10	−0·04	−0·21	−0·16	+0·023	+0·112	−0·060	292
302	22	255·181	255·075	c	1·0	−2·55	−0·39	+0·68	−2·26	+0·51	−3·14	+3·00	−2·92	−0·30	+2·24	+3·10	−0·09	0·00	−0·01	−0·23	+0·494	+0·467	+0·060	302
341	29	190·840	191·158	c	0·25	−1·91	−0·52	+0·54	−1·96	+0·22	−3·66	+3·28	−3·16	+0·29	+1·77	+3·78	−0·11	+0·04	−0·06	−0·30	+0·318	+0·464	−0·121	341
355	Oct. 4	428·781	428·783	c	0·5	−4·29	−0·23	−1·07	+1·08	−0·85	+1·38	−4·44	+0·37	−0·31	+0·67	−3·65	0·00	+0·02	−0·05	+0·03	+0·002	−0·101	+0·113	355
367	9	221·124	220·539	c	0·5	−2·21	−0·45	−1·78	+1·64	−1·36	+4·60	−4·80	+3·34	−0·68	+2·01	+5·72	−0·07	−0·11	−0·23	−0·24	−0·585	−0·425	−0·108	367
381	10	221·005	220·400	c	1·0	−2·21	−0·45	−1·43	+1·21	−1·11	+2·86	−2·73	+2·13	−0·47	−1·26	+1·77	0·00	+0·01	0·00	−0·04	−0·605	−0·472	−0·115	381
389	13	145·565	146·044	c	1·0	−1·46	−0·68	+0·75	−1·82	+0·14	−4·26	+3·61	−3·22	+0·38	+1·58	+4·31	−0·14	+0·07	−0·14	−0·36	+0·479	+0·478	+0·021	389
464	Nov. 5	171·079	171·424	c	1·0	−1·71	−0·58	+0·93	−1·39	+0·12	−2·45	+3·76	−0·24	−0·49	+0·93	−2·80	+0·02	−0·02	−0·05	+0·04	+0·345	+0·418	−0·085	464
480	6	140·127	140·336	c	1·0	−1·40	−0·71	+1·30	−1·88	+1·20	−4·67	+5·41	−1·74	−0·64	−1·75	−5·66	−0·16	+0·05	−0·40	−0·09	+0·209	+0·330	−0·159	480
516	27	301·771	302·208	c	1·0	−3·02	−0·33	+1·04	−0·95	+0·48	−4·05	+4·59	−0·62	+0·34	+0·11	−4·36	−0·01	+0·03	−0·09	+0·02	+0·437	+0·507	−0·091	516
550	Dec. 14	376·156	375·956	c	0·5	−3·76	−0·27	−1·41	+1·48	−0·16	+2·61	−2·51	+0·78	+0·15	−1·27	−0·77	−0·01	0·00	−0·02	+0·02	−0·200	−0·332	+0·134	550

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of														O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z_r.	x_1.	Δh_1.	Δk_1.	x_3.	Δh_3.	Δk_3.	y_3.	$\Delta\epsilon_1$.	$\Delta\epsilon_3$.	Δp_1.	Δq_1.	Δp_3.	Δq_3.				
	1891.																						
241	Sept. 18	365·217	364·686	c	1·0	−3·65	−0·27	−1·19	+1·16	−1·17	+3·76	−3·91	+3·67	+0·40	+1·21	+4·94	0·00	−0·07	−0·08	−0·17	−0·531	−0·427	−0·050	241
365	Oct. 6	144·763	145·287	ψ	1·0	−1·45	0·00	+0·56	−1·70	+0·14	−4·06	+3·55	−3·26	+0·72	+1·36	+4·24	−0·13	+0·08	−0·13	−0·37	+0·524	+0·487	+0·065	365
406	14	403·958	404·017	c	1·0	−4·04	−0·25	+1·25	−2·23	+0·50	−1·92	+2·99	−0·92	+0·43	−2·24	−0·68	−0·02	0·00	−0·03	+0·05	+0·059	+0·018	+0·052	406
430	16	170·705	170·310	c	0·5	−1·71	−0·59	−1·88	+1·78	−1·31	+4·78	−5·00	+3·12	−0·60	+2·14	+5·82	−0·11	−0·13	−0·32	−0·28	−0·395	−0·589	+0·243	430
434	17	228·763	228·154	c	1·0	−2·29	−0·44	−1·78	+1·52	−1·28	+3·39	−2·83	+2·48	−0·30	−1·84	+2·55	+0·01	+0·01	0·00	−0·04	−0·609	−0·696	+0·110	434
442	19	56·149	56·047	c	1·0	−0·56	−1·78	+0·89	−1·08	+0·48	−3·81	+5·62	−1·65	+0·72	+0·07	+5·82	+0·13	−0·33	−0·89	−0·15	−0·102	−0·179	+0·080	442
524	Nov. 30	322·312	321·966	c	0·0	−3·22	−0·31	−1·83	+1·92	−0·44	+2·88	−3·25	+0·36	−0·43	−1·93	−2·21	−0·03	0·00	−0·06	+0·06	−0·346	−0·513	[+0·164]	524
526	Dec. 8	81·818	81·352	c	1·0	−0·82	−1·23	−1·88	+1·87	−0·50	+4·63	−4·68	+1·09	−0·63	+1·90	−4·68	+0·08	0·00	−0·21	0·00	−0·466	−0·502	+0·012	526

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TABLE XVI.—*continued.*

SATELLITES I. AND IV.—DISTANCES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of μ.	z_G.	x_1.	Δh_1.	Δk_1.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_1$.	$\Delta\epsilon_4$.	Δp_1.	Δq_1.	Δp_4.	Δq_4.	O—C.	First Approximation.	Residuals.	Rot. No.
	1891.																						
1	Aug. 17	23".127	22".344	ψ	1·0	−0·23	0·00	−0·39	+0·66	−0·70	−5·03	+2·40	+0·07	+0·85	+5·56	−0·59	−2·11	−9·12	−3·70	−0".783	−0".739	−0".014	1
18	19	519·484	519·377	c	1·0	−5·19	−0·19	−0·17	+1·11	−0·60	−6·74	+7·09	+0·40	−0·08	+9·21	+0·03	−0·05	−0·11	−0·24	−0·107	−0·321	+0·289	18
38	20	417·873	417·836	c	1·0	−4·18	−0·24	+0·28	+1·63	−0·30	−5·00	+5·88	+0·70	+1·24	+6·06	−0·05	+0·03	−0·01	−0·25	−0·037	−0·039	+0·060	38
60	24	440·153	440·281	c	1·0	−4·40	−0·23	−1·00	+1·71	−1·57	−9·48	+2·22	+0·25	+2·25	−9·18	+0·03	+0·05	−0·27	+0·03	+0·128	−0·014	+0·125	60
83	25	31·925	31·369	ψ	1·0	−0·32	0·00	−0·80	+0·95	−1·11	−8·17	+3·41	+0·07	−1·20	−8·78	−0·15	−1·51	−6·47	−1·87	−0·556	−0·666	+0·078	83
93	29	724·391	724·290	c	1·0	−7·24	−0·14	−0·11	−2·03	+0·37	+4·71	−4·37	−0·46	−1·87	+3·45	0·00	0·00	0·00	0·00	−0·101	−0·083	+0·022	93
125	31	547·026	547·161	c	1·0	−5·47	−0·18	+1·04	−1·99	+1·48	+7·15	−1·21	−0·14	−2·45	−5·15	−0·04	−0·04	−0·24	+0·16	+0·135	+0·243	−0·102	125
131	Sept. 1	519·351	520·345	c	0·0	−5·19	−0·19	+1·15	−1·78	+1·52	+9·09	−1·64	−0·03	+2·27	−8·36	+0·03	+0·04	−0·21	+0·06	+0·994	+0·909	[+0·085]	131
151	5	323·294	322·893	c	0·5	−3·23	−0·31	+0·01	+1·74	−0·24	−6·34	+6·66	−0·11	+1·39	+8·32	−0·09	+0·05	−0·15	−0·43	−0·401	−0·166	−0·189	151
196	10	172·383	171·716	c	1·0	−1·72	−0·58	−1·12	+1·18	−1·22	−10·00	+2·47	+0·30	−1·30	−9·97	−0·02	−0·26	−1·14	+0·01	−0·667	−0·529	−0·179	196
214	13	301·120	301·134	c	1·0	−3·01	−0·33	+0·20	−1·97	+0·28	+7·03	−6·45	+0·37	+1·76	+8·87	−0·11	+0·04	−0·22	−0·48	+0·014	+0·202	−0·131	214
227	14	633·703	633·634	c	1·0	−6·34	−0·16	+0·45	−2·26	+0·51	+5·36	−5·36	+0·67	−2·23	+5·80	+0·01	0·00	0·00	−0·04	−0·069	−0·162	+0·156	227
259	19	299·613	300·442	c	1·0	−3·00	−0·33	+0·64	−1·12	+0·60	+10·49	−3·76	+0·13	+0·12	−11·07	−0·05	+0·12	−0·58	−0·13	+0·829	+0·748	+0·050	259
267	20	220·906	220·303	c	1·0	−2·21	−0·45	−0·43	+1·28	−0·37	−9·78	+5·63	−0·14	−0·47	+11·26	+0·10	−0·13	−0·63	−0·40	−0·603	−0·723	+0·165	267
296	21	220·182	219·859	c	1·0	−2·20	−0·45	−0·37	+2·00	−0·25	−7·80	+6·57	−0·32	+1·81	+9·80	−0·17	+0·06	−0·45	−0·65	−0·323	−0·299	+0·014	296
310	22	556·815	556·275	c	1·0	−5·57	−0·18	−0·81	+2·31	−0·63	−5·82	+5·92	−0·46	−2·35	+7·03	+0·02	0·00	−0·02	−0·08	−0·540	−0·477	−0·028	310
345	Oct. 4	354·639	355·255	c	0·5	−3·55	−0·28	+1·22	−1·11	+1·00	+8·36	−0·75	−0·45	−0·96	−7·26	0·00	−0·09	−0·36	+0·17	+0·616	+0·565	+0·029	345
377	9	650·395	650·233	c	1·0	−6·50	−0·15	−1·70	+1·46	−1·33	−5·12	+4·78	+0·66	+1·79	+5·05	−0·01	−0·02	−0·01	−0·12	−0·162	−0·048	−0·040	377
398	13	396·784	396·193	c	1·0	−3·97	−0·25	−0·87	+1·98	−0·21	−9·56	+1·58	+0·27	−1·83	−9·30	−0·09	+0·03	−0·42	+0·05	−0·591	−0·548	−0·078	398
474	Nov. 5	461·801	462·100	c	1·0	−4·62	−0·22	+1·08	−1·66	+0·10	+5·54	−0·85	+0·75	+1·40	−2·75	+0·01	−0·01	−0·05	+0·06	+0·299	+0·584	−0·248	474
486	6	541·140	541·546	c	0·5	−5·41	−0·18	+1·43	−1·98	+0·27	+7·28	−0·40	+0·54	−1·90	−6·05	−0·05	+0·01	−0·18	+0·11	+0·406	+0·448	−0·023	486
512	22	561·364	561·619	c	0·5	−5·61	−0·18	+1·82	−1·99	+0·48	+5·62	−0·65	−0·40	−2·01	−3·44	−0·03	0·00	−0·10	+0·10	+0·255	+0·314	−0·051	512
514	27	198·661	198·094	c	1·0	−1·99	−0·51	−1·05	+0·94	−0·50	−6·16	+5·28	+0·26	−0·17	+7·62	−0·02	+0·07	−0·17	−0·29	−0·567	−0·444	−0·078	514
548	Dec. 14	431·067	430·564	c	1·0	−4·31	−0·23	−1·37	+1·44	−0·15	−5·27	+5·08	−0·06	−1·21	+6·69	+0·02	−0·01	−0·04	−0·09	−0·503	−0·464	+0·005	548

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of μ.	z_F.	x_1.	Δh_1.	Δk_1.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_1$.	$\Delta\epsilon_4$.	Δp_1.	Δq_1.	Δp_4.	Δq_4.	O—C.	First Approximation.	Residuals.	Rot. No.
	1891.																						
84	Aug. 27	463".555	463".311	c	0·25	−4·64	−0·22	+0·31	−1·11	+0·62	+7·69	−6·81	−0·32	+0·03	+9·89	+0·04	−0·06	−0·18	−0·28	−0".244	+0".029	−0".226	84
361	Oct. 6	57·302	56·289	ψ	1·0	−0·57	0·00	−0·53	+1·55	−0·14	−10·03	+3·94	+0·13	−1·14	+10·76	+0·43	−0·34	−2·34	−0·75	−1·013	−0·954	−0·018	361
404	14	36·204	35·981	ψ	1·0	−0·36	0·00	−1·17	+2·12	−0·45	−9·66	+3·27	+0·13	+2·11	−10·16	+0·69	−0·02	−2·94	−0·77	−0·223	−0·232	−0·030	404
426	16	297·173	296·979	c	1·0	−2·97	−0·34	+1·90	−1·86	+1·29	+7·33	−5·74	−0·34	−2·21	+8·79	+0·04	+0·04	−0·12	−0·19	−0·194	−0·170	+0·004	426
436	17	601·890	602·148	c	1·0	−6·02	−0·17	+1·75	−1·48	+1·27	+5·56	−4·99	−0·47	+1·78	+6·02	−0·01	−0·03	−0·03	−0·14	+0·258	+0·303	−0·008	436
450	19	743·675	744·079	c	1·0	−7·44	−0·13	+0·80	−1·16	+0·35	+5·39	−1·32	−0·33	−0·26	−1·76	−0·01	+0·01	−0·04	+0·06	+0·404	+0·369	+0·059	450
530	Dec. 8	560·240	560·333	c	1·0	−5·60	−0·18	+1·92	−1·89	+0·56	+4·29	−1·63	−0·30	−1·94	−0·57	−0·02	0·00	−0·04	+0·07	+0·093	+0·109	+0·004	530

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TABLE XVI.—*continued.*

SATELLITES II. AND III.—DISTANCES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of															O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z.	x_2.	Δh_2.	Δk_2.	y_2.	x_3.	Δh_3.	Δk_3.	y_3.	$\Delta\epsilon_2$.	$\Delta\epsilon_3$.	Δp_2.	Δq_2.	Δp_3.	Δq_3.				
	1891.																							
12	Aug. 17	87″811	88″374	c	1·0	−0·88	−1·13	+0·51	−2·48	+0·46	−0·20	−2·07	+2·98	−3·44	+0·71	+1·77	+3·75	−0·31	+0·21	−0·06	−0·60	+0″563	+0″258	+0″186	12
22	19	331·528	331·985	c	1·0	−3·32	−0·30	+0·47	−3·06	+0·66	+0·01	+0·37	+5·65	−1·73	+0·56	−2·78	−5·72	−0·14	+0·04	−0·24	−0·01	+0·457	+0·537	+0·054	22
36	20	325·732	325·340	c	1·0	−3·26	−0·31	+1·33	+1·89	−2·11	+0·04	+2·11	−4·64	+3·93	−0·72	+2·31	+5·96	−0·02	−0·19	−0·19	−0·24	−0·386	−0·535	+0·038	36
52	24	13·507	13·628	d	0·5	−0·14	0·00	−0·12	−1·60	+0·62	−0·08	−2·08	+3·03	−3·11	+0·30	+0·36	+3·96	−1·28	+1·87	−1·21	−3·39	+0·121	+0·523	−0·409	52
74	25	436·223	436·649	c	0·5	−4·36	−0·23	−1·44	−3·13	+2·42	−0·39	−0·59	+2·89	−1·45	+0·38	+3·92	−0·25	−0·04	−0·04	+0·05	−0·08	+0·426	+0·221	+0·006	74
105	29	122·381	122·139	c	1·0	−1·22	−0·82	+1·41	+1·69	−1·85	−0·19	−0·19	−3·95	+0·74	−0·54	−1·69	−2·77	+0·02	−0·27	−0·34	+0·25	−0·242	−0·167	+0·021	105
109	31	30·682	30·630	ψ	0·25	−0·31	0·00	+0·78	+1·66	−1·15	+0·03	+2·91	−4·04	+3·83	−0·32	+0·49	−5·30	−0·34	+0·87	−0·71	−1·30	−0·052	−0·117	+0·011	109
140	Sept. 1	407·907	408·184	c	1·0	−4·08	−0·25	−1·52	−3·40	+2·22	−0·40	−1·14	+2·78	−1·71	+0·17	+4·05	+0·41	−0·06	−0·05	+0·05	−0·11	+0·277	+0·212	−0·138	140
154	5	167·492	167 141	c	1·0	−1·67	−0·60	+1·88	+1·96	−2·18	−0·25	+0·32	−3·54	+0·83	+0·22	−2·44	−1·92	−0·03	−0·18	−0·20	+0·20	−0·351	−0·097	−0·108	154
172	7	56·208	56·130	ψ	1·0	−0·56	0·00	+1·26	+1·58	−1·46	−0·01	+3·26	−4·50	+3·81	+0·80	+1·06	−5·71	−0·12	+0·47	−0·45	−0·63	−0·078	−0·135	−0·044	172
175	8	322·284	322·573	c	1·0	−3·22	−0·31	−1·18	−3·74	+1·58	−0·40	+0·11	+2·71	−2·28	−0·48	+4·05	+1·66	−0·11	−0·04	+0·04	−0·18	+0·289	+0·180	−0·097	175
186	10	33·422	33·108	ψ	1·0	−0·36	0·00	+1·63	+2·87	−1·86	+0·08	+1·92	−5·15	+2·32	+0·66	+3·39	+5·64	−1·49	−1·32	−2·96	−1·24	−0·314	−0·577	+0 060	186
202	13	48·015	47·920	c	1·0	−0·48	−2·09	+0·42	+3·23	−0·55	−0·03	+0·94	−5·55	+1·17	+0·47	+3·00	−5·39	+0·42	−0·11	−0·69	+0·07	−0·095	−0·239	−0·107	202
217	14	92·787	92·806	ψ	0·5	−0·93	0·00	+1·84	+1·91	−1·89	−0·07	+3·40	−5·12	+3·55	+0·72	+2·05	−6·16	+0·02	+0·25	−0·29	−0·28	+0·019	−0·156	+0·005	217
253	19	266·794	266·398	c	1·0	−2·67	−0·38	+2·47	+2·92	−2·36	−0·35	+1·55	−2·88	+1·44	+0·38	−3·65	+0·08	−0·05	−0·06	−0·05	+0·11	−0·396	−0·082	−0·074	253
279	20	57·040	57·041	ψ	1·0	−0·57	0·00	+0·52	+2·96	−0·38	−0·04	+1·09	−5·28	+0·83	+0·78	+2·56	−4·95	+0·32	−0·14	−0·56	+0·13	+0·001	−0·143	−0·041	279
284	21	113·225	113·215	c	1·0	−1·13	−0·88	+2·21	+2·18	−2·09	−0·10	+3·54	−5·46	+3·20	+0·66	+2·60	−6·30	+0·05	+0·18	−0·24	−0·17	−0·010	−0·194	−0·041	284
304	22	213·135	213·547	c	1·0	−2·13	−0·47	−1·06	−3·63	+0·79	−0·36	−3·11	+2·99	−2·90	−0·29	+3·59	+3·07	−0·18	+0·01	−0·02	−0·29	+0·412	+0·242	−0·013	304
321	23	560·592	560·974	c	1·0	−5·61	−0·18	−1·65	−1·71	+1·51	−0·16	−0·89	+3·75	−0·56	−0·53	+1·24	−2·34	0·00	0·00	0·00	0·00	+0·382	+0·298	+0·043	321
343	Oct. 4	82·662	82·333	ψ	0·5	−0·83	0·00	+0·93	+2·07	−0·47	−0·07	+1·35	−4·31	+0·37	−0·30	+0·92	−3·44	+0·14	−0·18	−0·32	+0·20	−0·329	−0·177	−0·221	343
379	9	379·026	378·338	c	0·0	−3·79	−0·26	+1·18	+1·84	−0·67	−0·06	+4·61	−4·68	+3·38	−0·67	−0·20	+5·62	+0·05	−0·09	−0·11	−0·12	−0·688	−0·566	[−0·062]	379
387	13	85·497	85·934	c	1·0	−0·86	−1·16	−1·23	−3·00	+0·22	−0·27	−4·25	+3·62	−3·21	+0·38	+2·65	+4·32	−0·38	+0·16	−0·24	−0·60	+0·437	+0·302	−0·006	387
478	Nov. 6	75·417	75·152	c	1·0	−0·76	−1·33	+2·86	+2·28	−1·85	+0·17	+4·61	−5·37	+1·69	+0·64	+2·70	+5·60	−0·25	−0·50	−0·86	−0·19	−0·265	−0·583	+0·108	478
490	8	148·079	147·790	c	1·0	−1·48	−0·68	+2·29	+1·71	−1·56	−0·16	+2·52	−2·63	+1·11	+0·36	−1·70	0·00	−0·01	−0·06	−0·04	+0·10	−0·289	−0·153	−0·046	490
505	22	177·855	177·678	c	1·0	−1·78	−0·56	+2·87	+2·16	−1·89	−0·20	+3·05	−2·41	+1·88	−0·41	−2·67	+1·58	−0·01	−0·02	0·00	+0·04	−0·177	−0·222	+0·210	505

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of															O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z_r.	x_2.	Δh_2.	Δk_2.	y_2.	x_3.	Δh_3.	Δk_3.	y_3.	$\Delta\epsilon_2$.	$\Delta\epsilon_3$.	Δp_2.	Δq_2.	Δp_3.	Δq_3.				
	1891.																							
243	Sept. 18	460″240	459″494	c	1·0	−4·60	−0·22	+0·43	+2·30	−0·38	−0·16	+3·76	−3·88	+3·67	+0·40	−1·34	+4·91	+0·05	−0·05	−0·04	−0·10	−0″746	−0″490	−0″105	243
410	Oct. 14	535·028	535·635	c	0·5	−5·35	−0·19	−2·37	−1·90	+1·82	−0·20	−1·89	+3·02	−0·88	+0·44	+2·07	−0·78	0·00	−0·02	+0·02	−0·03	+0·607	+0·227	+0·302	410
418	15	280·907	281·163	c	0·25	−2·81	−0·36	−1·98	−3·50	+0·75	−0·11	−2·64	+5·32	−0·75	+0·79	−3·51	−5·09	−0·15	0·00	−0·23	+0·04	+0·256	+0·417	+0·071	418
422	16	313·263	312·528	c	1·0	−3·13	−0·32	+1·66	+1·70	−1·11	+0·02	+4·76	−5·09	+3·07	−0·62	+0·76	+5·88	+0·03	−0·14	−0·18	−0·15	−0·735	−0·673	−0·077	422
444	19	160·657	160·625	c	1·0	−1·61	−0·62	+3·00	+3·00	−1·94	−0·16	+3·89	−5·68	+1·70	−0·73	+3·50	−5·89	+0·06	+0·06	−0·12	−0·02	−0·032	−0·385	+0·082	444
458	23	239·142	238·511	c	1·0	−2·39	−0·42	+2·13	+1·78	−1·48	+0·08	+4·82	−5·35	+2·66	+0·03	+1·56	+5·96	0·00	−0·18	−0·26	−0·15	−0·631	−0·679	−0·036	458
538	Dec. 8	238·223	238·078	c	0·5	−2·38	−0·42	+2·55	+1·94	−1·76	−0·19	+4·73	−4·73	+1·21	−0·63	+2·38	−4·79	+0·01	+0·03	−0·05	0·00	−0·145	−0·278	−0·049	538

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TABLE XVI.—*continued.*

SATELLITES II. AND IV.—DISTANCES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of														O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z_0.	x_2.	Δh_2.	Δk_2.	y_2.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_2$.	$\Delta\epsilon_4$.	Δp_2.	Δq_2.	Δp_4.	Δq_4.				
	1891.	"	"																	"	"	"	
6	Aug. 17	132·012	131·038	c	1·0	−1·32	−0·76	−0·44	+2·34	−0·49	+0·18	−9·96	+5·07	+0·16	−1·53	+11·16	+0·39	−0·30	−1·28	−0·54	−0·972	−0·762	−0·073	6
24	19	225·428	225·317	c	1·0	−2·25	−0·44	−0·47	+3·11	−0·68	−0·01	−6·63	+7·03	+0·41	+2·86	+9·06	−0·26	+0·07	−0·31	−0·71	−0·111	−0·350	+0·119	24
30	20	715·963	715·802	c	1·0	−7·16	−0·14	+1·39	+2·00	−2·21	+0·05	−5·06	+5·96	+0·68	+2·54	+6·23	−0·01	−0·07	−0·01	−0·20	−0·161	−0·107	−0·161	30
58	24	603·529	603·311	c	1·0	−6·04	−0·17	+0·12	+1·95	−0·72	+0·11	−9·48	+2·21	+0·25	−0·58	−9·16	−0·06	+0·07	−0·26	+0·03	−0·218	−0·247	+0·050	58
78	25	81·324	80·588	ψ	1·0	−0·81	0·00	+1·44	+2·94	−2·36	+0·37	−9·80	+3·98	+0·09	−3·72	−10·50	−0·78	−0·85	−3·06	−0·86	−0·736	−0·496	−0·076	78
95	29	828·999	829·140	c	1·0	−8·29	−0·12	−1·53	−1·76	+2·00	+0·22	+4·71	−4·35	−0·46	+1·98	+3·40	0·00	−0·04	+0·02	−0·16	+0·141	+0·076	−0·023	95
115	31	361·095	361·622	c	0·25	−3·61	−0·28	−0·66	−1·75	+1·05	−0·05	+7·04	−1·23	−0·15	−0·25	−4·96	+0·05	−0·11	−0·29	+0·21	+0·527	+0·455	+0·093	115
138	Sept. 1	479·848	480·813	c	1·0	−4·80	−0·21	−1·50	−3·42	+2·20	−0·40	+9·16	−1·69	−0·01	+4·06	−8·47	+0·05	+0·04	−0·17	+0·04	+0·965	+0·662	+0·077	138
158	5	263·007	262·403	c	1·0	−2·63	−0·38	+1·85	+1·92	−2·14	−0·24	−6·27	+6·63	−0·11	−2·35	+8·21	+0·01	+0·08	−0·07	−0·22	−0·604	−0·219	−0·091	158
168	7	873·434	873·501	c	1·0	−8·73	−0·11	+1·21	+1·72	−1·42	−0·01	−4·91	+2·96	−0·39	+1·06	+0·67	0·00	−0·01	+0·01	−0·02	+0·067	−0·056	+0·077	168
177	8	649·578	649·168	c	1·0	−6·49	−0·15	+1·22	+3·74	−1·62	+0·40	−6·29	+1·42	−0·50	−4·06	−3·48	−0·08	−0·03	−0·16	+0·16	−0·410	−0·160	−0·129	177
192	10	369·963	370·161	c	1·0	−3·70	−0·27	+1·96	+3·20	−2·22	+0·10	−10·04	+2·43	−0·31	+3·84	−9·99	+0·07	+0·07	−0·28	0·00	+0·198	−0·210	+0·088	192
212	13	539·796	539·591	c	1·0	−5·40	−0·10	−0·51	−3·40	+0·65	+0·03	+7·05	−6·48	+0·37	−3·25	+8·99	+0·06	−0·01	−0·07	−0·14	−0·205	+0·058	+0·008	212
223	14	354·529	354·465	c	0·25	−3·54	−0·28	−1·75	−1·87	+1·81	+0·06	+5·38	−5·39	+0·66	−1·88	+5·86	0·00	+0·02	0·00	−0·07	−0·064	−0·013	+0·132	223
230		352·568	352·525	c	1·0	−3·52	−0·28	−1·65	−1·82	+1·70	+0·05	+5·34	−5·33	+0·67	−1·66	+5·73	0·00	+0·02	0·00	−0·07	−0·043	+0·066	+0·116	230
261	19	266·806	267·659	c	1·0	−2·67	−0·37	−2·48	−2·78	+2·38	+0·33	+10·51	−3·78	+0·12	+3·52	−11·09	+0·09	+0·13	−0·42	−0·09	+0·853	+0·630	−0·034	261
277	20	91·149	91·505	c	1·0	−0·91	−1·09	−0·51	−2·89	+0·37	+0·04	+9·57	−5·63	+0·15	−2·48	−11·08	−0·69	+0·32	−1·82	−1·20	+0·356	+0·459	−0·001	277
288	21	494·328	494·086	c	1·0	−4·94	−0·20	+2·18	+2·14	−2·05	−0·09	−7·88	+6·58	−0·31	+2·52	+9·89	−0·03	−0·14	−0·23	−0·32	−0·242	−0·374	+0·009	288
312	22	604·160	603·609	c	1·0	−6·04	−0·17	+1·16	+3·67	−0·89	+0·37	−5·81	+5·92	−0·46	−3·68	+7·01	+0·02	0·00	−0·01	−0·06	−0·551	−0·244	−0·054	312
323	23	394·868	394·600	c	1·0	−3·95	−0·25	+1·61	+1·71	−1·47	+0·15	−4·82	+4·10	−0·47	−1·17	+3·32	0·00	−0·01	0·00	+0·02	−0·268	−0·157	−0·043	323
353	Oct. 4	692·940	693·504	c	0·5	−6·93	−0·14	−0·87	−2·20	+0·38	+0·06	+8·43	−0·75	−0·44	−1·22	−7·37	−0·04	+0·05	−0·17	+0·08	+0·564	+0·695	−0·047	353
375	9	778·325	777·967	c	1·0	−7·78	−0·13	+1·21	+1·82	−0·71	−0·05	−5·13	+4·79	+0·66	−0·10	+5·08	+0·01	−0·03	−0·01	−0·08	−0·358	−0·177	−0·107	375
396	13	463·021	462·561	c	1·0	−4·63	−0·22	+1·33	+3·11	−0·28	+0·28	−9·54	+1·56	+0·27	−2·83	−9·28	−0·12	+0·04	−0·35	+0·04	−0·460	−0·351	+0·023	396
472	Nov. 5	736·559	736·997	c	1·0	−7·36	−0·14	−1·61	−2·46	+0·15	−0·09	+5·52	−0·86	+0·75	−1·95	−2·71	−0·02	+0·02	−0·05	+0·07	+0·438	+0·447	+0·185	472
492	8	297·518	298·272	c	1·0	−2·98	−0·34	−2·27	−1·70	+1·54	+0·15	+9·49	−3·01	+0·13	+1·66	−9·91	+0·01	+0·10	−0·29	−0·06	+0·754	+0·642	−0·006	492
511	22	647·904	648·316	c	0·5	−6·48	−0·15	−2·78	−2·07	+1·86	+0·19	+5·60	−0·65	−0·40	+2·54	−3·42	0·00	+0·01	−0·02	+0·02	+0·412	+0·368	−0·090	511
544	Dec. 14	163·287	162·972	c	1·0	−1·63	−0·61	+1·72	+1·74	−0·34	−0·10	−5·30	+5·08	−0·06	+0·89	+6·73	−0·08	+0·09	−0·13	−0·30	−0·315	−0·314	−0·040	544

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of														O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z_0.	x_2.	Δh_2.	Δk_2.	y_2.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_2$.	$\Delta\epsilon_4$.	Δp_2.	Δq_2.	Δp_4.	Δq_4.				
	1891.	"	"																	"	"	"	
88	Aug. 27	171·478	170·694	c	0·25	−1·71	−0·58	−1·68	−2·31	+2·35	−0·07	+7·61	−6·81	−0·32	−3·01	+9·82	+0·06	+0·17	−0·26	−0·43	−0·784	−0·179	−0·357	88
359	Oct. 6	106·573	105·185	c	1·0	−1·06	−0·95	+0·93	+2·90	−0·21	+0·26	−10·22	+3·99	+0·14	−2·48	+10·95	+0·42	−0·21	−1·25	−0·40	−1·388	−1·101	−0·111	359
402	14	83·642	83·749	ψ	1·0	−0·84	0·00	−2·43	−1·96	+1·85	−0·21	+9·87	−3·24	−0·14	+2·22	+10·35	−0·18	−0·79	−2·20	−0·57	+0·107	+0·055	−0·002	402
416	15	224·533	224·175	c	0·5	−2·24	−0·45	−1·95	−3·49	+0·72	−0·10	+9·20	−5·07	−0·18	−3·48	+10·44	+0·15	0·00	−0·34	−0·25	−0·358	+0·067	−0·128	416
420	16	131·895	131·505	c	1·0	−1·32	−0·76	−1·68	−1·70	+1·13	+0·02	+7·37	−5·73	−0·34	−0·81	+8·83	−0·04	+0·21	−0·31	−0·50	−0·390	−0·149	−0·151	420
454	19	508·237	508·219	c	1·0	−5·08	−0·20	−2·99	−2·86	+1·98	+0·16	+5·41	−1·29	−0·33	−3·37	−1·83	−0·04	−0·05	−0·10	+0·17	−0·018	+0·227	+0·014	454
456	23	232·854	232·245	c	1·0	−2·33	−0·43	+2·16	+1·79	−1·51	+0·08	−9·55	+4·54	+0·02	+1·61	+10·57	−0·01	−0·26	−0·66	−0·33	−0·609	−0·796	+0·101	456
536	Dec. 8	397·866	397·950	c	0·5	−3·98	−0·25	−2·57	−1·96	+1·77	+0·19	+4·31	−1·58	−0·30	−2·41	−0·68	−0·02	−0·05	−0·07	+0·13	+0·084	+0·110	+0·158	536

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TABLE XVI.—*continued.*

SATELLITES III. AND IV.—DISTANCES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of														O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z_0.	x_3.	Δh_3.	Δk_3.	y_3.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_3$.	$\Delta\epsilon_4$.	Δp_3.	Δq_3.	Δp_4.	Δq_4.				
	1891.																						
8	Aug. 17	211″773	211″302	c	1·0	−2·12	−0·47	−2·09	+3·02	−3·49	+0·71	−9·97	+5·10	+0·16	+3·85	+11·18	−0·10	−0·72	−1·19	−0·50	−0″471	−0″428	+0″018	8
16	19	552·878	553·184	c	1·0	−5·53	−0·18	+0·46	+5·60	−1·63	+0·54	−6·77	+7·08	+0·40	−5·62	+9·24	+0·04	0·00	−0·03	−0·06	+0·306	+0·214	+0·123	16
40	20	403·154	403·585	c	1·0	−4·03	−0·25	−2·15	+4·60	−3·96	+0·72	−4·99	+5·87	+0·70	−5·94	+6·04	−0·03	−0·04	0·00	+0·09	+0·431	+0·420	+0·025	40
46	21	284·504	284·673	c	0·25	−2·84	−0·35	−1·66	+2·61	−2·67	+0·50	−4·54	+3·92	+0·86	−2·17	+2·28	+0·01	−0·07	−0·04	+0·11	+0·169	+0·161	+0·026	46
54	24	622·721	622·575	c	1·0	−6·23	−0·16	−2·61	+3·61	−3·84	+0·35	−9·45	+2·18	+0·26	+4·82	−9·11	+0·02	+0·06	−0·11	+0·01	−0·146	+0·027	−0·170	54
70	25	528·483	528·232	c	0·5	−5·29	−0·19	−0·64	+2·86	−1·50	+0·37	−10·23	+4·04	+0·10	−0·16	−10·92	−0·10	+0·17	−0·33	−0·09	−0·251	−0·004	−0·275	70
97	29	965·002	965·126	c	1·0	−9·65	−0·10	−0·16	−3·87	+0·76	−0·53	+4·70	−4·33	−0·46	−2·61	+3·36	0·00	0·00	0·00	+0·01	+0·124	−0·001	+0·167	97
113	31	387·528	387·902	c	0·25	−3·88	−0·26	+2·97	−4·09	+3·90	−0·32	+7·01	−1·24	−0·15	−5·35	−4·92	−0·14	−0·27	−0·44	+0·32	+0·374	+0·199	+0·158	113
127	Sept. 1	88·326	88·825	ψ	1·0	−0·88	0·00	+1·33	−2·66	+1·89	−0·15	+8·89	−1·62	−0·03	−0·79	−8·15	+0·42	−1·06	−1·95	+0·54	+0·499	+0·382	+0·098	127
147	5	83·780	83·296	ψ	0·5	−0·84	0·00	−0·37	+3·40	−0·87	−0·21	−6·33	+6·57	−0·11	+1·66	+8·28	−0·66	+0·72	−0·59	−1·62	−0·484	−0·239	−0·199	147
170	7	817·089	817·234	c	1·0	−8·17	−0·12	−3·26	+4·56	−3·82	−0·80	−4·92	+2·94	−0·39	+5·76	+0·63	−0·04	−0·06	+0·05	−0·11	+0·145	+0·166	+0·035	170
178	8	976·335	976·357	c	1·0	−9·76	−0·10	+0·14	+2·71	−2·24	−0·48	−6·31	+1·40	−0·50	+1·56	−3·53	−0·01	+0·02	−0·03	+0·03	+0·022	+0·048	−0·008	178
181		976·292	*976·221	c	2·0	−9·76	−0·10	+0·18	+2·70	−2·19	−0·48	−6·34	+1·38	−0·50	+1·47	−3·60	−0·01	+0·03	−0·03	+0·03	−0·071	+0·046	−0·099	181
189	10	329·369	329·856	c	1·0	−3·29	−0·30	−2·28	+5·85	−2·75	−0·77	−9·99	+2·41	−0·31	−6·46	−9·93	−0·46	−0·20	−0·88	+0·02	+0·487	+0·272	+0·133	189
210	13	590·380	590·203	c	1·0	−5·90	−0·17	+1·03	−5·66	+1·26	+0·51	+7·07	−6·48	+0·37	−5·57	+8·92	+0·03	0·00	−0·02	−0·04	−0·177	−0·109	−0·027	210
225	14	442·697	442·684	c	0·25	−4·43	−0·23	+3·46	−5·04	+3·60	+0·72	+5·36	−5·39	+0·66	−6·11	+5·83	−0·04	−0·04	+0·01	+0·10	−0·013	−0·213	+0·224	225
229		438·992	438·834	c	1·0	−4·39	−0·23	+3·48	−4·99	+3·62	+0·72	+5·34	−5·34	+0·67	−6·07	+5·76	−0·04	−0·04	+0·01	+0·10	−0·158	−0·211	+0·077	229
257	19	525·130	525·691	c	1·0	−5·25	−0·19	+1·51	−2·90	+1·40	+0·39	+10·51	−3·74	+0·13	−0·02	−11·08	−0·08	+0·17	−0·32	−0·07	+0·561	+0·568	−0·023	257
275	20	150·986	151·492	c	1·0	−1·51	−0·66	+1·07	−5·21	+0·81	+0·77	+9·65	−5·65	+0·15	−4·86	−11·16	−0·93	+0·23	−1·42	−0·93	+0·506	+0·371	+0·064	275
290	21	386·144	385·918	c	1·0	−3·86	−0·26	−3·57	+5·42	−3·24	−0·66	−7·85	+6·58	−0·31	+6·28	+9·86	−0·30	−0·22	−0·38	−0·54	−0·226	−0·155	+0·010	290
306	22	825·760	825·653	c	1·0	−8·26	−0·12	−3·10	+2·98	−2·88	−0·29	−5·84	+5·94	−0·46	+3·04	+7·08	0·00	−0·10	−0·04	−0·18	−0·107	+0·014	−0·046	306
319	23	956·443	956·620	c	1·0	−9·56	−0·10	−0·90	+3·72	−0·58	−0·52	−4·83	+4·15	−0·47	−2·28	+3·41	0·00	0·00	0·00	0·00	+0·177	+0·153	+0·060	319
324		955·067	955 278	c	1·0	−9·55	−0·10	−0·88	+3·80	−0·54	−0·53	−4·82	+4·08	−0·47	−2·43	+3·29	0·00	0·00	0·00	+0·01	+0·211	+0·162	+0·084	324
337	29	34·957	35·080	ψ	1·0	−0·35	0·00	−3·68	+3·28	−3·18	+0·29	−8·43	+5·94	0·00	+3·85	−10·08	+0·25	+1·08	−1·31	−1·45	+0·123	+0·004	+0·096	337
351	Oct. 4	775·000	775·634	c	0·5	−7·75	−0·13	+1·38	−4·42	+0·38	−0·30	+8·42	−0·76	−0·44	−3·61	−7·35	−0·11	+0·07	−0·21	+0·10	+0·574	+0·520	+0·048	351
371	9	406·413	406·788	c	1·0	−4·06	−0·25	−4·62	+4·77	−3·37	+0·68	−5·15	+4·83	+0·65	−5·70	+5·15	−0·04	−0·05	+0·01	+0·11	+0·375	+0·390	−0·002	371
383	10	276·339	276·433	c	1·0	−2·76	−0·30	−2·83	+2·73	−2·11	+0·47	−4·80	+2·82	+0·83	−1·72	+1·22	+0·01	−0·09	−0·04	+0·15	+0·094	+0·107	0 000	383
394	13	564·702	564·694	c	1·0	−5·65	−0·18	−4·24	+3·57	−3·22	+0·38	−9·53	+1·53	+0·27	+4·27	−9·27	+0·02	+0·06	−0·12	+0·01	−0·008	−0·010	+0·002	394
476	Nov. 5	285·282	285·200	c	1·0	−2·85	−0·35	+2·55	−3·95	+0·22	+0·50	+5·56	−0·84	+0·75	+3·13	−2·79	+0·03	−0·02	−0·04	+0·05	−0·082	+0·062	−0·109	476
488	6	422·914	422·903	c	0·5	−4·23	−0·24	+4·76	−5·41	+1·83	+0·65	+7·30	−0·40	+0·54	+5·70	−6·08	+0·02	0·00	−0·02	+0·01	−0·011	+0·043	−0·009	488
496	8	439·506	439·852	c	1·0	−4·40	−0·23	+2·49	−2·66	+1·05	+0·36	+9·49	−3·05	+0·13	−0·15	−9·92	−0·06	+0·14	−0·26	−0·05	+0·286	+0·519	−0 251	496
499	9	111·927	112·554	c	0·5	−1·12	−0·89	+3·35	−4·65	+0·49	+0·71	+8·66	−4·69	+0·14	−4·32	−9·82	−0·80	+0·24	−1·25	−0·79	+0·627	+0·320	+0·234	499
507	22	819·764	820·040	c	1·0	−8·20	−0·12	+3·03	−2·41	+1·85	−0·40	+5·57	−0·67	−0·41	+1·55	−3·35	0·00	+0·02	−0·02	+0·03	+0·276	+0·186	+0·124	507
518	27	500·974	500·821	c	1·0	−5·01	−0·20	−4·06	+4·61	−0·63	+0·35	−6·14	+5·29	+0·26	−4·38	+7·59	+0·02	0·00	−0·02	−0·03	−0·153	+0·111	−0·230	518
540	Dec. 14	44·386	44·250	ψ	1·0	−0·44	0·00	−2·55	+2·41	−0·85	−0·15	−5·29	+5·03	−0·06	+0·54	+6·70	−0·39	+0·64	−0·53	−1·20	−0·136	−0·201	+0·101	540

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of														O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z_0.	x_3.	Δh_3.	Δk_3.	y_3.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_3$.	$\Delta\epsilon_4$.	Δp_3.	Δq_3.	Δp_4.	Δq_4.				
	1891.																						
86	Aug. 27	401″390	400″218	c	0·25	−4·01	−0·25	+2·18	−5·21	+3·64	−0·59	+7·64	−6·79	−0·32	+6·31	+9·84	−0·30	−0·26	−0·37	−0·60	−1″172	−0″630	−0″450	86
245	Sept. 18	616·245	616·435	c	1·0	−6·16	−0·16	+3·76	−3·85	+3·67	+0·40	+9·77	−1·82	+0·30	+4·88	−9·39	+0·02	+0·06	−0·11	+0·02	+0·190	+0·221	+0·003	245
357	Oct. 6	178·827	178·342	c	1·0	−1·79	−0·56	−4·11	+3·65	−3·30	+0·73	−10·21	+3·97	+0·14	+4·41	+10·94	−0·28	−0·70	−1·28	−0·40	−0·485	−0·607	+0·185	357
408	14	438·665	438·584	c	0·5	−4·39	−0·23	−1·90	+3·00	−0·90	+0·43	−10·08	+3·41	+0·13	−0·73	−10·61	−0·10	+0·17	−0·33	−0·09	−0·081	−0·088	−0·025	408
414	15	71·566	71·766	ψ	0·5	−0·72	0·00	−2·49	+5·06	−0·69	+0·75	−8·88	+4·91	+0·17	−4·81	−10·10	−1·63	+0·34	−2·36	−1·70	+0·200	+0·160	−0·027	414
432	16	466·369	465·637	c	0·5	−4·66	−0·21	+4·79	−5·00	+3·13	−0·61	+7·27	−5·73	−0·31	+5·82	+8·72	−0·18	−0·16	−0·22	−0·36	−0·732	−0·545	−0·101	432
438	17	830·482	830·261	c	0·5	−8·30	−0·12	+3·34	−2·81	+2·43	−0·30	+5·55	−4·98	−0·47	+2·46	+5·99	0·00	−0·07	−0·02	−0·12	−0·221	−0·225	+0·069	438
452	19	674·224	674·211	c	1·0	−6·74	−0·15	+3·99	−5·68	+1·80	−0·73	+5·40	−1·30	−0·33	−5·93	−1·79	−0·11	−0·02	−0·11	+0·17	−0·013	+0·040	−0·050	452
460	23	5·944	4·588	ψ	0·0	−0·06	0·00	+3·46	−3·76	+1·96	+0·03	+6·77	−3·33	0·00	+4·23	−7·54	+3·65	+2·16	−6·68	−3·39	−1·356	−0·503	[−0·855]	460
534	Dec. 8	638·263	638·356	c	0·5	−6·38	−0·16	+4·70	−4·72	+1·17	−0·63	+4·31	−1·59	−0·30	−4·76	−0·66	−0·07	0·00	−0·06	+0·11	+0·093	−0·005	+0·104	534

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TABLE XVI.—*continued.*

SATELLITES II. AND IV.—DISTANCES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of														O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z_0.	x_2.	Δh_2.	Δk_2.	y_2.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_2$.	$\Delta\epsilon_4$.	Δp_2.	Δq_2.	Δp_4.	Δq_4.				
	1891.																						
6	Aug. 17	132·012	131·038	c	1·0	−1·32	−0·76	−0·44	+2·34	−0·49	+0·18	−9·96	+5·07	+0·16	−1·53	+11·16	+0·39	−0·30	−1·28	−0·54	−0·972	−0·762	−0·073	6
24	19	225·428	225·317	c	1·0	−2·25	−0·44	−0·47	+3·11	−0·68	−0·01	−6·63	+7·03	+0·41	+2·86	+9·06	−0·26	+0·07	−0·31	−0·71	−0·111	−0·350	+0·119	24
30	20	715·963	715·802	c	1·0	−7·16	−0·14	+1·39	+2·00	−2·21	+0·05	−5·06	+5·96	+0·68	+2·54	+6·23	−0·01	−0·07	−0·01	−0·20	−0·161	−0·107	−0·161	30
58	24	603·529	603·311	c	1·0	−6·04	−0·17	+0·12	+1·95	−0·72	+0·11	−9·48	+2·21	+0·25	−0·58	−9·16	−0·06	+0·07	−0·26	+0·03	−0·218	−0·247	+0·050	58
78	25	81·324	80·588	♆	1·0	−0·81	0·00	+1·44	+2·94	−2·36	+0·37	−9·80	+3·98	+0·09	−3·72	−10·50	−0·78	−0·85	−3·06	−0·86	−0·736	−0·496	−0·076	78
95	29	828·999	829·140	c	1·0	−8·29	−0·12	−1·53	−1·76	+2·00	+0·22	+4·71	−4·35	−0·46	+1·98	+3·40	0·00	−0·04	+0·02	−0·16	+0·141	+0·076	−0·023	95
115	31	301·095	301·622	c	0·25	−3·61	−0·28	−0·66	−1·75	+1·05	−0·05	+7·04	−1·23	−0·15	−0·25	−4·96	+0·05	−0·11	−0·29	+0·21	+0·527	+0·455	+0·093	115
138	Sept. 1	479·848	480·813	c	1·0	−4·80	−0·21	−1·50	−3·42	+2·20	−0·40	+9·16	−1·69	−0·01	+4·06	−8·47	+0·05	+0·04	−0·17	+0·04	+0·965	+0·662	+0·077	138
158	5	263·007	262·403	c	1·0	−2·63	−0·38	+1·85	+1·92	−2·14	−0·24	−6·27	+6·63	−0·11	−2·35	+8·21	+0·01	+0·08	−0·07	−0·22	−0·604	−0·219	−0·091	158
168	7	873·434	873·501	c	1·0	−8·73	−0·11	+1·21	+1·72	−1·42	−0·01	−4·91	+2·96	−0·39	+1·06	+0·67	0·00	−0·01	+0·01	−0·02	+0·067	−0·056	+0·077	168
177	8	649·578	649·168	c	1·0	−6·49	−0·15	+1·22	+3·74	−1·62	+0·40	−6·29	+1·42	−0·50	−4·06	−3·48	−0·08	−0·03	−0·16	+0·16	−0·410	−0·160	−0·129	177
192	10	369·963	370·161	c	1·0	−3·70	−0·27	+1·96	+3·20	−2·22	+0·10	−10·04	+2·43	−0·31	+3·84	−9·99	+0·07	+0·07	−0·28	0·00	+0·198	−0·210	+0·088	192
212	13	539·796	539·591	c	1·0	−5·40	−0·19	−0·51	−3·40	+0·65	+0·03	+7·05	−6·48	+0·37	−3·25	+8·90	+0·06	−0·01	−0·07	−0·14	−0·205	+0·058	+0·008	212
223	14	354·529	354·405	c	0·25	−3·54	−0·28	−1·75	−1·87	+1·81	+0·06	+5·38	−5·39	+0·66	−1·88	+5·86	0·00	+0·02	0·00	−0·07	−0·064	−0·013	+0·132	223
230		352·568	352·525	c	1·0	−3·52	−0·28	−1·65	−1·82	+1·70	+0·05	+5·34	−5·33	+0·67	−1·66	+5·73	0·00	+0·02	0·00	−0·07	−0·043	+0·006	+0·116	230
261	19	266·806	267·659	c	1·0	−2·67	−0·37	−2·48	−2·78	+2·38	+0·33	+10·51	−3·78	+0·12	+3·52	−11·09	+0·09	+0·13	−0·42	−0·09	+0·853	+0·630	−0·034	261
277	20	91·149	91·505	c	1·0	−0·91	−1·09	−0·51	−2·89	+0·37	+0·04	+9·57	−5·63	+0·15	−2·48	−11·08	−0·69	+0·32	−1·82	−1·20	+0·356	+0·459	−0·001	277
288	21	494·328	494·086	c	1·0	−4·94	−0·20	+2·18	+2·14	−2·05	−0·09	−7·88	+6·58	−0·31	+2·52	+9·89	−0·03	−0·14	−0·23	−0·32	−0·242	−0·374	+0·009	288
312	22	604·160	603·609	c	1·0	−6·04	−0·17	+1·16	+3·67	−0·89	+0·37	−5·81	+5·92	−0·46	−3·68	+7·01	+0·02	0·00	−0·01	−0·06	−0·551	−0·244	−0·054	312
323	23	394·868	394·600	c	1·0	−3·95	−0·25	+1·61	+1·71	−1·47	+0·15	−4·82	+4·10	−0·47	−1·17	+3·32	0·00	−0·01	0·00	+0·02	−0·268	−0·157	−0·043	323
353	Oct. 4	692·940	693·504	c	0·5	−6·93	−0·14	−0·87	−2·20	+0·38	+0·06	+8·43	−0·75	−0·44	−1·22	−7·37	−0·04	+0·05	−0·17	+0·08	+0·564	+0·695	−0·047	353
375	9	778·325	777·967	c	1·0	−7·78	−0·13	+1·21	+1·82	−0·71	−0·05	−5·13	+4·79	+0·66	−0·10	+5·08	+0·01	−0·03	−0·01	−0·08	−0·358	−0·177	−0·107	375
396	13	403·021	462·561	c	1·0	−4·63	−0·22	+1·33	+3·11	−0·28	+0·28	−9·54	+1·56	+0·27	−2·83	−9·28	−0·12	+0·04	−0·35	+0·04	−0·460	−0·351	+0·023	396
472	Nov. 5	736·559	736·997	c	1·0	−7·36	−0·14	−1·61	−2·46	+0·15	−0·09	+5·52	−0·86	+0·75	−1·95	−2·71	−0·02	+0·02	−0·05	+0·07	+0·438	+0·447	+0·185	472
492	8	297·518	298·272	c	1·0	−2·98	−0·34	−2·27	−1·70	+1·54	+0·15	+9·49	−3·01	+0·13	+1·66	−9·91	+0·01	+0·10	−0·29	−0·06	+0·754	+0·642	−0·006	492
511	22	647·904	648·316	c	0·5	−6·48	−0·15	−2·78	−2·07	+1·86	+0·19	+5·60	−0·65	−0·40	+2·54	−3·42	0·00	+0·01	−0·02	+0·02	+0·412	+0 368	−0·090	511
544	Dec. 14	163·287	162·972	c	1·0	−1·63	−0·61	+1·72	+1·74	−0·34	−0·10	−5·30	+5·08	−0·06	+0·89	+6·73	−0·08	+0·09	−0·13	−0·30	−0·315	−0·314	−0·040	544

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of														O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z_0.	x_2.	Δh_2.	Δk_2.	y_2.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_2$.	$\Delta\epsilon_4$.	Δp_2.	Δq_2.	Δp_4.	Δq_4.				
	1891.																						
88	Aug. 27	171·478	170·694	c	0·25	−1·71	−0·58	−1·68	−2·31	+2·35	−0·07	+7·61	−6·81	−0·32	−3·01	+9·82	+0·06	+0·17	−0·26	−0·43	−0·784	−0·179	−0·357	88
359	Oct. 6	106·573	105·185	c	1·0	−1·06	−0·95	+0·93	+2·90	−0·21	+0·26	−10·22	+3·99	+0·14	−2·48	+10·95	+0·42	−0·21	−1·25	−0·40	−1·388	−1·101	−0·111	359
402	14	83·642	83·749	♆	1·0	−0·84	0·00	−2·43	−1·96	+1·85	−0·21	+9·87	−3·24	−0·14	+2·22	+10·35	−0·18	−0·79	−2·20	−0·57	+0·107	+0·055	−0·002	402
416	15	224·533	224·175	c	0·5	−2·24	−0·45	−1·95	−3·49	+0·72	−0·10	+9·20	−5·07	−0·18	−3·48	+10·44	+0·15	0·00	−0·34	−0·25	−0·358	+0·067	−0·128	416
420	16	131·895	131·505	c	1·0	−1·32	−0·76	−1·68	−1·70	+1·13	+0·02	+7·37	−5·73	−0·34	−0·81	+8·83	−0·04	+0·21	−0·31	−0·50	−0·390	−0·149	−0·151	420
454	19	508·237	508·219	c	1·0	−5·08	−0·20	−2·99	−2·86	+1·98	+0·16	+5·41	−1·29	−0·33	−3·37	−1·83	−0·04	−0·05	−0·10	+0·17	−0·018	+0·227	+0·014	454
456	23	232·854	232·245	c	1·0	−2·33	−0·43	+2·16	+1·79	−1·51	+0·08	−9·55	+4·54	+0·02	+1·61	+10·57	−0·01	−0·26	−0·66	−0·33	−0·609	−0·796	+0·101	456
536	Dec. 8	397·866	397·950	c	0·5	−3·98	−0·25	−2·57	−1·96	+1·77	+0·19	+4·31	−1·58	−0·30	−2·41	−0·68	−0·02	−0·05	−0·07	+0·13	+0·084	+0·110	+0·158	536

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TABLE XVI.—*continued.*

SATELLITES III. AND IV.—DISTANCES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of														O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z_0.	x_3.	Δh_3.	Δk_3.	y_3.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_3$.	$\Delta\epsilon_4$.	Δp_3.	Δq_3.	Δp_4.	Δq_4.				
	1891.																						
8	Aug. 17	211″·773	211″·302	c	1·0	−2·12	−0·47	−2·09	+3·02	−3·49	+0·71	−9·97	+5·10	+0·16	+3·85	+11·18	−0·10	−0·72	−1·19	−0·50	−0″·471	−0″·428	+0″·018	8
16	19	552·878	553·184	c	1·0	−5·53	−0·18	+0·46	+5·60	−1·63	+0·54	−6·77	+7·08	+0·40	−5·62	+9·24	+0·04	0·00	−0·03	−0·06	+0·306	+0·214	+0·123	16
40	20	403·154	403·585	c	1·0	−4·03	−0·25	−2·15	+4·60	−3·96	+0·72	−4·99	+5·87	+0·70	−5·94	+6·04	−0·03	−0·04	0·00	+0·09	+0·431	+0·420	+0·025	40
46	21	284·504	284·673	c	0·25	−2·84	−0·35	−1·66	+2·61	−2·67	+0·50	−4·54	+3·92	+0·86	−2·17	+2·28	+0·01	−0·07	−0·04	+0·11	+0·169	+0·161	+0·026	46
54	24	622·721	622·575	c	1·0	−6·23	−0·16	−2·61	+3·61	−3·84	+0·35	−9·45	+2·18	+0·26	+4·82	−9·11	+0·02	+0·06	−0·11	+0·01	−0·146	+0·027	−0·170	54
70	25	528·483	528·232	c	0·5	−5·29	−0·19	−0·64	+2·86	−1·50	+0·37	−10·23	+4·04	+0·10	−0·16	−10·92	−0·10	+0·17	−0·33	−0·09	−0·251	−0·004	−0·275	70
97	29	965·002	965·126	c	1·0	−9·65	−0·10	−0·16	−3·87	+0·76	−0·53	+4·70	−4·33	−0·46	−2·61	+3·36	0·00	0·00	0·00	+0·01	+0·124	−0·001	+0·167	97
113	31	387·528	387·902	c	0·25	−3·88	−0·26	+2·97	−4·09	+3·90	−0·32	+7·01	−1·24	−0·15	−5·35	−4·92	−0·14	−0·27	−0·44	+0·32	+0·374	+0·199	+0·158	113
127	Sept. 1	88·326	88·825	☿	1·0	−0·88	0·00	+1·33	−2·66	+1·89	−0·15	+8·89	−1·62	−0·03	−0·79	−8·15	+0·42	−1·06	−1·95	+0·54	+0·499	+0·382	+0·098	127
147	5	83·780	83·296	☿	0·5	−0·84	0·00	−0·37	+3·40	−0·87	−0·21	−6·33	+6·57	−0·11	+1·66	+8·28	−0·66	+0·72	−0·59	−1·62	−0·484	−0·239	−0·199	147
170	7	817·089	817·234	c	1·0	−8·17	−0·12	−3·26	+4·56	−3·82	−0·80	−4·92	+2·94	−0·39	+5·76	+0·63	−0·04	−0·06	+0·05	−0·11	+0·145	+0·166	+0·035	170
178	8	976·335	976·357	c	1·0	−9·76	−0·10	+0·14	+2·71	−2·24	−0·48	−6·31	+1·40	−0·50	+1·56	−3·53	−0·01	+0·02	−0·03	+0·03	+0·022	+0·048	−0·008	178
181		976·292	*976·221	c	2·0	−9·76	−0·10	+0·18	+2·70	−2·19	−0·48	−6·34	+1·38	−0·50	+1·47	−3·60	−0·01	+0·03	−0·03	+0·03	−0·071	+0·046	−0·099	181
189	10	329·369	329·856	c	1·0	−3·29	−0·30	−2·28	+5·85	−2·75	−0·77	−9·99	+2·41	−0·31	−6·46	−9·93	−0·46	−0·20	−0·88	+0·02	+0·487	+0·272	+0·133	189
210	13	590·380	590·203	c	1·0	−5·90	−0·17	+1·03	−5·66	+1·26	+0·51	+7·07	−6·48	+0·37	−5·57	+8·92	+0·03	0·00	−0·02	−0·04	−0·177	−0·109	−0·027	210
225	14	442·697	442·684	c	0·25	−4·43	−0·23	+3·46	−5·04	+3·60	+0·72	+5·36	−5·39	+0·66	−6·11	+5·83	−0·04	−0·04	+0·01	+0·10	−0·013	−0·213	+0·224	225
229		438·992	438·834	c	1·0	−4·39	−0·23	+3·48	−4·99	+3·62	+0·72	+5·34	−5·34	+0·67	−6·07	+5·76	−0·04	−0·04	+0·01	+0·10	−0·158	−0·211	+0·077	229
257	19	525·130	525·691	c	1·0	−5·25	−0·19	+1·51	−2·90	+1·40	+0·39	+10·51	−3·74	+0·13	−0·02	−11·08	−0·08	+0·17	−0·32	−0·07	+0·561	+0·568	−0·023	257
275	20	150·986	151·492	c	1·0	−1·51	−0·66	+1·07	−5·21	+0·81	+0·77	+9·65	−5·65	+0·15	−4·86	−11·16	−0·93	+0·23	−1·42	−0·93	+0·506	+0·371	+0·064	275
290	21	386·144	385·918	c	1·0	−3·86	−0·26	−3·57	+5·42	−3·24	−0·66	−7·85	+6·58	−0·31	+6·28	+9·86	−0·30	−0·22	−0·38	−0·54	−0·226	−0·155	+0·010	290
306	22	825·760	825·653	c	1·0	−8·26	−0·12	−3·10	+2·98	−2·88	−0·29	−5·84	+5·94	−0·46	+3·04	+7·08	0·00	−0·10	−0·04	−0·18	−0·107	+0·014	−0·046	306
319	23	956·443	956·620	c	1·0	−9·56	−0·10	−0·90	+3·72	−0·58	−0·52	−4·83	+4·15	−0·47	−2·28	+3·41	0·00	0·00	0·00	0·00	+0·177	+0·153	+0·060	319
324		955·067	955·278	c	1·0	−9·55	−0·10	−0·88	+3·80	−0·54	−0·53	−4·82	+4·08	−0·47	−2·43	+3·29	0·00	0·00	0·00	+0·01	+0·211	+0·162	+0·084	324
337	29	34·957	35·080	☿	1·0	−0·35	0·00	−3·68	+3·28	−3·18	+0·29	−8·43	+5·94	0·00	+3·85	−10·08	+0·25	+1·08	−1·31	−1·45	+0·123	+0·004	+0·096	337
351	Oct. 4	775·060	775·634	c	0·5	−7·75	−0·13	+1·38	−4·42	+0·38	−0·30	+8·42	−0·76	−0·44	−3·61	−7·35	−0·11	+0·07	−0·21	+0·10	+0·574	+0·520	+0·048	351
371	9	406·413	406·788	c	1·0	−4·06	−0·25	−4·62	+4·77	−3·37	+0·68	−5·15	+4·83	+0·65	−5·70	+5·15	−0·04	−0·05	+0·01	+0·11	+0·375	+0·390	−0·002	371
383	10	276·339	276·433	c	1·0	−2·76	−0·30	−2·83	+2·73	−2·11	+0·47	−4·80	+2·82	+0·83	−1·72	+1·22	+0·01	−0·09	−0·04	+0·15	+0·094	+0·107	0·000	383
394	13	564·702	564·694	c	1·0	−5·65	−0·18	−4·24	+3·57	−3·22	+0·38	−9·53	+1·53	+0·27	+4·27	−9·27	+0·02	+0·06	−0·12	+0·01	−0·008	−0·010	+0·002	394
476	Nov. 5	285·282	285·200	c	1·0	−2·85	−0·35	+2·55	−3·95	+0·22	+0·50	+5·56	−0·84	+0·75	+3·13	−2·79	+0·03	−0·02	−0·04	+0·05	−0·082	+0·062	−0·109	476
488	6	422·914	422·903	c	0·5	−4·23	−0·24	+4·76	−5·41	+1·83	+0·65	+7·30	−0·40	+0·54	+5·70	−6·08	+0·02	0·00	−0·02	+0·01	−0·011	+0·043	−0·009	488
496	8	439·566	439·852	c	1·0	−4·40	−0·23	+2·49	−2·66	+1·05	+0·36	+9·49	−3·05	+0·13	−0·15	−9·92	−0·06	+0·14	−0·26	−0·05	+0·286	+0·519	−0 251	496
499	9	111·927	112·554	c	0·5	−1·12	−0·89	+3·35	−4·65	+0·49	+0·71	+8·66	−4·69	+0·14	−4·32	−9·82	−0·80	+0·24	−1·25	−0·79	+0·627	+0·320	+0·234	499
507	22	819·764	820·040	c	1·0	−8·20	−0·12	+3·03	−2·41	+1·85	−0·40	+5·57	−0·67	−0·41	+1·55	−3·35	0·00	+0·02	−0·02	+0·03	+0·276	+0·186	+0·124	507
518	27	500·974	500·821	c	1·0	−5·01	−0·20	−4·06	+4·61	−0·63	+0·35	−6·14	+5·29	+0·26	−4·38	+7·59	+0·02	0·00	−0·02	−0·03	−0·153	+0·111	−0·230	518
540	Dec. 14	44·386	44·250	♆	1·0	−0·44	0·00	−2·55	+2·41	−0·85	−0·15	−5·29	+5·03	−0·06	+0·54	+6·70	−0·39	+0·64	−0·53	−1·20	−0·136	−0·201	+0·101	540

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Distance.	Observed Distance.	Method.	Weight.	Coefficients of														O—C.	First Approximation.	Residuals.	Rot. No.
						μ.	z_F.	x_3.	Δh_3.	Δk_3.	y_3.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_3$.	$\Delta\epsilon_4$.	Δp_3.	Δq_3.	Δp_4.	Δq_4.				
	1891.																						
86	Aug. 27	401″·390	400″·218	c	0·25	−4·01	−0·25	+2·18	−5·21	+3·64	−0·59	+7·64	−6·79	−0·32	+6·31	+9·84	−0·30	−0·26	−0·37	−0·60	−1″·172	−0″·630	−0″·450	86
245	Sept. 18	616·245	616·435	c	1·0	−6·16	−0·16	+3·76	−3·85	+3·67	+0·40	+9·77	−1·82	+0·30	+4·88	−9·39	+0·02	+0·06	−0·11	+0·02	+0·190	+0·221	+0·003	245
357	Oct. 6	178·827	178·342	c	1·0	−1·79	−0·56	−4·11	+3·65	−3·30	+0·73	−10·21	+3·97	+0·14	+4·41	+10·94	−0·28	−0·70	−1·28	−0·40	−0·485	−0·607	+0·185	357
408	14	438·665	438·584	c	0·5	−4·39	−0·23	−1·90	+3·00	−0·90	+0·43	−10·08	+3·41	+0·13	−0·73	−10·61	−0·10	+0·17	−0·33	−0·09	−0·081	−0·088	−0·025	408
414	15	71·566	71·766	☿	0·5	−0·72	0·00	−2·49	+5·06	−0·69	+0·75	−8·88	+4·91	+0·17	−4·81	−10·10	−1·63	+0·34	−2·36	−1·70	+0·200	+0·160	−0·027	414
432	16	466·369	465·637	c	0·5	−4·66	−0·21	+4·79	−5·00	+3·13	−0·61	+7·27	−5·73	−0·31	+5·82	+8·72	−0·18	−0·16	−0·22	−0·36	−0·732	−0·545	−0·101	432
438	17	830·482	830·261	c	0·5	−8·30	−0·12	+3·34	−2·81	+2·43	−0·30	+5·55	−4·98	−0·47	+2·46	+5·99	0·00	−0·07	−0·02	−0·12	−0·221	−0·225	+0·069	438
452	19	674·224	674·211	c	1·0	−6·74	−0·15	+3·99	−5·68	+1·80	−0·73	+5·40	−1·30	−0·33	−5·93	−1·79	−0·11	−0·02	−0·11	+0·17	−0·013	+0·040	−0·050	452
460	23	5·944	4·588	♆	0·0	−0·06	0·00	+3·46	−3·76	+1·96	+0·03	+6·77	−3·33	0·00	+4·23	−7·54	+3·65	+2·16	−6·68	−3·39	−1·356	−0·503	[−0·855]	460
534	Dec. 8	638·263	638·356	c	0·5	−6·38	−0·16	+4·70	−4·72	+1·17	−0·63	+4·31	−1·59	−0·30	−4·76	−0·66	−0·07	0·00	−0·06	+0·11	+0·093	−0·005	+0·104	534

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TABLE XVII.

SATELLITES I. AND II. POSITION-ANGLES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of Δp_1.	Δq_1.	Δp_2.	Δq_2.	x_1.	Δh_1.	Δk_1.	x_2.	Δh_2.	Δk_2.	y_2.	$\Delta\epsilon_1$.	$\Delta\epsilon_2$.	O—C.	(O—C) × s sin 1″.	First Approximation.	Residuals.	Rot. No.
	1891.	° ′ ″	° ′ ″															′ ″	″	″	″	
4	Aug. 17	66 58 38	66 57 10	1·0	−0·50	−2·48	−3·15	+2·51	+0·03	+0·06	+0·01	+0·11	−0·16	−0·06	−0·02	0·00	+0·16	− 1 28	−0·046	+0·031	−0·089	4
19	19	245 39 11	245 35 8	1·0	−1·46	+2·06	−3·81	+1·19	−0·08	−0·06	−0·06	−0·08	+0·12	+0·05	0·00	+0·08	+0·13	− 4 3	−0·367	−0·288	−0·085	19
33	20	64 29 54	64 32 12	1·0	−2·08	+1·45	+0·46	+3·98	+0·06	+0·03	+0·06	+0·09	−0·02	−0·09	+0·01	+0·06	+0·09	+ 2 18	+0·211	+0·254	−0·072	33
43	21	85 59 22	*84 22 42	1·0	+2·20	−0·84	−3·79	+0·01	+0·16	+0·73	−0·09	+0·16	−1·39	+0·32	+0·14	−0·64	+1·36	− 96 40	−0·131	−0·243	+0·027	43
49		84 8 15	*81 41 57	2·0	+2·33	−0·51	−3·83	−0·26	+0·13	+0·72	−0·11	+0·10	−1·28	+0·33	+0·13	−0·68	+1·29	−146 18	−0·197	−0·273	−0·004	49
63	24	66 18 58	66 19 37	1·0	−1·28	−2·20	−2·59	+3·13	+0·03	+0·04	+0·02	+0·10	−0·12	−0·07	−0·01	+0·01	+0·13	+ 39	+0·029	+0·081	−0·070	63
79	25	70 29 13	69 47 2	1·0	+0·39	+2·50	−2·72	−3·02	−0·03	+0·13	−0·06	−0·09	−0·32	+0·19	−0·04	−0·08	+0·35	− 42 11	−0·481	−0·512	+0·029	79
102	29	241 39 41	241 51 14	1·0	+2·54	−0·24	−0·21	+4·05	−0·04	+0·10	−0·07	−0·18	−0·06	+0·20	+0·02	+0·09	+0·19	+ 11 33	+0·433	+0·440	−0·014	102
120	31	65 55 18	66 0 19	1·0	−1·89	−1·73	−1·93	+3·58	+0·03	+0·04	+0·02	+0·09	−0·09	−0·07	−0·01	+0·01	+0·11	+ 5 1	+0·288	+0·136	+0·130	120
141	Sept. 1	68 26 24	67 48 43	1·0	+1·03	+2·33	−3·23	−2·52	−0·01	+0·10	−0·03	−0·04	−0·23	+0·09	−0·02	−0·07	+0·24	− 37 41	−0·547	−0·475	−0·066	141
144	5	234 40 37	235 19 27	0·5	+2·02	−1·50	+1·06	+3·87	−0·07	+0·24	−0·10	−0·43	−0·36	+0·49	+0·05	+0·13	+0·56	+ 38 50	+0·450	+0·520	−0·106	144
163	7	65 27 56	65 30 53	1·0	−2·51	−0·53	−0·74	+4·00	+0·03	+0·04	+0·03	+0·08	−0·05	−0·08	−0·01	+0·03	+0·09	+ 2 57	+0·225	+0·202	−0·006	163
193	10	62 50 59	62 59 0	1·0	+0·23	+2·54	+2·84	+2·86	+0·12	−0·04	+0·12	+0·13	+0·10	−0·14	+0·01	+0·12	+0·17	+ 8 1	+0·457	+0·408	+0·026	193
203	13	246 35 28	246 29 40	1·0	−2·22	+1·23	−3·90	+0·93	−0·05	−0·08	−0·04	−0·04	+0·15	+0·04	0·00	+0·09	+0·15	− 5 48	−0·452	−0·324	−0·140	203
220	14	65 13 22	65 16 19	0·5	−2·53	+0·36	+0·16	+4·04	+0·04	+0·03	+0·04	+0·08	−0·03	−0·08	−0·01	+0·04	+0·08	+ 2 57	+0·246	+0·247	−0·027	220
248	19	71 14 15	70 36 30	1·0	−0·57	+2·46	−2·62	−3·07	−0·02	+0·13	−0·02	−0·21	−0·34	+0·20	+0·04	+0·01	+0·36	− 37 45	−0·534	−0·547	0·000	248
268	20	245 57 56	245 57 15	1·0	−1·56	+1·99	−3·54	+1·85	−0·06	−0·05	−0·06	−0·05	+0·10	+0·06	0·00	+0·08	+0·12	− 41	−0·064	−0·213	+0·139	268
293	21	65 7 32	65 11 43	1·0	−2·38	+0·87	+0·75	+3·95	+0·04	+0·03	+0·04	+0·08	−0·02	−0·08	−0·01	+0·04	+0·07	+ 4 11	+0·351	+0·276	+0·046	293
299	22	66 3 53	65 38 34	1·0	+2·51	−0·17	−4·05	+0·29	+0·02	+0·06	+0·03	+0·02	−0·11	−0·03	−0·01	−0·06	+0·11	− 25 19	−0·324	−0·164	−0·142	299
328	24	347 45 21	*353 54 21	0·0	+0·43	+0·35	+0·85	+0·26	+1·38	−1·91	+1·20	+1·62	+3·36	−1·29	+0·08	+2·24	+3·59	+369 0	+1·078	+0·365	[+0·459]	328
332	28	64 49 9	64 51 59	0·5	−1·83	+1·72	+1·68	+3·60	+0·05	+0·02	+0·06	+0·08	+0·01	−0·08	−0·01	+0·05	+0·08	+ 2 50	+0·231	+0·313	−0·108	332
334	29	66 2 23	65 52 35	1·0	+2·23	−1·12	−3·82	+1·23	+0·03	+0·06	+0·04	+0·03	−0·10	−0·05	−0·01	−0·07	+0·11	− 9 48	−0·125	−0·056	−0·057	334
346	Oct. 4	245 23 6	245 21 41	1·0	−0·15	+2·47	−2·50	+3·00	−0·06	−0·01	−0·07	−0·06	+0·05	+0·07	0·00	+0·07	+0·09	− 1 25	−0·144	+0·036	−0·154	346
368	9	66 14 35	66 17 5	1·0	−1·22	−2·13	−1·51	+3·62	+0·03	+0·02	+0·03	+0·05	−0·06	−0·07	+0·01	−0·02	+0·09	+ 2 30	+0·103	+0·200	−0·118	368
384	13	65 35 40	65 38 4	1·0	+2·12	−1·16	−3·56	+1·60	+0·03	+0·03	+0·04	+0·03	−0·07	−0·05	−0·01	−0·05	+0·08	+ 2 24	+0·039	−0·008	+0·059	384
465	Nov. 5	246 14 48	246 14 48	1·0	−1·72	+1·47	−2·79	+2·24	−0·01	−0·04	−0·04	0·00	+0·07	+0·05	0·00	+0·06	+0·09	0	0·000	−0·119	+0·108	465
481	6	65 25 45	65 31 49	1·0	−2·19	+0·58	+1·53	+3·27	+0·02	+0·01	+0·03	+0·04	0·00	−0·06	0·00	+0·02	+0·05	+ 6 4	+0·386	+0·284	+0·076	481
497	9	7 41 27	10 52 5	0·0	+0·78	+0·98	+1·96	+0·22	+1·64	−1·36	+1·00	+2·28	+2·71	−0·70	−0·13	+1·63	+2·77	+190 38	+0·425	+0·397	[−0·167]	497
502	22	239 13 59	239 35 14	1·0	+2·12	−0·26	+1·64	+2·97	−0·19	+0·19	−0·06	−0·33	−0·23	+0·25	+0·02	+0·18	+0·33	+ 21 15	+0·357	+0·466	−0·126	502

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of Δp_1.	Δq_1.	Δp_2.	Δq_2.	x_1.	Δh_1.	Δk_1.	x_2.	Δh_2.	Δk_2.	y_2.	$\Delta\epsilon_1$.	$\Delta\epsilon_2$.	O—C.	(O—C) × s sin 1″.	First Approximation.	Residuals.	Rot. No.
	1891.	° ′ ″	° ′ ″															′ ″	″	″	″	
234	Sept. 18	66 39 50	66 40 41	1·0	−0·35	−2·51	−2·69	+3·05	+0·04	+0·04	+0·04	+0·07	−0·12	−0·07	+0·01	−0·03	+0·13	+ 51	+0·026	+0·113	−0·102	234
399	Oct. 14	241 50 22	242 3 47	1·0	+2·41	−0·23	+0·92	+3·73	−0·10	+0·12	−0·06	−0·22	−0·11	+0·19	−0·02	+0·11	+0·21	+ 13 25	+0·425	+0·494	−0·077	399
423	16	66 5 3	66 6 17	1·0	−1·71	−1·70	−0·83	+3·76	+0·02	+0·01	+0·03	+0·04	−0·05	−0·06	+0·01	−0·01	+0·07	+ 1 14	+0·059	+0·231	−0·194	423
445	19	64 33 23	64 38 38	1·0	−1·02	+2·17	+2·62	+2·73	+0·06	0·00	+0·07	+0·08	+0·04	−0·08	−0·01	+0·06	+0·08	+ 5 15	+0·340	+0·330	−0·010	445
521	Nov. 30	246 47 19	246 42 26	0·0	−2·09	+0·03	−2·92	+1·54	+0·04	−0·06	−0·02	+0·06	+0·10	+0·04	0·00	+0·06	+0·10	− 4 53	−0·283	−0·163	[−0·133]	521
527	Dec. 8	65 10 5	65 15 37	1·0	−2·03	−0·18	+1·48	+2·90	+0·01	0·00	+0·02	+0·02	0·00	−0·05	0·00	0·00	+0·04	+ 5 32	+0·250	+0·277	−0·051	527

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TABLE XVII.—*continued.*

SATELLITES I. AND III.—POSITION-ANGLES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of												O—C.	(O—C) × s sin 1″	First Approximation.	Residuals.	Rot. No.
					Δp_1.	Δq_1.	Δp_3.	Δq_3.	x_1.	Δh_1.	Δk_1.	x_3.	Δh_3.	Δk_3.	y_3.	$\Delta\epsilon_1$.	$\Delta\epsilon_3$.					
	1891.	° ′ ″	° ′ ″															′ ″	″	″	″	
9	Aug. 17	63 31 39	63 27 8	1·0	−0·23	−2·51	+0·78	+6·35	+0·09	−0·01	+0·10	−0·21	+0·04	−0·24	+0·04	−0·10	+0·25	− 4 31	−0·229	−0·384	+0·146	9
25	19	87 37 18	87 28 43	1·0	+1·59	−1·70	−5·93	−0·40	+0·09	+0·54	−0·11	−0·22	−2·27	+0·62	−0·21	−0·29	+2·31	− 8 35	−0·056	−0·080	+0·047	25
27	20	97 45 10	96 49 5	1·0	−1·53	+1·48	−3·56	−4·07	+0·12	+0·78	−0·15	+1·00	−2·62	+2·02	−0·39	+0·45	+3·25	− 56 5	−0·357	−0·087	−0·249	27
61	24	63 40 18	63 28 38	1·0	−1·33	−2·17	+1·92	+6·14	+0·08	−0·03	+0·09	−0·21	+0·05	−0·23	0·00	−0·09	+0·24	− 11 40	−0·618	−0·429	−0·206	61
71	25	64 41 1	64 39 13	0·5	+0·67	+2·45	−3·32	+5·54	+0·07	0·00	+0·07	−0·19	−0·08	−0·17	−0·02	+0·07	+0·19	− 1 48	−0·248	−0·407	+0·156	71
100	29	245 13 12	245 8 32	1·0	+2·54	−0·31	−5·18	+3·87	−0·04	−0·04	−0·04	+0·16	+0·13	+0·13	+0·03	−0·05	+0·18	− 4 40	−0·342	−0·149	−0·182	100
122	31	63 39 58	63 30 26	1·0	−1·85	−1·76	+2·74	+5·86	+0·07	−0·04	+0·08	−1·21	+0·08	−0·23	0·00	−0·09	+0·24	− 9 32	−0·507	−0·437	−0·086	122
132	Sept. 1	64 27 12	64 24 47	1·0	+1·40	+2·13	−2·49	+5·98	+0·07	−0·01	+0·07	−0·20	−0·05	−0·19	−0·02	+0·07	+0·20	− 2 25	−0·309	−0·397	+0·091	132
155	5	245 12 1	245 7 47	1·0	+2·36	−0·99	−4·60	+4·56	−0·05	−0·04	−0·04	+0·16	+0·11	+0·14	+0·02	−0·06	+0·18	− 4 14	−0·297	−0·193	−0·098	155
165	7	63 39 6	63 32 2	1·0	−2·48	−0·63	+3·83	+5·21	+0·05	−0·04	+0·06	−0·20	+0·10	−0·22	−0·04	−0·05	+0·24	− 7 4	−0·407	−0·454	+0·028	165
187	10	68 30 40	68 33 28	1·0	+0·49	+2·50	−5·97	−2·54	0·00	+0·08	−0·01	+0·05	−0·38	+0·08	+0·03	−0·03	+0·38	+ 2 48	+0·115	+0·112	+0·012	187
205	13	247 24 18	247 22 12	1·0	−2·27	+1·14	−6·42	+0·55	−0·04	−0·11	−0·04	+0·05	+0·30	+0·04	−0·01	+0·11	+0·30	− 2 6	−0·189	−0·127	−0·073	205
218	14	63 25 16	63 20 24	0·5	−2·51	+0·45	+4·66	+4·45	+0·05	−0·04	+0·05	−0·21	+0·14	−0·21	−0·03	−0·03	+0·25	− 4 52	−0·279	−0·428	+0·132	218
250	19	245 24 19	245 18 38	1·0	+0·69	−2·44	−2·69	+5·85	−0·07	−0·02	−0·07	+0·16	+0·08	+0·16	−0·02	−0·07	+0·18	− 5 41	−0·372	−0·320	−0·056	250
270	20	246 42 19	246 39 55	1·0	−1·61	+1·94	−6·20	+1·63	−0·05	−0·07	−0·05	+0·06	+0·22	+0·08	−0·04	+0·08	+0·23	− 2 24	−0·265	−0·208	−0·066	270
291	21	63 9 6	63 3 11	1·0	−2·35	+0·95	+5·12	+3·83	+0·05	−0·04	+0·05	−0·23	+0·18	−0·22	−0·02	−0·02	+0·28	− 5 55	−0·308	−0·389	+0·062	291
301	22	63 24 55	63 19 45	1·0	+2·52	−0·08	+0·40	+6·39	+0·06	−0·05	+0·05	−0·26	+0·07	−0·26	0·00	+0·05	+0·26	− 5 10	−0·384	−0·320	−0·057	301
340	29	62 47 56	62 44 16	0·25	+2·34	−0·86	+1·32	+6·19	+0·06	−0·06	+0·05	−0·31	+0·13	−0·29	+0·03	+0·04	+0·31	− 3 40	−0·204	−0·292	+0·096	340
354	Oct. 4	246 10 35	246 8 28	1·0	−0·36	+2·45	−5·42	+3·16	−0·05	−0·03	−0·06	+0·07	+0·14	+0·10	−0·01	+0·06	+0·17	− 2 7	−0·264	−0·280	+0·012	354
366	9	242 37 17	242 31 0	0·5	+1·28	+2·09	+4·25	+4·55	−0·14	+0·08	−0·12	+0·33	−0·24	+0·27	−0·04	+0·15	+0·36	− 6 17	−0·403	−0·354	−0·043	366
380	10	245 22 55	245 16 59	1·0	−0·32	−2·43	−0·77	+6·16	−0·06	0·00	−0·06	+0·14	+0·03	+0·16	−0·01	−0·06	+0·16	− 5 56	−0·381	−0·391	+0·001	380
388	13	62 6 55	62 1 57	1·0	+2·17	−1·05	+2·23	+5·72	+0·07	−0·08	+0·05	−0·37	+0·21	−0·32	+0·02	+0·05	+0·38	− 4 58	−0·210	−0·258	+0·055	388
463	Nov. 5	66 27 58	66 23 58	1·0	+1·66	−1·54	−4·53	+3·57	+0·01	+0·04	+0·04	0·00	−0·12	−0·10	+0·01	−0·06	+0·15	− 4 0	−0·199	−0·137	−0·059	463
479	6	69 56 14	69 54 23	1·0	−2·17	+0·64	−5·60	−1·27	−0·09	+0·16	+0·02	+0·32	−0·44	+0·07	+0·05	+0·16	+0·44	− 1 51	−0·075	−0·031	−0·055	479
515	27	66 41 41	66 41 8	1·0	−0·75	+1·98	−5·25	+1·13	0·00	+0·04	+0·04	+0·09	−0·15	−0·05	0·00	+0·05	+0·15	− 33	−0·048	−0·155	+0·104	515
549	Dec. 14	245 50 5	245 48 22	0·5	−1·81	+0·84	−2·86	+4·20	+0·02	−0·04	−0·03	−0·03	+0·08	+0·10	+0·01	+0·04	+0·12	− 1 43	−0·377	−0·187	+0·175	549

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of												O—C.	(O—C) × s sin 1″	First Approximation.	Residuals.	Rot. No.
					Δp_1.	Δq_1.	Δp_3.	Δq_3.	x_1.	Δh_1.	Δk_1.	x_3.	Δh_3.	Δk_3.	y_3.	$\Delta\epsilon_1$.	$\Delta\epsilon_3$.					
	1891.	° ′ ″	° ′ ″															′ ″	″	″	″	
240	Sept. 18	243 47 9	243 44 29	1·0	+0·11	+2·53	+2·64	+5·89	+0·10	+0·02	−0·10	−0·23	−0·09	+0·23	0·00	+0·10	+0·25	− 2 40	−0·283	−0·474	+0·082	240
364	Oct. 6	62 8 12	61 57 58	1·0	+2·07	−1·32	+2·03	+5·90	+0·07	−0·07	+0·06	−0·37	+0·20	−0·33	+0·06	+0·03	+0·38	− 10 14	−0·432	−0·269	−0·068	364
405	14	66 16 51	66 11 29	1·0	−2·43	−0·02	−3·05	+5·33	+0·01	+0·05	+0·02	−0·08	−0·10	−0·13	−0·02	+0·05	+0·16	− 5 22	−0·631	−0·459	−0·083	405
429	16	241 46 25	241 40 35	0·5	+1·53	+1·85	+4·61	+4·01	−0·17	+0·12	−0·13	+0·40	−0·33	+0·30	−0·03	+0·18	+0·44	− 5 50	−0·290	−0·308	+0·016	429
433	17	245 23 50	245 17 37	1·0	−1·07	−2·15	+0·16	+6·10	−0·05	+0·01	−0·06	+0·13	+0·02	+0·15	+0·01	−0·05	+0·15	− 6 13	−0·414	−0·426	−0·025	433
441	19	57 15 54	57 7 25	0·0	−0·86	+2·21	+5·90	+0·98	+0·18	−0·14	+0·13	−0·61	+0·79	−0·31	+0·10	+0·07	+0·83	− 8 29	−0·138	−0·122	[−0·012]	441
523	Nov. 30	246 32 39	246 33 10	0·0	−2·09	−0·05	−3·98	+3·51	+0·03	−0·05	−0·02	−0·05	+0·12	+0·09	+0·01	+0·05	+0·15	+ 31	+0·048	−0·312	[+0·155]	523
525	Dec. 8	247 55 48	247 55 18	1·0	+2·04	+0·12	−5·16	0·00	+0·08	−0·10	0·00	−0·21	+0·25	+0·01	+0·04	−0·09	+0·25	− 30	−0·012	−0·103	−0·048	525

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TABLE XVII.—*continued.*

SATELLITES I. AND IV.—POSITION-ANGLES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of												O—C.	(O—C) × $s \sin 1''$.	First Approximation.	Residuals.	Rot. No.
					Δp_1.	Δq_1.	Δp_4.	Δq_4.	x_1.	Δh_1.	Δk_1.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_1$.	$\Delta\epsilon_4$.					
	1891.	° ′ ″	° ′ ″														′ ″	″	″	″	
2	Aug. 17	191 34 53	190 48 51	1·0	+0·37	+1·45	+ 6·23	+ 2·54	−0·75	+1·04	−1·24	−8·05	+4·17	+0·14	+1·46	+9·04	−46 2	−0·309	−0·031	−0·223	2
17	19	243 41 59	243 42 15	1·0	−1·32	+2·15	+ 4·75	+10·25	−0·08	−0·01	−0·08	−0·14	+0·39	+0·01	+0·08	+0·41	+ 16	+0·040	+0·056	−0·008	17
37	20	243 47 6	243 46 32	1·0	+2·17	−1·31	+ 0·56	+11·20	−0·06	−0·01	−0·06	−0·06	+0·40	+0·02	−0·04	+0·40	− 34	−0·069	+0·007	−0·056	37
59	24	246 30 11	246 23 7	1·0	+1·38	+2·13	−11·18	+ 1·18	−0·03	−0·05	−0·01	+0·35	+0·09	−0·01	−0·02	+0·36	− 7 4	−0·903	−0·714	−0·186	59
82	25	282 0 20	281 32 7	1·0	−0·21	−2·03	− 8·67	− 2·50	+0·51	−0·73	+0·74	+6·18	−2·38	−0·06	+0·82	+6·59	−28 13	−0·262	−0·239	−0·008	82
92	29	65 10 32	65 9 18	1·0	−2·48	+0·63	− 2·19	+11·22	+0·05	+0·04	+0·04	−0·08	−0·28	0·00	+0·05	+0·29	− 1 14	−0·260	−0·420	+0·152	92
124	31	66 36 37	66 32 15	1·0	−1·81	−1·80	− 9·40	+ 6·60	+0·02	+0·06	+0·01	−0·33	−0·18	+0·01	+0·04	+0·37	− 4 22	−0·694	−0·742	+0·033	124
130	Sept. 1	66 15 29	66 9 36	1·0	+1·45	+2·09	−11·10	+ 3·01	+0·03	+0·04	+0·02	−0·30	−0·13	+0·01	−0·01	+0·33	− 5 53	−0·890	−0·772	−0·115	130
150	5	242 58 50	242 59 57	0·5	+2·23	−1·26	+ 3·77	+10·76	−0·06	+0·02	−0·06	−0·24	+0·50	−0·02	−0·01	+0·55	+ 1 7	+0·105	+0·212	−0·083	150
161	7	244 59 39	244 58 34	1·0	+2·52	+0·47	− 4·52	+10·40	−0·04	−0·02	−0·04	+0·08	+0·28	−0·01	−0·01	+0·29	− 1 5	−0·191	−0·326	+0·155	161
195	10	251 7 47	250 55 20	1·0	−0·16	−2·53	−11·23	+ 0·08	+0·05	−0·14	+0·06	+1·11	−0·12	+0·02	+0·07	+1·11	−12 27	−0·623	−0·610	−0·019	195
213	13	62 40 30	62 42 45	1·0	+2·39	−0·85	+ 4·77	+10·29	+0·05	−0·05	+0·06	+0·33	−0·53	+0·02	+0·03	+0·62	+ 2 15	+0·197	+0·284	−0·064	213
226	14	65 8 32	65 7 4	1·0	−2·55	+0·13	+ 0·82	+11·36	+0·04	+0·03	+0·04	−0·01	−0·28	+0·01	+0·03	+0·28	− 1 28	−0·270	−0·218	−0·059	226
258	19	68 25 44	68 16 47	1·0	−1·00	+2·33	−11·13	− 2·41	+0·03	+0·09	+0·04	−0·60	+0·07	−0·01	+0·07	+0·60	− 8 57	−0·779	−0·821	+0·030	258
266	20	241 41 59	241 46 36	1·0	−1·50	+2·03	+ 9·56	+ 6·11	−0·09	+0·05	−0·09	−0·63	+0·52	0·00	+0·04	+0·81	+ 4 37	+0·296	+0·376	−0·071	266
295	21	241 29 31	241 35 42	1·0	+2·40	−0·82	+ 6·43	+ 9·28	−0·06	+0·10	−0·06	−0·55	+0·66	−0·01	+0·08	+0·85	+ 6 11	+0·397	+0·387	+0·036	295
309	22	245 5 8	245 5 3	1·0	−2·51	−0·19	+ 2·56	+10·79	−0·04	−0·01	−0·04	−0·02	+0·29	0·00	+0·01	+0·29	− 5	−0·014	−0·076	+0·056	309
344	Oct. 4	67 41 10	67 35 17	1·0	+0·06	−2·47	−10·09	+ 4·74	+0·01	+0·06	+0·03	−0·41	−0·13	+0·01	−0·03	+0·43	− 5 53	−0·607	+0·681	+0·067	344
376	9	245 4 21	245 4 5	1·0	+0·96	+2·25	+ 0·74	+10·88	−0·07	+0·01	−0·07	−0·03	+0·29	+0·01	+0·06	+0·29	− 16	−0·050	−0·111	+0·076	376
397	13	247 59 3	247 51 58	0·5	−2·31	+0·72	−10·66	+ 1·19	+0·01	−0·11	−0·03	+0·45	+0·06	−0·01	+0·11	+0·45	− 7 5	−0·816	−0·778	−0·054	397
473	Nov. 5	66 13 51	66 11 10	1·0	+1·98	−1·10	− 6·34	+ 8·00	0·00	+0·04	+0·04	−0·12	−0·17	0·00	−0·05	+0·21	− 2 41	−0·361	−0·422	+0·070	473
485	6	67 2 0	66 57 23	0·5	−2·23	+0·41	− 8·75	+ 5·22	−0·02	+0·07	+0·02	−0·24	−0·13	−0·01	+0·07	+0·27	− 4 37	−0·726	−0·755	+0·014	485
513	27	243 40 41	243 47 53	1·0	+0·70	−2·00	+ 4·74	+ 8·17	−0·05	+0·02	−0·07	−0·23	+0·34	+0·01	−0·06	+0·41	+ 7 12	+0·417	+0·258	+0·171	513
547	Dec. 14	244 53 19	244 52 18	1·0	−1·78	+0·91	+ 3·48	+ 8·23	0·00	−0·01	−0·03	−0·06	+0·21	−0·01	+0·03	+0·21	− 1 1	−0·127	+0·018	−0·146	547

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of												O—C.	(O—C) × $s \sin 1''$.	First Approximation.	Residuals.	Rot. No.
					Δp_1.	Δq_1.	Δp_4.	Δq_4.	x_1.	Δh_1.	Δk_1.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_1$.	$\Delta\epsilon_4$.					
	1891.	° ′ ″	° ′ ″														′ ″	″	″	″	
85	Aug. 27	63 27 56	63 27 40	0·25	−1·20	+2·25	+ 6·00	+ 9·61	+0·09	0·00	+0·09	+0·22	−0·41	0·00	+0·08	+0·46	− 16	−0·041	+0·147	−0·182	85
360	Oct. 6	232 50 53	233 13 36	1·0	−1·89	+1·47	+10·30	+ 3·31	−0·16	+0·31	−0·08	−2·25	+1·02	+0·04	−0·20	+2·46	+22 43	+0·379	+0·314	+0·077	360
403	14	262 15 41	261 30 3	1·0	+2·33	−0·06	− 9·94	− 2·61	+0·33	−0·65	+0·10	+2·88	−0·84	−0·05	−0·65	+3·00	−45 38	−0·482	−0·472	+0·007	403
425	16	64 32 31	64 33 36	1·0	−1·66	−1·75	+ 5·65	+ 9·11	+0·07	−0·04	+0·07	+0·17	−0·31	0·00	−0·07	+0·35	+ 1 5	+0·094	+0·204	−0·112	425
435	17	64 58 24	64 57 18	1·0	+1·00	+2·18	+ 2·03	+10·54	+0·07	−0·02	+0·07	+0·07	−0·29	0·00	+0·07	+0·30	− 1 6	−0·193	−0·028	−0·152	435
449	19	66 8 25	66 4 55	1·0	−1·17	+2·09	− 5·77	+ 9·05	+0·03	+0·04	+0·05	−0·12	−0·20	0·00	+0·06	+0·23	− 3 30	−0·758	−0·603	−0·161	449
529	Dec. 8	66 6 9	66 2 24	1·0	−2·03	−0·24	− 4·36	+ 8·06	−0·02	+0·04	+0·02	−0·10	−0·15	0·00	+0·03	+0·18	− 3 45	−0·612	−0·474	−0·146	529

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TABLE XVII.—*continued.*

SATELLITES II. AND III.—POSITION-ANGLES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of														O—C.	(O—C) × s sin 1″.	First Approximation.	Residuals.	Rot. No.
					Δp_2.	Δq_2.	Δp_3.	Δq_3.	x_2.	Δh_2.	Δk_2.	y_2.	x_3.	Δh_3.	Δk_3.	y_3.	$\Delta\epsilon_2$.	$\Delta\epsilon_3$.					
	1891.	° ′ ″	° ′ ″																′ ″	″	″	″	
11	Aug. 17	59 36 26	59 17 11	1·0	+3·33	−2·25	+0·69	+6·33	−0·05	−0·15	+0·12	−0·01	−0·35	+0·24	−0·48	+0·09	+0·06	+0·50	− 19 15	−0·492	−0·363	−0·141	11
21	19	67 12 43	67 14 37	1·0	−3·84	+1·08	−6·42	−0·31	−0·10	+0·20	+0·03	0·00	−0·12	−0·31	−0·01	−0·02	+0·20	+0·31	+ 1 54	+0·184	−0·173	+0·354	21
35	20	242 20 45	242 20 58	1·0	+0·39	+3·99	+3·99	+5·04	+0·15	+0·06	−0·17	+0·01	+0·21	−0·19	+0·29	−0·04	+0·18	+0·34	+ 13	+0·021	−0·026	+0·022	35
51	24	31 6 25	28 56 45	1·0	+1·90	−2·77	+1·80	+5·02	−0·20	−1·00	+0·52	−0·05	−1·58	+2·01	−2·26	+0·19	+0·11	+2·81	−129 40	−0·510	−0·303	−0·211	51
73	25	64 13 21	64 13 38	0·5	+2·86	+2·91	−3·35	+5·52	−0·07	−0·01	+0·08	−0·01	−0·19	−0·06	−0·18	−0·02	+0·06	+0·19	+ 17	+0·036	+0·078	−0·066	73
104	29	248 58 18	248 45 48	1·0	+0·29	−4·04	−5·22	+3·78	0·00	−0·16	+0·04	+0·01	+0·17	+0·39	+0·08	+0·06	+0·03	+0·36	− 12 30	−0·443	−0·560	+0·120	104
108	31	258 29 49	258 44 31	0·25	−1·45	+3·68	−3·02	−5·52	−0·09	−0·45	+0·18	−0·02	−0·56	+0·98	−0·78	+0·09	−0·01	+1·16	+ 14 42	+0·131	+0·660	−0·523	108
139	Sept. 1	64 6 22	64 8 26	1·0	+3·26	+2·48	−2·69	+5·89	−0·07	−0·02	+0·08	−0·01	−0·20	−0·04	−0·20	−0·02	+0·06	+0·19	+ 2 4	+0·246	+0·048	+0·174	139
153	5	247 48 31	247 31 49	1·0	−0·65	−4·02	−4·57	+4·58	−0·01	−0·12	+0·03	+0·01	+0·15	+0·27	+0·11	+0·01	+0·04	+0·27	− 16 42	−0·812	−0·692	−0·119	153
171	7	252 9 57	252 44 53	1·0	−0·97	+3·92	−3·70	−5·25	−0·05	−0·25	+0·08	−0·01	−0·28	+0·57	−0·35	−0·08	−0·02	+0·61	+ 34 56	+0·572	+0·663	−0·090	171
174	8	63 40 5	63 38 9	1·0	+3·90	+1·29	−1·39	+6·33	−0·07	−0·05	+0·08	−0·01	−0·16	+0·01	−0·23	−0·04	+0·07	+0·22	− 1 56	−0·181	+0·039	−0·157	174
185	10	215 46 9	216 0 31	1·0	+2·62	+2·32	+5·21	+2·17	+0·97	+1·59	−1·10	+0·05	+1·18	−2·86	+1·41	+0·38	+1·92	+3·18	+ 14 22	+0·141	−0·036	+0·048	185
201	13	251 33 29	251 44 6	1·0	+3·86	−1·00	−6·38	+0·67	+0·01	−0·43	+0·01	0·00	−0·01	+0·70	−0·04	−0·05	−0·41	+0·69	+ 10 37	+0·149	+0·187	−0·019	201
216	14	248 56 25	249 10 25	0·5	+0·28	+4·03	−4·68	−4·41	−0·04	−0·15	+0·04	0·00	−0·12	+0·35	−0·13	−0·04	−0·06	+0·34	+ 14 0	+0·379	+0·695	−0·316	216
252	19	246 31 30	246 19 12	1·0	−2·50	−3·19	−2·74	+5·82	0·00	−0·08	0·00	+0·01	+0·13	+0·14	+0·13	−0·03	+0·05	+0·18	− 12 18	−0·952	−0·854	−0·094	252
278	20	250 35 54	250 31 54	1·0	+3·63	−1·62	−6·22	+1·48	+0·01	−0·34	−0·02	0·00	−0·01	+0·58	+0·02	−0·09	−0·32	+0·57	− 4 0	−0·066	+0·053	−0·105	278
283	21	248 5 51	247 58 17	0·0	+1·02	+3·88	−5·22	−3·70	−0·04	−0·13	+0·03	0·00	−0·08	+0·29	−0·06	−0·04	−0·06	+0·27	− 7 34	−0·250	+0·682	[−0·932]	283
303	22	62 53 2	62 53 18	1·0	+4·05	−0·17	+0·36	+6·39	−0·08	−0·09	+0·08	−0·01	−0·28	+0·10	−0·28	0·00	+0·09	+0·29	+ 16	+0·016	−0·152	+0·148	303
320	23	65 27 57	65 28 55	1·0	−0·51	+3·98	−4·67	+4·39	−0·08	+0·04	+0·08	0·00	−0·13	−0·10	−0·14	+0·02	+0·08	+0·17	+ 58	+0·158	+0·004	+0·137	320
342	Oct. 4	249 5 38	248 45 52	1·0	+2·40	−3·08	−5·29	+3·36	0·00	−0·18	−0·04	0·00	0·00	+0·36	+0·09	+0·01	−0·15	+0·35	− 19 46	−0·476	−0·296	−0·172	342
378	9	244 11 39	244 9 49	1·0	−1·76	+3·50	+4·06	+4·72	+0·09	0·00	−0·09	+0·01	+0·20	−0·10	+0·18	−0·02	+0·08	+0·20	− 1 50	−0·202	−0·190	−0·027	378
386	13	59 42 17	59 31 55	1·0	+3·56	−1·55	+2·26	+5·68	−0·15	−0·25	+0·07	−0·02	−0·55	+0·36	−0·46	+0·03	+0·20	+0·56	− 10 22	−0·259	−0·239	−0·042	386
467	Nov. 5	66 21 19	66 15 8	0·0	−2·85	+2·16	−4·61	+3·46	+0·01	+0·08	+0·05	0·00	−0·01	−0·11	−0·10	+0·01	+0·09	+0·15	− 6 11	−0·831	−0·258	[−0·544]	467
477	6	236 56 56	237 11 0	0·0	+1·60	+3·18	+5·56	+1·22	+0·47	+0·34	−0·33	+0·03	+0·73	−0·79	+0·32	+0·09	+0·45	+0·84	+ 14 4	+0·310	+0·208	[+0·060]	477
489	8	246 51 14	246 31 55	1·0	−0·36	−3·54	−2·21	+5·26	−0·02	−0·05	−0·03	0·00	+0·01	+0·10	+0·10	−0·02	−0·02	+0·13	− 19 19	−0·831	−0·685	−0·142	489
504	22	246 9 41	245 56 10	1·0	−1·60	−3·01	−0·62	+5·42	−0·01	−0·04	−0·02	0·00	+0·02	+0·05	+0·10	−0·01	0·00	+0·11	− 13 31	−0·698	−0·748	+0·055	504

OBSERVER—FINLAY.

Rot. No	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of														O—C.	(O—C) × s sin 1″.	First Approximation.	Residuals.	Rot. No.
					Δp_2.	Δq_2.	Δp_3.	Δq_3.	x_2.	Δh_2.	Δk_2.	y_2.	x_3.	Δh_3.	Δk_3.	y_3.	$\Delta\epsilon_2$	$\Delta\epsilon_3$.					
	1891.	° ′ ″	° ′ ″																′ ″	″	″	″	
242	Sept. 18	244 25 50	244 22 40	1·0	−2·88	+2·87	+2·58	+5·91	+0·08	−0·03	−0·08	+0·01	+0·19	−0·04	+0·19	0·00	+0·08	+0·19	− 3 10	−0·422	−0·398	−0·036	242
409	Oct. 14	65 22 17	65 22 29	0·5	+0·61	+3·80	−3·14	+5·28	−0·06	+0·01	+0·07	0·00	−0·11	−0·06	−0·14	−0·01	+0·07	+0·15	+ 12	+0·031	+0·005	+0·007	409
417	15	67 57 8	67 52 28	0·25	+3·80	−0·04	−6·03	+1·13	+0·07	+0·19	0·00	0·00	+0·06	−0·29	−0·05	−0·05	+0·19	+0·29	− 4 40	−0·381	+0·189	−0·591	417
421	16	243 43 0	243 41 55	1·0	−0·78	+3·77	+4·74	+3·88	+0·11	+0·02	−0·11	+0·01	+0·25	−0·17	+0·20	−0·02	+0·11	+0·26	− 1 5	−0·099	−0·047	−0·068	421
443	19	246 58 34	247 9 13	1·0	+2·66	+2·70	−5·95	−1·05	−0·04	−0·09	+0·01	0·00	−0·05	+0·18	+0·03	+0·03	−0·07	+0·17	+ 10 39	+0·499	+0·517	−0·026	443
457	23	242 59 1	242 59 22	1·0	+0·11	+3·76	+5·20	+3·00	+0·15	+0·06	−0·14	+0·01	+0·28	−0·23	+0·20	+0·01	+0·14	+0·30	+ 21	+0·024	+0·072	−0·067	457
537	Dec. 8	246 10 1	246 16 19	0·5	+1·22	+3·02	−5·16	−0·26	−0·03	−0·04	−0·02	0·00	−0·07	+0·10	+0·04	+0·02	0·00	+0·10	+ 6 18	+0·438	+0·380	+0·050	537

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TABLE XVII.—*continued.*

SATELLITES II. AND IV.—POSITION-ANGLES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of Δp_2.	Δq_2.	Δp_4.	Δq_4.	x_2.	Δh_2.	Δk_2.	y_2.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_2$.	$\Delta\epsilon_4$.	O—C.	(O—C) × s sin 1.″	First Approximation.	Residuals.	Rot. No.
	1891.	° ′ ″	° ′ ″															′ ″	″	″	″	
5	Aug. 17	238 0 29	238 11 37	1·0	−3·17	+2·45	+10·38	+ 4·34	+0·05	+0·20	−0·14	+0·01	−1·17	+0·76	+0·03	−0·08	+1·37	+11 8	+0·428	+0·509	−0·060	5
23	19	241 8 17	241 20 3	1·0	+3·86	−0·98	+ 4·55	+10·31	+0·04	+0·13	−0·10	0·00	−0·43	+0·71	+0·03	+0·10	+0·82	+11 46	+0·770	+0·332	+0·442	23
29	20	244 3 4	244 3 49	1·0	+0·67	+3·95	+ 0·77	+11·25	+0·10	0·00	−0·11	+0·01	−0·04	+0·37	+0·01	+0·11	+0·38	+ 45	+0·156	+0·286	−0·136	29
57	24	246 25 8	246 22 24	1·0	−2·47	+3·22	−11·18	+ 1·21	+0·10	−0·12	−0·07	−0·01	+0·34	+0·09	−0·01	+0·12	+0·35	− 2 44	−0·480	−0·622	+0·131	57
77	25	261 28 52	260 52 33	1·0	−2·66	−2·87	−10·41	− 2·91	−0·37	−0·90	+0·65	−0·11	+2·96	−1·04	−0·03	+1·09	+3·13	−36 19	−0·858	−0·997	+0·145	77
94	29	64 38 36	64 39 1	1·0	+0·04	+4·07	− 2·22	+11·21	−0·10	+0·02	+0·10	+0·01	−0·04	−0·32	0·00	+0·10	+0·32	+ 25	+0·100	+0·041	+0·050	94
114	31	66 57 37	66 47 56	0·25	+1·71	−3·69	− 9·28	+ 6·76	−0·08	+0·11	+0·06	+0·01	−0·37	−0·17	+0·01	−0·10	+0·40	− 9 41	−1·017	−0·907	−0·106	114
137	Sept. 1	66 4 54	66 2 0	1·0	+3·30	+2·44	−11·14	+ 2·86	−0·02	+0·10	0·00	+0·01	−0·28	−0·13	+0·01	−0·08	+0·30	− 2 54	−0·405	−0·330	−0·095	137
157	5	244 4 57	244 0 34	1·0	−0·54	−4·04	+ 3·60	+10·82	+0·12	0·00	−0·12	−0·01	−0·11	+0·38	−0·01	−0·12	+0·40	− 4 23	−0·334	−0·292	−0·018	157
167	7	245 9 22	245 9 36	1·0	−0·86	+3·97	− 4·61	+10·35	+0·09	−0·04	−0·09	−0·01	+0·10	+0·27	0·00	+0·10	+0·29	+ 14	+0·059	−0·147	+0·200	167
176	8	246 25 21	246 19 28	1·0	−3·88	−1·35	− 8·02	+ 7·97	+0·01	−0·13	0·00	−0·01	+0·26	+0·20	0·00	+0·12	+0·33	− 5 53	−1·112	−0·998	−0·105	176
191	10	246 44 9	246 43 15	1·0	+2·88	+2·82	−11·29	+ 0·14	−0·01	−0·11	+0·01	0·00	+0·34	+0·08	−0·01	−0·09	+0·34	− 54	−0·097	−0·261	+0·145	191
211	13	64 33 48	64 33 39	1·0	−3·96	+0·66	+ 4·80	+10·29	−0·06	+0·03	+0·06	0·00	+0·10	−0·32	+0·01	+0·04	+0·33	− 9	−0·024	−0·074	+0·067	211
222	14	65 1 10	64 56 4	0·25	−0·09	−4·05	+ 0·89	+11·36	−0·09	+0·02	+0·09	+0·01	0·00	−0·30	+0·01	−0·09	+0·29	− 5 6	−0·526	−0·460	−0·050	222
260	19	67 36 14	67 33 39	1·0	+2·29	+3·35	−11·13	− 2·45	+0·05	+0·13	−0·04	−0·01	−0·45	+0·01	−0·01	−0·11	+0·44	− 2 35	−0·200	−0·263	+0·043	260
276	20	76 28 5	76 5 5	1·0	−3·56	+1·64	− 9·32	− 6·14	+0·05	+0·64	−0·02	0·00	−1·88	+0·94	−0·04	+0·58	+2·08	−23 0	−0·610	−0·759	+0·141	276
287	21	243 28 26	243 33 1	1·0	+0·92	+3·91	+ 6·56	+ 9·22	+0·14	+0·05	−0·14	−0·01	−0·28	+0·43	0·00	+0·15	+0·51	+ 4 35	+0·660	+0·684	−0·025	287
311	22	245 10 11	245 8 3	1·0	−4·06	−0·04	+ 2·54	+10·98	+0·04	−0·05	−0·04	0·00	−0·02	+0·28	+0·01	+0·05	+0·27	− 2 8	−0·375	−0·279	−0·079	311
322	23	245 38 24	245 33 22	1·0	+0·58	−3·96	− 1·60	+11·11	+0·07	−0·04	−0·08	0·00	+0·07	+0·25	0·00	−0·08	+0·26	− 5 2	−0·577	−0·533	−0·030	322
352	Oct. 4	66 34 43	66 31 7	1·0	−2·63	+2·89	−10·16	+ 4·61	−0·04	+0·10	+0·06	0·00	−0·25	−0·14	0·00	+0·11	+0·29	− 3 36	−0·726	−0·682	−0·050	352
374	9	245 16 0	245 15 38	1·0	−1·68	+3·55	+ 0·76	+10·88	+0·06	−0·03	−0·08	+0·01	−0·01	+0·27	+0·01	+0·09	+0·27	− 22	−0·083	+0·085	−0·168	374
395	13	247 37 28	247 31 38	1·0	−3·66	+1·33	−10·65	+ 1·25	−0·02	−0·17	−0·04	−0·02	+0·39	+0·07	−0·01	+0·17	+0·39	− 5 50	−0·784	−0·830	+0·042	395
471	Nov. 5	66 18 48	66 16 21	1·0	−2·96	+2·01	− 6·30	+ 8·04	+0·01	+0·08	+0·05	0·00	−0·13	−0·17	0·00	+0·09	+0·21	− 2 27	−0·525	−0·572	+0·046	471
491	8	67 29 32	67 28 10	1·0	+0·30	+3·55	− 9·92	− 1·96	+0·04	+0·07	+0·01	0·00	−0·30	−0·01	−0·01	0·00	+0·29	− 1 22	−0·118	−0·311	+0·177	491
510	22	65 55 3	65 53 4	0·5	+1·43	+3·09	− 6·73	+ 6·94	0·00	+0·03	+0·03	0·00	−0·10	−0·15	0·00	+0·02	+0·17	− 1 59	−0·374	−0·172	−0·214	510
543	Dec. 14	243 22 51	243 25 42	1·0	+2·10	−2·36	+ 3·55	+ 8·20	+0·04	+0·02	−0·06	0·00	−0·20	+0·34	−0·01	−0·03	+0·39	+ 2 51	+0·136	+0·064	+0·082	543

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of Δp_2.	Δq_2.	Δp_4.	Δq_4.	x_2.	Δh_2.	Δk_2.	y_2.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_2$.	$\Delta\epsilon_4$.	(O—C).	(O—C) × s sin 1.″	First Approximation.	Residuals.	Rot. No.
	1891.	° ′ ″	° ′ ″															′ ″	″	″	″	
89	Aug. 27	62 34 18	62 29 51	0·25	−1·36	−3·79	+ 5·89	+ 9·68	−0·14	−0·07	+0·17	−0·01	+0·34	−0·52	0·00	−0·18	+0·62	− 4 27	−0·222	−0·185	−0·007	89
358	Oct. 6	238 53 39	239 12 35	1·0	−3·50	+1·79	+10·50	+ 3·33	+0·15	+0·27	−0·08	+0·02	−1·19	+0·60	+0·03	−0·21	+1·31	+18 56	+0·585	+0·440	+0·168	358
401	14	53 30 11	54 14 7	1·0	+0·85	+3·66	+10·15	+ 2·61	−0·57	−0·40	+0·46	−0·05	+2·11	−0·82	−0·02	+0·53	+2·24	+43 56	+1·070	+1·033	+0·047	401
415	15	63 30 4	63 32 58	0·5	−3·80	+0·03	+ 8·65	+ 6·32	−0·09	−0·08	+0·06	0·00	+0·37	−0·34	0·00	−0·08	+0·50	+ 2 54	+0·190	+0·197	+0·017	415
419	16	62 38 10	62 34 21	1·0	+0·73	−3·78	+ 5·73	+ 9·05	−0·14	−0·06	+0·13	−0·01	+0·42	−0·49	−0·01	−0·12	+0·64	− 3 49	−0·146	−0·025	−0·098	419
453	19	66 49 9	66 41 45	1·0	−2·43	−2·90	− 5·83	+ 9·01	+0·03	+0·08	0·00	0·00	−0·18	−0·18	+0·01	+0·05	+0·25	− 7 24	−1·093	−0·934	−0·148	453
455	23	241 51 24	242 4 33	1·0	+0·16	+3·76	+ 9·48	+ 4·71	+0·19	+0·10	−0·17	+0·01	−0·66	+0·44	−0·02	+0·17	+0·78	+13 9	+0·892	+0·834	+0·055	455
535	Dec. 8	66 28 27	66 22 37	0·5	−1·26	−3·01	− 4·46	+ 8·00	+0·04	+0·05	+0·01	0·00	−0·13	−0·14	+0·01	+0·01	+0·19	− 5 50	−0·676	−0·755	+0·092	535

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TABLE XVII.—*continued.*

SATELLITES III. AND IV.—POSITION-ANGLES.

OBSERVER—GILL.

Rot. No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of													O—C.	(O—C) × s sin 1″.	First Approximation.	Residuals.	Rot. No.
					Δp_3.	Δq_3.	Δp_4.	Δq_4.	x_3.	Δh_3.	Δk_3.	y_3.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_3$.	$\Delta\epsilon_4$.					
	1891.	° ′ ″	° ′ ″															′ ″	″	″	″	
7	Aug. 17	238 29 15	238 33 8	1·0	+0·83	+6·30	+10·37	+ 4·40	−0·40	+0·30	−0·55	+0·10	−1·08	+0·72	+0·03	+0·59	+1·28	+ 3 53	+0·240	+0·127	+0·124	7
15	19	244 43 5	244 44 28	1·0	−6·44	−0·07	+ 4·79	+10·24	−0·11	−0·07	−0·09	+0·01	−0·02	+0·27	+0·01	+0·07	+0·25	+ 1 23	+0·222	+0·195	+0·037	15
39	20	245 31 57	245 34 33	1·0	−3·92	−5·11	+ 0·53	+11·26	−0·09	−0·07	−0·07	0·00	+0·09	+0·22	0·00	−0·02	+0·21	+ 2 36	+0·304	+0·269	+0·048	39
45	21	245 41 52	245 41 42	0·25	+1·12	−6·35	− 3·46	+10·71	−0·16	−0·10	−0·14	0·00	+0·15	+0·23	−0·01	−0·15	+0·27	− 10	−0·014	+0·090	−0·093	45
53	24	245 39 9	245 31 35	1·0	+2·11	+6·08	−11·17	+ 1·30	−0·11	−0·07	−0·10	−0·01	+0·21	+0·12	−0·01	+0·07	+0·23	− 7 34	−1·371	−1·145	−0·240	53
69	25	246 51 4	246 45 10	0·5	−3·28	+5·56	−10·88	− 2·90	−0·16	−0·19	−0·12	−0·03	+0·38	+0·01	−0·01	+0·20	+0·37	− 5 54	−0·906	−1·026	+0·105	69
96	29	65 12 28	65 10 54	1·0	−5·12	+3·94	− 2·28	+11·20	+0·16	+0·13	+0·13	+0·03	−0·08	−0·28	0·00	+0·18	+0·29	− 1 34	−0·440	−0·574	+0·131	96
112	31	67 54 30	67 52 3	0·25	−2·99	−5·72	− 9·25	+ 6·79	−0·02	+0·22	−0·06	+0·03	−0·49	−0·15	+0·01	+0·16	+0·48	− 2 27	−0·276	−0·307	+0·034	112
126	Sept. 1	75 21 9	75 2 29	1·0	+2·33	−5·94	−10·91	+ 3·04	−0·06	+0·56	−0·17	+0·05	−1·73	+0·13	+0·02	−0·05	+1·64	− 18 40	−0·480	−0·424	−0·053	126
146	5	236 36 52	236 48 3	0·5	+4·32	−4·73	+ 3·84	+10·61	−0·22	+0·41	−0·28	−0·05	−0·95	+1·24	−0·03	+0·07	+1·49	+ 11 11	+0·272	+0·402	−0·109	146
169	7	244 41 13	244 38 54	1·0	+3·76	+5·26	− 4·64	+10·34	−0·15	+0·02	−0·15	−0·02	+0·06	+0·30	−0·01	+0·13	+0·29	− 2 19	−0·550	−0·797	+0·246	169
182	8	245 32 22	245 29 23	1·0	−1·62	+6·27	− 8·12	+ 7·86	−0·17	−0·08	−0·16	−0·02	+0·17	+0·22	−0·01	+0·17	+0·27	− 2 59	−0·847	−1·042	+0·186	182
209	13	65 6 41	65 7 26	1·0	−6·44	+0·44	+ 4·84	+10·25	+0·09	+0·07	+0·09	+0·01	+0·03	−0·25	+0·01	+0·08	+0·24	+ 45	+0·129	+0·148	−0·010	209
224	14	65 51 33	65 53 55	0·25	−4·55	−4·57	+ 0·85	+11·36	+0·06	+0·07	+0·06	0·00	−0·08	−0·22	0·00	+0·01	+0·21	+ 2 22	+0·305	+0·220	+0·095	224
256	19	67 5 48	66 58 4	1·0	−2·82	+5·78	−11·15	− 2·37	+0·11	+0·17	+0·12	−0·03	−0·36	−0·02	−0·01	+0·18	+0·35	− 7 44	−1·183	−1·106	−0·092	256
274	20	74 1 57	73 47 23	1·0	−6·15	+1·53	− 9·40	− 6·17	−0·07	+0·89	−0·03	−0·14	−1·47	+0·70	+0·01	+0·86	+1·61	− 14 34	−0·641	−0·687	+0·042	274
289	21	242 8 17	242 7 57	1·0	+5·14	+3·80	+ 6·52	+ 9·24	−0·29	+0·28	−0·27	−0·03	−0·46	+0·58	−0·01	+0·39	+0·74	− 20	−0·037	+0·010	−0·037	289
305	22	244 33 54	244 33 0	1·0	+0·32	+6·39	+ 2·63	+10·96	−0·20	+0·01	−0·20	+0·01	−0·08	+0·34	0·00	+0·20	+0·35	− 54	−0·216	−0·416	+0·203	305
318	23	245 31 44	245 29 14	1·0	−4·63	+4·44	− 1·50	+11·12	−0·13	−0·10	−0·14	+0·02	+0·06	+0·26	0·00	+0·17	+0·20	− 2 30	−0·698	−0·533	−0·164	318
336	29	255 42 19	254 25 13	1·0	+1·42	+6·07	− 7·32	− 8·14	+0·53	−0·61	+0·43	−0·04	+1·49	−0·88	+0·02	−0·56	+1·66	− 77 6	−0·786	−0·801	+0·004	336
350	Oct. 4	66 50 0	66 45 55	1·0	−5·40	+3·20	−10·15	+ 4·64	+0·05	−0·19	+0·10	−0·01	−0·29	−0·14	0·00	+0·22	+0·32	− 4 5	−0·921	−0·966	+0·039	350
370	9	246 17 34	246 19 17	1·0	−4·20	−4·61	+ 0·84	+10·88	−0·03	−0·07	−0·06	0·00	+0·08	+0·19	0·00	0·00	+0·18	+ 1 43	+0·203	+0·246	−0·030	370
382	10	246 32 0	246 33 1	1·0	+0·83	−6·15	− 3·21	+10·37	−0·08	−0·09	+0·12	0·00	+0·14	+0·20	−0·01	−0·13	+0·23	+ 1 1	+0·082	+0·084	+0·008	382
393	13	246 21 21	246 15 12	1·0	+2·15	+5·76	−10·66	+ 1·29	−0·06	−0·06	−0·08	−0·01	+0·17	+0·11	0·00	+0·06	+0·18	− 6 9	−1·011	−1·094	+0·069	393
475	Nov. 5	66 8 35	66 4 11	1·0	+4·76	−3·26	− 6·37	+ 7·98	+0·01	+0·10	+0·09	−0·01	−0·11	−0·17	0·00	−0·14	+0·20	− 4 24	−0·364	−0·308	−0·053	475
487	6	65 58 48	65 53 51	0·5	+5·57	+1·47	− 8·77	+ 5·19	0·00	+0·06	+0·05	−0·01	−0·10	−0·14	0·00	−0·05	+0·16	− 4 57	−0·609	−0·757	+0·143	487
495	8	67 19 7	67 12 13	1·0	−2·35	+5·20	− 9·90	− 2·03	−0·02	+0·14	+0·09	−0·02	−0·27	−0·03	−0·01	+0·14	+0·26	− 6 54	−0·882	−0·982	+0·086	495
506	22	65 57 54	65 53 34	1·0	−0·67	+5·41	− 6·67	+ 6·99	+0·03	+0·05	+0·11	−0·01	−0·10	−0·15	0·00	+0·11	+0·18	− 4 20	−1·035	−0·910	−0·131	506
517	27	245 29 51	245 30 27	1·0	−5·26	+1·09	+ 4·69	+ 8·20	0·00	−0·05	−0·06	+0·01	−0·03	+0·17	0·00	+0·06	+0·17	+ 36	+0·087	+0·127	−0·033	517
539	Dec. 14	237 0 6	237 25 7	1·0	+2·62	−4·29	+ 3·58	+ 8·09	−0·36	+0·30	−0·23	−0·03	−0·79	+0·90	−0·02	−0·04	+1·14	+ 25 1	+0·323	+0·394	−0·053	539

OBSERVER—FINLAY.

Rot. No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight	Coefficients of													O—C.	(O—C) × s sin 1″.	First Approximation.	Residuals.	Rot. No.
					Δp_3.	Δq_3.	Δp_4.	Δq_4.	x_3.	Δh_3.	Δk_3.	y_3.	Δh_4.	Δk_4.	y_4.	$\Delta\epsilon_3$.	$\Delta\epsilon_4$.					
	1891.	° ′ ″	° ′ ″															′ ″	″	″	″	
87	Aug. 27	61 35 46	61 43 49	0·0	+4·86	+4·27	+ 5·93	+ 9·64	+0·24	−0·28	+0·32	−0·02	+0·47	−0·63	−0·01	+0·42	+0·78	+ 8 3	+0·942	−0·109	[+1·061]	87
244	Sept. 18	65 59 24	65 50 46	1·0	+2·53	+5·95	−11·30	+ 1·75	+0·09	+0·06	+0·09	−0·01	−0·20	−0·12	−0·01	+0·06	+0·22	− 8 38	−1·549	−1·186	−0·378	244
356	Oct. 6	238 43 2	238 43 20	1·0	+2·27	+5·77	+10·51	+ 3·29	−0·62	+0·43	−0·52	+0·11	−1·22	+0·61	+0·03	+0·64	+1·34	+ 18	+0·016	+0·177	−0·150	356
407	14	247 34 6	247 26 49	0·5	−3·10	+5·31	−10·34	− 2·79	−0·04	−0·17	−0·11	−0·03	+0·35	+0·01	−0·01	+0·18	+0·33	− 7 17	−0·931	−0·978	+0·035	407
413	15	261 27 14	261 18 3	0·5	−5·79	+1·20	− 8·36	− 6·05	+0·66	−1·52	+0·11	−0·23	+2·49	−1·24	−0·06	+1·46	+2·75	− 9 11	−0·191	−0·603	+0·420	413
431	16	63 31 51	63 32 31	0·5	+4·59	+4·05	+ 5·56	+ 9·16	+0·25	−0·17	+0·20	−0·02	+0·30	−0·41	0·00	+0·26	+0·51	+ 40	+0·090	−0·094	+0·193	431
437	17	65 5 51	65 4 46	0·5	+0·06	+6·10	+ 1·99	+10·55	+0·15	0·00	+0·16	+0·01	+0·06	−0·28	0·00	+0·16	+0·29	− 1 5	−0·262	−0·458	+0·195	437
451	19	66 51 17	66 50 2	1·0	−5·92	−1·25	− 5·80	+ 9·03	−0·04	+0·16	+0·03	+0·03	−0·19	−0·18	+0·01	+0·15	+0·25	− 1 15	−0·245	−0·402	+0·157	451
459	23	110 49 51	103 8 46	0·0	+3·66	+2·18	− 6·71	− 3·40	−3·38	+3·78	−1·85	−0·01	−6·74	+3·14	+0·02	−4·20	+7·43	−461 5	−0·798	−0·586	[−0·218]	459
533	Dec. 8	66 20 52	66 18 20	0·5	−5·16	−0·19	− 4·43	+ 8·02	−0·09	+0·12	+0·03	+0·02	−0·12	−0·14	0·00	+0·12	+0·18	− 2 32	−0·470	−0·372	−0·095	533

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CHAPTER III.

Normal Equations and Solution. Discussion of Results.

Normal Equations. — From the equations of condition normal equations were formed for each pair of satellites, and these partial normals were added to form the complete normal equations. For the distances the observations by the two observers were treated separately. Thus, if it appears by the residuals that the observations of one of the observers are affected by some personality, it will be easier to form a judgment about the influence of that error on the final result, than it would be if the observations of both observers had been treated collectively. For the position-angles, as has been remarked before, there is no reason to suspect the existence of appreciable personal differences between the two observers. Accordingly the observations by the two observers were not separated.

The products were multiplied out to their full extent, so as to make the check by sums absolutely rigorous. The computations were done by Messrs S. L. Goodman and V. A. Löwinger of the Cape Observatory, partly with the aid of a Brunsviga calculating machine. The normals thus derived are given below, in Table XVIII. The equations C are the sums of the corresponding equations A and B; similarly set E is derived by adding the corresponding equations of C and D. (*N.B.* These additions were done before the last decimals were dropped.)

The absolute terms are only given in symbolical form. The numerical values, so far as they are required for the final solution, will be given in Table XIX. In consequence of the fact that the earth is nearly in the fundamental plane, the coefficients of Δp_i, Δq_i in the normals from the distances, and those of the other unknowns in the normals from the position-angles, are comparatively very small, and the same thing is true of the corresponding absolute terms. The absolute terms are denoted by capital letters corresponding to the symbol of the corresponding unknown. Thus the absolute terms of the normals in μ, . . . x_1, . . . p_1, . . . are called M, . . . X_1, . . . P_1, . . . respectively. In the distance equations derived from the observations of one observer only these letters are further distinguished by the initial of the observer. Thus $(M)_a$, . . . $(X)_a$ . . . etc. The terms P_i and Q_i are provided with a dash (P_i', Q_i') in the distance equations, and the others (X_i', H_i', K_i', Y_i') similarly in the position-angle

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normals. Finally, the absolute terms in the complete normals are put in parentheses. Thus, *e.g.*, we have

$$M=(M)_0+(M)_{''} \quad . \quad . \quad . \quad X_1=(X_1)_0+(X_1)_{''} \quad . \quad . \quad . \quad (X_1)=X_1+X_1', \quad . \quad . \quad . \quad \text{etc.}$$

From these normals two entirely independent sets of solutions were made.

I. At the Cape rigorous solutions were made, under the direction of Mr S. S. Hough, of the normals E. The absolute terms used in these solutions are not given here, as they have no interest whatever for the final solution.

Two different solutions of the normals E were made, viz. :

(α) The rigorous conditions

(1) $$\Delta\epsilon_1-3\Delta\epsilon_2+2\Delta\epsilon_3=0$$

(2) $$y_2=1{\cdot}468y_4$$

were *not* introduced. The values of $\Delta\epsilon_1$, $\Delta\epsilon_2$, $\Delta\epsilon_3$, y_2 and y_4 alone were derived from this solution, with their respective weights.

(β) The conditions (1) and (2) were introduced into the normals and all the unknowns were solved with their weights. The differences between the values of $\Delta\epsilon_1$, $\Delta\epsilon_2$, $\Delta\epsilon_3$, y_2 and y_4 from this solution and those from solution α were of the same order of magnitude as the probable errors.

As will be explained below, the absolute terms used in these solutions were not final, and required several corrections. It would have been very laborious to carry these through, and no further use has been made of the Cape solutions. The final weights have, however, been taken from them, and also they have served as a control on the applicability of my method of solution to the case in hand.

II. Another solution was made by myself. This was based on the normals C and D, in which the conditions (1) and (2) were *not* introduced. The method by which this solution was made, and further details, will be explained below.

In this solution the same absolute terms, on which the Cape solution was based, were first used. The agreement between my results and those derived by the Cape computers proved to be so satisfactory, that I felt no hesitation in making my own solution the basis of all further approximations.

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TABLE XVIII.

NORMAL EQUATIONS.

A. DISTANCES—GILL.

Δh_4.	Δk_4.	y_2.	y_3.	y_4.	Δp_1.	Δq_1.	Δp_2.	Δq_2.	Δp_3.	Δq_3.	Δp_4.	Δq_4.	n.
+ 587·4	− 475·5	− 2·6	+14·1	− 1·2	+ 3·0	+ 0·4	+ 6·7	+ 3·2	+17·0	+ 9·3	+ 56·1	+23·5	$(M)_G$
+ 22·6	− 21·9	+ 1·0	− 3·5	− 2·5	+ 1·0	− 0·6	+ 2·2	+ 1·8	+ 6·1	+ 2·7	+ 8·4	+ 4·6	Z_G
− 26·0	+ 16·4	+ 2·5	+ 0·6	+ 3·7	− 0·9	− 0·7	− 5·8	− 3·7	+ 1·3	+ 1·4	+ 1·6	− 3·7	$(E_1)_G$
+ 83·8	+ 6·2	−10·2	+ 6·8	− 1·6	− 5·9	− 3·9	+ 1·1	− 1·2	−16·5	−13·4	+ 15·0	+ 3·3	$(E_2)_G$
− 210·2	+ 108·0	0·0	−43·6	−16·6	+ 3·4	− 2·2	−18·5	− 9·6	+24·3	− 7·1	+ 4·2	−10·1	$(E_3)_G$
−1077·4	+ 943·8	− 9·4	− 0·3	+15·6	− 2·0	+ 2·4	+17·6	− 2·6	+ 5·8	−14·2	+114·4	−33·8	$(E_4)_G$
+ 109·9	− 48·9	+ 1·1	− 1·1	− 0·6	− 0·5	+ 2·1	− 2·5	− 0·5	− 0·9	− 1·0	+ 10·2	+ 4·2	$(X_1)_G$
− 229·3	+ 134·2	− 1·3	+ 4·0	− 0·2	− 2·2	− 2·4	+ 5·4	− 0·2	+ 7·3	+ 6·4	− 14·9	− 7·2	$(H_1)_G$
+ 98·2	− 47·2	+ 1·2	− 0·3	− 2·0	+ 0·4	+ 3·6	− 3·2	− 0·8	− 2·9	− 2·3	+ 15·0	+ 5·7	$(K_1)_G$
− 228·9	+ 121·1	+ 0·6	+ 7·1	− 4·0	+ 0·1	− 0·7	− 7·3	− 6·1	−10·7	− 1·7	− 2·7	− 1·4	$(X_2)_G$
− 452·4	+ 241·5	+11·5	+ 6·8	− 5·0	+ 2·5	− 2·0	−10·8	−14·2	−15·8	+ 3·8	− 8·8	− 3·5	$(H_2)_G$
+ 238·1	− 132·9	− 2·0	− 4·7	+ 2·2	− 1·6	− 0·1	+ 8·3	+ 7·8	+10·7	+ 1·3	+ 7·3	+ 3·2	$(K_2)_G$
+ 461·6	− 251·3	+ 2·2	+18·4	+ 0·8	+ 2·3	− 3·2	+ 0·4	− 5·7	− 4·3	+ 8·2	+ 5·2	+10·6	$(X_3)_G$
− 842·7	+ 483·5	− 2·8	−42·0	− 8·0	+ 0·1	+ 1·6	− 0·5	+13·5	+ 4·5	−16·7	− 0·8	−10·3	$(H_3)_G$
+ 458·6	− 249·0	+ 3·4	+ 9·0	+ 1·2	+ 1·9	− 3·3	+ 1·5	− 5·8	+ 5·6	+15·3	+ 7·6	+10·2	$(K_3)_G$
+4164·1	−2098·2	− 9·3	+23·8	+16·5	+ 3·8	+26·2	+ 2·6	+15·8	− 0·3	− 7·9	+146·3	+83·9	$(H_4)_G$
	+1494·9	+ 1·2	− 9·8	− 6·1	− 2·0	−11·4	− 0·6	− 7·2	+ 0·6	+ 4·6	− 73·4	−55·0	$(K_4)_G$
		+ 3·1	− 0·1	− 0·3	− 0·3	+ 0·2	+ 0·2	− 0·8	− 0·1	+ 0·4	− 1·7	− 0·4	$(Y_2)_G$
			+21·0	+ 6·3	− 0·4	+ 0·3	− 1·3	− 0·1	− 6·5	− 3·2	− 1·7	− 0·9	$(Y_3)_G$
				+13·3	− 0·1	− 0·3	− 0·2	0·0	+ 0·1	− 0·1	− 2·6	− 1·1	$(Y_4)_G$
					+ 5·1	+ 1·8	+ 3·6	+ 1·3	+ 0·7	+ 2·5	+ 6·5	+ 2·6	$(P_1')_G$
						+10·7	+ 4·7	+ 0·7	− 0·5	− 2·4	+ 29·3	+10·6	$(Q_1')_G$
							+17·4	+ 3·6	+ 5·1	+ 4·8	+ 3·2	+ 1·5	$(P_2')_G$
								+ 6·8	+ 3·1	− 2·4	+ 2·3	+ 0·5	$(Q_2')_G$
									+25·7	+12·3	+ 1·8	+ 2·2	$(P_3')_G$
										+20·2	+ 0·6	− 3·4	$(Q_3')_G$
											+154·3	+56·9	$(P_4')_G$
												+28·7	$(Q_4')_G$

B. DISTANCES—FINLAY.

Δh_4.	Δk_4.	y_2.	y_3.	y_4.	Δp_1.	Δq_1.	Δp_2.	Δq_2.	Δp_3.	Δq_3.	Δp_4.	Δq_4.	n.
− 267·2	+ 133·0	+ 1·4	+ 3·2	+14·9	− 0·1	+ 0·8	− 0·3	+ 2·9	+ 6·2	+ 4·2	+ 16·2	+ 5·0	$(M)_F$
+ 0·7	+ 4·2	+ 0·3	0·0	+ 0·6	− 0·2	+ 0·6	− 0·4	+ 0·5	+ 2·5	+ 0·9	+ 2·8	+ 1·3	Z_F
− 24·9	+ 9·6	− 0·1	− 0·7	+ 0·7	+ 0·4	+ 0·1	− 0·1	− 0·5	− 1·0	− 1·1	− 3·2	− 0·8	$(E_1)_F$
− 18·8	+ 14·8	− 3·9	− 5·3	+ 1·7	− 0·3	− 0·1	− 1·6	− 2·6	− 0·8	− 0·4	− 1·4	− 0·4	$(E_2)_F$
+ 24·6	− 25·8	+ 1·9	+20·0	+ 2·3	− 0·5	− 2·3	+ 0·2	− 2·6	− 5·0	−10·9	− 0·9	− 1·0	$(E_3)_F$
+ 52·0	− 151·5	+ 0·6	− 5·1	−15·2	− 2·0	− 3·4	+ 3·3	−10·9	+ 4·0	−12·3	− 46·5	−23·8	$(E_4)_F$
+ 53·4	− 30·3	− 0·2	+ 3·4	− 2·6	− 0·8	+ 0·1	0·0	+ 0·2	− 0·1	+ 0·3	+ 4·3	+ 0·8	$(X_1)_F$
− 74·4	+ 37·6	+ 0·1	− 4·7	+ 2·8	+ 1·9	− 0·4	0·0	− 0·4	+ 0·5	+ 0·2	− 9·4	− 2·4	$(H_1)_F$
+ 27·8	− 18·2	− 0·1	+ 1·3	− 1·4	− 0·2	0·0	0·0	+ 0·1	− 0·1	+ 0·3	+ 1·4	0·0	$(K_1)_F$
− 100·1	+ 44·6	− 0·7	− 5·5	+ 2·8	+ 0·5	− 0·2	+ 1·1	+ 1·0	− 1·3	− 0·7	+ 4·1	+ 0·9	$(X_2)_F$
− 118·3	+ 53·6	− 0·3	− 4·6	+ 3·0	+ 0·4	− 0·2	+ 1·9	+ 0·1	− 1·1	− 0·8	+ 1·1	+ 0·3	$(H_2)_F$
+ 65·3	− 29·9	+ 0·4	+ 3·6	− 1·9	− 0·4	+ 0·2	− 0·6	− 0·9	+ 0·9	+ 0·4	− 3·7	− 0·9	$(K_2)_F$
+ 160·7	− 66·5	− 1·4	−27·8	− 3·5	+ 0·2	+ 0·4	+ 0·7	− 1·3	+ 1·2	+ 0·5	+ 6·7	+ 3·5	$(X_3)_F$
− 187·8	+ 79·5	+ 1·4	+33·8	+ 4·4	+ 0·1	− 1·0	− 1·0	+ 1·4	− 4·0	+ 0·8	− 8·8	− 5·3	$(H_3)_F$
+ 113·9	− 47·1	− 0·7	−12·4	− 1·6	+ 0·2	− 0·1	+ 0·4	− 1·0	+ 0·1	+ 0·7	+ 3·8	+ 1·1	$(K_3)_F$
+1164·6	− 526·5	− 4·7	−18·3	−26·0	−10·8	+ 3·7	− 5·7	− 1·7	+ 8·7	+ 4·0	+ 64·1	+20·6	$(H_4)_F$
	+ 286·3	+ 2·0	+ 9·6	+15·3	+ 3·8	− 1·4	+ 2·0	− 0·8	− 4·0	− 0·7	− 24·8	− 4·6	$(K_4)_F$
		+ 0·4	+ 0·1	0·0	0·0	0·0	+ 0·2	+ 0·1	0·0	0·0	+ 0·1	0·0	$(Y_2)_F$
			+ 5·9	+ 0·9	0·0	− 0·2	− 0·1	0·0	− 1·1	− 0·5	− 1·7	− 0·9	$(Y_3)_F$
				+ 1·4	+ 0·1	0·0	+ 0·1	0·0	− 0·1	0·0	− 0·6	− 0·1	$(Y_4)_F$
					+ 0·7	− 0·2	0·0	0·0	− 0·1	0·0	− 3·0	− 0·9	$(P_1')_F$
						+ 0·2	0·0	0·0	+ 0·3	0·0	+ 0·8	+ 0·3	$(Q_1')_F$
							+ 0·2	+ 0·1	0·0	0·0	− 0·1	− 0·1	$(P_2')_F$
								+ 0·9	+ 0·1	0·0	+ 2·1	+ 0·5	$(Q_2')_F$
									+ 2·5	+ 0·3	+ 2·3	+ 1·5	$(P_3')_F$
										+ 0·9	+ 0·5	+ 0·1	$(Q_3')_F$
											+ 25·7	+ 8·8	$(P_4')_F$
												+ 4·0	$(Q_4')_F$

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TABLE XVIII.—*continued.*

Normal Equations.

C. Distances.—Gill and Finlay.

μ.	z_G.	z_F.	$\Delta\epsilon_1$.	$\Delta\epsilon_2$.	$\Delta\epsilon_3$.	$\Delta\epsilon_4$.	x_1.	Δh_1.	Δk_1.	x_2.	Δh_2.	Δk_2.	x_3.	Δh_3.	Δk_3.
+2994·5	+125·2	+30·0	+ 83·2	− 36·8	+ 8·2	− 214·9	−44·8	+ 74·4	−32·0	− 52·2	− 52·7	+ 61·4	− 18·4	− 98·0	+132·7
	+ 35·6	−	+ 6·2	− 12·4	+ 3·7	− 0·7	− 5·6	+ 8·6	− 8·4	− 14·3	− 16·1	+ 15·7	+ 12·5	+ 1·1	+ 19·8
		+10·5	− 0·5	− 5·0	− 10·9	− 34·2	− 1·1	+ 1·0	− 0·9	− 4·8	− 6·2	+ 2·6	− 10·6	+ 9·0	− 5·2
			+227·8	− 27·6	− 35·4	+ 7·9	−34·4	+ 66·0	33·2	− 27·3	− 8·0	+ 23·5	+ 25·8	− 38·6	+ 25·8
				+575·1	+ 22·7	− 50·5	+ 0·4	+ 4·5	− 6·5	− 0·8	− 43·1	+ 25·6	+ 25·4	− 59·6	− 1·9
					+1701·6	− 43·1	+ 1·5	− 61·3	+ 2·3	− 13·8	− 43 0	+ 10·3	−139·4	+ 219·4	−162·5
						+5756·3	+ 2·3	+ 24·1	+13·1	+ 60·7	+111·9	− 48·9	−168·2	+ 230·9	− 84·1
							+88·2	−115·4	+60·4	+ 39·6	+ 54·9	− 31·3	− 78·2	+ 98·7	− 55·1
								+237·0	−92·4	− 67·1	−115·2	+ 62·5	+123·2	− 191·2	+105·0
									+59·8	+ 23·7	+ 47·2	− 26·7	− 58·0	+ 85·5	− 53·8
										+252·6	+296·8	−207·4	+113·2	− 168·4	+ 78·6
											+554·8	−282·1	+146·4	− 270·9	+136·8
												+204·6	− 88·6	+ 156·5	− 77·0
													+758·4	− 942·5	+550·3
														+1622·8	−806·5
															+547·1

D. Position-Angles.—Gill and Finlay.

Δp_1.	Δq_1.	Δp_2.	Δq_2.	Δp_3.	Δq_3.	Δp_4.	Δq_4.	$\Delta\epsilon_1$.	$\Delta\epsilon_2$.	$\Delta\epsilon_3$.	$\Delta\epsilon_4$.
+281·6	− 38·9	− 21·2	− 52·8	− 20·6	+ 19·8	− 30·2	− 36·0	− 6·5	+12·2	− 1·2	+ 4·4
	+245·2	+ 10·2	− 27·0	− 44·1	− 62·0	+ 36·7	+ 38·9	+ 8·6	+ 2·0	+ 1·7	+ 3·0
		+591·8	− 40·4	− 42·7	+ 88·5	+ 42·9	− 129·8	+12·0	−18·8	+22·6	− 24·1
			+783·8	+ 42·6	− 41·0	− 101·4	+ 65·2	+ 4·6	+18·2	+ 4·1	+ 16·3
				+1484·0	+ 21·2	+ 137·5	− 186·2	− 1·1	+16·3	−41·5	− 6·9
					+1927·4	− 301·0	+ 75·6	− 2·2	+11·0	+71·0	+ 30·9
						+5106·5	+ 511·6	+ 5·1	−16·1	− 4·0	− 74·7
							+5604·4	+ 6·4	+ 0·7	+16·6	+187·6
								+11·6	+ 7·8	+ 0·9	+ 16·5
									+29·1	+ 6·3	+ 5·8
										+42·5	+ 5·0
											+192·7

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TABLE XVIII.—*continued.*

NORMAL EQUATIONS.

C. DISTANCES—GILL AND FINLAY.

Δh_4.	Δk_4.	y_2.	y_3.	y_4.	Δp_1.	Δq_1.	Δp_2.	Δq_2.	Δp_3.	Δq_3.	Δp_4.	Δq_4.	n.
+ 320·2	− 342·5	− 1·2	+17·3	+13·7	+2·9	+ 1·2	+ 6·4	+ 6·1	+23·2	+13·5	+ 72·3	+ 28·5	M
+ 22·6	− 21·9	+ 1·0	− 3·5	− 2·5	+1·0	− 0·6	+ 2·2	+ 1·8	+ 6·1	+ 2·7	+ 8·4	+ 4·6	Z_G
+ 0·7	+ 4·2	+ 0·3	0·0	+ 0·6	−0·2	+ 0·6	− 0·4	+ 0·5	+ 2·5	+ 0·9	+ 2·8	+ 1·3	Z_F
− 50·9	+ 26·0	+ 2·4	− 0·1	+ 4·4	−0·5	− 0·6	− 5·9	− 4·2	+ 0·3	+ 0·4	− 1·6	− 4·5	E_1
+ 65·0	+ 21·0	−14·1	+ 1·6	+ 0·1	−6·2	− 4·0	− 0·5	− 3·8	−17·3	−13·7	+ 13·6	+ 2·9	E_2
− 185·6	+ 82·2	+ 1·9	−23·5	−14·3	+3·0	− 4·5	−18·3	−12·3	+19·2	−18·0	+ 3·2	− 11·1	E_3
−1025·4	+ 792·3	− 8·8	− 5·4	+ 0·4	−4·0	− 1·1	+20·9	−13·5	+ 9·8	−26·5	+ 67·9	− 57·6	E_4
+ 163·3	− 79·2	+ 0·9	+ 2·3	− 3·2	−1·3	+ 2·2	− 2·5	− 0·3	− 1·0	− 0·8	+ 14·5	+ 5·1	X_1
− 303·7	+ 171·8	− 1·2	− 0·6	+ 2·7	−0·2	− 2·9	+ 5·4	− 0·7	+ 7·8	+ 6·5	− 24·4	− 9·6	H_1
+ 125·9	− 65·4	+ 1·1	+ 1·0	− 3·4	+0·2	+ 3·6	− 3·2	− 0·7	− 2·9	− 2·0	+ 16·4	+ 5·7	K_1
− 328·9	+ 165·6	− 0·1	+ 1·6	− 1·2	+0·6	− 0·9	− 6·2	− 5·1	−12·0	− 2·3	+ 1·4	− 0·5	X_2
− 570·7	+ 295·1	+11·2	+ 2·2	− 2·0	+2·9	− 2·2	− 8·9	−14·1	−16·9	+ 3·0	− 7·7	− 3·2	H_2
+ 303·4	− 162·8	− 1·6	− 1·2	+ 0·3	−2·0	0·0	+ 7·7	+ 6·8	+11·6	+ 1·7	+ 3·6	+ 2·2	K_2
+ 622·3	− 317·7	+ 0·9	− 9·4	− 2·7	+2·5	− 2·8	+ 1·1	− 7·0	− 3·1	+ 8·7	+ 11·9	+ 14·0	X_3
−1030·4	+ 563·0	− 1·3	− 8·2	− 3·6	+0·2	+ 0·6	− 1·4	+14·9	+ 0·4	−15·9	− 9·6	− 15·6	H_3
+ 572·5	− 296·1	+ 2·6	− 3·4	− 0·4	+2·1	− 3·4	+ 1·9	− 6·8	+ 5·7	+16·1	+ 11·5	+ 11·4	K_3
+5328·7	−2624·6	−14·0	+ 5·4	− 9·5	−7·0	+29·9	− 3·1	+14·1	+ 8·4	− 3·9	+210·4	+104·5	H_4
	+1781·2	+ 3·2	− 0·2	+ 9·2	+1·8	−12·8	+ 1·4	− 8·0	− 3·4	+ 3·9	− 98·3	− 50·7	K_4
		+ 3·5	0·0	− 0·3	−0·3	+ 0·2	+ 0·4	− 0·7	− 0·1	+ 0·4	− 1·6	− 0·3	Y_2
			+26·8	+ 7·2	−0·4	+ 0·1	− 1·4	− 0·1	− 7·6	− 3·7	− 3·4	− 1·8	Y_3
				+14·7	0·0	− 0·3	− 0·1	0·0	0·0	0·0	− 3·2	− 1·1	Y_4
					+5·8	+ 1·6	+ 3·6	+ 1·4	+ 0·6	+ 2·6	+ 3·4	+ 1·8	P_1'
						+10·9	+ 4·7	+ 0·7	− 0·2	− 2·4	+ 30·1	+ 10·8	Q_1'
							+17·7	+ 3·6	+ 5·1	+ 4·8	+ 3·1	+ 1·4	P_2'
								+ 7·7	+ 3·1	− 2·3	+ 4·4	+ 1·0	Q_2'
									+28·2	+12·5	+ 4·1	+ 3·7	P_3'
										+21·1	+ 1·2	− 3·3	Q_3'
											+180·0	+ 65·7	P_4'
												+ 32·7	Q_4'

D. POSITION-ANGLES—GILL AND FINLAY.

x_1.	Δh_1.	Δk_1.	x_2.	Δh_2.	Δk_2.	x_3.	Δh_3.	Δk_3.	Δh_4.	Δk_4.	y_2.	y_3.	y_4.	n.
+ 1·5	+ 2·6	− 1·4	− 0·6	− 8·5	+ 4·3	− 2·6	+ 0·8	− 1·8	+ 7·9	+ 1·0	+ 0·9	+ 0·8	− 0·1	P_1
− 2·3	+ 1·6	− 3·8	+ 0·3	+ 5·8	0·0	+ 3·1	− 2·4	+ 4·2	− 31·0	+ 13·9	− 0·3	− 0·6	+ 0·2	Q_1
+ 1·7	−12·0	+ 4·2	+ 2·8	+20·2	− 8·0	− 6·4	+ 5·4	− 7·1	+ 2·2	− 2·0	− 1·3	+ 1·3	0·0	P_2
+ 0·9	+ 0·7	+ 1·4	+ 6·8	+13·9	− 9·2	+ 6·9	−18·1	− 8·6	− 14·3	+ 8·3	+ 0·6	− 0·5	0·0	Q_2
+ 1·3	− 6·0	+ 2·4	+ 8·9	+14·8	− 9·1	− 4·9	− 2·9	−11·0	− 5·0	+ 6·9	− 0·3	+ 6·2	− 0·1	P_3
+ 1·9	− 5·8	+ 1·9	+ 1·0	− 4·8	+ 0·2	−22·8	+18·7	−29·4	+ 8·1	− 2·6	− 0·1	+ 3·6	+ 0·1	Q_3
−16·5	+22·3	−19·4	− 0·1	+ 5·3	− 6·0	−16·1	+ 5·5	−16·6	−196·2	+125·8	+ 1·3	+ 5·2	+ 4·6	P_4
− 6·2	+ 9·5	− 6·7	+ 4·4	+ 0·5	− 6·1	−21·6	+13·7	−15·5	−112·6	+ 86·3	+ 0·2	+ 2·9	+ 2·3	Q_4
+ 3·2	− 5·3	+ 2·4	+ 5·1	+12·4	− 4·0	+ 0·6	+ 0·6	+ 0·8	− 8·1	+ 4·5	− 0·2	− 0·1	+ 0·2	E_1'
+ 7·6	− 5·5	+ 5·2	+10·3	+12·4	− 5·1	+ 2·0	− 5·9	+ 2·4	+ 3·5	− 0·9	+ 0·6	+ 0·8	− 0·1	E_2'
+ 0·6	+ 3·7	− 0·7	+ 2·4	+ 1·1	− 1·9	+ 0·4	−14·6	+ 4·0	− 1·9	+ 1·2	0·0	− 0·2	0·0	E_3'
− 2·9	+ 3·4	− 6·3	− 1·8	− 1·6	+ 2·4	− 0·7	+ 1·0	− 1·5	− 28·5	+ 24·9	− 0·4	− 0·4	+ 0·7	E_4'
+ 4·6	− 4·8	+ 4·0	+ 4·3	+ 6·3	− 2·4	− 0·2	− 0·4	0·0	+ 10·7	− 5·1	+ 0·1	− 0·1	− 0·2	X_1'
	+ 9·5	− 5·3	− 4·5	−11·4	+ 3·8	+ 0·9	− 3·6	+ 2·0	− 15·9	+ 7·0	+ 0·2	− 0·4	+ 0·2	H_1'
		+ 4·4	+ 3·2	+ 5·8	− 2·2	− 0·4	+ 0·7	− 0·7	+ 15·2	− 7·5	0·0	+ 0·1	− 0·2	K_1'
			+ 7·5	+10·7	+ 5·2	+ 1·8	− 3·4	+ 2·2	− 2·9	+ 1·5	+ 0·2	+ 0·3	0·0	X_2'
				+26·5	−10·0	+ 3·8	− 7·8	+ 4·9	− 5·8	+ 2·2	− 0·2	+ 0·4	0·0	H_2'
					+ 5·1	− 2·4	+ 4·4	− 3·1	+ 3·4	− 1·8	− 0·1	− 0·3	0·0	K_2'
						+ 8·8	−11·3	+10·4	+ 3·4	− 2·7	+ 0·2	− 0·4	0·0	X_3'
							+31·0	−17·3	− 6·8	+ 3·2	− 0·3	+ 0·9	+ 0·1	H_3'
								+15·0	+ 2·6	− 2·2	+ 0·2	− 1·0	0·0	K_3'
									+156·4	− 70·0	− 0·5	− 0·4	− 2·0	H_4'
										+ 39·1	+ 0·2	+ 0·1	+ 0·9	K_4'
											+ 0·1	0·0	0·0	Y_2''
												+ 0·5	0·0	Y_3''
													0·0	Y_4''

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TABLE XVIII.—*continued.*

NORMAL EQUATIONS.

E. DISTANCES AND POSITION-ANGLES—GILL AND FINLAY.

Δp_1.	Δq_1.	Δp_2.	Δq_2.	Δp_3	Δq_3.	Δp_4.	Δq_4.	μ.	z_{ii}.	z_{ij}.	$\Delta\epsilon_1$.	$\Delta\epsilon_2$.	$\Delta\epsilon_3$.	$\Delta\epsilon_4$.
+287·4	− 37·3	− 17·6	− 51·5	− 20·0	+ 22·4	− 26·8	− 34·3	+ 2·9	+ 1·0	− 0·2	− 6·9	+ 6·0	+ 1·8	− 0·4
	+256·2	+ 14·8	− 26·3	− 44·3	− 64·4	+ 66·9	+ 49·8	+ 1·2	− 0·6	+ 0·6	+ 8·0	− 2·0	− 2·8	+ 1·9
		+609·4	− 36·7	− 37·5	+ 93·2	+ 46·0	− 128·4	+ 6·4	+ 2·2	− 0·4	+ 6·2	− 19·3	+ 4·3	− 3·2
			+791·5	+ 45·7	− 43·2	− 97·0	+ 66·1	+ 6·1	+ 1·8	+ 0·5	+ 0·4	+ 14·4	− 8·2	+ 2·8
				+1512·2	+ 33·7	+ 141·6	− 182·5	+ 23·2	+ 6·1	+ 2·5	− 0·8	− 1·0	− 22·3	+ 2·9
					+1948·5	− 299·8	+ 72·3	+ 13·5	+ 2·7	+ 0·9	− 1·8	− 2·7	+ 53·0	+ 4·4
						+5286·4	+ 577·3	+ 72·3	+ 8·4	+ 2·8	+ 3·5	− 2·5	− 0·8	− 6·8
							+5637·1	+ 28·5	+ 4·6	+ 1·3	+ 1·9	+ 3·5	+ 5·4	+ 130·1
								+2994·5	+125·2	+30·0	+ 83·2	− 36·8	+ 8·2	− 214·9
									+ 35·6	−	+ 6·2	− 12·4	+ 3·7	− 0·7
										+10·5	− 0·5	− 5·0	− 10·9	− 34·2
											+239·4	− 19·8	− 34·5	+ 24·4
												+604·2	+ 29·0	− 44·7
													+1744·1	− 38·1
														+5949·0

The final solution, as has already been mentioned and will be more fully explained below, depends on the normals C and D. Of the normals C the last eight were omitted, and of the normals D only the first eight were used. For the derivation of the absolute terms of these normals the residuals remaining after the substitution of the first approximation in the equations of condition are used. Calling these residuals n, we have

n = O—C—first approximation,

n = (O—C) . s sin 1″—first approximation,

for the distances and position-angles respectively.

Using these values of n as right-hand members of the equations of condition, we find the right-hand members of the normal equations given in Table XIX. below.

The seven observations 87, 283, 328, 329, 467, 497, 498, which were included in the first solution, but rejected in the final solution, require a special treatment. As it was desirable that the-left hand members of the normal equations should not be changed, the values of the right-hand members of the equations of condition resulting from the

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TABLE XVIII.—*continued.*

NORMAL EQUATIONS.

E. DISTANCES AND POSITION-ANGLES—GILL AND FINLAY.

x_1.	Δh_1.	Δk_1	x_2.	Δh_2.	Δk_2.	x_3.	Δh_3.	Δk_3.	Δh_4.	Δk_4.	y_2.	y_3.	y_4.	n.
+ 0·2	+ 2·3	− 1·2	− 0·1	− 5·6	+ 2·3	− 0·1	+ 1·0	+ 0·3	+ 0·9	+ 2·7	+ 0·6	+ 0·3	0·0	(P_1)
− 0·1	− 1·2	− 0·2	− 0·6	+ 3·6	0·0	+ 0·3	− 1·7	+ 0·8	− 1·2	+ 1·1	− 0·1	− 0·5	0·0	(Q_1)
− 0·8	− 6·6	+ 1·0	− 3·3	+ 11·3	− 0·3	− 5·3	+ 3·9	− 5·2	− 0·9	− 0·6	− 0·9	− 0·1	− 0·1	(P_2)
+ 0·7	0·0	+ 0·7	+ 1·7	− 0·2	− 2·3	− 0·2	− 3·2	+ 1·8	− 0·2	+ 0·3	− 0·1	− 0·6	0·0	(Q_2)
+ 0·2	+ 1·7	− 0·5	− 3·1	− 2·1	+ 2·4	− 8·0	− 2·5	− 5·3	+ 2·8	+ 3·5	+ 0·2	− 1·3	0·0	(P_3)
+ 1·1	+ 0·7	0·0	− 1·4	− 1·8	+ 1·9	− 14·1	+ 2·8	− 13·3	+ 4·2	+ 1·2	+ 0·3	− 0·2	0·0	(Q_3)
− 2·0	− 2·1	− 3·0	+ 1·3	− 2·4	− 2·4	− 4·2	− 4·2	− 5·1	+ 14·3	+ 27·5	− 0·3	+ 1·8	+ 1·4	(P_4)
− 1·1	− 0·1	− 0·9	+ 3·9	− 2·8	− 3·9	− 7·5	− 1·9	− 4·1	− 8·1	+ 26·6	− 0·1	+ 1·1	+ 1·2	(Q_4)
−44·8	+ 74·4	−32·0	− 52·2	− 52·7	+ 61·4	− 18·4	− 98·0	+132·7	+ 320·2	− 342·4	− 1·2	+17·3	+13·7	M
− 5·6	+ 8·6	− 8·4	− 14·3	− 16·1	+ 15·7	+ 12·5	+ 1·1	+ 19·8	+ 22·6	− 21·9	+ 1·0	− 3·5	− 2·5	Z_G
− 1·1	+ 1·0	− 0·9	− 4·8	− 6·2	+ 2·6	− 10·6	+ 9·0	− 5·2	+ 0·7	+ 4·2	+ 0·3	0·0	+ 0·6	Z_F
−31·2	+ 60·8	−30·8	− 22·2	+ 4·3	+ 19·5	+ 26·4	− 39·2	+ 26·6	− 59·0	+ 30·5	+ 2·2	− 0·3	+ 4·6	(E_1)
+ 8·0	− 1·1	− 1·4	+ 9·5	− 30·7	+ 20·5	+ 27·5	− 65·5	+ 0·5	+ 68·5	+ 20·1	−13·5	+ 2·4	0·0	(E_2)
+ 2·1	− 57·5	+ 1·6	− 11·4	− 42·0	+ 8·4	−139·0	+ 204·8	−158·4	− 187·5	+ 83·3	+ 1·9	−23·7	−14·3	(E_3)
− 0·6	+ 27·6	+ 6·9	+ 59·0	+110·2	− 46·5	−168·8	+ 232·0	− 85·5	−1053·9	+ 817·0	− 9·2	− 5·8	+ 1·1	(E_4)
+92·8	−120·3	+64·3	+ 43·9	+ 61·2	− 33·7	− 78·4	+ 98·3	− 55·0	+ 174·0	− 84·3	+ 1·0	+ 2·2	− 3·3	(X_1)
	+246·5	−97·7	− 71·6	−126·6	+ 66·3	+124·2	− 194·7	+107·0	− 319·6	+ 178·8	− 1·0	− 1·0	+ 2·9	(H_1)
		+64·2	+ 26·9	+ 53·0	− 28·8	− 58·4	+ 86·2	− 54·5	+ 141·1	− 72·8	+ 1·1	+ 1·1	− 3·6	(K_1)
			+260·0	+307·6	−212·5	+115·0	− 171·8	+ 80·8	− 331·8	+ 167·2	+ 0·1	+ 1·9	− 1·2	(X_2)
				+581·4	−292·1	+150·2	− 278·7	+141·6	− 576·6	+ 297·3	+11·0	+ 2·7	− 2·0	(H_2)
					+209·8	− 91·1	+ 160·9	− 80·1	+ 306·8	− 164·6	− 1·7	− 1·5	+ 0·2	(K_2)
						+767·2	− 953·8	+560·7	+ 625·6	− 320·4	+ 1·0	− 9·8	− 2·7	(X_3)
							+1653·8	−823·8	−1037·2	+ 566·1	− 1·6	− 7·3	− 3·5	(H_3)
								+562·1	+ 575·1	− 298·4	+ 2·9	− 4·4	− 0·4	(K_3)
									+5485·0	−2694·6	−14·5	+ 5·0	−11·5	(H_4)
										+1820·3	+ 3·4	− 0·1	+10·0	(K_4)
											+ 3·6	0·0	− 0·3	(Y_2)
												+27·4	+ 7·2	(Y_3)
													+14·8	(Y_4)

substitution of the first approximation were adopted instead of the actual (O—C)'s, in other words n was taken equal to zero. As the final solution differs very little from the first approximation, these equations will have a residual practically equal to zero after substitution of the final values of the unknowns, and thus the fact that they were not rejected will not influence the values of the unknowns, but only have a (very small) effect on the weights.

Solution of Normals.—In the Cape Solutions absolute terms of the normal equations were used, which did not contain most of the corrections enumerated in the preceding chapter. The agreement between my own solution based on these same absolute terms, and the Cape Solution was, however, so good, that it did appear useless to correct also the Cape Solution. The leading principle of my solution was the following :— In the normals D, which are derived from the position-angles, the coefficients of the unknowns μ, z, $\Delta\epsilon$, x, Δh, Δk, y are very small compared to those of the p's and q's. Consequently a very satisfactory first approximation for these last unknowns may be derived from the normals D in which all the other unknowns are neglected. Then these values of p and q are substituted in the normals C, and the unknowns μ, z, $\Delta\epsilon$, x, Δh,

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Δk, y are solved. The resulting values are again substituted in the normals D, and a second approximation for the values of p and q is derived, which is then again substituted in the normals C, and so on.

TABLE XIX.

Absolute Terms of Normal Equations.

Pair.	M.	Z_0.	Z_F.	E_1.	E_2.	E_3.	E_4.	X_1.	H_1.	K_1.
I.–II.	−1″·972	−0″·428	−0″·309	−2″·361	+11″·962	″ ...	″ ...	′ +0·951	−1″·313	′ +0·370
I.–III.	+ ·957	+ ·300	− ·258	+ ·493	...	+0·046	...	− ·199	+ ·579	− ·068
I.–IV.	+2·813	+ ·218	+ ·029	− ·351	...	...	− 3·422	− ·173	+ ·625	− ·243
II.–III.	−2·096	−1·178	− ·213	...	+10·623	− ·155	...	...	...	...
II.–IV.	+5·062	+ ·211	+ ·611	...	+14·364	...	−18·812	...	...	...
III.–IV.	+4·986	− ·062	− ·010	...	...	−6·710	− 4·949	...	...	...
Normals C	+9·750	−0·939	−0·150	−2·219	+36·949	−6·819	−27·183	+0·579	−0·109	+0·059

Pair.	X_2.	H_2.	K_2.	X_3.	H_3.	K_3.	H_4.	K_4.	Y_2.	Y_3.	Y_4.
I.–II.	″ +2·124	″ +1·904	″ −1·095	″ ...	″ ...	″ ...	″ ...	″ ...	″ −0·233	″ ...	″ ...
I.–III.	...	...	...	+0·683	−1·280	+0·026	...	...	...	+0·116	...
I.–IV.	...	...	...	...	...	...	−2·376	+0·955	...	...	−0·165
II.–III.	+ ·684	− ·547	− ·096	+1·661	−3·166	+ ·059	...	...	− ·071	+ ·227	...
II.–IV.	− ·336	−1·319	+ ·887	...	...	...	+3·790	− ·596	− ·379	...	+ ·085
III.–IV.	...	...	...	+ ·467	−1·411	+ ·799	+3·047	−2·107	...	+ ·050	− ·079
Normals C	+2·472	+0·038	−0·304	+2·811	−5·857	+0·884	+4·461	−1·748	−0·683	+0·393	−0·159

Pair.	P_1.	Q_1.	P_2.	Q_2.	P_3.	Q_3.	P_4.	Q_4.
I.–II.	″ −0·153	″ +0·918	″ +0·913	″ −1·561	″ ...	″ ...	″ ...	″ ...
I.–III.	− ·464	+ ·807	...	...	+3·471	+0·623	...	...
I.–IV.	− ·118	−1·814	...	...	...	...	−0·914	−4·390
II.–III.	...	...	−2·004	+2·316	+1·479	− ·343	...	...
II.–IV.	...	...	+1·336	+2·413	...	...	−4·265	−2·589
III.–IV.	...	...	...	...	−3·518	+2·587	− ·512	+3·020
Normals D	−0·735	−0·089	+0·245	+3·168	+1·432	+2·867	−5·691	−3·959

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For this purpose a general solution was made of the two sets of normals, giving the values of the unknowns in terms of the right-hand members of the normal equations. Consequently the whole process of successive approximation is reduced to substituting the successive values of these right-hand members in the general solution. A symmetrical arrangement of the computations necessary for the construction of such a general solution, which has proved very convenient in practice, will be explained in the Appendix.

The normals as given above were somewhat transformed before they were used for the solution. In the normals D, in order to make the coefficients of all the unknowns of the same order of magnitude, I introduced $\frac{1}{3}\Delta p_1$, $\frac{1}{3}\Delta q_1$, $\frac{1}{2}\Delta p_2$, and $\frac{1}{2}\Delta q_2$ as unknowns instead of Δp_1, Δq_1, Δp_2, and Δq_2. These normals thus become:

NORMALS P.

$\frac{1}{3}\Delta p_1$.	$\frac{1}{3}\Delta q_1$.	$\frac{1}{2}\Delta p_2$.	$\frac{1}{2}\Delta q_2$.	Δp_3.	Δq_3.	Δp_4.	Δq_4.	
+2534	− 350	− 127	− 317	− 62	+ 59	− 91	− 108	$= 3P_1$
	+2207	+ 61	− 162	− 132	− 186	+ 110	+ 117	$= 3Q_1$
		+2367	− 161	− 85	+ 177	+ 86	− 260	$= 2P_2$
			+3135	+ 85	− 82	− 203	+ 130	$= 2Q_2$
				+1484	+ 21	+ 138	− 186	$= P_3$
					+1927	− 301	+ 76	$= Q_3$
						+5106	+ 512	$= P_4$
							+5604	$= Q_4$

of which the solution is:

SOLUTION P.

$3P_1$.	$3Q_1$.	$2P_2$.	$2Q_2$.	P_3.	Q_3.	P_4.	Q_4.	
+ 412	+ 68	+ 25	+ 46	+ 23	− 6	+ 5	+ 7	$= \frac{1}{3}\Delta p_1$
	+ 474	− 9	+ 30	+ 42	+ 45	− 5	− 8	$= \frac{1}{3}\Delta q_1$
		+ 432	+ 20	+ 28	− 43	− 12	+ 23	$= \frac{1}{2}\Delta p_2$
			+ 328	− 16	+ 16	+ 15	− 8	$= \frac{1}{2}\Delta q_2$
				+ 686	− 12	− 23	+ 26	$= \Delta p_3$
					+ 534	+ 34	− 14	$= \Delta q_3$
						+ 201	− 21	$= \Delta p_4$
							+ 183	$= \Delta q_4$

where the coefficients are given in units of the *sixth* decimal place, and where:

n.	$\Delta\epsilon_4$.	Δh_4.	Δk_4.	$\frac{1}{2}x_3$.	$\frac{1}{2}\Delta k_3$.	Δh_3.	$\frac{1}{3}x_2$.	$\frac{1}{2}\Delta h_2$.	$\frac{1}{3}\Delta k_2$.	$\frac{1}{3}x_1$.	$\frac{1}{3}\Delta h_1$.	$\frac{1}{3}\Delta k_1$.	$\frac{1}{2}\Delta\epsilon_2$.	$\frac{1}{3}\Delta\epsilon_1$.	$\Delta\epsilon_3$.	y_2.	y_3.	y_4.
$3P_1 = -2''\cdot21$	− 13	− 24	− 3	+16	+11	− 2	+ 6	+51	−39	−23	−23	+20	−73	+58	+ 4	−3	−2	0
$3Q_1 = -0\cdot27$	− 9	+ 93	− 42	−19	−25	+ 7	− 3	−35	0	+34	−14	+58	−12	−78	− 5	+1	+2	−1
$2P_2 = +0\cdot49$	+ 48	− 4	+ 4	+26	+29	−11	−17	−81	+48	−17	+72	−42	+75	−72	−45	+3	−3	0
$2Q_2 = +6\cdot33$	− 33	+ 29	− 17	−27	+34	+36	−41	−56	+55	− 9	− 4	−14	−73	−28	− 8	−1	+1	0
$P_3 = +1\cdot43$	+ 7	+ 6	− 7	+10	+22	+ 3	−27	−30	+27	− 6	+18	−12	−33	+ 3	+41	0	−6	0
$Q_3 = +2\cdot87$	− 31	− 8	+ 3	+46	+59	−19	− 3	+10	− 1	− 9	+18	−10	−22	+ 7	−71	0	−4	0
$P_4 = -5\cdot69$	+ 75	+196	−126	+32	+33	− 5	0	−11	+18	+82	−67	+97	+32	−15	+ 4	−1	−5	−5
$Q_4 = -3\cdot96$	−188	+113	− 86	+43	+31	−14	−13	− 1	+19	+31	−28	+33	− 1	−19	−17	0	−3	−2

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In the normals C $\frac{1}{7}z_G$, $\frac{1}{10}z_F$, $\frac{1}{3}\Delta\epsilon_1$, $\frac{1}{2}\Delta\epsilon_2$, $\frac{1}{5}x_1$, $\frac{1}{3}\Delta h_1$, $\frac{1}{5}\Delta k_1$, $\frac{1}{3}x_2$, $\frac{1}{2}\Delta h_2$, $\frac{1}{3}\Delta k_2$, $\frac{1}{2}x_3$, and $\frac{1}{2}\Delta k_3$ were introduced as unknowns instead of z_G, z_F, $\Delta\epsilon_1$ . . . etc., and the order of the unknowns was altered for the convenience of the solution (*viz.* so that no "multipliers" [see Appendix] larger than unity should appear). The unknowns y_2, y_3, and y_4 were neglected in first approximation. I thus derived the following normal equations:

Normals S.

$\Delta\epsilon_4$.	Δh_4.	Δk_4.	$\frac{1}{2}x_3$.	$\frac{1}{2}\Delta k_3$.	Δh_3.	$\frac{1}{3}x_2$.	$\frac{1}{2}\Delta h_2$.	$\frac{1}{3}\Delta k_2$.	$\frac{1}{5}x_1$.	$\frac{1}{3}\Delta h_1$.	$\frac{1}{5}\Delta k_1$.	$\frac{1}{2}\Delta\epsilon_2$.	$\frac{1}{3}\Delta\epsilon_1$.	$\Delta\epsilon_3$.	μ.	$\frac{1}{7}z_G$.	$\frac{1}{10}z_F$.	
+5756	−1025	+ 792	− 336	− 168	+ 231	+ 182	+ 224	− 147	+ 11	+ 72	+ 66	− 101	+ 24	− 43	− 215	− 5	− 342	$= E_4$
	+5329	−2625	+1245	+1145	−1030	− 987	−1141	+ 910	+ 817	− 911	+ 630	+ 130	− 153	− 186	+ 320	+ 158	+ 7	$= H_4$
		+1781	− 635	− 592	+ 563	+ 497	+ 590	− 488	− 396	+ 516	− 327	+ 42	+ 78	+ 82	− 343	− 154	+ 42	$= K_4$
			+3034	+2201	−1885	+ 679	+ 586	− 532	− 782	+ 740	− 580	+ 102	+ 155	− 279	− 37	+ 175	− 211	$= 2X_3$
				+2188	−1613	+ 472	+ 547	− 462	− 551	+ 630	− 538	− 8	+ 155	− 325	+ 265	+ 276	− 103	$= 2K_3$
					+1623	− 505	− 542	+ 469	+ 493	− 573	+ 427	− 119	− 116	+ 219	− 98	+ 8	+ 90	$= H_3$
						+2273	+1781	−1866	+ 593	− 604	+ 356	− 5	− 246	− 41	− 157	− 301	− 143	$= 3X_2$
							+2219	−1693	+ 549	− 691	+ 472	− 172	− 48	− 86	− 105	− 225	− 123	$= 2H_2$
								+1842	− 469	+ 562	− 400	+ 154	+ 212	+ 31	+ 184	+ 329	+ 77	$= 3K_2$
									+2205	−1732	+1510	+ 4	− 516	+ 7	− 224	− 197	− 54	$= 5X_1$
										+2133	−1386	+ 27	+ 594	− 184	+ 223	+ 181	+ 31	$= 3H_1$
											+1495	− 65	− 498	+ 11	− 160	− 295	− 44	$= 5K_1$
												+2300	− 165	+ 45	− 74	− 173	− 100	$= 2E_2$
													+2050	− 106	+ 250	+ 130	− 16	$= 3E_1$
														+1702	+ 8	+ 26	− 109	$= E_3$
															+2995	+ 876	+ 300	$= M$
																+1744	...	$= 7Z_G$
																	+1053	$= 10Z_F$

of which the solution is:

Solution S.

E_4.	H_4.	K_4.	$2X_3$.	$2K_3$.	H_3.	$3X_2$.	$2H_2$.	$3K_2$.	$5X_1$.	$3H_1$.	$5K_1$.	$2E_2$.	$3E_1$.	E_3.	M.	$7Z_G$.	$10Z_F$.	
+20	+ 2	− 18	+ 7	− 9	+ 12	− 7	+ 10	− 9	+ 6	+ 10	− 10	+ 2	− 2	+ 2	0	− 1	+ 8	$= \Delta\epsilon_4$
	+124	+ 66	− 7	− 34	+ 96	+ 20	+ 98	− 4	− 10	+ 95	− 23	+ 2	−15	+ 5	− 4	+ 5	− 4	$= \Delta h_4$
		+270	− 26	+ 22	−129	+ 36	− 88	+ 57	− 37	−111	+ 41	−22	+12	− 7	+19	+ 4	− 24	$= \Delta k_4$
			+206	−128	+123	−110	+ 35	− 88	+ 89	+ 26	− 82	+ 3	− 8	− 2	+22	−17	+ 12	$= \frac{1}{2}x_3$
				+289	+ 64	+ 88	− 69	+ 90	− 70	− 58	+ 98	+ 3	+14	+13	−15	−18	− 1	$= \frac{1}{2}\Delta k_3$
					+539	− 39	+215	− 78	+ 50	+235	− 80	+32	−32	+16	− 7	−33	+ 18	$= \Delta h_3$
						+379	− 48	+341	−130	− 15	+139	−19	+14	+ 5	−18	+10	+ 11	$= \frac{1}{3}x_2$
							+335	+ 63	+ 39	+203	− 79	+19	−43	+15	−13	− 5	+ 15	$= \Delta\frac{1}{2}h_2$
								+513	−110	− 52	+131	−26	+ 3	+ 7	−13	−11	+ 13	$= \Delta\frac{1}{3}k_2$
									+248	+ 96	−187	+ 1	−12	+ 7	+14	−17	+ 3	$= \frac{1}{5}x_1$
										+352	0	+20	−38	+25	−19	+ 9	+ 16	$= \frac{1}{3}\Delta h_1$
											+342	0	+29	+ 8	−17	+31	+ 6	$= \frac{1}{5}\Delta k_1$
												+49	+ 2	0	− 3	+ 5	+ 6	$= \frac{1}{2}\Delta\epsilon_2$
													+61	+ 1	− 3	+ 2	+ 2	$= \frac{1}{3}\Delta\epsilon_1$
														+64	− 3	− 2	+ 10	$= \Delta\epsilon_3$
															+46	−23	− 14	$= \mu$
																+79	+ 6	$= \frac{1}{7}z_G$
																	+107	$= \frac{1}{10}z_F$

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where the coefficients are given in units of the *fifth* decimal place, and where:

n.	$\frac{1}{8}\Delta p_1$.	$\frac{1}{8}\Delta q_1$.	$\frac{1}{2}\Delta p_2$.	$\frac{1}{2}\Delta q_2$.	Δp_3.	Δq_3.	Δp_4.	Δq_4.	y_2.	y_3.	y_4.
$E_4=-27''.18$	+12	+ 1	−42	+27	−10	+27	− 68	+ 58	+ 9	+ 5	0
$H_4=+\ 4.46$	+21	−90	+ 6	−28	− 8	+ 4	−211	−105	+14	− 5	+10
$K_4=-\ 1.75$	− 5	+38	− 3	+16	+ 3	− 4	+ 98	+ 60	− 3	0	− 9
$2X_3=+\ 5.62$	−15	+17	− 4	+28	+ 6	−17	− 24	− 28	− 2	+19	+ 5
$2K_3=+\ 1.77$	−12	+21	− 8	+27	−11	−32	− 23	− 23	− 5	+ 7	+ 1
$H_3=-\ 5.86$	− 1	− 2	+ 3	−30	0	+16	+ 10	+ 16	+ 1	+ 8	+ 4
$3X_2=+\ 7.42$	− 5	+ 8	+37	+31	+36	+ 7	− 4	+ 2	0	− 5	+ 4
$2H_2=+\ 0.08$	−18	+13	+36	+56	+34	− 6	+ 15	+ 6	−22	− 4	+ 4
$3K_2=-\ 0.91$	+18	0	−46	−41	−35	− 5	− 11	− 7	+ 5	+ 4	− 1
$5X_1=+\ 2.90$	+20	−32	+25	+ 3	+ 5	+ 4	− 72	− 25	− 5	−12	+16
$3H_1=-\ 0.33$	+ 2	+26	−32	+ 4	−23	−20	+ 73	+ 29	+ 4	+ 2	− 8
$5K_1=+\ 0.30$	− 2	−54	+32	+ 7	+15	+10	− 82	− 29	− 6	− 5	+17
$2E_2=+73.90$	+37	+24	+ 2	+15	+35	+27	− 27	− 6	+28	− 3	0
$3E_1=-\ 6.22$	+ 4	+ 5	+35	+26	− 1	− 1	+ 5	+ 13	− 7	0	−13
$E_3=-\ 6.82$	− 9	+13	+37	+25	−19	+18	− 3	+ 11	− 2	+24	+14
$M=+\ 9.75$	− 9	− 4	−13	−12	−23	−13	− 72	− 29	+ 1	−17	−14
$7Z_0=-\ 6.57$	−21	+12	−30	−25	−43	−19	− 58	− 32	− 7	+25	+17
$10Z_F=-\ 1.50$	+ 5	−19	+ 7	−10	−25	− 9	− 28	− 13	− 3	0	− 6

Finally, there remain the normals for the determination of y_2, y_3, and y_4, which were taken from the set C without alteration. We have thus:

NORMALS Y.

y_2.	y_3.	y_4.	
+ 3.5	0.0	− 0.3	$= Y_2$
	+26.8	+ 7.2	$= Y_3$
		+14.7	$= Y_4$

SOLUTION Y.

Y_2.	Y_3.	Y_4.	
+0.287	−0.002	+0.007	$= y_2$
	+0.043	−0.021	$= y_3$
		+0.078	$= y_4$

where:

n	$\Delta\epsilon_4$.	Δh_4.	Δk_4.	$\frac{1}{2}x_3$.	$\frac{1}{2}\Delta k_3$.	Δh_3.	$\frac{1}{3}x_2$.	$\frac{1}{2}\Delta h_2$.	$\frac{1}{3}\Delta k_2$.	$\frac{1}{5}x_1$.	$\frac{1}{3}\Delta h_1$.	$\frac{1}{5}\Delta k_1$.	$\frac{1}{2}\Delta\epsilon_2$.
$Y_2=-0''.683$	+9	+14	−3	− 2	−5	+1	0	−22	+5	− 5	+4	− 7	+28
$Y_3=+0.393$	+5	− 5	0	+19	+7	+8	−5	− 4	+4	−12	+2	− 5	− 3
$Y_4=-0.159$	0	+10	−9	+ 5	+1	+4	−4	+ 4	− 1	+16	−8	+17	0

	$\frac{1}{3}\Delta\epsilon_1$.	$\Delta\epsilon_3$.	μ.	$\frac{1}{7}z_0$.	$\frac{1}{10}z_F$.	$\frac{1}{3}\Delta p_1$.	$\frac{1}{3}\Delta q_1$.	$\frac{1}{2}\Delta p_2$.	$\frac{1}{2}\Delta q_2$.	Δp_3.	Δq_3.	Δp_4.	Δq_4.
Y_2	− 7	− 2	+ 1	− 7	−3	+1	−1	−1	+1	0	0	+2	0
Y_3	0	+24	−17	+25	0	+1	0	+3	0	+8	+4	+3	+2
Y_4	−13	+14	−14	+17	−6	0	+1	0	0	0	0	+3	+1

A solution was first made by this method using absolute terms which contained only partially the corrections enumerated in Chapter II. The effect of these corrections on the values of the unknowns was then approximately computed and added to the values found from the original solution. Thus the *First Approximation* mentioned on page 115 is derived, to which corrections must now be found from the normal equations given above.

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It will be noticed that the absolute terms of the normals in μ, $\Delta\epsilon_2$, and $\Delta\epsilon_4$ are comparatively large. This is due to the fact that the correction given by formula (9) of Chapter II. and the correction depending on the arguments ζ_2 η_2 (see page 92) are only very incompletely allowed for in the First Approximation. Accordingly the following first corrections were substituted in the normal equations—

$$(a) \qquad \delta_0(\mu) = +0{\cdot}0034, \qquad \delta_0(\tfrac{1}{2}\Delta\epsilon_2) = +0{\cdot}0339, \qquad \delta_0(\Delta\epsilon_4) = -0{\cdot}0040.$$

The absolute terms thus become

	n.		n.		n.
E_4 :	$-0''{\cdot}01$	$3X_2$:	$+8''{\cdot}85$	$2E_2$:	$-4''{\cdot}22$
H_4 :	$-5{\cdot}14$	$2H_2$:	$+7{\cdot}17$	$3E_1$:	$-1{\cdot}38$
K_4 :	$+1{\cdot}17$	$3K_2$:	$-7{\cdot}35$	E_3 :	$-8{\cdot}55$
$2X_3$:	$+0{\cdot}95$	$5X_1$:	$+3{\cdot}56$	M :	$+1{\cdot}22$
$2K_3$:	$+0{\cdot}47$	$3H_1$:	$-1{\cdot}72$	$7Z_G$:	$-3{\cdot}71$
H_3 :	$-0{\cdot}58$	$5K_1$:	$+3{\cdot}30$	$10Z_F$:	$-0{\cdot}50$

	n.		n.		n.
$3P_1$:	$-4''{\cdot}63$	P_3 :	$+0''{\cdot}28$	Y_2 :	$+0''{\cdot}239$
$3Q_1$:	$-0{\cdot}63$	Q_3 :	$+2{\cdot}24$	Y_3 :	$+0{\cdot}207$
$2P_2$:	$+2{\cdot}84$	P_4 :	$-4{\cdot}91$	Y_4 :	$-0{\cdot}211$
$2Q_2$:	$+3{\cdot}99$	Q_4 :	$-3{\cdot}24$		

Now the normals P are first solved, *i.e.* the above values of $3P_1$, $3Q_1$, $2P_2$ . . . are substituted in the solution P. This gives corrections $\delta_1(\frac{1}{3}\Delta p_1)$, $\delta_1(\frac{1}{3}\Delta q_1)$, $\delta_1(\frac{1}{2}\Delta p_2)$ . . . , etc. These are substituted in the values given above for E_4, H_4, K_4 . . . and in Y_2, Y_3, Y_4. The resulting values of E_4, K_4 . . . , etc., are substituted in the solution S, giving corrections $\delta_1(\Delta e_4)$, $\delta_1(\Delta h_4)$ . . . , etc. These are then also substituted in Y_2, Y_3, Y_4, and the resulting values in the solution Y, giving corrections $\delta_1(y_2)$, $\delta_1(y_3)$, $\delta_1(y_4)$. The corrections $\delta_1(\Delta e_4)$ . . . , $\delta_1(y_1)$ . . . are then substituted in $3P_1$, $3Q_1$, $2P_2$ . . . , and the resulting values are substituted in the solution P, giving corrections $\delta_2(\frac{1}{3}\Delta p_1)$, $\delta_2(\frac{1}{3}\Delta q_1)$ . . . . These are then again substituted in E_4, H_4 . . . , and Y_2, Y_3, Y_4, and so on. This process is repeated until the corrections have become so small that the effect on the values of $3P_1$, $3Q_1$ . . . , E_4, H_4 . . . , etc., is inappreciable. This process of approximation is very convergent, the third correction $\delta_3(\frac{1}{3}\Delta p_1)$ . . . , etc., being already entirely negligible.

The final corrections to the First Approximation are thus the sum of the corrections (a) given above, and the successive corrections $\delta_1(\mu)$, $\delta_1(z_G)$ . . . $\delta_2(\mu)$ . . . , etc. We find in this way :—

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$\delta(\frac{1}{3}\Delta p_1) = -0{\cdot}00182$
$\delta(\frac{1}{3}\Delta q_1) = - \cdot00025$
$\delta(\frac{1}{2}\Delta p_2) = + \cdot00125$
$\delta(\frac{1}{2}\Delta q_2) = + \cdot00112$
$\delta(\Delta p_3) = - \cdot00012$
$\delta(\Delta q_3) = + \cdot00117$
$\delta(\Delta p_4) = - \cdot00083$
$\delta(\Delta q_4) = - \cdot00049$

$\delta(y_2) = +0{\cdot}0734$
$\delta(y_3) = + \cdot0059$
$\delta(y_4) = - \cdot0209$

$\delta(\Delta\epsilon_4) = -0{\cdot}00400$
$\delta(\Delta h_4) = - \cdot00051$
$\delta(\Delta k_4) = - \cdot00280$

$\delta(\frac{1}{3}x_3) = + \cdot00087$
$\delta(\frac{1}{2}\Delta k_3) = - \cdot00133$
$\delta(\Delta h_3) = + \cdot00294$

$\delta(\frac{1}{3}x_2) = + \cdot00414$
$\delta(\frac{1}{2}\Delta h_2) = + \cdot00014$
$\delta(\frac{1}{3}\Delta k_2) = - \cdot00088$

$\delta(\frac{1}{3}x_1) = +0{\cdot}00053$
$\delta(\frac{1}{3}\Delta h_1) = + \cdot00280$
$\delta(\frac{1}{3}\Delta k_1) = + \cdot00062$

$\delta(\frac{1}{2}\Delta\epsilon_2) = + \cdot03266$
$\delta(\frac{1}{3}\Delta\epsilon_1) = - \cdot00070$
$\delta(\Delta\epsilon_3) = - \cdot00514$

$\delta(\mu) = + \cdot00484$
$\delta(z_c) = - \cdot00245$
$\delta(z_r) = - \cdot00099$

Returning now to the original unknowns and adding the corrections just found to the First Approximation, we find the following final values :—

	Weights.	Probable Errors.		Weights.	Probable Errors.
$\Delta p_1 = +0{\cdot}0360$	276	$\pm0{\cdot}0045$	$\Delta h_1 = +0{\cdot}0309$	34	$\pm0{\cdot}0129$
$\Delta q_1 = -0{\cdot}0273$	245	·0048	$\Delta k_1 = -0{\cdot}0636$	13	·0207
$\Delta p_2 = +0{\cdot}0752$	594	·0031	$\Delta h_2 = +0{\cdot}0098$	79	·0084
$\Delta q_2 = +0{\cdot}0867$	770	·0028	$\Delta k_2 = -0{\cdot}0560$	23	·0156
$\Delta p_3 = -0{\cdot}0029$	1482	·0020	$\Delta h_3 = +0{\cdot}0618$	195	·0054
$\Delta q_3 = -0{\cdot}0681$	1889	·0017	$\Delta k_3 = -0{\cdot}0517$	90	·0079
$\Delta p_4 = +0{\cdot}0630$	5115	·0011	$\Delta h_4 = +0{\cdot}0614$	826	·0026
$\Delta q_4 = -0{\cdot}0137$	5483	·0010	$\Delta k_4 = +0{\cdot}0254$	389	·0038
$\mu = -0{\cdot}0072$	2133	$\pm0{\cdot}0016$	$x_1 = +0{\cdot}0773$	17	$\pm0{\cdot}0182$
$z_c = +0{\cdot}0253$	26	·0147	$x_2 = -0{\cdot}0047$	31	·0135
$z_r = +0{\cdot}2981$	9	·0250	$x_3 = -0{\cdot}0043$	123	·0068
$\Delta\epsilon_1 = +0{\cdot}1003$	194	$\pm0{\cdot}0054$	$y_2 = -0{\cdot}044$	3	$\pm0{\cdot}043$
$\Delta\epsilon_2 = +0{\cdot}0651$	503	·0033	$y_3 = +0{\cdot}051$	23	·016
$\Delta\epsilon_3 = -0{\cdot}0307$	1568	·0019	$y_4 = -0{\cdot}040$	12	·022
$\Delta\epsilon_4 = -0{\cdot}0248$	5205	·0010			

The weights derived from the Cape solution have been adopted, those of $\Delta\epsilon_1$, $\Delta\epsilon_2$, $\Delta\epsilon_3$, y_2, and y_4 being taken from the solution a, in which no rigorous conditions were introduced.

Substituting the adopted values of the unknowns in the individual equations of condition, we get the residuals v which are given in the last columns of the Tables XVI. and XVII. at the end of Chapter II.

Probable Errors.—From these residuals we derive the probable error of weight unity by the formula

$$\text{Probable error of observation of weight unity} \quad \rho = 0{\cdot}6745\sqrt{\frac{[pvv]}{n-m}}.$$

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Here n is the number of equations of condition, and m is the number of unknowns in each equation. Now the latter number varies for the distance-equations between 14 and 16 and for the position-angles between 12 and 14 (see the little table on p. 115). I have, however, taken $m = 15$ for the distances and $m = 13$ for the position-angles throughout. We find then the following values :—

		[*pcr*].	*n*.	ρ.
GILL . . .	Distances alone . . .	1·5011	162	±0″·0682
	Position-angles alone . .	2·0296	156	·0803
	Distances and position-angles	3·5307	318	·0728
FINLAY . . .	Distances	·3766	42	±0 ·080
	Position-angles . . .	·6654	41	·104
	Distances and position-angles	1·0420	83	·084
GILL and FINLAY .	Distances	1·8777	204	±0 ·0672
	Position-angles . . .	2·6950	197	·0816
	Distances and position-angles	4·5727	401	·0734

The finally adopted value, used for computing the probable errors of the unknowns which have already been given above, is

$$\rho = \pm 0''{\cdot}075.$$

It would appear that the observations by Finlay were of a smaller weight than those by Gill. Quite probably there is reason to accept this conclusion from the probable errors. The observations by Finlay are, however, so few in number, that the probable errors derived from them are not very trustworthy. I have therefore not thought it advisable to make a new solution, assigning a smaller weight to observations by Finlay. The effect on the resulting values of the unknowns would have been inappreciable.

The position-angles appear to give a larger probable error than the distances. This is due to the presence of a greater number of large residuals. If we reject the six largest residuals* from the position-angles, which are all larger than 0″·350, we find

		[*pcr*].	*n*.	ρ.
GILL and FINLAY .	Position-angles . . .	1·9875	191	±0″·0713
	Distances and position-angles	3·8652	395	·0681

Amongst the residuals in distance there are only three† which exceed 0″·350 two of which have the weight $\frac{1}{4}$, and the other $\frac{1}{2}$.

* These are the rotation numbers 21, 23, 108, 244, 413, 417. It is noticeable that 21 and 23 are on the same night, and similarly 413 and 417.

† Viz. the rotation numbers 52, 86, 88.

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There can hardly be any doubt that observations of position-angles are more liable to small mistakes (such as 10′, or the recording of 42 instead of 24, and many other errors of a similar origin) than those of distances. In observing position-angles the microscope readings are read out to an assistant and recorded by him; in the distance observations they are printed automatically, so that mistakes are practically excluded. Take, *e.g.*, the observation 21 of the Satellites II. and III. on 19th August (see Table II., p. 21).

The observations, as they are recorded, run :—

21^h	24^m	$2^s{\cdot}5$	$67°$	$50'$	$50''{\cdot}5$
	24	52	67	51·5	52
	42	48	67	53	54
	44	30	67	46	47

Now the relative motion of the satellites in this case *decreases* the position-angle. It is thus probable that the first reading after the reversing of the segments should be

$$67°\ 43'\ 44'.$$

If this is assumed, the residual is diminished from $+0''{\cdot}354$ to $+0''{\cdot}113$.

In this particular case the existence of a mistake can be pointed out with considerable probability, though by no means with certainty. In other cases the origin of the mistake may not be so evident, and it would certainly be unwarranted to suppose that *all*, or even the majority, of the large residuals were due to mistakes. Still it is very probable that the true accidental errors of observation are, on the average, not larger in the case of position-angles than for the distances. We would thus come to the conclusion that, for Gill, the true probable error of one observation is not more than $\pm 0''{\cdot}070$.

Systematic Errors.—A very important question is, whether after correction for κ, or z, the observations of distance are really free from systematic errors. To investigate this question the observations were arranged in order of distance, and for every *five* consecutive distances the mean residual was taken (no weights being applied). The observations of half and double distances, and the observations in which the amount of crossing was measured, were treated separately. In the following table these mean distances and mean residuals are given in order of distance. The means which depend on observations of the two just mentioned exceptional kinds are distinguished by a prefixed letter.

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TABLE XX.

MEAN RESIDUALS.—DISTANCES.

GILL.						FINLAY.	
Mean Distance.	Mean Residual.	Mean Distance.	Mean Residual.	Mean Distance.	Mean Residual.	Mean Distance.	Mean Residual.
d 8‡	−0″·139	197″	−0″·021	417″	+0″·005	ψ 79″	−0″·002
ψ 26	− ·006	217	+ ·029	437	− ·008	97	− ·030
ψ 38	− ·015	227	− ·029	456	+ ·004	165	− ·043
ψ 49	+ ·038	260	·000	504	+ ·024	218	+ ·073
63	− ·007	275	− ·014	542	− ·054	274	− ·017
ψ 81†	− ·072	291	− ·076	567	− ·012	401	− ·063
103	+ ·096	305	− ·062	622	− ·022	474*	− ·104
133	− ·027	322	− ·041	686	− ·071	578*	+ ·075
148	+ ·044	343	+ ·106	785	+ ·057	722*	+ ·046
170	− ·092	368	+ ·059	871*	+ ·023		
183	+ ·030	394	+ ·014	968*	+ ·030		

d Observations of double distances. ψ Observations in which ψ was recorded.
† *Six* observations. * *Four* observations. ‡ *Three* observations.

These residuals show no sign of any systematic error depending on the distance. The first value may seem to show that the distances measured by the method of double distances are systematically measured too short, but it is very doubtful whether any legitimate conclusion can be based on the mean of only three observations. The three separate residuals are +0″·001, −0″·010, −0″·409 ($p=\frac{1}{2}$). The residuals of the two rejected observations 329 (half distance) and 498 (double distance) are +0″·276 and −0″·375 respectively. If these had been included, the mean residual would have been −0″·078.

If there were a systematic error proportional to the distance, and *common to both observers*, the amount of this error would be included in the determination of μ, the coefficient of which is proportional to s. But such an error is eliminated *a priori* by the use of the standard stars for the determination of the scale-value, the true distance of which is known within $\frac{1}{100000}$ part of its amount (see Chapter I., p. 50). A systematic error which follows any other law, whether common to both observers or not, would of necessity appear in the mean residuals of Table XX. It can thus be asserted with great confidence that, *after correction for* κ, *the heliometer-observations of distances up to 1000″ made by* Gill *and* Finlay, *are free from systematic error*, provided the scale-value be determined by the observation of standard stars of which the distance is well known.

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It will be remembered that z_G and z_F are the corrections on account of "crossing through" to observations by each of the observers, and on account of the distance of optical centres. Thus

$$z = \frac{\kappa^2 + \delta^2}{200}.$$

From the values of z_G and z_F, which were found by the solution of the equations, we derive—

Assuming $\delta = 1''$	$\kappa_G = 2''{\cdot}02 \pm 0''{\cdot}45$	$\kappa_F = 7''{\cdot}66 \pm 0''{\cdot}33$
„ $\delta = 0$	$\kappa_G = 2{\cdot}25 \pm 0{\cdot}40$	$\kappa_F = 7{\cdot}72 \pm 0{\cdot}32$.

From the distances in which κ was directly observed we found (Chapter I., Table IX., p. 58)

$$\kappa_G = 3''{\cdot}04 \qquad \kappa_F = 6''{\cdot}67.$$

The agreement, though not exact within the probable errors, is as near as might reasonably be expected. It is certainly near enough to justify the assertion that, at least for Gill and Finlay, the systematic error inversely proportional to the distance finds its true explanation in the "crossing through," which is a necessary consequence of the method of observing with a modern heliometer, and must not be considered as a purely empirical correction derived from the observations. The fact that the agreement is not absolute, however, makes it unsafe to assert that κ is the same for all distances. It will thus be necessary in order to realise the extreme possible accuracy, always to record the amount of "crossing through" for short distances, say under 700″ or 800″.

For the position-angles the assumption was made (Chapter I., p. 64) that they are not affected by any systematic error. It was pointed out there that the only kind of error which would be of importance for the present work, would be one depending on the distance. To see whether any such error does exist I again arranged the observations in order of distance and took the mean of every *five* consecutive residuals. The observations by the two observers were treated separately; the means are given in Table XXI., arranged in order of distance. The observations in which the amount of crossing was recorded have in Table XXI. not been distinguished from the others, since the method of observing the position-angles is exactly the same for these observations as it is in the ordinary method.

When these mean residuals are considered, the first thing which is remarked is that they are much larger than the corresponding means for the distances. Taking account of accidental errors only, the probable mean residual of five observations is ±0″·034. In Table XX. there are 20 means exceeding this amount, and 22 smaller than this probable value. This is entirely in agreement with the theory of accidental

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errors. In Table XXI., however, taking Gill alone, there are 23 means exceeding the probable value, and only 10 smaller than 0″·034. Now this would point to the existence of systematic errors. However, considering first Gill alone, and leaving out of consideration the mean residuals belonging to distances not exceeding 100″, it is evident that there is no trace whatever of any regularity in the signs of the mean residuals. There are 16 variations of sign and 10 permanences. Also, though the average means are larger than for the distances, this is due more to a deficiency of small values than to an excess of large ones, none of the means amounting to as much as ±0″·100.

TABLE XXI.

Mean Residuals—Position-angles.

GILL.						FINLAY.		
Mean Distance.	Mean Residual.		Mean Distance.	Mean Residual.		Mean Distance.	Mean Residual.	
	Arc of Great Circle.	Angle.		Arc of Great Circle.	Angle.		Arc of Great Circle.	Angle.
d 8‡	−0″·063	−27′·1	305	−0″·018	−0′·20	79	+0″·097	+ 4′·2
26	− ·191	−25·2	322	− ·030	−0·32	101	− ·031	− 1·0 b
39	− ·033	− 2·9	348*	+ ·054	+0·53	165	− ·052	− 1·1
51	− ·049	− 3·3	373	+ ·004	+0·03	218	− ·023	− 0·4
58	− ·051	− 3·0	394	− ·008	−0·07	274	− ·158	− 2·0
86	− ·105	− 4·3	417	+ ·049	+0·40	402*	+ ·032	+ 0·27
103*	+ ·008	+ 0·3	437	− ·044	−0·35	474*	− ·043	− 0·32
135*	− ·029	− 0·7	464	+ ·054	+0·40	578*	− ·167	− 0·98
148	− ·047	− 1·1	512	− ·055	−0·37	722*	+ ·024	+ 0·12
170	+ ·006	+ 0·1	542	+ ·051	+0·32			
183	− ·015	− 0·3	575	+ ·077	+0·47			
197	+ ·097	+ 1·8	623	− ·087	−0·48			
217	+ ·063	+ 1·0	686*	− ·013	−0·07			
227	+ ·049	+ 0·8	776*	+ ·041	+0·18			
260	− ·044	− 0·6	837*	+ ·080	+0·32			
275	− ·085	− 1·1	966‡	+ ·051	+0·18			
291	+ ·050	+ 0·6						

d Observations of double distances. * *Four* observations. ‡ *Three* observations.

It is possible that owing to slight irregularities and local deficiencies of straightness of the slides carrying the semi-lenses, small errors are introduced into the position-angles, which would thus be always the same for any particular very short range of distances, but would never reach large amounts, and vary very irregularly with the distance. Obviously it is practically impossible to determine such errors, or to take account of them, but also they cannot have any effect on the values of the unknowns.

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Gill's residuals for short distances are systematically negative. How this apparent systematic error in the position-angles of short distances must be explained is not easy to say.* If the origin were instrumental, Finlay's means should also show it, which they do not. However, for our purpose a small systematic error in the position-angles of short distances is practically of no importance. The coefficients of Δp_i and Δq_i in the equations of condition belonging to these observations will, as a rule, be small, while those of $\Delta\epsilon_i$, x_i, Δh_i, Δk_i are large. Now by my method of solution the latter have no part in the determination of the unknowns, as they only enter into those of the normals D, which have not been used. The influence of these observations on the determination of Δp_i and Δq_i on the other hand is small in consequence of the smallness of the coefficients. The question whether the position-angles of very short distances observed by Gill are affected by a systematic error is thus of no importance for our purpose.†

Turning now to Finlay, we find that the mean residuals are systematically negative, but apart from this there is no evidence of a systematic error varying with the distance. Taking the mean of all residuals, we find for Finlay :—

In arc of great circle	$-0''\cdot036$
In position-angle	$-8''$.

This last value agrees very well with the systematic difference Gill—Finlay $= +6''\cdot0$, which was found from the Victoria triangulation (see these *Annals*, vol. vi. p. 173), assuming Gill's observations to be free from systematic error. The amount of this systematic difference is too small to be of any importance for our present purpose, but the fact that it has been possible from observations of distances below $900''$ to detect this minute systematic error testifies very strongly for the excellent quality of the observations, especially if we remember that the position-circle was only read to the nearest half minute.

Mass of Jupiter's System.—From the conclusion arrived at just now, viz. that the observations of distance are free from systematic error, it follows further that the reciprocal of the mass of Jupiter's system, which is determined exclusively from the distances, is also free from systematic error, except in so far as it is affected by the

* From the formulas given in the footnote on p. 62, we find easily $\frac{1}{2}(\phi_2 - \phi_1) = -\frac{\kappa\delta^2}{S^3}$. This is the error in the mean position-angle due to the crossing and the distance of optical centres. It is easily verified that it can hardly amount to as much as a minute of arc even for the very short distances, and can thus not explain the systematic error in question.

† With regard to the first mean, depending on observations of double distances, it may be remarked that if the two rejected observations 328 (half distance) and 497 (double distance) had been included, the mean would have been $+0''\cdot021$ in arc of great circle, or $+5'\cdot5$ in position-angle.

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remaining uncertainty in the adopted distance of the standard stars. This uncertainty corresponds to a probable error (see Chapter I., p. 50) of

$$\pm 3 \frac{1047{\cdot}7}{6705{\cdot}4} \times 0{\cdot}065 = \pm 0{\cdot}030.$$

On the other hand, the probable error resulting from the errors of observation is

$$\pm 3 \frac{1047{\cdot}7}{100} \times 0{\cdot}0016 = \pm 0{\cdot}050.$$

The resulting true probable error is $\pm$ 0·058.

The correction to the adopted reciprocal of the mass is

$$\Delta M = \frac{3M}{100} \mu = -0{\cdot}226.$$

We have seen (Chapter I., p. 48), that the fact that the observations were not corrected for parallax introduces a slight systematic error in the distances, depending on the hour-angle. The hour-angle has always been between $\pm$ 1^h and $\pm$ 4^h. If it is desired to take this error into account, it will be sufficiently accurate to adopt a constant correction $\frac{\delta s}{s} = -0{\cdot}0000075$, which gives

$$\delta M = +0{\cdot}024.$$

The final value of the reciprocal of the mass of Jupiter's system, derived from these observations, is thus

$$M = 1047{\cdot}498 \pm 0{\cdot}058.$$

The two determinations by Mr Cookson (this volume, Part II.) are

1901	1047·46$\pm$0·090
1902	1047·25$\pm$0·060.

The probable errors do not contain the effect of the uncertainty of the distance of the standard stars. If we estimate this uncertainty as equivalent to a probable error of $\pm$ 0″·1 (which is certainly not too large), the corresponding probable errors in the value of the mass will be for the two years $\pm$0·050 and $\pm$0·070 respectively. The total probable errors thus become

1901	$\pm$0·103
1902	$\pm$0·092.

The relative weights of the three series would thus be about 10, 3, and 4.

In the *Proceedings of the Academy of Sciences at Amsterdam*, vol. x. p. 717

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(March 1908), I have combined these three determinations with the best results from perturbations of planets. The different determinations there considered are

TABLE XXII.

RECIPROCAL OF MASS OF SYSTEM.

Authority.	Observed Values.	Weight.	Residuals.
KRÜGER, perturbations of Themis	1047·54 ± 0·19	5	+0·14
HILL, ,, ,, Saturn	·38 ·12	7	− ·02
NEWCOMB, ,, ,, Polyhymnia	·34 ·06	20	− ·06
GILL-DE SITTER, satellites, 1891	·50 ·06	10	+ ·10
COOKSON, satellites, 1901	·46 ·10	4	+ ·06
,, ,, 1902	·25 ·09	6	. − ·15

The weights of the first three values have been adopted from Newcomb (*Astronomical Papers of the American Ephemeris*, vol. v., part 5). For Cookson's two series the weights 3 and 4, as determined above, would perhaps have been better. It is, however, of little importance which weights are used.

Newcomb in his determination of the mass of Jupiter also includes the old observations of eclipses and the determination from Faye's and Winnecke's comet. The old satellite-observations have here been entirely rejected on account of the uncertainty of the scale value. The determinations based on observations of comets have been rejected because they involve the assumption, which seems to me very unsafe, that the relative position of the centre of gravity and the centre of light, which is observed, remains the same throughout the period or periods of observation.

The weighted mean of all six determinations is . . . . 1047·394.
The mean without weights is . . . . . 1047·412.
The mean from the planets alone is . . . . . 1047·380,
and from the satellites alone . . . . . 1047·417.

All these values agree within their probable errors. The finally adopted value is

$$M = 1047{\cdot}40 \pm 0{\cdot}03.$$

This leaves the residuals given in the last column of Table XXII. Of these residuals three are smaller than, two are larger than, and one is equal to, the probable error of the corresponding observed values. Their distribution is thus in satisfactory agreement with the theory of errors. The probable error of the final value of M has been derived from these residuals. It seems certain that it can be considered as a true measure of the real accuracy.

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The mass of Jupiter is thus here determined with a probable error of $\frac{1}{3500}$ of its amount. Not only is there no other mass in the solar system, but there are very few astronomical (or physical) constants indeed (mean motions excepted) which are known with the same accuracy.

Longitudes at Epoch.—It must now be investigated in how far the other unknowns can be assumed to be determined free from systematic errors.

If there is a personality in the observer's calling out the moment of observation, or in the assistant's recording this moment from the chronometer, or if there is an error in the adopted longitude of the Cape Observatory, or in the adopted constant of aberration, the sum of all these effects will be a constant error τ in all the observed times. This will produce an error in the computed distance of

$$\tau \frac{ds}{dt} = \tau\left(n_1 \frac{\partial s}{\partial \epsilon_1} + n_2 \frac{\partial s}{\partial \epsilon_2}\right)$$

and similarly for the position-angles.

It is thus not possible to determine the longitudes at the epoch free from systematic error, but what we can get from the observations is

$$\Delta\epsilon_1 - n_1\tau \qquad \Delta\epsilon_2 - n_2\tau \qquad \Delta\epsilon_3 - n_3\tau \qquad \Delta\epsilon_4 - n_4\tau$$

and the other unknowns are not affected by this source of error. The error τ is unknown, and it is impossible to determine it. If we estimate its probable amount as $\pm$ 0^s·5, which is almost certainly very much too large, the errors introduced in the unknowns will be

$\Delta\epsilon_1$	$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\Delta\epsilon_4$
$\pm$0°·00118	$\pm$0°·00058	$\pm$0°·00029	$\pm$0°·00012.

Adding these to the probable errors found from the solution, we find for the values of the unknowns with their true probable errors :—

$\Delta\epsilon_1 = +0°·1003$	$\pm$0°·0055
$\Delta\epsilon_2 = +0\cdot0651$	·0034
$\Delta\epsilon_3 = -0\cdot0307$	·0019
$\Delta\epsilon_4 = -0\cdot0248$	·0010.

It may be remarked that this systematic error τ disappears from the condition

(1) $$\Delta\epsilon_1 - 3\Delta\epsilon_2 + 2\Delta\epsilon_3 = 0.$$

From the values of the unknowns we find

$$\Delta\epsilon_1 - 3\Delta\epsilon_2 + 2\Delta\epsilon_3 = -0°·1564 \pm 0°·0120.$$

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The probable error has been computed by the formula for a linear function of the unknowns.* This quantity is more than twelve times its probable error, so that it is undoubtedly real, and the inequality which is called *libration* by Laplace is not, as Delambre found it to be, entirely insensible.†

The libration is one of the elements of the theory of the satellites, which are most difficult to determine from the observations. An attempt of such a determination has been made by the writer in No. 17 of the *Publications of the Astronomical Laboratory at Gronigen.* This was based on the observations here discussed, those made by Mr Cookson in 1901 and 1902, and on photographic plates made at the Cape in 1902 and in 1904, and at Helsingfors and Pulkowa in the years 1892 to 1898. The great difficulty is that the libration is, in observations made near the epochs of conjunction, intimately bound up with the inequalities of long period, which depend on the excentricities and are still very incompletely known. (See *Gron. Publ.*, **17**, p. 7; *Proceedings, Amsterdam Academy*, **10**, p. 711, 712.) The only means to get a really reliable determination of the libration seems to be to make strong series of observations not at the epochs of conjunction, but from two to three months before and after each conjunction. Photographic plates for this purpose are now being made at the Cape Observatory.‡

Equations of the Centre and Inequalities.—We now come to the unknowns x, Δh, Δk. These too depend almost exclusively on the observations of distance, and are therefore not affected by any systematic error, at least in so far as such a thing can ever be asserted of any quantity determined by one instrument and one method only. It will be remarked that the weights of these unknowns are very small compared with those of the p's and q's, although the coefficients in the equations of condition are of the same order of magnitude. This is not accidental, but a consequence of the nature of the problem.

* See *Chauvenet* II., p. 541–3. It is easy to show that the formula (83) there given is equivalent to the following rule: *to find the mean error of a function*

$$F = Ax + By + Cz + \ldots,$$

substitute A, B, C . . . *for* $[an]$, $[bn]$, $[cn]$ . . . *in the absolute terms of the normal equations. Let the solution of these altered normals be* $\alpha, \beta, \gamma, \ldots$, *then the required mean error will be given by the formula*

$$\epsilon_r^2 = \{A\alpha + B\beta + C\gamma + \ldots\}\epsilon^2,$$

where ϵ is the mean error of an observation of weight unity.

† It may be remembered that Delambre actually found for the epoch 1750·0 $\theta = l_1 - 3l_2 + 2l_3 = 180°{\cdot}0176$ (Laplace, *Mécanique Céleste*, t. IV., p. 136). The deviation from 180° was neglected by him as being within the uncertainties of the determination.

‡ The first two series of these plates, made in 1913, have already been received in Leiden, and partly measured. So far as can be judged at present, the results will prove very satisfactory.

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If we neglect the excentricities and inclinations, and if we suppose the earth to be in the fundamental plane, we have

$$v = l \qquad \rho = 1 \qquad B = 0 \qquad p - P = \pm 90^\circ,$$

and the differential coefficients [formula (13), p. 107] now become

$$\frac{\partial s}{\partial v_1} = -K_1 \cos (l_1 - L)$$

$$\frac{\partial s}{\partial \rho_1} = -\frac{K_1}{\sin 1^\circ} \sin (l_1 - L), \tag{3}$$

$$\text{where } K_1 = \frac{a_1'' \rho_1}{\Delta_1'} \sin 1^\circ \sin (p - P).$$

Further, we have

$$\frac{\partial s}{\partial h_1} = K_1 [\cos l_1 \cos (l_1 - L) + \tfrac{1}{2} \sin l_1 \sin (l_1 - L)]$$

$$\frac{\partial s}{\partial k_1} = -K_1 [\sin l_1 \cos (l_1 - L) - \tfrac{1}{2} \cos l_1 \sin (l_1 - L)] \tag{4}$$

$$\frac{\partial s}{\partial x_1} = -K_1 [\sin a_1 \cos (l_1 - L) - \tfrac{1}{2} \cos a_1 \sin (l_1 - L)]$$

and the same formulæ hold for the other satellites, if the suffixes are altered, with the only difference that for Satellite III. γ is substituted for a in accordance with Marth's notation (see formula II., p. 113), and for Satellite IV. only the first two equations must be considered.

Now we have

$$a_1 = 2(l_1 - l_2) = l_1 + c$$
$$a_2 = l_1 - l_2 + 180^\circ = l_2 + c + 180^\circ$$
$$\gamma_3 = l_2 - l_3 + 180^\circ = l_3 + c,$$

where

$$c = l_1 - 2l_2 = l_2 - 2l_3 + 180^\circ.$$

Introducing these values, the equations (4) become after a simple transformation—

$$\frac{\partial s}{\partial h_i} = \tfrac{3}{4} K_i \cos L [1 + \tfrac{1}{3} \cos (2l_i - L)]$$

$$\frac{\partial s}{\partial k_i} = -\tfrac{3}{4} K_i \sin L [1 + \tfrac{1}{3} \sin (2l_i - L)] \tag{5}$$

$$\frac{\partial s}{\partial x_i} = \mp \tfrac{3}{4} K_i [\sin (L + c) + \tfrac{1}{3} \sin (2l_i + c - L)],$$

where $i = 1, 2, 3$, or 4, and where the lower sign in the last formula must be taken for Satellite II.

If we neglect the small periodic terms within the square brackets, the coefficients become

$$\frac{\partial s}{\partial h_i} = \tfrac{3}{4} K_i \cos L$$

$$\frac{\partial s}{\partial k_i} = -\tfrac{3}{4} K_i \sin L \tag{6}$$

$$\frac{\partial s}{\partial x_i} = \mp \tfrac{3}{4} K_i \sin (L + c)$$

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The angles L and c change very slowly, especially L, the variation during the time from 17th August to 14th December 1891 being for L 7°, and for c 89°. If they were absolutely constant, it would evidently not be possible to determine the *three* unknowns Δh, Δk, and x from the observations, but only *one* linear function of them, viz. :—

$$F_i = \cos L\,\Delta h_i - \sin L\,\Delta k_i \mp \sin (L+c)x_i, \tag{7}$$

where again the lower sign must be taken for Satellite II., and the last term must be dropped for Satellite IV.

If we adopt for L and c their mean values, viz. :—

$$L = 205°{\cdot}7 \qquad c = 11°{\cdot}5,$$

we find for the values of the functions F and their probable errors (computed by the rule, p. 155, footnote) :—

$$\begin{aligned}
F_1 &= -0{\cdot}901\,\Delta h_1 + 0{\cdot}434\,\Delta k_1 + 0{\cdot}605 x_1 = -0°{\cdot}0084 \pm 0°{\cdot}0124 \\
F_2 &= -0{\cdot}901\,\Delta h_2 + 0{\cdot}434\,\Delta k_2 - 0{\cdot}605 x_2 = -0\ {\cdot}0301 \pm\ {\cdot}0078 \\
F_3 &= -0{\cdot}901\,\Delta h_3 + 0{\cdot}434\,\Delta k_3 + 0{\cdot}605 x_3 = -0\ {\cdot}0806 \pm\ {\cdot}0045 \\
F_4 &= -0{\cdot}901\,\Delta h_4 + 0{\cdot}434\,\Delta k_4 \qquad\qquad = -0\ {\cdot}0442 \pm\ {\cdot}0025.
\end{aligned}$$

The probable errors, though smaller than those of the separate values of Δh, Δk, and x, are still very large. The reason of this must be sought in the circumstance that the satellites are not observed separately, but in pairs. If the same simplifications are introduced as before, the equations of condition become, as far as the quantities here considered are involved,

$$\tfrac{3}{4} K_1 F_1 - \tfrac{3}{4} K_2 F_2 = O - C. \tag{8}$$

If for the moment we introduce the new unknowns

$$f_i = \tfrac{3}{4} \frac{a_i''}{\Delta} F_i \qquad (i = 1, 2, 3, 4),$$

and if in the expression for K_i we take

$$\rho_i = 1, \qquad \Delta_i' = \Delta, \qquad \sin (p - P) = \pm 1,$$

the equation (8) becomes

$$f_1 - f_2 = \pm (O - C).$$

It is thus evident that in the simplified problem here considered we cannot determine the separate unknowns f_1, f_2, f_3, f_4, but only their differences. It is easy to see that for a similar reason it would not be possible to determine the unknowns Δh, Δk, and x for each satellite separately, even if the angles L and c changed so much that the three unknowns referring to one satellite could be separated.

In the actual problem, however, the neglected terms in formula (5) have some influence, and consequently it is possible to derive values for F_1, F_2, F_3, and F_4. Also

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the angles L and c do vary, and the values of Δh, Δk, and x can be determined separately, but their determination is subject to relatively large uncertainties. It is one of the advantages of the method of least squares that inherent uncertainties of this kind are automatically taken into account by a corresponding reduction of the weights. With the exception of Δh_1, Δh_2, x_2, and x_3, however, even these separate values exceed their probable errors many times, and must thus be considered as real corrections. If they are added to the adopted values, we get for the true values at Greenwich mean noon 2nd October 1891—

$$h_1 = +0^\circ\cdot0279 \pm 0^\circ\cdot0129 \qquad k_1 = -0^\circ\cdot0670 \pm 0^\circ\cdot0207$$
$$h_2 = +0\cdot0157 \pm \cdot0084 \qquad k_2 = -0\cdot0771 \pm \cdot0156$$
$$h_3 = +0\cdot0430 \pm \cdot0054 \qquad k_3 = -0\cdot2244 \pm \cdot0079$$
$$h_4 = +0\cdot5249 \pm \cdot0026 \qquad k_4 = -0\cdot6559 \pm \cdot0038.$$

From these values we derive the following instantaneous excentricities and perijoves (not to be confounded with the *excentricités et périjoves propres* of Laplace and Souillart):—

$$E_1 = 0\cdot00064 \pm 0\cdot00019 \qquad \Omega_1 = 158^\circ\cdot \pm 10^\circ\cdot6$$
$$E_2 = 0\cdot00068 \pm 0\cdot00014 \qquad \Omega_2 = 168\cdot5 \pm 7\cdot0$$
$$E_3 = 0\cdot00199 \pm 0\cdot00007 \qquad \Omega_3 = 169\cdot2 \pm 1\cdot45$$
$$E_4 = 0\cdot00733 \pm 0\cdot00003 \qquad \Omega_4 = 141\cdot36 \pm 0\cdot23.$$

(The probable errors are only roughly computed.)

The longitudes are reckoned in the fundamental plane from the point 0, of which the adopted longitude is 135°·3421 (equinox and epoch 1891·75).

Turning now to the x's, and adding the corrections x to the adopted coefficients of the great inequalities, we find the values for these coefficients, which are given below. The theoretical expressions in terms of the masses have been derived by me, on the basis of Souillart's theory, in *Gron. Publ.*, **17**, p. 85. If we neglect second orders in the masses, we have

		Theoretical Values.	Observed Values.
Satellite	I.:	$0^\circ\cdot4091 - 0\cdot0034\nu_1 + 0\cdot4016\nu_2 - 0\cdot0146\nu_3$	$= 0^\circ\cdot5092 \pm 0^\circ\cdot0182,$
„	II.:	$1\cdot0517 + \cdot1922 - \cdot0128 + \cdot8035$	$= 1\cdot0211 \pm \cdot0135,$
„	III.:	$0\cdot0601 - \cdot0014 + \cdot0594 - \cdot0064$	$= 0\cdot0592 \pm \cdot0068.$

The values of the masses, implied by the theoretical values, are

$$m_1 = 0\cdot0000400\ (1+\nu_1)$$
$$m_2 = 0\cdot0000220\ (1+\nu_2)$$
$$m_3 = 0\cdot0000800\ (1+\nu_3).$$

It is not possible to derive the three masses, *i.e.* the values of the three unknowns ν_1, ν_2, ν_3, from the three equations given. They must, as has already been remarked (see Chapter II., p. 114), be combined with data derived from other observations. An

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attempt at such a combination and derivation of the masses has been made by the writer in *Proceedings, Amsterdam Academy*, **10**, p. 718.

From the values of y_2 and y_4 we find

$$y_2 - 1{\cdot}468\, y_4 = +0{\cdot}015 \pm 0{\cdot}051.$$

Thus the rigorous condition (2) is satisfied as nearly as can be expected. The probable errors are, however, so large that it is very doubtful whether the values found can be considered as real corrections. It would perhaps have been better to neglect these unknowns altogether. The representation of the equations of condition would not be sensibly altered by such a course. I have actually made a solution in which for y_i I took the values resulting from the masses found in the discussion quoted above. It does not seem necessary here to relate the result of this solution. The values of all unknowns were identical to those found above within one or two units of the last decimal place.

Latitudes.—Lastly, we come to the unknowns Δp and Δq. These are determined almost exclusively from the position-angles. It has been shown that the residuals give no evidence of any systematic error. We must thus here also consider the probable errors derived from the residuals as a true measure of the accuracy of the determination of the unknowns.

Adding the corrections found to the adopted values, we find

$p_1 = +0^\circ{\cdot}0436 \pm 0^\circ{\cdot}0045$	$q_1 = -0^\circ{\cdot}0299 \pm 0^\circ{\cdot}0048$
$p_2 = +0\ {\cdot}4914 \pm 0\ {\cdot}0031$	$q_2 = -0\ {\cdot}1747 \pm 0\ {\cdot}0028$
$p_3 = +0\ {\cdot}1137 \pm 0\ {\cdot}0020$	$q_3 = -0\ {\cdot}1378 \pm 0\ {\cdot}0017$
$p_4 = +0\ {\cdot}2324 \pm 0\ {\cdot}0010$	$q_4 = +0\ {\cdot}2891 \pm 0\ {\cdot}0010.$

These quantities determine the inclinations and nodes of the orbital planes of the satellites referred to the adopted fundamental plane at the mean epoch of the observations. It is not necessary to derive here these inclinations and nodes. The above values have been combined with those resulting from the heliometer-observations by Mr Bryan Cookson in the years 1901 and 1902 [see Part II. of this volume], from photographic plates taken in 1902 and discussed by Mr Cookson [Part IV.], and from plates of the years 1903 and 1904, measured and discussed by me [Part III.]. The derivation of the inclinations and nodes from all these data is contained in Part III., but the results from 1902 used there were only provisional. A new discussion and derivation of final values of the inclinations and nodes and of the position of the fundamental plane is given in Part V., to which the reader is referred for details. Here it will suffice to mention that the final residuals as compared with the probable errors derived from each series separately and independently has shown the observations to be of a very high standard of accuracy, and in particular to be entirely free from systematic errors.

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APPENDIX.

ON THE SOLUTION OF NORMAL EQUATIONS.

If a system of m linear equations with m unknowns be given of the form

N

$$\begin{array}{l}
[aa]\,x+[ab]\,y+[ac]\,z+[ad]\,u+\ \ldots\ =X\\
[ba]\,x+[bb]\,y+[bc]\,z+[bd]\,u+\ \ldots\ =Y\\
[ca]\,x+[cb]\,y+[cc]\,z+[cd]\,u+\ \ldots\ =Z\\
[da]\,x+[db]\,y+[dc]\,z+[dd]\,u+\ \ldots\ =U\\
\ldots\ldots\ldots\ldots\ldots\ldots
\end{array}$$

where the quantities $[kl]$ denote any numbers, such that $[kl]=[lk]$, then the solution of these equations will again be of the same form, thus :—

S

$$\begin{array}{l}
[\alpha\alpha]\,X+[\alpha\beta]\,Y+[\alpha\gamma]\,Z+[\alpha\delta]\,U+\ \ldots\ =x\\
[\beta\alpha]\,X+[\beta\beta]\,Y+[\beta\gamma]\,Z+[\beta\delta]\,U+\ \ldots\ =y\\
[\gamma\alpha]\,X+[\gamma\beta]\,Y+[\gamma\gamma]\,Z+[\gamma\delta]\,U+\ \ldots\ =z\\
[\delta\alpha]\,X+[\delta\beta]\,Y+[\delta\gamma]\,Z+[\delta\delta]\,U+\ \ldots\ =u\\
\ldots\ldots\ldots\ldots\ldots\ldots
\end{array}$$

where $[\kappa\lambda]=[\lambda\kappa]$.

The method of deriving the set of equations S from the set N, which was used for the solution of the normal equations occurring in the present work, is essentially the same as that which is called *Zweites Auflösungsverfahren* by Helmert (*Die Ausgleichungsrechnung nach der Methode der kleinsten Quadrate*). The practical value of this method appears not to have found the general appreciation which it, in my opinion, deserves. Also my attention was only called to Helmert's exposition of the method after I had developed it independently (starting from Gauss's method as given by Oppolzer, *Lehrbuch zur Bahnbestimmung von Kometen und Planeten*, vol. ii.). The symmetrical arrangement of the computations, which is explained below, has, since its first publication in my inaugural dissertation, been largely used by myself and others. (See, *e.g.*, Cookson, these *Annals*, vol. xii., Part II., p. 163.) In the Astronomical Laboratory at Groningen it has now been in constant use for several years. We find that even inexperienced computers learn to use the method correctly very quickly, and owing to the many checks occurring as the work proceeds, errors are

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extremely rare. After a little experience the solution of a set of six normals (including the preparing of the forms for computation, the derivation of the weights, and the final substitution of the unknowns in the normal equations) never takes more than three hours, and is often accomplished in two hours and a half. It is thus with increased confidence that I now again call the attention of practical computers to this method.*

The first step is the formation of the *equations of elimination.* (*Eliminationsgleichungen, Reducirte Normalgleichungen, Intermediate normals.*) If we solve x from the first equation N, and if we put

$$-\frac{[ab]}{[aa]}=a_b \qquad -\frac{[ac]}{[aa]}=a_c \qquad -\frac{[ad]}{[aa]}=a_d \quad . \quad . \quad . \quad .$$

we get

$$x=\frac{1}{[aa]}X+a_b\,y+a_c\,z+a_d\,u+ \quad . \quad . \quad . \quad .$$

Substituting this value in the other $m-1$ equations N, and introducing the well-known notations:

$$\begin{array}{ll} [bb]+a_b[ba]=[bb1] & \text{or} \quad [bb]+a_b[ab]=[bb1] \\ [bc]+a_c[ba]=[bc1] & \quad\;\; [bc]+a_b[ac]=[bc1] \\ [bd]+a_d[ba]=[bd1] & \quad\;\; [bd]+a_b[ad]=[bd1] \end{array}$$

and generally

$$[kl]+a_k[al]=[kl1]$$

we get $m-1$ equations with $m-1$ unknowns:

$$\begin{array}{l} [bb1]\,y+[bc1]\,z+[bd1]\,u+ \quad . \quad . \quad . \quad . = Y+a_bX \\ [cb1]\,y+[cc1]\,z+[cd1]\,u+ \quad . \quad . \quad . \quad . = Z+a_cX \\ [db1]\,y+[dc1]\,z+[dd1]\,u+ \quad . \quad . \quad . \quad . = U+a_dX \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \end{array}$$

* It may not be without interest to remark that the same method can be used for the determination of the value of a determinant consisting of many columns and rows, a process which even so experienced a practical mathematician as Sir George Darwin "found it singularly difficult to work correctly." (*Acta Mathematica*, vol. xxi., page 165). In fact, the required determinant is the product

$$[aa]\,.\,[bb1]\,.\,[cc2]\,.\,[dd3] \quad . \quad . \quad . \quad .$$

If the computation is carried out by the method given below, arithmetical errors will be found to be very rare indeed. Prof. Darwin's method is the same in principle, the difference being that some intermediate operations are performed mentally instead of being written down explicitly, and that the checks by sums are absent. The mathematician, who understands what he is doing, always has a tendency to omit writing unnecessary figures and to do two or more simple operations at a time. The professional computer, on the other hand, whether understanding or not, works on a fixed scheme. When only a small number of operations is required, as in solving a set of three (or even four) normals, or determining the value of a determinant of a few columns and rows, the mathematician's methods will generally be preferable; with a long calculation, however, there is a decided advantage in the "cut-and-dried" methods of the professional computer. In this case the combining of simple operations and the performing them without writing down the intermediate steps, thus excluding the possibility of systematic application of sum-checks, gives rise to a feeling of growing uncertainty as the computations proceed to greater length, which is reflected in Sir George Darwin's words quoted above.

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These are again of the normal form, and can be treated in the same way as the equations N. If we put

$$-\frac{[bc1]}{[bb1]}=b_c' \qquad -\frac{[bd1]}{[bb1]}=b_d' \quad . \quad . \quad . \quad .$$

$$[kl2]=[kl1]+b_k'[bl1]$$

$$a_c+b_c'a_b=a_c' \qquad a_d+b_d'a_b=a_d' \quad . \quad . \quad . \quad .$$

we will get $m-2$ equations:

$$[cc2]\,z+[cd2]\,u+\quad .\quad .\quad .\quad .\quad =Z+b_c'\,Y+a_c'\,X$$
$$[dc2]\,z+[dd2]\,u+\quad .\quad .\quad .\quad .\quad =U+b_d'\,Y+a_d'\,X$$
$$.\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .$$

These are again treated in the same manner, introducing the notations:

$$-\frac{[cd2]}{[cc2]}=c_d'' \quad . \quad . \quad . \quad .$$

$$[kl3]=[kl2]+c_k''[cl2]$$

$$a_d'+c_d''a_c'=a_d'' \quad . \quad . \quad .$$

$$b_d'+c_d''b_c'=b_d'' \quad . \quad . \quad . \quad .$$

We then get $m-3$ equations:

$$[dd3]\,u+\quad .\quad .\quad .\quad .\quad =U+c_d''Z+b_d''Y+a_d''X$$
$$.\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .$$

which are again reduced to $m-4$, and these to $m-5$, and so on till there is only one equation with one unknown left. For the present purpose it will be sufficient to suppose the number of unknowns to be four.

The equations of elimination are then:

E.

$$[aa]\,x+[ab]\,y+[ac]\,z+[ad]\,u= \qquad X$$
$$[bb1]\,y+[bc1]\,z+[bd1]\,u= \qquad Y+a_b\,X$$
$$[cc2]\,z+[cd2]\,u= \qquad Z+b_c'\,Y+a_c'\,X$$
$$[dd3]\,u=U+c_d''Z+b_d''Y+a_d''X$$

Recapitulating, we have, for the formation of the equations E, to compute the following quantities:

Ia. $$-\frac{[ab]}{[aa]}=a_b \qquad -\frac{[ac]}{[aa]}=a_c \qquad -\frac{[ad]}{[aa]}=a_d \qquad +\frac{[as]}{[aa]}=a_s$$

IIa. $$[bb]+a_b[ab]=[bb1] \qquad [bc]+a_b[ac]=[bc1] \qquad [bd]+a_b[ad]=[bd1] \qquad [bs]+a_b[as]=[bs1]$$

IIIa. $$-\frac{[bc1]}{[bb1]}=b_c' \qquad -\frac{[bd1]}{[bb1]}=b_d' \qquad +\frac{[bs1]}{[bb1]}=b_s'$$

IVa. $$[cc]+a_c[ac]+b_c'[bc1]=[cc2] \qquad [cd]+a_c[ad]+b_c'[bd1]=[cd2] \qquad [cs]+a_c[as]+b_c'[bs1]=[cs2]$$

Va. $$-\frac{[cd2]}{[cc2]}=c_d'' \qquad +\frac{[cs2]}{[cc2]}=c_s''$$

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VIa. $[dl]+a_d[al]+b_d'[bd1]+c_d''[cd2]=[dd3]$ $[ds]+a_d[as]+b_d'[bs1]+c_d''[cs2]=[ds3]$

VIIa. $a_b=\alpha_b$ $1+a_b=s_b$

VIIIa. $b_c'=\beta_c'$ $a_c+b_c'a_b=\alpha_c'$ $1+a_c+b_c's_b=s_c'$

IXa. $c_d''=\gamma_d''$ $b_d'+c_d''b_c'=\beta_d''$ $a_d+b_d'a_b+c_d''a_c'=\alpha_d''$ $1+a_d+b_d's_b+c_d''s_c'=s_d''$

I have also included the quantities which are necessary for the formation of the check-equations.

We put, as is usual:

$$[aa]+[ab]+[ac]+[ad]=[as]$$
$$[ba]+[bb]+[bc]+[bd]=[bs]$$
$$[ca]+[cb]+[cc]+[cd]=[cs]$$
$$[da]+[db]+[dc]+[dd]=[ds]$$

We then have the following check-equations, all of which are very easily verified, and may be given here without proof:

Ib. $\alpha_b+\alpha_c+\alpha_d+\alpha_s=+1$

IIb. $[bb1]+[bc1]+[bd1]=[bs1]$

IIIb. $\beta_c'+\beta_d'+\beta_s'=+1$

IVb. $[cc2]+[cd2]=[cs2]$

Vb. $\gamma_d''+\gamma_s''=+1$

VIb. $[dd3]=[ds3]$

VIIb. $1+\alpha_b=s_b$

VIIIb. $1+\beta_c'+\alpha_c'=s_c'$

IXb. $1+\gamma_d''+\beta_d''+\alpha_d''=s_d''$

There is another very important check on the second members of the equations Ib. If they are multiplied respectively by α_s, β_s', γ_s'' and 1, and then added, the coefficients of U, Z, Y and X in the sum will each be equal to unity. In other words, we have the following relations:

X.
$$+1=+1$$
$$\gamma_s''+\gamma_d''=+1$$
$$\beta_s'+\gamma_s''\beta_c'+\beta_d''=+1$$
$$\alpha_s+\beta_s'\alpha_b+\gamma_s''\alpha_c'+\alpha_d''=+1$$

which can easily be proved with the aid of the check-equations Ib, IIIb and Vb and the definitions VIIIa and IXa of the quantities α_c', β_d'' and α_d''.

If we add the four equations X we get

XI. $$\alpha_s+\beta_s's_b+\gamma_s''s_c'+s_d''=4=\textit{number of unknowns.}$$

It will be sufficient in practice to apply the test XI, and only to use X to localise the error, in the cases when XI is not satisfied.

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We now come to the second step, viz. the derivation of the coefficients of the solution S from the equations of elimination E.

To begin with, we write these equations thus:

E'.
$$\begin{aligned} x - a_b y - a_c z - a_d u &= \frac{1}{[aa]} X \\ y - b_c' z - b_d' u &= \frac{1}{[bb1]} Y + \frac{a_b}{[bb1]} X \\ z - c_d'' u &= \frac{1}{[cc2]} Z + \frac{b_c'}{[cc2]} Y + \frac{a_c'}{[cc2]} X \\ u &= \frac{1}{[dd3]} U + \frac{c_d''}{[dd3]} Z + \frac{b_d''}{[dd3]} Y + \frac{a_d''}{[dd3]} X \end{aligned}$$

The last equation E' gives at once the last equation S; we have thus:

$$[\delta\delta] = \frac{1}{[dd3]} \qquad [\delta\gamma] = \frac{c_d''}{[dd3]} \qquad [\delta\beta] = \frac{b_d''}{[dd3]} \qquad [\delta a] = \frac{a_d''}{[dd3]}$$

and

(1) $$u = [\delta\delta] U + [\delta\gamma] Z + [\delta\beta] Y + [\delta a] X.$$

Multiplying this value of u by c_d'' and adding it to the last equation but one of E', we get:

$$z = c_d''[\delta\delta]\, U + \left\{ \frac{1}{[cc2]} + c_d''\,[\delta\gamma] \right\} Z + \left\{ \frac{b_c'}{[cc2]} + c_d''\,[\delta\beta] \right\} Y + \left\{ \frac{a_c'}{[cc2]} + c_d''\,[\delta a] \right\} X$$

or

(2) $$z = [\gamma\delta]\, U + [\gamma\gamma]\, Z + [\gamma\beta]\, Y + [\gamma a]\, X,$$

which is the last equation but one of S.

It is easily verified that $[\gamma\delta] = [\delta\gamma]$, as it should be.

Again adding $b_d' u + b_c' z$ [where for u and z their values (1) and (2) must be taken] to the second equation E', we get:

$$y = \ldots\ldots + \left\{ \frac{1}{[bb1]} + b_d'\,[\delta\beta] + b_c'\,[\gamma\beta] \right\} Y + \left\{ \frac{a_b}{[bb1]} + b_d'\,[\delta a] + b_c'\,[\gamma a] \right\} X.$$

It is not necessary to compute the coefficients of U and Z, as we know that they will be $[\beta\delta] = [\delta\beta]$ and $[\beta\gamma] = [\gamma\beta]$, which have been computed before. We have thus:

(3) $$y = [\beta\delta]\, U + [\beta\gamma]\, Z + [\beta\beta]\, Y + [\beta a]\, X.$$

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Lastly, adding to the first equation E′ the sum $\alpha_d u + \alpha_c z + \alpha_b y$ [where for u, z and y we take their values (1), (2) and (3)], we get:

$$x = \ldots\ldots + \left\{ \frac{1}{[aa]} + a_d[\delta a] + a_c[\gamma a] + a_b[\beta a] \right\} X,$$

where it is unnecessary to compute other coefficients than that of X.

Recapitulating, we have, for the solution of the equations E, to compute the following quantities, among which I include those necessary for the formation of the check-equations:

XII*a*.
$$\begin{array}{lllll} \frac{1}{[dd3]} = [\delta\delta] & \frac{c_d''}{[dd3]} = [\delta\gamma] & \frac{b_d''}{[dd3]} = [\delta\beta] & \frac{a_d''}{[dd3]} = [\delta a] & \frac{s_d''}{[dd3]} = [\delta\sigma] \\ & \frac{1}{[cc2]} & \frac{b_c'}{[cc2]} & \frac{a_c'}{[cc2]} & \frac{s_c'}{[cc2]} \\ & & \frac{1}{[bb1]} & \frac{a_b}{[bb1]} & \frac{s_b}{[bb1]} \\ & & & \frac{1}{[aa]} & \frac{1}{[aa]} \end{array}$$

XIII*a*. $\frac{1}{[cc2]} + c_d''[\delta\gamma] = [\gamma\gamma]$ $\frac{b_c'}{[cc2]} + c_d''[\delta\beta] = [\gamma\beta]$ $\frac{a_c'}{[cc2]} + c_d''[\delta a] = [\gamma a]$ $\frac{s_c'}{[cc2]} + c_d''[\delta\sigma] = [\gamma\sigma]$

XIV*a*. $\frac{1}{[bb1]} + b_d'[\delta\beta] + b_c'[\gamma\beta] = [\beta\beta]$ $\frac{a_b}{[bb1]} + b_d'[\delta a] + b_c'[\gamma a] = [\beta a]$ $\frac{s_b}{[bb1]} + b_d'[\delta\sigma] + b_c'[\gamma\sigma] = [\beta\sigma]$

XV*a*. $\frac{1}{[aa]} + a_d[\delta a] + a_c[\gamma a] + a_b[\beta a] = [aa]$ $\frac{1}{[aa]} + a_d[\delta\sigma] + a_c[\gamma\sigma] + a_b[\beta\sigma] = [a\sigma]$

and we have the following check-equations:

XII*b*.
$$\begin{array}{r} [\delta\delta] + [\delta\gamma] + [\delta\beta] + [\delta a] = [\delta\sigma] \\ \frac{1}{[cc2]} + \frac{b_c'}{[cc2]} + \frac{a_c'}{[cc2]} = \frac{s_c'}{[cc2]} \\ \frac{1}{[bb1]} + \frac{a_b}{[bb1]} = \frac{s_b}{[bb1]} \end{array}$$

XIII*b*. $[\gamma\delta] + [\gamma\gamma] + [\gamma\beta] + [\gamma a] = [\gamma\sigma]$

XIV*b*. $[\beta\delta] + [\beta\gamma] + [\beta\beta] + [\beta a] = [\beta\sigma]$

XV*b*. $[a\delta] + [a\gamma] + [a\beta] + [aa] = [a\sigma]$

The first three of these are self-evident. To prove XIII*b*, remark that it can be written:

$$\left.\begin{array}{l} c_d''[\delta\delta] + c_d''[\delta\gamma] + c_d''[\delta\beta] + c_d''[\delta a] \\ + \frac{1}{[cc2]} + \frac{b_c'}{[cc2]} + \frac{a_c'}{[cc2]} \end{array}\right\} = \left\{\begin{array}{l} c_d''[\delta\sigma] \\ + \frac{s_c'}{[cc2]} \end{array}\right.$$

which is a combination of the first and second equation XII*b*. The two remaining equations, XIV*b* and XV*b*, are proved by a similar reasoning.

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If the values S of the unknowns are substituted in the equations N, and then these equations are added, we find by comparing the coefficients of X, Y, Z, U in the left- and right-hand members of the sum :

$$\begin{aligned}
&[as] \cdot [a\alpha] + [bs] \cdot [a\beta] + [cs] \cdot [a\gamma] + [ds] \cdot [a\delta] = +1\\
&[as] \cdot [\beta a] + [bs] \cdot [\beta\beta] + [cs] \cdot [\beta\gamma] + [ds] \cdot [\beta\delta] = +1\\
&[as] \cdot [\gamma a] + [bs] \cdot [\gamma\beta] + [cs] \cdot [\gamma\gamma] + [ds] \cdot [\gamma\delta] = +1\\
&[as] \cdot [\delta a] + [bs] \cdot [\delta\beta] + [cs] \cdot [\delta\gamma] + [ds] \cdot [\delta\delta] = +1
\end{aligned}$$

Adding these four equations, we get as a final control :

XVI.* $$[s\sigma] = [as]\,[a\sigma] + [bs]\,[\beta\sigma] + [cs]\,[\gamma\sigma] + [ds]\,[\delta\sigma] = 4 = \textit{number of unknowns.}$$

I now proceed to explain a symmetrical arrangement of the computations, which I have found very convenient in practice.

The first step is the formation of the equations of elimination (E) and the "*multipliers*" a_b, a_c, . . b_c' . . etc. The computation is arranged in two sets of $m+1$ vertical columns and $\frac{m[m+3]}{2} - 1$ horizontal spaces, one set for the left-hand and one for the right-hand members of the equations, the last column of each set being reserved for the checks. The first horizontal space is included between two horizontal lines, then two spaces are left blank, then one included between two lines, then three left blank, etc. The arrangement will be seen from the example which is appended. The normal equations are written in the horizontal spaces marked with N_1, N_2, N_3, N_4, in the example, viz. the first normal in the top space ; the second normal, without the first term, immediately below it ; the third normal, without its first two terms, in the space below the second double horizontal line, . . etc. In the right-hand set of columns are written the coefficients of U, Z, Y, X in the normals, and their sums, *i.e.* $+1$ in each case.

I. We now proceed to compute the multipliers a_b, a_c, a_d.† These are written in the column under $[aa]$, opposite the blank spaces immediately below the normal equations. a_s is written on the bottom line of the form. The check

$$a_b + a_c + a_d + a_s = 1$$

is applied as soon as these quantities are computed.

* This equation is also given by Prof. A. S. Flint, *Annals of Mathematics*, vol. iv., No. 6.

† This computation can be somewhat simplified by using the reciprocals of $[aa]$, $[bb1]$, etc. ; thus $a_b = -\frac{1}{[aa]} \cdot [ab]$, . . . $b_c' = -\frac{1}{[bb1]} \cdot [bc1]$, etc. These reciprocals may be written immediately under the quantities $[aa]$, $[bb1]$, etc., preferably in red ink.

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II. We now take in Crelle's tables the page of the multiplier a_b and write its products by each of the terms of the first normal equation (excepting the first term) in the blank space opposite a_b, *i.e.* immediately below the corresponding terms of the second normal. Adding the numbers which are in the same vertical column, we get the second equation E, which finds its place in the space between the two horizontal lines, in the example marked with E_2. The check

$$[bb1]+[bc1]+[bd1]=[bs1]$$

is then applied.

VII. The same operation is carried through the right-hand members, which here consists in simply writing a_b in the columns headed X and σ. For the right-hand member of equation E_2 we get by addition the coefficients 1 and a_b, and the check-quantity $s_b = 1 + a_b$.

III. Next compute the multipliers b_c', b_d' and the check-quantity b_s', which are written vertically under $[bb1]$ opposite the second blank spaces below the third and fourth normals. Apply the check

$$b_c'+b_d'+b_s'=+1$$

IV. and VIII. Then take in Crelle the page of a_c and write its products by the terms of E_1 (excepting the first two) under the corresponding terms of N_3 in both left- and right-hand members. Next, taking the page of b'_c in Crelle, write its products by the terms of E_2 in the next line, and add. This gives the equation E_3. Apply the checks

$$[cc2]+[cd2]=[cs2]$$
$$1+b_c'+a_c'=s_c'$$

V. Form the quantities c_d'' and c_s'', which are written in the column of $[cc2]$, and apply the check

$$c_d''+c_s''=+1$$

VI. and IX. Form, by the aid of Crelle, the products of a_d by the last term and the check-quantity of E_1, similarly of b_d' by the terms of E_2, and of c_d'' by the terms of E_3. Addition gives the equation E_4. Apply the checks

$$[dd3]=[ds3]$$
$$1+c_d''+b_d''+a_d''=s_d''$$

If it is desired to apply the check-equations X, this can be done in a separate computation, as is shown in the example. If we wish only to use XI, which will generally be the case in practice, the products of 1, s_b, s_c', and s_d'' by a_s, b_s', c_s'' and 1 can be conveniently written in the margin, which is also done in the example.

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We now come to the second step, viz. the determination of the solution S from the equations E. For this part of the computation a form is arranged, in every respect similar to that for the left-hand members of the equations E.

XII. In the first place, the equations E must be transformed to the equations E′, which are written in the same places that were occupied by the normal equations in the first part of the computation. Thus the five quantities

$$1 \quad c_d'' \quad b_d'' \quad a_d'' \quad s_d''$$

are multiplied by $\frac{1}{[dd3]}$ and the products are written in the space between the first pair of horizontal lines. The check

$$[\delta\delta]+[\delta\gamma]+[\delta\beta]+[\delta a]=[\delta\sigma]$$

is applied.

The four coefficients of the right-hand member of E_3 are similarly multiplied by $\frac{1}{[cc2]}$ and the products written on the line, which in the example is marked with E_3'. We have the check

$$\frac{1}{[cc2]}+\frac{b_c'}{[cc2]}+\frac{a_c'}{[cc2]}=\frac{s_c'}{[cc2]}$$

The coefficients of E_2 are multiplied by $\frac{1}{[bb1]}$ and the products written in the space E_2'. The check is:

$$\frac{1}{[bb1]}+\frac{a_b}{[bb1]}=\frac{s_b}{[bb1]}$$

Lastly, in the space marked with E_1' we write the coefficient of X in the equation E_1', viz. $\frac{1}{[aa]}$, and the same quantity is placed in the check-column.

The mechanism of the computation is precisely similar to that of the first part; with the exception that the multipliers are taken from that first part. They are written in the same places which they occupied there, but in a different order, so that the quantities which were in the first part in a line inclined towards the right now come in a vertical column, and *vice versa*, while the quantity which was in the left-hand top corner now comes in the right-hand bottom corner.

XIII. The first multiplier thus being c_d'', its products with the terms of S_4, which is the same as E_4', are written below the corresponding terms of E_3' and added to them. This gives the terms of S_3. Now the check

$$[\gamma\delta]+[\gamma\gamma]+[\gamma\beta]+[\gamma a]=[\gamma\sigma]$$

must be applied.

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The last three of the four quantities in the left-hand member of this equation are in the same horizontal line with $[\gamma\sigma]$, but $[\delta\gamma]$ must be taken from the horizontal line of S_4, in the column of $[\gamma\gamma]$.

XIV. The equation S_2 is formed by adding to E_2' the products of b_d' by the terms of S_4, and of b_c' by those of S_3. The check equation is:

$$[\beta\delta]+[\beta\gamma]+[\beta\beta]+[\beta\alpha]=[\beta\sigma],$$

where $[\beta\alpha]$ and $[\beta\beta]$ are taken from the horizontal space S_2, then going upwards along the vertical column of $[\beta\beta]$ we get $[\gamma\beta]$ from the equation S_3 and $[\beta\delta]$ from equation S_4.

XV. The formation of the last term $[\alpha\alpha]$ of equation S_1 is exactly similar to the formation of $[dd3]$ in the first part of the computation; the quantities required for the check-equation

$$[\alpha\delta]+[\alpha\gamma]+[\alpha\beta]+[\alpha\alpha]=[\alpha\sigma]$$

are all in the same vertical column as $[\alpha\alpha]$ in the horizontal spaces S_4, S_3, S_2, S_1, respectively.

XVI. For the final check $[s\sigma]=4$, the products $[\delta\sigma]\cdot[ds]$, $[\gamma\sigma]\cdot[cs]$, $[\beta\sigma]\cdot[bs]$, $[\alpha\sigma]\cdot[as]$ can be written in the margin, as is shown in the example.

For the multipliers a_b, a_c . . b_c' . . it will be always advisable to take *three* decimal places, and no more. I have sometimes been in the habit of taking into account halves, or even quarters of the last decimal, by a small interpolation which is done at sight while using Crelle's tables. By this device it is possible through a very long series of computations with Crelle always to keep three significant figures correct. The number of decimal places to be taken in the equations E′ and in the solution must depend on the size of the coefficients of the normal equations. Four significant figures in the quantities $\frac{1}{[aa]}, \frac{1}{[bb1]}$ . ., will generally be most convenient, and in the final solution the last figure can be dropped without any real loss of accuracy. The uncertainties of the resulting values of the unknowns will then very seldom be more than one or two units of the third significant figure. The method works most quickly and conveniently when the diagonal coefficients of the normals (the quantities $[aa]$, $[bb]$, etc.) are between 10 and 20, two decimal places being given. The reciprocals and the diagonal coefficients of the solution will then be taken to four decimal places, and will either have three significant figures or four, of which the first is 1; the other coefficients of the solution will have three significant figures. It will generally well repay the labour before solving a set of normals to reduce the

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diagonal coefficients to this size by introducing suitable new unknowns, and to arrange them so that $[aa]$ shall be the largest of the diagonal terms, $[bb]$ the second largest, and so on.

A greater accuracy than three significant figures will seldom be required. In the rare cases where this accuracy is wanted, recourse can, of course, be had to logarithms or a calculating machine, but this seems not advisable, as an almost unlimited degreé of refinement can very easily be obtained by our method with Crelle. If the values of the unknowns are substituted in the normal equations N, these will not be rigorously satisfied, but the absolute terms resulting from the substitution will be $X-\delta X$, $Y-\delta Y$. . . . If, then, δX, δY . . . are substituted in the solution S, we will get corrections δx, δy to the first values of x, y. . . . The relative uncertainty in the corrections δx, δy . . . will be the same as in the original values x, y . . ., but the uncertainty in the final values $x+\delta x$, $y+\delta y$ . . . is equal to the *absolute* uncertainty in δx, δy. . . . Consequently, at least two more significant figures will be correct, than in the first values of x, y. . . . This process of successive approximation, which can of course, if necessary, be continued, requires very little additional labour, certainly incomparably less than the excess of the labour involved in a logarithmic computation over that necessary for a solution with Crelle. And in addition to this saving of labour, the liability to mistakes is also smaller than with the use of logarithms.

In order to demonstrate the accuracy which can be attained by this method, working with Crelle, I will here give the different checks, employed in the solution of the system of 18 normal equations which was treated in Chapter III., and there denoted by S (see p. 142). The sums of the multipliers, such as

$$a_b+a_c+a_d \ldots\ldots\ldots +a_s=1$$

gave the 17 results:

1·001 1·001 0·999 1·001 0·999 0·997 1·000 1·001 1·000 1·000 0·999 0·997 1·000 1·000 0·999 1·000 1·000.

The check-equations X. gave the following 18 sums:

1·000 1·000 1·000 0·999 1·000 0·999 1·001 1·000 1·001 1·002 1·000 0·999 0·998 1·001 1·001 1·001 1·001 1·000.

The equation XI. gave 18·003.

The final check gave $[s\sigma]=17{\cdot}96$.

The whole of this solution took me, single-handed, about fifty hours, of which about 15 were spent in searching for and correcting a mistake, which had arisen through changing the sign of one of the multipliers in the right-hand member of one of the equations E. At that time I had not yet found the check-equations X. and XI.,

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and the error was only discovered by means of the check on the corresponding equation S. The equation XI. would have shown it at a much earlier stage of the computations.

I do not think it necessary to dwell upon the general advantages of a solution in terms of the right-hand members of the normal equations. A practical example of its usefulness is furnished by the investigation described in the preceding paper.

It needs hardly be remembered that the weights of the unknowns are given by the same solution, viz. :

$$p_\alpha = \frac{1}{[\alpha\alpha]}, \qquad p_\beta = \frac{1}{[\beta\beta]} \cdot \quad . \quad . \quad . \quad \text{etc.}$$

The first part of the method here advocated, viz. the formation of the left-hand members of the equation E, is the same as in Oppolzer's method, only with the abbreviations which are the natural outcome of the substitution of Crelle's tables for the use of logarithms. The determination of the unknowns *and their weights* according to Oppolzer's method is certainly not shorter than it is in my method, while the fact that this second part of the computation is arranged in a precisely similar form to the first part, seems to me to be a great advantage.

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Example Showing Arrangement of Computations.

(The Normal equations are marked with N_1, N_2, N_3, N_4; the equations of elimination with E_1, E_2, E_3, E_4; their transformations with E_1', E_2', E_3' E_4'; the solution with S_1, S_2, S_3, S_4. An asterisk (*) refers to a check-equation.)

Normal Equations.

$$[aa]x+[ab]y+[ac]z+[ad]u=X$$
$$[bb]+[bc]+[bd]=Y$$
$$+[cc]+[cd]=Z$$
$$+[dd]=U$$

Left-hand members. *Right-hand members.*

	x	y	z	u	s	U	Z	Y	X	σ	
$N_1=E_1$	$[aa]$	$[ab]$	$[ac]$	$[ad]$	$[as]$				$+1$	$+1$	a_s
N_2		$[bb]$	$[bc]$	$[bd]$	$[bs]$			$+1$		$+1$	
$-\frac{[ab]}{[aa]}=a_b$		$a_b[ab]$	$a_b[ac]$	$a_b[ad]$	$a_b[as]$				a_b	a_b	add.
E_2		$[bb1]$	$[bc1]$	$[bd1]$	$[bs1]$			$+1$	a_b	s_b	* $b_s's_b$
N_3			$[cc]$	$[cd]$	$[cs]$		$+1$			$+1$	
$-\frac{[ac]}{[aa]}=a_c$			$a_c[ac]$	$a_c[ad]$	$a_c[as]$				a_c	a_c	
$-\frac{[bc1]}{[bb1]}=b_c'$			$b_c'[bc1]$	$b_c'[bd1]$	$b_c'[bs1]$			b_c'	$b_c'a_b$	$b_c's_b$	add.
E_3			$[cc2]$	$[cd2]$	$[cs2]$		$+1$	b_c'	a_c'	s_c'	* $c_s''s_c'$
N_4				$[dd]$	$[ds]$	$+1$				$+1$	
$-\frac{[ad]}{[aa]}=a_d$				$a_d[ad]$	$a_d[as]$				a_d	a_d	
$-\frac{[bd1]}{[bb1]}=b_d'$				$b_d'[bd1]$	$b_d'[bs1]$			b_d'	$b_d'a_b$	$b_d's_b$	
$-\frac{[cd2]}{[cc2]}=c_d''$				$c_d''[cd2]$	$b_d''[cs2]$		c_d''	b_c'	$c_d''a_c'$	$c_d''s_c'$	add.
E_4				$[dd3]$	$[ds3]$	$+1$	c_d''	b_d''	a_d''	s_d''	* s_d''
$+\frac{[as]}{[aa]}=a_s$		$+\frac{[bs1]}{[bb1]}=b_s'$	$+\frac{[cs2]}{[cc2]}=c_s''$								Sum $=m^*$
Sum $=+1$		Sum $=+1$	Sum $=+1$		*						

Check-equations X.

			a_s	a_s	
	c_s''	b_s'	$b_s'a_b$	$b_s's_b$	
	c_d''	$c_s''b_c'$	$c_s''a_c'$	$c_s''s_c'$	
$+1$		b_d''	a_d''	s_d''	
$+1$	$+1$	$+1$	$+1$	m	add.
					*

Solution.

Weights.

$$[aa]X+[a\beta]Y+[a\gamma]Z+[a\delta]U=x \quad \frac{1}{[aa]}$$
$$[\beta\beta]+[\beta\gamma]+[\beta\delta]=y \quad \frac{1}{[\beta\beta]}$$
$$[\gamma\gamma]+[\gamma\delta]=z \quad \frac{1}{[\gamma\gamma]}$$
$$[\delta\delta]=u \quad \frac{1}{[\delta\delta]}=[dd3]$$

$E_4'=S_4$	$\frac{1}{[dd3]}=[\delta\delta]$	$\frac{c_d''}{[dd3]}=[\delta\gamma]$	$\frac{b_d''}{[dd3]}=[\delta\beta]$	$\frac{a_d''}{[dd3]}=[\delta\alpha]$	$\frac{s_d''}{[dd3]}=[\delta\sigma]$	* $[ds][\delta\sigma]$
E_3'		$\frac{1}{[cc2]}$	$\frac{b_c'}{[cc2]}$	$\frac{a_c'}{[cc2]}$	$\frac{s_c'}{[cc2]}$	*
c_d''		$c_d''[\delta\gamma]$	$c_d''[\delta\beta]$	$c_d''[\delta\alpha]$	$c_d''[\delta\sigma]$	
S_3		$[\gamma\gamma]$	$[\gamma\beta]$	$[\gamma\alpha]$	$[\gamma\sigma]$	add. * $[cs][\gamma\sigma]$
E_2'			$\frac{1}{[bb1]}$	$\frac{a_b}{[bb1]}$	$\frac{s_b}{[bb1]}$	*
b_d'			$b_d'[\delta\beta]$	$b_d'[\delta\alpha]$	$b_d'[\delta\sigma]$	
b_c'			$b_c'[\gamma\beta]$	$b_c'[\gamma\alpha]$	$b_c'[\gamma\sigma]$	
S_2			$[\beta\beta]$	$[\beta\alpha]$	$[\beta\sigma]$	add. * $[bs][\beta\sigma]$
E_1'				$\frac{1}{[aa]}$	$\frac{1}{[aa]}$	
a_d				$a_d[\delta\alpha]$	$a_d[\delta\sigma]$	
a_c				$a_c[\gamma\alpha]$	$a_c[\gamma\sigma]$	
a_b				$a_b[\beta\alpha]$	$a_b[\beta\sigma]$	
S_1				$[a\alpha]$	$[a\sigma]$	add. * $[as][a\sigma]$
						Sum $=[s\sigma]=m^*$

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Numerical Example.

(The values printed in smaller type are those resulting from the check-equations.)

Normal Equations.

$$\begin{aligned} +10x + 3y + 2z - 3u &= X \\ +12 \quad +4 \quad -2 \quad &= Y \\ +8 \quad -1 \quad &= Z \\ +7 \quad &= U \end{aligned}$$

Left-hand members.

x	y	z	u	s	
+10·00	+ 3·00	+ 2·00	−3·00	+12·00	
	+12·00	+ 4·00	−2·00	+17·00	
−0·300	− ·90	− ·60	+ ·90	− 3·60	
	+11·10	+ 3·40	−1·10	+13·40	+13·40
		+ 8·00	−1·00	+13·00	
−0·200		− ·40	+ ·60	− 2·40	
	− 0·306	− 1·04	+ ·34	− 4·10	
		+ 6·56	−0·06	+ 6·50	+ 6·50
			+7·00	+ 1·00	
+0·300			− ·90	+ 3·60	
	+ 0·099		− ·11	+ 1·33	
		+ 0·009	− 0	+ 6	
			+5·99	+ 5·99	
+1·200	+ 1·207	+ 0·991			
+1·000	+ 1·000	+ 1·000			

Right-hand members.

U	Z	Y	X	σ		
			+1·000	+1·000		+1·200
		+1		+1·000		
			−0·300	− ·300		
		+1·000	−0·300	+0·700	+0·700	+0·845
	+1			+1·000		
			−0·200	− ·200		
		−0·306	+ 92	− ·214		
	+1·000	−0·306	−0·108	+0·586	+0·586	+0·581
+1				+1·000		
			+0·300	+ ·300		
		+0·099	− 30	+ 69		
	+0·009	− 3	− 1	+ 5		
+1·000	+0·009	+0·096	+0·269	+1·374	+1·374	+1·374
						+4·000

+0·1669	+0·0015	+0·0160	+0·0449	+0·2293	+0·2293	+0·229
	+0·1524	−0·0466	−0·0165	+0·0893	+0·0893	
+0·009	+ 0	+ 1	+ 4	+ 21		
	+0·1524	−0·0465	−0·0161	+0·0914	+0·0913	+1·188
		+0·0901	−0·0270	+0·0631	+0·0631	
+0·099		+ 16	+ 44	+ 227		
	−0·306	+ 142	+ 49	− 280		
		+0·1059	−0·0177	+0·0578	+0·0577	+0·983
			+0·1090	+0·1000		
+0·300			+ 135	+ 688		
	−0·200		+ 32	− 183		
		−0·300	+ 53	− 173		
			+0·1220	+0·1332	+0·1331	+1·598
					[$s\sigma$]	= +3·998

Check-equations X.

			+1·200	+1·200
		+1·207	−0·362	+0·845
	+0·991	−0·303	−0·107	+0·581
+1·000	+0·009	+0·096	+0·269	+1·374
+1·000	+1·000	+1·000	+1·000	+4·000

Solution.

				Weights.
+0·1220X	−0·0177Y	−0·0161Z	+0·0449U = x	8·20
+0·1059	−0·0465	+0·0160	= y	9·44
	+0·1524	+0·0015	= z	6·56
		+0·1669	= u	5·99

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ANNALS OF THE CAPE OBSERVATORY.

VOL. XII. PART II.

DETERMINATION OF THE MASS OF JUPITER AND ORBITS OF THE SATELLITES,

FROM OBSERVATIONS MADE WITH THE CAPE HELIOMETER.

By BRYAN COOKSON, M.A.

UNDER THE DIRECTION OF

Sir DAVID GILL, K.C.B., LL.D., D.Sc., F.R.S., Etc.,

HIS MAJESTY'S ASTRONOMER AT THE CAPE OF GOOD HOPE.

Published by Order of the Lords Commissioners of the Admiralty in obedience to His Majesty's Command.

EDINBURGH:
PRINTED FOR HIS MAJESTY'S STATIONERY OFFICE
By NEILL & CO., Limited, Bellevue.

And to be purchased, either directly or through any Bookseller, from
WYMAN & SONS, Limited, Fetter Lane, E.C ; or
OLIVER & BOYD, Edinburgh; or
E. PONSONBY, 116 Grafton Street, Dublin.

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CHAPTER I.

Introduction.

This paper contains the reduction and discussion of two series of observations of Jupiter's satellites made by the writer at the Cape of Good Hope in the years 1901 and 1902. Ten years previously, Sir David Gill had made a fine series of observations of the satellites, and it was on his suggestion that the work was repeated, so as to form a continuation and in some sense a completion of the earlier series. The object was two-fold: first, a determination of the mass of Jupiter, and second, a determination of the elements of the orbits of the satellites, a comparison of which with the values derived by Dr. de Sitter from the observations of 1891, would provide means of finding some of the secular perturbations of the elements.

The instrument with which the observations were made was the 7-inch heliometer, and the quantities observed were the mutual distance and position-angle of each pair of satellites. A heliometer was first employed for an investigation of the mass of Jupiter by Bessel during the years 1832–1839, next by E. Luther in 1856, and again by Schur in 1874–1880. In these older series the positions of the satellites were measured from the limbs of Jupiter, and the difficulty of placing the image of a satellite on an ill-defined object like the limb of the planet led to results which were not very satisfactory. Thus Bessel found that the probable error of an observation of the satellites was about $\pm 0''\cdot 2$, which is just twice what he found for a pair of stars. Had he, however, observed the satellites relatively to one another instead of relatively to the planet, he would have attained the accuracy of star observations, for the images of the satellites are ideal objects to observe. This method of observing the co-ordinates of a satellite relatively to the other satellites was first put into practice by H. Struve in 1884, in his observations of the satellites of Saturn with a micrometer attached to an equatorial; it was a great improvement so far as accuracy of observation is concerned. Sir D. Gill adopted the same method in his observations of the satellites of Jupiter, and reached a wonderfully high degree of accuracy, probably the limit attainable with a 7-inch heliometer.

The method of reduction adopted in the present paper is practically the same as that adopted by Dr. de Sitter in his reduction of Sir D. Gill's observations: this in its turn is on the lines indicated by Marth in his ephemeris for the satellites in 1891. It is an improvement on the methods of the older investigators, chiefly owing to the

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simplification gained by adopting the plane of Jupiter's equator as plane of reference instead of the earth's equator.

The best tables at present at the disposal of astronomers, namely those of Souillart, are not capable of giving the positions of the satellites with an accuracy corresponding to that attained by observation, as will be apparent on a glance at the columns printed on the pages which contain the difference between the calculated and the observed co-ordinates. These differences will be seen to amount not infrequently to a second of arc, which is ten times the probable error of their observation. If, then, theory is to keep pace with observation, what is now required is a new set of numerical tables, but for the preparation of them it is necessary first to obtain from observation new values of the elements of the orbits and of the masses of the satellites. It is hoped that the present work may be of some assistance in providing material out of which a new numerical theory may be built.

The algebraical development of the theory was first given by Laplace, and a full account of it, together with numerical expressions for the co-ordinates of the satellites, was published in the fourth volume of the *Mécanique Céleste*. The theory was further developed along the same lines by Souillart, who discovered many inequalities undetected by his predecessors, and embodied all his results in numerical form. Adams also in his lectures gave in outline the method he proposed to adopt in attacking the problem, but much additional work would be required before his investigations could be made use of by the practical astronomer. As already mentioned, the best numerical theory we have at present is that of Souillart and it is with his theory that the writer's observations have been compared.

The special value of observations made with the heliometer as compared with those made at eclipses lies in accuracy of the measurement of position-angle, which in general corresponds to the latitude of the satellites; and for the determination of the nodes and inclinations of the orbits, the heliometer in all probability is superior to even the best photometric observations of eclipses. The want of accurate measurements of the latitudes has long been felt. Adams suggested that in addition to eclipse observations the difference in declination between the limbs of the planet and the shadow of a satellite in transit should be measured with a filar micrometer, and Marth on several occasions drew attention to the great want of accurate determinations of latitude. But none were forthcoming till Sir D. Gill made his observations at the opposition of Jupiter in 1891. These were reduced by Dr. de Sitter in 1898: the results of his discussion were published in the form of an inaugural dissertation in 1901 (J. B. Wolters, Groningen).

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When the present writer went out to the Cape in 1901 at Sir D. Gill's invitation, he undertook to make and to reduce another series of observations similar to those made in 1891, and the following paper gives a detailed account of the work. A short summary has already been published in the June number of the "Monthly Notices, R.A.S.," 1904. The numerical computations connected with this investigation have been long and heavy, and every possible care has been taken throughout to ensure accuracy; every figure has been checked in one or more ways. In this connection it is a pleasure to record how much assistance was received from Mr. V. A. Löwinger of the Cape Observatory. At the beginning of the work he offered to help in his spare time, and throughout my stay at the Cape I had the benefit of his skill as a computer and also the pleasure of working with an enthusiastic man and a good friend. I wish to offer him my most sincere thanks.

On July 22, 1905, I left England on another visit to the Cape, as a member of the official party of the British Association, and returned to England on October 7. The six weeks I had in South Africa were utilised in making observations at the Cape Observatory to determine whether there was a systematic difference in scale value between distances as large as those of the standard stars which were used for the determination of scale value and distances as small as the average distance of the satellites. Such a systematic difference had recently been shown by Mr. Hough to exist in the observations made for the triangulation of stars about the South Pole (*Cape Annals*, xi., i., pp. 111–2), and it was essential to discover whether a similar difference existed in the case of my own observations and if so, to measure it. This was satisfactorily accomplished and the correction was found to be practically zero. It would be interesting to follow up the question as to how far this correction depends on the habit of observing referred to on p. 21. In any case, so far as the mass of Jupiter from the present investigation depends on instrumental scale value, the value found at the end of this paper may be taken as correct.

Finally, it is a pleasant duty to record that I owe much to Mr. Hough, with whom I lived whilst at the Cape, and I wish to thank him sincerely for his unfailing courtesy and many acts of kindness.

Above all, I wish to express my obligations to Sir David Gill, not only for the opportunity he afforded me for making the observations at the Cape Observatory, but also for numberless acts of kindness, encouragement and help throughout the work.

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CHAPTER II.

The Observations.

The observations here discussed were all made by the writer with the 7-inch heliometer, the quantities observed being the mutual distances and position-angles of every pair of satellites. In March 1901 the heliometer, of which a full description may be found in vol. vii of these Annals, was completely dismounted and thoroughly overhauled by Mr. Miller the optical fitter attached to the Observatory. Every part of it was carefully examined, repolished or repainted as was necessary, and a few small improvements were added. The work was pushed on as fast as was consistent with thoroughness, and the instrument was ready for use in the middle of June 1901, to all intents and purposes as good as new.

As every screw had been examined, it was first of all necessary to investigate the instrumental adjustments. The proper observations for this purpose were made on June 19 and 20, and gave

$$\xi = +26''{\cdot}6 \qquad e = +\ 3''{\cdot}7$$
$$\eta = +10''{\cdot}6 \qquad \psi = \pm 21''{\cdot}2 \begin{cases} \text{East hour-angles.} \\ \text{West} \quad ,, \quad ,, \end{cases}$$

where ξ and η are the distances of the instrumental pole from the six-hour circle and from the meridian respectively; e is the maximum flexure of the telescope tube, and ψ is the maximum amount of rotation of the tube about the point at which it is secured to the declination axis (*Chauvenet*, ii., p. 430).

The correction to be applied to a position-angle observed at an hour-angle T may be put into the form

$$I + a \cos \tau + b \sin \tau$$

in which a and b are coefficients involving the foregoing instrumental constants and the declination of the object observed; I varies with the declination, but for a given star has the same value at all hour-angles on the same side of the meridian: it changes its value, however, on passing through the meridian.

For the declination of Jupiter we have

$$\left.\begin{matrix} a = +\ 5'' \\ b = -30'' \end{matrix}\right\} \text{East hour-angles.} \qquad \left.\begin{matrix} a = +19'' \\ b = +30'' \end{matrix}\right\} \text{West hour-angles.}$$

where τ the hour-angle must be taken without sign.

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Observations of a well-known pair of "Standard Stars" were made every night to determine I.

The coefficients of sin τ and cos τ were also determined in another way, which is, however, of small comparative weight, but which serves as a useful control on the preceding values. From every observation of the Standard Stars we can get an equation of condition of the form

$$I + a \cos \tau + b \sin \tau = C - O,$$

and equations of this type may be solved by least squares to give I, a and b; in this way the following values were found from the standards of 1901 :

$a = +\ 8''$, $b = -25''$ } East hour-angles. $\qquad$ $a = +22''$, $b = +25''$ } West hour-angles.

The coefficients first given have been adopted; practically it is a matter of indifference which are chosen, because as the standards were never more than two or three degrees distant from Jupiter the effect of mal-adjustment of the instrument would be nearly the same on both standards and satellites, if, as was actually the case, they were observed at about the same hour-angle. The observed position-angle of the standards was corrected for refraction, aberration and for the instrumental adjustments as described; the difference between the observation thus corrected and the assumed position-angle of the standards is the index error I to be applied to each observed position-angle of the satellites. The observations of distance of the standards in a similar manner control the scale value; if $\sigma_\circ$ is the adopted distance of the standards, σ the observed distance corrected for refraction and aberration, the correction to any other observed distance s is

$$\frac{\sigma_\circ - \sigma}{\sigma} s.$$

The standards were observed on each night, as a rule immediately before or after observation of the satellites, but often both before and after. An observation of either their distance or position-angle consisted of two pointings in each position of the semi-lenses, *i.e.*, four pointings in each co-ordinate; and in order to rotate the field of view through 180° after each pointing, the reversing prism in front of the eye-piece was rotated through 90°. The times of these pointings, necessary only for the computation of refraction, were recorded on a small chronograph registering to 0·1 minute, and the pointings were made symetrically so that the mean epoch of the pointings in distance was always very near the mean of those in position-angle.

The standards for 1901 were 24 and 26 Sagittarii, and their adopted distance and position angle were

$$6591''{\cdot}100 \quad \text{and} \quad 84^\circ\ 20'\ 36'';$$

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those for 1902 were 21 and 23 (θ) Capricorni, the adopted distance and position-angle being

$$4486''\cdot930 \text{ and } 76^\circ\ 31'\ 9''.$$

The discussion of the distances and position-angles of these stars is postponed till a later chapter, for the effect of any corrections to them can be allowed for at the end of the work. A change in the adopted distance affects only the mass of Jupiter, whilst a change in the adopted position-angle is equivalent to a change in the fundamental plane to which the orbits of the satellites are referred.

The following tables contain a complete list of the observations of the standards. The meanings of the various columns are sufficiently explained by the headings, the quantities involved being expressed in arc with the aid of the tables A, B, C, *Cape Annals*, vol. vii., pp. 46–53. The refraction was read off to $0''\cdot001$ for distances, and $0''\cdot1$ for position-angles, from a large scale chart, on which the mean refraction for the two pairs of stars was plotted with the hour-angles as abscissæ: this was found very much quicker than a separate computation for each observation, and also much less liable to error. The tables used in the construction of the chart were Jacoby's "Tables for the Correction of Micrometric Measures for Refraction," which are based on the Poulkova tables. The barometer and thermometers were always read immediately before and after making the observations, and from these readings the "Meteorological Factor" was computed for the time of observation: the mean refraction from the chart multiplied by this factor is the refraction for the observed state of the atmosphere. The aberration correction is necessary to reduce to mean distance or position-angle: the correction for precession and nutation in the case of the position-angles is included in the aberration correction. The column headed C—O contains the differences between the observed and adopted distances or position-angles of the standards. When two observations were made on one night the mean has been adopted as the correction to be applied to all observations of the satellites made on that night. During the year 1901 the focus of the heliometer eye-piece was not changed; but in 1902 a change was made, and the effect of it is shown in the observed distances of the standards—a fact which emphatically confirms the necessity of having control or standard stars, and justifies the care always taken to observe the satellites with the same focussing of the eye-piece as was used in observing the standards. A line across the column C—O marks a change of focus. The index correction of the position-angle circle for west hour-angles differs from that for east hour-angles because in this correction are included, as explained above, the other corrections for instrumental adjustments which do not vary with the hour-angle.

In the last column the differences from the mean scale correction for a given focus

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of the eye-piece are given: these differences include errors of observation and real changes due to temperature effects and to variations in the focus of the observer's eye. In the case of distance observations the probable error of an observation of the satellites will be shown in chapter v. to be less than $\pm 0''\cdot100$, but the probable error for the standards computed from the residuals in this table is in 1901 $\pm 0''\cdot158$, and in 1902 $\pm 0''\cdot167$, or of all together $\pm 0''\cdot164$: hence of this probable error about $\pm 0''\cdot13$, is due to real changes in the instrument and observer.

For the observations in position-angle, the probable error of a single observation is

1901	For East hour-angles	(27 obs.)	$\pm$ 10″·2 or	$\pm$ 0″·33 of great circle.
	For West „	(38 obs.)	$\pm$ 6·8 or	$\pm$ 0·22 „
	For both together	(65 obs.)	$\pm$ 8·5 or	$\pm$ 0·27 „

It appears from the observations of 1902 that a change of focus has no effect on the measurement of position-angle. The probable errors for this year are

1902	For East hour-angles	(47 obs.)	$\pm$ 7″·1 or	$\pm$ 0″·15 of great circle.
	For West „	(15 obs.)	$\pm$ 7·6 or	$\pm$ 0·17 „
	For both together	(62 obs.)	$\pm$ 7·2 or	$\pm$ 0·16 „

The only explanation of the unduly large probable errors in 1901 is that these were the first observations that were made by the observer with a heliometer and that more experience is needed in making measurements of position-angle than of distance. In 1902 the greatest possible care was taken with every pointing in position-angle and a much more satisfactory degree of accuracy was reached.

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TABLE I.

Observed Distances of Standards, 1901.

Scales Equivalent 6595″·559.

Rotation No.	Date.	Sidereal Time.	Micrometer.	Run.	Refraction.	Aberration.	Observed Distance 6580″.	C—O or $\sigma_o - \sigma$.	Adopted $\sigma_o - \sigma$.	Difference from Mean.
	1901.	h m	″	″	″	″	″	″	″	′
1	June 24	17 12·3	− 7·844	− 0·028	+ 2·037	+ 0·050	+ 9·774	+ 1·326	+1·110	− 0·207
9		19 44·3	− 7·333	− 0·026	1·956	+ 0·050	10·206	0·894		
11	25	15 3·0	− 9·086	− 0·027	3·647	+ 0·037	10·130	0·970	0·970	− 0·347
17	29	20 28·6	− 8·069	− 0·021	2·180	+ 0·010	9·659	1·441	1·592	+ 0·275
25		21 28·1	− 8·946	− 0·027	2·760	+ 0·010	9·356	1·744		
27	July 2	16 9·3	− 8·655	− 0·010	2·562	− 0·037	9·419	1·681	1·665	+ 0·348
41		17 21·5	− 8·099	− 0·010	2·038	− 0·037	9·451	1·649		
43	8	15 22·4	− 8·633	− 0·052	3·237	− 0·100	10·007	1·093	1·335	+ 0·018
57		16 54·7	− 8·042	− 0·052	2·158	− 0·100	9·523	1·577		
59	10	15 20·2	− 9·139	− 0·014	3·247	− 0·122	9·531	1·569	1·405	+ 0·088
73		16 53·8	− 7·728	− 0·012	2·162	− 0·122	9·859	1·241		
75	11	17 0·0	− 7·478	− 0·037	2·108	− 0·134	10·018	1·082	0·756	− 0·561
89		20 9·4	− 6·806	− 0·034	2·084	− 0·134	10·669	0·431		
91	12	15 29·0	− 7·237	− 0·035	3·119	− 0·145	[11·261]		1·454	+ 0·137
105		16 42·0	− 7·976	− 0·040	2·248	− 0·145	9·646	1·454		
107	14	14 56·7	− 9·478	+ 0·019	3·799	− 0·165	9·734	1·366	1·200	− 0·117
115		15 43·6	− 8·172	+ 0·016	2·829	− 0·165	10·067	1·033		
117	16	15 9·0	− 9·211	− 0·009	3·529	− 0·184	9·684	1·416	1·088	− 0·229
131		16 26·4	− 7·388	− 0·007	2·357	− 0·184	10·339	0·761		
139	18	16 2·9	− 6·834	− 0·029	2·646	− 0·206	[11·136]		1·300	− 0·017
141	22	15 11·2	− 9·101	− 0·018	3·453	− 0·246	9·647	1·453	1·453	+ 0·136
157	27	15 24·2	− 8·565	+ 0·004	3·162	− 0·295	9·865	1·235	1·208	− 0·109
171		16 45·7	− 7·543	+ 0·003	2·196	− 0·295	9·920	1·180		
173	29	15 45·4	− 8·358	0·000	2·818	− 0·316	9·703	1·397	1·162	− 0·155
181		16 35·8	− 7·333	0·000	2·264	− 0·316	10·174	0·926		
183	Aug. 10	16 0·3	− 7·964	− 0·034	2·609	− 0·425	9·745	1·355	1·514	+ 0·197
197		17 10·1	− 7·711	− 0·033	2·036	− 0·425	9·426	1·674		
199	13	20 6·0	− 7·306	− 0·014	2·080	− 0·446	9·873	1·227	1·054	− 0·263
213		21 35·8	− 7·802	− 0·016	2·923	− 0·446	10·218	0·882		
215	14	15 50·5	− 8·067	− 0·024	2·744	− 0·455	9·757	1·343	1·290	− 0·027
223		16 33·9	− 7·498	− 0·022	2·279	− 0·455	9·863	1·237		
225	15	20 12·9	− 7·531	− 0·020	2·103	− 0·463	9·648	1·452	1·452	+ 0·135
229	19	20 6·2	− 7·275	− 0·031	2·041	− 0·494	9·800	1·300	1·300	− 0·017
245	24	20 17·3	− 7·478	− 0·035	2·129	− 0·527	9·648	1·452	1·380	+ 0·063
259		21 31·8	− 8·024	− 0·038	2·823	− 0·527	9·793	1·307		
261	26	20 19·2	− 7·172	− 0·031	2·174	− 0·540	9·990	1·110	1·110	− 0·207
273	27	20 10·8	− 7·395	− 0·014	2·062	− 0·545	9·687	1·413	1·349	+ 0·032
287		21 21·9	− 7·862	− 0·016	2·679	− 0·545	9·815	1·285		
289	28	20 28·2	− 7·496	− 0·017	2·204	− 0·551	9·699	1·401	1·072	− 0·245
297		21 10·3	− 7·207	− 0·016	2·572	− 0·551	10·357	0·743		
299	29	20 24·9	− 7·787	− 0·021	2·181	− 0·557	9·375	1·725	1·499	+ 0·182
313		21 45·0	− 8·216	− 0·023	3·064	− 0·557	9·827	1·273		
315	31	20 33·1	− 7·621	− 0·012	2·150	− 0·570	9·506	1·594	1·594	+ 0·277
319	Sept. 1	20 33·6	− 7·538	− 0·010	2·209	− 0·576	9·644	1·456	+1·338	+ 0·021
333		21 53·0	− 8·249	− 0·010	+ 3·156	− 0·576	+ 9·880	+ 1·220		

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TABLE I.—*continued.*

Rotation No.	Date.	Sidereal Time.	Micrometer.	Run.	Refraction.	Aberration.	Observed Distance 6580″.	C—O or $\sigma_o-\sigma$.	Adopted $\sigma_o-\sigma$.	Difference from Mean.
	1901.	h m	″	″	″	″	″	″	″	″
335	Sept. 3	20 18·9	− 7·398	− 0·019	+ 2·125	− 0·585	+ 9·682	+ 1·418	+1·246	− 0·071
349		21 41·1	− 7·892	− 0·020	2·964	− 0·585	10·026	1·074		
351	4	20 25·4	− 6·974	− 0·009	2·193	− 0·591	10·178	0·922	0·942	− 0·375
365		21 42·1	− 7·839	− 0·010	3·018	− 0·591	10·137	0·963		
367	9	20 25·9	− 7·451	− 0·028	2·188	− 0·613	9·655	1·445	1·540	+ 0·223
375		21 18·0	− 8·109	− 0·031	2·660	− 0·613	9·466	1·634		
377	10	20 19·4	− 7·999	− 0·004	2·142	− 0·616	9·082	2·018	2·018	+ 0·701
387	14	20 21·6	− 7·603	− 0·015	2·140	− 0·630	9·451	1·649	1·182	− 0·135
401		21 59·1	− 7·822	− 0·015	3·293	− 0·630	10·385	0·715		
403	16	20 40·5	− 7·346	− 0·004	2·271	− 0·635	9·845	1·255	1·144	− 0·173
417		22 11·4	− 8·420	− 0·004	3·566	− 0·635	10·066	1·034		
419	17	20 22·4	− 7·533	− 0·030	2·158	− 0·638	9·516	1·584	1·349	+ 0·032
433		21 48·7	− 8·019	− 0·032	3·116	− 0·638	9·986	1·114		
435	19	20 20·2	− 7·396	− 0·019	2·150	− 0·644	9·650	1·450	1·321	+ 0·004
449		21 48·6	− 8·067	− 0·020	3·080	− 0·644	9·908	1·192		
451	23	20 32·0	− 7·313	− 0·035	2·167	− 0·650	9·728	1·372	1·372	+ 0·055
455	25	21 3·0	− 7·653	− 0·017	2·438	− 0·652	9·665	1·435	1·435	+ 0·118
461	27	19 59·2	− 7·310	− 0·011	+ 1·977	− 0·654	+ 9·561	+ 1·539	+1·539	+ 0·222

Mean $\sigma_o-\sigma$ = + 1″·317.

Nos. 25, 117 : Wrong pair of wires in micrometer for reading scales was used: corrected accordingly. On July 26 these superfluous wires were removed.

Nos. 91, 139 : Wrong wires used, but impossible to correct observations with any certainty: rejected

No. 107 : Wrong scale division read: corrected accordingly.

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TABLE II.

Observed Position-Angles of Standards, 1901.

Rotation No.	Date.	Sidereal Time.	Circle Reading.	Refraction.	Aberration and Precession.	Adjustments.	Observed P–A.	Index Error.	Adopted Index Error.	Difference from Mean.
	1901.	h m	° ′ ″	″	″	″	° ′ ″	′ ″	′ ″	″
2	June 24	17 13·2	84 54 25	− 11	+ 25	− 5	84 54 34	−33 58	−33 58	+ 29
10		19 43·9	84 54 41	+ 10	25	+ 28	84 55 44	−35 8	−35 8	+ 4
12	25	15 3·9	84 54 57	− 43	25	− 21	84 54 18	−33 42	−33 42	+ 13
18	29	20 28·1	84 54 52	+ 18	25	+ 31	84 56 6	−35 30	−35 15	+ 26
26		21 28·0	84 54 0	+ 36	25	+ 34	84 55 35	−34 59		− 5
28	July 2	16 9·2	84 54 11	− 22	25	− 13	84 54 1	−33 25	−33 34	− 4
42		17 21·6	84 54 8	− 10	25	− 4	84 54 19	−33 43		+ 14
44	8	15 23·6	84 54 45	− 36	25	− 18	84 54 16	−33 40	−33 26	+ 11
58		16 55·1	84 53 45	− 13	25	− 8	84 53 49	−33 13		− 16
60	10	15 19·6	84 54 45	− 36	25	− 18	84 54 16	−33 40	−33 30	+ 11
74		16 53·6	84 53 53	− 13	25	− 8	84 53 57	−33 21		− 8
76	11	17 0·6	84 53 19	− 12	25	− 6	84 53 26	−32 50	−32 50	− 39
90		20 9·5	84 54 23	+ 15	25	+ 30	84 55 33	−34 57	−34 57	− 7
92	12	15 28·7	84 54 41	− 33	25	− 17	84 54 16	−33 40	−33 20	+ 11
106		16 41·5	84 53 38	− 16	25	− 10	84 53 37	−33 1		− 28
108	14	14 56·7	84 54 41	− 46	25	− 22	84 53 58	−33 22	−33 25	− 7
116		15 43·6	84 54 23	− 28	25	− 16	84 54 4	−33 28		− 1
118	16	15 9·0	84 54 56	− 42	26	− 20	84 54 20	−33 44	−33 36	+ 15
132		16 26·4	84 54 8	− 18	26	− 12	84 54 4	−33 28		− 1
140	18	16 3·0	84 54 0	− 24	26	− 14	84 53 48	−33 12	−33 12	− 17
142	22	15 10·8	84 54 45	− 40	26	− 20	84 54 11	−33 35	−33 35	+ 6
151	23	15 19·7	84 54 30	− 36	26	− 18	84 54 2	−33 26	−33 13	− 3
156		16 22·6	84 53 41	− 19	26	− 12	84 53 36	−33 0		− 29
158	27	15 23·6	84 54 33	− 35	26	− 18	84 54 6	−33 30	−33 34	+ 1
172		16 45·3	84 54 13	− 15	26	− 9	84 54 15	−33 39		+ 10
174	29	15 45·6	84 54 28	− 28	26	− 16	84 54 10	−33 34	−33 32	+ 5
182		16 35·5	84 54 6	− 17	26	− 10	84 54 5	−33 29		0
184	Aug. 10	16 0·4	84 54 28	− 24	25	− 14	84 54 15	−33 39	−33 39	+ 10
198		17 10·1	84 54 6	− 11	25	− 5	84 54 15	−33 39		+ 10
200	13	20 6·8	84 54 48	+ 15	25	+ 29	84 55 57	−35 21	−35 18	+ 17
214		21 35·4	84 54 14	+ 38	25	+ 34	84 55 51	−35 15		+ 11
216	14	15 50·3	84 54 30	− 27	25	− 15	84 54 13	−33 37	−33 33	+ 8
224		16 33·7	84 54 8	− 17	25	− 11	84 54 5	−33 29		0
226	15	20 13·1	84 55 3	+ 16	25	+ 29	84 56 12	−35 36	−35 36	+ 32
230	19	20 6·4	84 54 46	+ 14	25	+ 29	84 55 54	−35 18	−35 18	+ 14
246	24	20 16·8	84 54 24	+ 17	25	+ 30	84 55 36	−35 0	−35 4	− 4
260		21 31·6	84 54 8	+ 38	25	+ 34	84 55 45	−35 9		+ 5
262	26	20 18·2	84 54 20	+ 17	24	+ 30	84 55 31	−34 55	−34 55	− 9
274	27	20 10·0	84 54 14	+ 15	24	+ 30	84 55 23	−34 47	−34 53	− 17
288		21 21·8	84 54 2	+ 34	24	+ 34	84 55 34	−34 58		− 6
290	28	20 28·4	84 54 23	+ 18	24	+ 32	84 55 37	−35 1	−35 2	− 3
298		21 10·4	84 54 10	+ 31	24	+ 33	84 55 38	−35 2		− 2
300	29	20 24·9	84 54 27	+ 18	24	+ 31	84 55 40	−35 4	−35 8	0
314		21 45·0	84 54 4	+ 44	24	+ 35	84 55 47	−35 11		+ 7
316	31	20 32·2	84 54 18	+ 19	24	+ 32	84 55 33	−34 57	−34 57	− 7
320	Sept. 1	20 34·8	84 54 21	+ 21	24	+ 32	84 55 38	−35 2	−35 10	− 2
334		21 52·6	84 54 8	+ 47	+ 24	+ 36	84 55 55	−35 19		+ 15

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TABLE II.—*continued.*

Rotation No.	Date.	Sidereal Time.	Circle Reading.	Refraction.	Aberration and Precession.	Adjust-ments.	Observed P–A.	Index Error.	Adopted Index Error.	Difference from Mean.
	1901.	h m	° ′ ″	″	″	″	° ′ ″	′ ″	′ ″	″
336	Sept. 3	20 18·8	84 54 17	+ 17	+ 24	+ 30	84 55 28	−34 52	−35 1	− 12
350		21 40·9	84 54 2	43	24	35	84 55 46	−35 10		+ 6
352	4	20 28·2	84 54 21	18	23	31	84 55 33	−34 57	−35 0	− 7
366		21 41·3	84 53 59	43	23	35	84 55 40	−35 4		0
368	9	20 25·6	84 54 17	18	23	31	84 55 29	−34 53	−34 56	− 11
376		21 17·7	84 54 4	33	23	34	84 55 34	−34 58		− 6
378	10	20 20·4	84 54 34	17	23	30	84 55 44	−35 8	−35 8	+ 4
388	14	20 21·4	84 54 24	17	22	31	84 55 34	−34 58	−34 58	− 6
402		21 55·1	84 53 47	49	22	36	84 55 34	−34 58		− 6
404	16	20 40·5	84 54 26	22	22	33	84 55 43	−35 7	−35 4	+ 3
418		22 11·0	84 53 37	60	22	37	84 55 36	−35 0		− 4
420	17	20 22·3	84 54 16	17	22	31	84 55 26	−34 50	−34 56	− 14
434		21 48·0	84 53 57	45	22	35	84 55 39	−35 3		− 1
436	19	20 20·2	84 54 29	17	22	30	84 55 38	−35 2	−35 4	− 2
450		21 46·5	84 53 58	47	22	35	84 55 42	−35 6		+ 2
452	23	20 32·7	84 54 16	19	21	32	84 55 28	−34 52	−34 52	− 12
456	25	21 2·7	84 54 8	28	21	33	84 55 30	−34 54	−34 54	− 10
462	27	19 59·0	84 54 36	+ 13	+ 21	+ 29	84 55 39	−35 3	−35 3	− 1

Mean Index-Correction , East hour-angles = − 33′ 29″
„ „ , West „ = − 35′ 4″

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TABLE III.

Observed Distances of Standards, 1902.

Scale Equivalent { + Micrometer reading 4470″·615.
− „ „ 4490″·533.

Rotation No.	Date.	Sidereal Time.	Micrometer.	Run.	Refraction.	Aberration.	Observed Distance 4480″.	C—O or $\sigma_o - \sigma$.	Difference from Mean.
	1902.	h m	″	″	″	″	″	″	″
1	June 10	17 11·2	+ 11·875	+ 0·032	+ 3·296	+ 0·354	+ 6·172	+ 0·758	− 0·058
11	11	17 33·7	+ 11·950	+ 0·039	2·728	+ 0·350	5·682	+ 1·248	+ 0·178
19		18 46·7	− 6·425	− 0·021	1·753	+ 0·350	6·190	+ 0·740	
21	16	17 47·2	− 7·313	− 0·024	2·439	+ 0·324	5·959	+ 0·971	+ 0·236
35		19 22·0	− 6·555	− 0·022	1·516	+ 0·324	5·796	+ 1·134	
37	19	17 18·0	− 7·333	− 0·036	3·090	+ 0·308	6·562	+ 0·368	− 0·448
45	21	17 53·6	− 6·991	− 0·044	2·346	+ 0·297	6·141	+ 0·789	− 0·027
49	24	17 42·3	− 7·132	− 0·040	2·551	+ 0·280	6·192	+ 0·738	− 0·078
69	30	18 54·9	− 6·405	− 0·037	1·687	+ 0·246	6·024	+ 0·906	+ 0·090
81	July 1	19 8·8	− 6·154	− 0·018	1·610	+ 0·240	6·211	+ 0·719	− 0·097
89	7	18 14·0	− 6·896	− 0·018	2·084	+ 0·202	5·905	+ 1·025	+ 0·209
101	8	23 24·0	− 6·129	− 0·008	1·605	+ 0·196	6·197	+ 0·733	− 0·041
115		1 20·8	+ 11·918	+ 0·015	3·369	+ 0·196	6·113	+ 0·817	
123	14	18 13·7	− 6·786	− 0·048	2·073	+ 0·155	5·927	+ 1·003	+ 0·187
125	15	17 28·8	− 7·234	− 0·016	2·817	+ 0·150	6·250	+ 0·680	− 0·273
139		19 18·0	− 5·689	− 0·013	1·543	+ 0·150	6·524	+ 0·406	
141	16	17 36·4	− 7·523	− 0·041	2·629	+ 0·142	5·740	+ 1·190	+ 0·374
153	18	18 15·0	− 6·881	− 0·026	2·053	+ 0·129	5·808	+ 1·122	+ 0·306
161	19	17 35·0	− 7·177	− 0·058	2·693	+ 0·120	6·1.1	+ 0·819	+ 0·003
171	20	18 39·7	− 6·014	− 0·029	1·822	+ 0·112	6·424	+ 0·506	− 0·310
179	24	17 23·2	− 7·556	− 0·055	2·974	+ 0·083	5·979	+ 0·951	+ 0·135
207	26	18 5·0	− 6·177	− 0·032	2·197	+ 0·070	6·591	+ 0·339	− 0·477
215	27	18 12·1	− 6·670	− 0·036	2·094	+ 0·063	5·984	+ 0·946	+ 0·130
217	Aug. 3	17 9·0	− 7·561	− 0·091	3·312	+ 0·010	6·203	+ 0·727	− 0·066
233		18 49·1	− 6·029	− 0·072	1·715	+ 0·010	6·157	+ 0·773	
235	4	23 17·2	− 5·498	− 0·027	1·559	+ 0·003	6·570	+ 0·360	+ 0·119
243	7	17 4·1	− 7·222	− 0·048	3·490	− 0·020	6·733	+ 0·197	− 0·070
259		18 57·0	− 5·388	− 0·037	1·677	− 0·020	6·765	+ 0·165	
261	16	17 26·1	+ 11·191	+ 0·008	2·901	− 0·085	4·630	+ 2·300	+ 0·145
271	17	17 19·9	+ 10·841	+ 0·038	3·054	− 0·094	4·454	+ 2·476	+ 0·321
289	18	18 40·0	− 7·787	− 0·044	1·796	− 0·101	4·397	+ 2·533	+ 0·378
297	19	17 30·4	+ 11·480	+ 0·077	2·788	− 0·109	4·851	+ 2·079	− 0·076
307	20	17 14·2	+ 11·103	+ 0·052	3·176	− 0·115	4·831	+ 2·099	− 0·056
315	21	17 57·7	− 7·670	− 0·005	2·256	− 0·122	4·992	+ 1·938	− 0·217
329	22	17 19·3	− 8·277	− 0·050	2·991	− 0·130	5·067	+ 1·863	− 0·292
351	24	23 18·9	− 6·949	− 0·021	1·555	− 0·143	4·975	+ 1·955	− 0·200
357		0 4·1	− 4·760	− 0·014	1·901	− 0·143	7·517	− 0·587	− 0·385
361	25	17 47·0	− 5·470	− 0·035	2·443	− 0·150	7·321	− 0·391	− 0·189
375	27	17 18·5	− 6·107	− 0·027	3·061	− 0·166	7·294	− 0·364	− 0·075
385		18 47·4	− 4·945	− 0·023	1·725	− 0·166	7·124	− 0·194	
393	28	17 0·5	− 6·913	− 0·016	3·603	− 0·174	7·033	− 0·103	+ 0·099
407	30	23 2·3	− 4·608	− 0·012	1·473	− 0·184	7·202	− 0·272	− 0·070
419	Sept. 1	17 24·0	− 6·234	− 0·009	2·931	− 0·200	7·021	− 0·091	+ 0·111
433	2	18 14·8	− 5·306	− 0·014	+ 2·047	− 0·206	+ 7·054	− 0·124	+ 0·078

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TABLE III.—*continued.*

Rotation No.	Date.	Sidereal Time.	Micrometer.	Run.	Refraction.	Aberration.	Observed Distance 4480″.	C—O or $\sigma_o - \sigma$.	Difference from Mean.
	1902.	h m							
435	Sept. 3	17 8·4	— 6·365	—0·013	+3·317	— 0·214	+ 7·258	— 0·328	— 0·139
441		17 48·3	— 5·428	—0·011	2·404	— 0·214	7·284	— 0·354	
451	4	22 43·8	— 4·561	—0·006	1·389	— 0·218	7·137	— 0·207	— 0·005
459	6	17 19·8	— 5·608	—0·014	2·993	— 0·232	7·672	— 0·742	— 0·398
467		18 14·7	— 4·925	—0·013	2·025	— 0·232	7·388	— 0·458	
477	7	17 15·4	— 6·182	—0·009	3·124	— 0·239	7·227	— 0·297	— 0·197
491		18 46·3	— 4·599	—0·007	1·143	— 0·239	7·431	— 0·501	
493	8	17 19·8	— 5·939	—0·030	2·998	— 0·246	7·316	— 0·386	— 0·028
507		18 29·5	— 5·130	—0·025	1·872	— 0·246	7·004	— 0·074	
509	10	23 5·7	— 4·809	—0·024	1·510	— 0·259	6·951	— 0·021	+ 0·028
523		0 32·4	— 5·257	—0·026	2·266	— 0·259	7·257	— 0·327	
525	12	18 3·9	— 5·453	—0·040	2·172	— 0·270	6·942	— 0·012	+ 0·190
529	16	22 24·7	— 4·451	—0·009	2·315	— 0·294	7·094	— 0·164	+ 0·038
533	26	22 55·1	— 5·265	—0·017	1·445	— 0·347	6·349	+ 0·581	+ 0·687
549		0 38·2	— 5·949	—0·020	2·324	— 0·347	6·541	+ 0·389	
551	Oct. 4	22 56·3	— 4·674	—0·012	1·438	— 0·382	6·903	+ 0·027	+ 0·256
563		0 6·2	— 5·180	—0·014	1·893	— 0·382	6·850	+ 0·080	
567	5	22 55·1	— 4·291	—0·011	+1·425	— 0·387	+ 7·269	— 0·339	— 0·137

Aug. 24—Changed focus during the night, and observed standards again; distances of Satellites observed after change of focus were reduced with second value of $\sigma_o - \sigma$

Nos. 1—233 : mean $\sigma_o - \sigma$ = + 0″·816.
Nos. 235—259 : mean $\sigma_o - \sigma$ = + 0″·241.
Nos. 261—351 : mean $\sigma_o - \sigma$ = + 2″·155.
Nos. 357—567 : mean $\sigma_o - \sigma$ = — 0″·202.

In the case of two observations on one night, the mean of the observed $\sigma_o - \sigma$ is the value adopted, except on Aug. 24.

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TABLE IV.

Observed Position-Angles of Standards, 1902.

Rotation No.	Date.	Sidereal Time.	Circle Reading.	Refraction.	Aberration and Precession.	Adjustments.	Observed P.-A.	Index Error.	Adopted Index Error.	Difference from Mean.
	1902.	h m	° ′ ″	″	″	″	° ′ ″	′ ″	′ ″	″
2	June 10	17 11·2	77 5 7	− 48	+ 7	− 23	77 4 3	−32 54	−32 54	− 4
12	11	17 33·7	77 4 48	− 38	7	− 20	77 3 57	−32 48	−32 56	− 10
20		18 48·9	77 4 36	− 18	7	− 12	77 4 13	−33 4		+ 6
22	16	17 47·2	77 4 56	− 33	8	− 18	77 4 13	−33 4	−33 2	+ 6
36		19 22·0	77 4 22	− 13	8	− 7	77 4 10	−33 1		+ 3
38	19	17 17·3	77 5 0	− 45	8	− 22	77 4 1	−32 52	−32 52	− 6
46	21	17 53·6	77 4 49	− 31	8	− 17	77 4 9	−33 0	−33 0	+ 2
50	24	17 42·3	77 5 3	− 35	9	− 19	77 4 18	−33 9	−33 9	+ 11
70	30	18 54·6	77 4 25	− 17	9	− 11	77 4 6	−32 57	−32 57	− 1
82	July 1	19 8·8	77 4 16	− 15	10	− 10	77 4 1	−32 52	−32 52	− 6
90	7	18 13·3	77 4 35	− 26	10	− 16	77 4 3	−32 54	−32 54	− 4
102	8	23 24·0	77 4 35	+ 28	10	+ 33	77 5 46	−34 37	−34 39	+ 3
116		1 20·8	77 3 1	+ 119	10	+ 40	77 5 50	−34 41		+ 7
124	14	18 13·1	77 4 37	− 26	11	− 16	77 4 6	−32 57	−32 57	− 1
126	15	17 28·8	77 4 50	− 39	11	− 21	77 4 1	−32 52	−33 2	− 6
140		19 18·0	77 4 31	− 14	11	− 8	77 4 20	−33 11		+ 13
142	16	17 36·4	77 4 55	− 36	11	− 20	77 4 10	−33 1	−33 1	+ 3
154	18	18 14·1	77 5 12	− 25	11	− 16	77 4 42	−33 33	−33 33	+ 35
162	19	17 35·0	77 4 56	− 37	11	− 20	77 4 10	−33 1	−33 1	+ 3
172	20	18 39·7	77 4 25	− 20	11	− 13	77 4 3	−32 54	−32 54	− 4
180	24	17 22·4	77 4 48	− 43	12	− 22	77 3 55	−32 46	−32 46	− 12
208	26	18 5·0	77 4 13	− 28	12	− 16	77 3 43	−32 34	−32 34	− 24
216	27	18 11·5	77 4 30	− 26	12	− 16	77 4 0	−32 51	−32 51	− 7
218	Aug. 3	17 8·7	77 5 22	− 49	12	− 22	77 4 23	−33 14	−33 7	+ 16
234		18 49·1	77 4 27	− 18	12	− 12	77 4 9	−33 0		+ 2
236	4	23 17·7	77 4 35	+ 25	12	+ 33	77 5 45	−34 36	−34 36	+ 2
244	7	17 4·1	77 5 0	− 52	12	− 23	77 3 57	−32 48	−32 48	− 10
260		18 55·7	77 4 14	− 17	12	− 11	77 3 58	−32 49		− 9
262	16	17 26·1	77 4 28	− 41	13	− 21	77 3 39	−32 30	−32 30	− 28
272	17	17 20·1	77 4 49	− 44	13	− 22	77 3 56	−32 47	−32 47	− 11
290	18	18 39·7	77 4 15	− 20	13	− 13	77 3 55	−32 46	−32 46	− 12
298	19	17 30·2	77 4 52	− 39	13	− 21	77 4 5	−32 56	−32 56	− 2
308	20	17 14·5	77 5 5	− 46	13	− 22	77 4 10	−33 1	−33 1	+ 3
316	21	17 57·2	77 4 39	− 29	13	− 17	77 4 6	−32 57	−32 57	− 1
330	22	17 19·6	77 4 50	− 43	13	− 22	77 3 58	−32 49	−32 49	− 9
352	24	23 18·0	77 4 42	+ 25	13	+ 33	77 5 53	−34 44	−34 44	+ 10
358		0 4·1	77 3 49	+ 44	13	+ 34	77 5 20	−34 11	−34 11	− 23
362	25	17 47·0	77 4 49	− 32	13	− 18	77 4 12	−33 3	−33 3	+ 5
376	27	17 17·9	77 5 16	− 44	13	− 21	77 4 24	−33 15	−32 58	+ 17
386		18 47·4	77 4 9	− 19	13	− 12	77 3 51	−32 42		− 16
394	28	16 59·8	77 5 3	− 55	13	− 24	77 3 57	−32 48	−32 48	− 10
408	30	23 2·5	77 4 37	+ 21	13	+ 32	77 5 43	−34 34	−34 34	− 0
420	Sept. 1	17 24·5	77 4 55	− 41	13	− 21	77 4 6	−32 57	−32 57	− 1
434	2	18 15·0	77 4 45	− 25	13	− 16	77 4 17	−33 8	−33 8	+ 10
436	3	17 8·1	77 5 11	− 49	13	− 22	77 4 13	−33 4	−32 58	+ 6
442		17 48·1	77 4 39	− 32	13	− 18	77 4 2	−32 53		− 5
452	4	22 43·8	77 4 41	+ 16	+ 13	+ 30	77 5 40	−34 31	−34 31	− 3

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TABLE IV.—*continued.*

Rotation No.	Date.	Sidereal Time.	Circle Reading.	Refraction.	Aberration and Precession.	Adjustments.	Observed P.-A.	Index Error.	Adopted Index Error.	Difference from Mean.
	1902.	h m	° ′ ″	″	″	″	° ′ ″	′ ″	′ ″	″
460	Sept. 6	17 19·5	77 5 3	− 43	+ 13	− 22	77 4 11	−33 2	−33 0	+ 4
468		18 15·4	77 4 36	− 25	13	− 16	77 4 8	−32 59		+ 1
478	7	17 15·8	77 5 7	− 45	13	− 22	77 4 13	−33 4	−33 0	+ 6
492		18 46·3	77 4 22	− 19	13	− 12	77 4 4	−32 55		− 3
494	8	17 19·8	77 5 2	− 43	13	− 22	77 4 10	−33 1	−32 58	+ 3
507		18 29·5	77 4 28	− 22	13	− 14	77 4 5	−32 56		− 2
510	10	23 5·7	77 4 36	+ 22	12	+ 33	77 5 44	−34 35	−34 22	+ 1
524		0 32·6	77 3 29	+ 62	12	+ 36	77 5 19	−34 10		− 24
526	12	18 3·5	77 4 40	− 28	12	− 17	77 4 7	−32 58	−32 58	0
530	16	22 24·3	77 4 52	+ 12	12	+ 29	77 5 45	−34 36	−34 36	+ 2
534	26	22 55·3	77 4 52	+ 19	12	+ 32	77 5 55	−34 46	−34 36	+ 12
550		0 38·0	77 3 42	+ 66	12	+ 36	77 5 36	−34 27		− 7
552	Oct. 4	22 56·5	77 4 44	+ 19	11	+ 32	77 5 46	−34 37	−34 44	+ 3
564		0 6·0	77 4 31	+ 44	11	+ 35	77 6 1	−34 52		+ 18
568	5	22 54·7	77 4 43	+ 18	+ 11	+ 32	77 5 44	−34 35	−34 35	+ 1

The mean Index-Errors for different Focussing of the Eye-Piece are as follows:—

	East Hour-Angles.	No. of Obs.	West Hour-Angles.	No. of Obs.
Nos. 2–234	−32′ 58″	23	−34′ 39″	2
236–260	−32 48	2	−34 36	1
262–352	−32 56	7	−34 44	1
358–568	−32 59	15	−34 32	11
All	−32 58	47	−34 34	15

This shows that measurement of position-angle is not affected by change of focus.

We now come to the observations of the satellites themselves and it will be convenient first to give a reference list of all the observations made. The numbers under the letters *s* and *p* at the top of each column of the following tables are the rotation numbers of the observations in distance and position-angle respectively. Excepting on July 23rd, 1901, when the printing micrometer of the heliometer was out of order and observations of distance could not be secured, odd rotation numbers always refer to distances, even numbers to position-angles. In the last column the letters Cl denote that observation was stopped on that night by clouds, the letter D by the definition becoming too bad.

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TABLE V.

List of Observations, 1901.

Date.	Standards.		I.–II.		I.–III.		I.–IV.		II.–III.		II.–IV.		III.–IV.		Standards.		
	s	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	
June 24	1	2	...		...		...		7	8	5	6	3	4	9	10	
25	11	12	...		...		...		...		15	16	13	14	...		Cl.
29	17	18	...		...		...		23	24	21	22	19	20	25	26	
July 2	27	28	31	32	33	34	29	30	37	38	35	36	39	40	41	42	
8	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	
10	59	60	63	64	61	62	65	66	67	68	69	70	71	72	73	74	
11	75	76	77	78	83	84	85	86	79	80	81	82	87	88	89	90	
12	91	92	93	94	99	100	101	102	95	96	97	98	103	104	105	106	
14	107	108	111	112	...		113	114	...		109	110	...		115	116	
16	117	118	121	122	119	120	123	124	127	128	125	126	129	130	131	132	
18	...		133	134	...		...		135	136	137	138	...		139	140	D.
22	141	142	149	150	145	146	143	144	...		...		147	148	...		D.
23		151		152		153	...			154	...		...			156	
27	157	158	159	160	167	168	165	166	161	162	163	164	169	170	171	172	
29	173	174	...		175	176	177	178	...		...		179	180	181	182	
Aug. 10	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	
13	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	
14	215	216	...		217	218	219	220	...		...		221	222	223	224	
15	225	226	..		...		...		...		227	228	...		...		
19	229	230	231	232	233	234	235	236	237	238	...		...		...		D.
23	...		...		...		...		239	240	241	242	243	244	...		Cl.
24	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	
26	261	262	263	264	265	266	267	268	269	270	271	272	...		...		D.
27	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	
28	289	290	...		291	292	293	294	...		...		295	296	297	298	
29	299	300	307	308	311	312	305	306	301	302	309	310	303	304	313	314	
31	315	316	...		...		...		317	318	...		...		...		Cl.
Sept. 1	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	
3	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	
4	351	352	353	354	355	356	357	358	359	360	363	364	361	362	365	366	
9	367	368	369	370	...		371	372	...		373	374	...		375	376	
10	377	378	381	382	385	386	...		383	384	379	380	...		...		
14	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	
16	403	404	413	414	411	412	415	416	405	406	407	408	409	410	417	418	
17	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	
19	435	436	439	440	443	444	447	448	437	438	441	442	445	446	449	450	
23	451	452	453	454	...		...		...		...		...		...		D.
25	455	456	457	458	459	460	...		...		...		...		...		D.
27	461	462	465	466	...		463	464	...		467	468	...		...		
Total	36	37	29	30	27	28	28	28	27	28	30	30	27	27	27	28	6 D. 3 Cl.

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TABLE VI.

List of Observations, 1902.

Date.	Standards.		I.–II.		I.–III.		I.–IV.		II.–III.		II.–IV.		III.–IV.		Standards.		
	s	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	
June 10	1	2	3	4	5	6	7	8	...		...		9	10	...		
11	11	12	...		...		...		13	14	15	16	17	18	19	20	
16	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
19	37	38	...		...		...		39	40	41	42	43	44	...		Cl.
21	45	46	...		47	48	...		...		...		...		...		
24	49	50	51	52	53	54	55	56	57	58	59	60	61	62	...		
30	...		63	64	...		65 / 71	66 / 72	...		67 / 73	68 / 74	...		69	70	
July 1	...		...		79	80	...		.		77	78	75	76	81	82	
3	...		83	84	85	86	...		...		...		...		...		Cl.
7	89	90	95	96	87	88	93	94	97	98	99	100	91	92	...		
8	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	
14	...		...		117	118	119	120	...		...		121	122	123	124	
15	125	126	127	128	129	130	131	132	137	138	135	136	133	134	139	140	
16	141	142	143	144	145	146	147	148	149	150	...		...		...		Cl.
18	153	154	159	160	...		157	158	...		151 / 155 / 155A	152 / 156	...		...		
19	161	162	163	164	165	166	...		...				...		...		D.
20	171	172	...		...		...		167	168	169	170	173 / 175 / 177	174 / 178	...		
24	179	180	181	182	183	184	185	186	187	188	189	190	191[Wt. 2]192		...		
26	...		193	194	197	198	199	200	201	202	203	204	195 / 205	196 / 206	207	208	
27	...		209	210	211	212	...		213	214	...		...		215	216	
Aug. 3	217	218	223	224	225	226	229	230	227	228	231	232	219 / 221	220 / 222	233	234	
4	235	236	237	238	241	242	...		239	240	...		...		...		
7	243	244	245	246	247	248	249	250	251	252	253	254	255 / 257	256 / 258	259	260	
16	261	262	265	266	267	268	...		269	270	...		263	264	...		Cl.
17	271	272	...		273	274	275	276	...		...		277 / 279	278 / 280	...		
18	289	290	291	292	293	294	295	296	281	282	285	286	283 / 287	284 / 288	...		
19	297	298	...		299	300	...		303	304	301 / 305	302 / 306	...		...		
20	307	308	309	310	...		...		...		...		...		...		
21	315	316	325	326	327	328	323	324	311 / 313 / 317	312 / 314 / 318	319	320	321	322	...		
22	329	330	331	332	333	334	341	342	337	338	339	340	335	336	...		
24	351	352	355	356	345	346	349	350	343	344	347	348	353	354	357	358	
25	361	362	371	372	369	370	359 / 373	360 / 374	363	364	367	368	365	366	...		
27	375	376	377	378	379	380	383 / 387	384 / 388	381	382	389	390	391	392	385	386	
28	393	394	399	400	397	398	395	396	405	406	401	402	403	404	...		
30	407	408	...		411	412	...		413	414	409	410	415	416	...		D.
Sept. 1	...		...		...		...		...		...		417[Wt. 2]418		419	420	
2	...		429	430	425	426	423	424	427	428	431	432	421	422	433	434	
3	435	436	445	446	437	438	447	448	443	444	439	440	449	450	441	442	
4	451	452	...		...		...		457	458	455	456	453	454	...		
6	459	460	475	476	469	470	473	474	463	464	465 / 471	466 / 472	461	462	467	468	
7	477	478	479	480	487	488	485	486	483	484	481	482	489	490	491	492	
8	493	494	495	496	499	500	497	498	501	502	503	504	505	506	507	508	
10	509	510	511	512	513	514	515	516	521	522	517	518	519	520	523	524	
12	525	526	527	528	...		...		...		...		...		...		
16	529	530	...		531	532	..		...		...		...		...		
26	533	534	539	540	541	542	535	536	543	544	545	546	537 / 547	538 / 548	549	550	
Oct. 4	551	552	557	558	555	556	553	554	565	566	561	562	559	560	563	564	
5	567	568	...		...		...		569	570	...		...		...		D.
Total	40	40	35	35	38	38	33	33	37	37	37	37	43	43	22	22	3 D. 4 Cl.

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The total number of observations is as follows :—

1901 & 1902.	Standards.		I.-II.		I.-III.		I.-IV.		II.-III.		II.-IV.		III.-IV.		Sum Total.
	s	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s*	*p*	*s* & *p*
Number of Observations.	125	127	64	65	66	67	62	62	63	64	66	66	70	70	1037

The following small table may also be inserted here :—

	Extent of Observing Season.	Number of Nights on which Observations were secured.	Percentage of Working Nights.	Observing stopped by Clouds.	Observing stopped by Definition.
1901	95 nights	39	41·1	3	6
1902	128 nights	48	37·5	4	3
Total	223 nights	87	39·0	7	9

The aim was to secure as many observations as possible within two months of the opposition of Jupiter; but owing to the heliometer being under repair in 1901, it was not possible to begin observing in that year till little more than a month before opposition; this accounts for the comparative shortness of the observing season in 1901. Under working nights are included all nights on which an observation of the satellites was secured. In 1901 there were 8 and in 1902 there were 7 clear nights on which the definition was so bad as to make observation impossible : this generally occurs when the "South-Easter" is blowing strong. As a rule, the North-West or winter wind is favourable to observation.

Before giving the tables of observations of the satellites, we will explain the method of procedure adopted in making the observations. The dome was generally opened up or the ventilators were lowered some time before commencing work. The first thing to be done on starting work was to compare the chronometer by which the time of every pointing was noted, with the Transit Circle Clock and then on entering the dome of the heliometer to read the barometer and the inside and outside thermometers. The small chronograph marking to 0·1 min. was next set going and then work with the heliometer could be started. An observation of either standard stars or satellites consisted of the following operations :

(i.) One pointing in distance, read micrometers and rotate reversing prism in front of eyepiece through 90°.

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(ii.) One pointing in distance, read and rotate prism through 45°.
(iii.) One pointing in position angle, read and rotate prism through 90°.
(iv.) One pointing in position angle, read and interchange the segments of the object glass.
(v.) (vi.) (vii.) (viii.) Repeat operations in reverse order.

In this way the mean epochs of the pointings in distance and position-angle were made very nearly the same. The moment of each pointing was marked by the observer saying "stop," an assistant watching the chronometer entered the time to the nearest second in the observing book, and opposite to this entry were set the readings of the micrometers called out by the observer at the instrument. The observer also registered the time of each pointing on the small chronograph to serve as a check on the time entered by the assistant. A complete observation in distance and position-angle, consisting of the 8 pointings described above, occupied very nearly 10 minutes, and a complete set of observations of standards and satellites could be made in about two hours. At the end of the night's work the meteorological instruments were again read and the chronometer compared with the Transit Circle Clock.

To vary the instrumental conditions of observing, the telescope-tube was rotated through 180° in position-angle generally every other night. Excepting in the case of a few double and half distances obtained in 1902, all the pointings were made in the usual way, viz., the images of the two objects were placed close together near the centre of the wire square in the focus of the object-glass, and by moving the proper handles the imaginary line joining the two images was made parallel to a side of the square: it was the writer's custom always to make this line appear horizontal, not vertical as is perhaps more usually done, by means of the reversing prism. When very small distances, however, were observed, the square was drawn aside out of the field and twice or half the distance of the objects was measured by making the intervals between the four images in the field of view appear equal; at the same time all four images were placed in a line by moving the position-angle handles. These two simultaneous operations gave a pointing both in distance and position-angle: four such were made in each position of the semi-lenses, two with the images horizontal, two with them vertical.

Both these classes of observation are affected by known systematic errors, which can be rigorously allowed for. Let us first take the second class or the case of very small distances; the error here arises from the non-coincidence of the optical centres of the semi-lenses when placed at their minimum distance. For suppose that the optical centres are at A B in the figure instead of lying on the line of motion $a\,b$ of the semi-lenses, their minimum distance is A A′ + B′ B.

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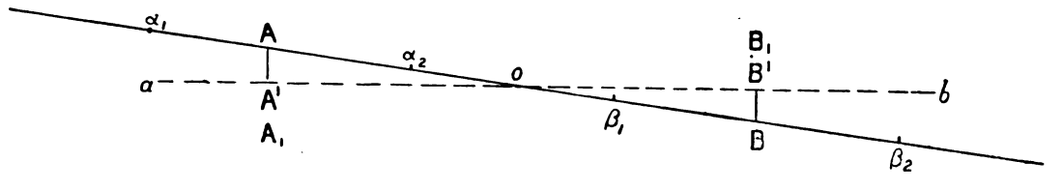
Now the measurement of a double distance is made with the four images α_1, α_2, β_1, β_2, in the field of view at the same time, and they are so placed that they lie in a straight line and that the distances $\alpha_1\alpha_2$, $\alpha_2\beta_1$, $\beta_1\beta_2$ are equal. The separation of the semi-lenses is now A′ B′ and not A B or $2s$ as it should be, *i.e.*, the measured distance is $2s \cos A O A'$ where $\sin A O A' = (A A' + B B')/2s$: the observed position-angle is $p + A O A'$. When the semi-lenses are crossed and their centres are at A_1 and B_1, the



measured distance is again $2s \cos A O A'$, but the position-angle is $p - A O A'$. The mean of the observed position-angles is therefore correct, and their difference gives $2 A O A'$: whence the necessary correction to the observed distance can be found. The same reasoning applies to the case of half distances, where the positions of α_2 and β_1 are interchanged. This correction has been applied to all observations of double or half distances.

Let us now consider the first class of observation, *i.e.*, the usual one: as this has already been dealt with by Sir David Gill in the *Cape Annals*, vi., pp. 232–236, very few words of explanation must here suffice. The error arises in this case from the fact that the observer does not superpose the two images in measuring a distance, but he places them side by side so that the imaginary line joining them is parallel to a side of the "square"; this brings the semi-lenses nearer together than they would be if the

images were placed exactly one over the other, and the measured distance is accordingly too small. The two pointings in each position of the semi-lenses were always made the first, as in (i.); the second, as in (ii.); and the position-circle was read for each pointing. The difference of the readings of the position-circle is twice the angular separation (2ψ) of the two images as seen from the centre of either semi-lens, and the correction to be applied to a measured distance is $s \sin \frac{1}{2}\psi$, or $\kappa^2/2s$ where κ is the distance of the artificial double star formed by the images a and b. It was the habit of the writer to bring the

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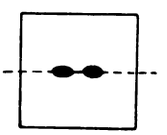
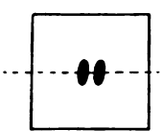
images a and b as close together as the sharpness of definition of the images would permit: this of course varied from night to night.

In the following table the values of κ are given for all observations of this class together with the remarks from the observer's note-book. The letter I refers to the sharpness of definition, the letter S to the steadiness of the images: and the scale adopted is such that 1 denotes very fine seeing and 4 very bad. No observations were made when the definition of the images or their steadiness was 4. The steadiness of the images is clearly an important point in heliometer observations; and since images of objects with an angular distance of 1000″ or more are brought into juxtaposition, the relative motion of the images is often considerable and rather troublesome. It will be noticed that the artificial double star formed by the images of the satellites had a dis-



Distance.



Position-Angle.

tance of about 2″·0 on good nights, and one of 3″·5 or even more on poor nights: the accuracy of setting depends therefore on the judgment of parallelism of two lines, one an imaginary line about 4″ in length passing through the centres of two discs with apparent diameters of about 2″, the other a side of a square nearly 50″ in length. In observations of distance the elliptical shape of the discs which is caused by the semi-circular figure of the object-glasses is of great assistance, but there is not this advantage in pointings for position-angle which the writer found rather the more difficult of the two. The accompanying diagrams, in which the appearance of the images is much exaggerated, will make this clear.

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TABLE VII.

VALUES OF κ, 1901.

Rotation No.	Date.	Distance.	κ	Remarks.	Rotation No.	Date.	Distance.	κ	Remarks.
	1901.	"	"			1901.	"	"	
3	June 24	270	2·08	I. 2–3 : S. 2–3.	137	July 18	704	2·76	I. 3 : S. 3–4.
5		819	2·35		143	22	124	3·54	I. 3–4 : S. 3–4.
7		554	2·61		145		365	3·14	
13	25	289	2·79		147		490	3·12	
15		652	2·21		149		68	3·12	
19	29	124	3·05	I. 2 : S. 2–3.	159	27	28	2·13	I. 2 : S. 2.
21		406	3·17		161		225	2·21	
23		281	2·88	I. 2 : S. 3–4.	163		682	2·24	
29	July 2	560	3·35		165		718	2·20	
31		52	2·84		167		186	2·37	I. 2 : S. 3.
33		361	2·99	I. 3–4 : S. 3–4.	169		906	2·61	
35		510	3·89		175	29	342	2·04	I. 2 : S. 2–3.
37		401	3·93		177		603	2·17	
39		913	3·49		179		261	2·63	
45	8	60	1·55	I. 2 : S. 2.	185	Aug. 10	101	1·85	I. 2 : S. 2.
47		361	2·34		187		279	2·03	
49		397	3·04		189		325	1·77	
51		435	2·56		191		185	2·00	I. 1 : S. 2.
53		478	2·42		193		416	2·59	
55		41	2·17		195		603	2·36	
61	10	62	2·16	I. 3 : S. 3–4.	201	13	312	1·76	I. 1: S. 2–3.
63		131	2·00		203		222	1·96	
65		545	2·55		205		477	2·21	
67		86	2·30		207		532	1·92	
69		414	2·01		209		782	2·25	I. 2 : S. 2–3.
71		506	2·72		211		250	2·36	
77	11	271	2·15	I. 3–4 : S. 3–4.	217	14	403	2·07	I. 1 : S. 2.
79		187	2·00		219		653	1·95	
81		566	2·44		221		253	2·45	
83		91	2·19		227	15	193	2·28	
85		738	2·66		231	19	195	2·31	I. 2–3 : S. 3–4.
87		831	2·90		233		256	2·34	
93	12	339	2·02		235		283	2·73	
95		135	2·61		237		83	2·77	
97		742	2·10	I. 1 : S. 1–2.	239	23	133	2·23	I. 2 : S. 3–4.
99		479	2·14		241		338	2·44	
101		395	2·61		243		203	2·78	
103		875	2·16		247	24	116	1·91	I. 1–2 : S. 2–3.
109	14	45	1·82	I. 2 : S. 2.	249		279	2·28	
111		91	1·49		251		212	2·28	
113		50	1·79		253		170	2·28	
119	16	290	2·03		255		105	2·27	
121		230	1·98		257		69	2·40	
123		322	2·22	I. 1–2 : S. 2–3.	263	26	215	2·10	I. 1 : S. 2.
125		102	1·87		265		196	2·22	
127		516	1·70		267		278	1·92	I. 2 : S. 3.
129		619	2·04		269		18	1·98	
133	18	191	2·57	I. 3 : S. 3–4.	271		80	2·83	I. 2 : S. 3–4.
135		273	2·39						

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TABLE VII.—*continued.*

Rotation No.	Date.	Distance.	κ	Remarks.
	1901.	"	"	
275	Aug. 27	313	2·90	I. 2–3 : S. 2–3.
277		161	2·90	
279		219	3·49	
281		479	4·08	
283		537	3·75	I. 2–3 : S. 3–4.
285		58	3·42	
291	28	407	1·93	I. 1–2 : S. 2–3.
293		599	2·16	
295		196	2·46	
301	29	117	2·30	I. 1–2 : S. 3–4.
303		470	3·09	
305		466	3·06	
307		117	2·69	
309		353	2·31	
317	31	142	2·92	I. 1 : S. 1–2.
321	Sept. 1	74	2·16	I. 1 : S. 1.
323		273	1·90	
325		256	2·10	
327		347	2·13	
329		178	2·99	
331		521	3·05	
337	3	281	2·41	I. 2 : S. 2–3.
339		143	3·03	
341		198	2·93	
343		431	3·00	
345		89	3·14	
347		349	3·44	
353	4	62	1·94	I. 1 : S. 1.
355		423	2·09	
357		152	3·77	
359		353	2·17	
361		575	2·13	
363		231	1·99	
369	9	212	3·21	I. 2 : S. 3–4.
371		311	2·85	
373	Sept. 9	521	3·64	I. 2 : S. 3–4.
379	10	62	2·98	I. 2 : S. 3–4.
381		232	2·47	
383		367	3·16	
385		129	3·23	
389	14	266	3·31	I. 2–3 : S. 3–4.
391		354	3·33	
393		410	4·03	
395		94	3·78	
397		671	4·68	
399		768	4·41	
405	16	238	1·95	I. 1 : S. 1–2.
407		330	2·03	
409		562	1·65	
411		31	2·16	
413		201	2·10	I. 1–2 : S. 1–2.
415		535	2·30	
421	17	161	2·14	
423		124	2·42	
425		344	2·54	
427		300	2·58	
429		519	3·43	I. 1–2 : S. 2–3.
431		214	2·89	
437	19	45	2·20	I. 1 : S. 1–2.
439		44	2·20	
441		91	2·69	
443		80	2·49	I. 2 : S. 2–3.
445		127	2·76	
447		53	2·90	
453	23	157	2·77	Very bad.
457	25	80	2·26	I. 2–3 : S. 3–4.
459		358	3·34	
463	27	44	3·22	I. 2 : S. 3–4.
465		221	3·19	I. 2 : S. 3.
467		257	3·08	

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TABLE VIII.

VALUES OF κ, 1902.

Rotation No.	Date.	Distance.	κ	Remarks.	Rotation No.	Date.	Distance.	κ	Remarks.
	1902.	"	"			1902.	"	"	
3	June 10	21	3·00		137	July 15	57	2·20	I. 1 : S. 1.
7		465	(assumed)	I. and S. 3–4.	143	16	82	2·94	
9		234			145		485	3·20	Haze : bad. Very faint.
13	11	493	2·92		147		662	3·34	
15		783	2·77	I. 2 : S. 3.	149		414	3·35	
17		292	2·64		157	18	244	3·26	
23	16	266	2·15		159		263	3·62	
25		176	2·33		163	19	66	3·00	Definition too bad to continue.
27		38	2·90		165		345	3·48	
29		91	2·62	I. 2 : S. 3.	167	20	162	2·78	I. 1–2 : S. 2.
31		315	2·60		169		122	3·06	
33		229	2·50		181	24	357	2·70	
39	19	191	3·94		183		180	2·75	
41		524	4·20	I. 3 : S. 4.	185		706	2·87	I. 1 : S. 2–3.
43		706	3·85		187		532	2·57	Long swings.
47	21	246	3·92	I. 2–3 : S. 4.	189		349	2·44	
51	24	39	3·03		191		874	2·48	
53		136	3·16		197	26	343	3·18	
55		27	3·16	I. 2 : S. 3–4.	199		323	2·94	I. 2 : S. 2–3 or 3.
57		107	2·99	Bad.	201		409	3·07	Not good.
59		55	3·22		203		382	3·20	
61		166	3·56		209	27	82	3·48	
63	30	191	2·47		211		284	3·14	I. 3 : S. 3.
65		537	2·60	I. and S. 1–2.	213		195	3·38	
67		335	2·29	Good.					
71		536	2·24	Haze.	219	Aug. 3	655	3·08	
73		313	2·21		221		654	2·82	
79	July 1	103	2·74		225		249	2·40	I. 1 : S. 2.
83	3	246	2·62		227		223	2·62	Very good.
85		118	3·00	I. 2 : S. 2.	229		394	2·89	
91	7	487	2·51		231		429	2·79	
93		470	2·46	I. and S. 2–3.	239	4	155	2·97	I. 3 : S. 3.
95		151	2·30		241		141	3·22	
97		127	2·13	I. and S. 1–2.	245	7	307	4·06	
99		624	2·44		247		269	3·75	
103	8	49	1·52		249		639	3·64	
105		134	1·96		251		579	3·63	I. 2 : S. 3–4.
107		544	1·98	Haze.	253		326	3·59	
109		95	2·03		255		906	3·49	
111		581	2·21		257		907	3·21	
113		685	2·28		263	16	709	5·54	
117	14	102	2·68		265		96	4·92	I. 3–4 : S. 3.
119		684	2·67	I. 2 : S. 3.	267		233	4·91	
121		773	3·10		269		329	5·61	
127	15	118	2·16		273	17	185	3·89	
129		43	2·20	I. 1 : S. 1.	275		778	5·00	
131		503	2·58		277		966	5·36	
133		457			279		968	5·48	
135		392	2·64	I. 2 : S. 3.	281		123	4·68	

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TABLE VIII.—*continued.*

Rotation No.	Date.	Distance.	κ	Remarks.
	1902.	″	″	
283	Aug. 18	892	4·39	
285		766	4·41	I. 3 : S. 3.
287		885	4·69	Very faint.
291		314	3·89	Satisfactory.
293		436	4·06	
295		444	4·24	
301	19	275	3·74	
303		227	2·88	I. 3 : S. 2.
305		264	3·38	
309	20	107	4·82	
311	21	547	4·98	
313		549	4·85	
317		557	4·79	
319		164	3·83	I. 2–3 : S. 2–3.
321		398	4·19	
323		79	3·57	
325		246	2·87	
327		315	4·01	
333	22	311	2·72	
335		503	3·24	
337		301	2·91	I. 2 : S. 2.
339		209	2·68	
341		181	3·27	
343	24	253	3·11	
345		200	3·28	I. 2–3 : S. 2–3.
347		514	3·15	
349		472	3·18	
353		264	2·84	I. 1 : S. 2–3.
359	25	771	2·54	
363		126	2·59	
365		·289	2·84	I. 1–2 : S. 2.
367		415	2·40	Very good night.
369		478	2·67	
371		350	2·56	
373		766	2·87	
377	27	96	2·36	I. 1 : S. 3–4.
379		81	2·63	
383		539	2·46	
387		522	2·36	I. 1 : S. 3.
389		624	2·40	
391		640	2·60	
395	28	232	2·62	
397		392	2·76	
399		122	2·60	I. 1–2 : S. 3.
401		110	2·96	
403		622	3·52	
405		518	2·61	I. 1–2 : S. 2–3.
411	30	208	2·99	Definition quite good.
413		297	3·10	I. and S. 3–4.
415		323	3·73	Too bad to continue.
417	Sept. 1	895	3·82	I. 2–3 : S. 3.

Rotation No.	Date.	Distance.	κ	Remarks.
	1902.	″	″	
421	Sept. 2	804	3·80	
423		758	3·55	
427		233	3·51	I. 2–3 : S. 3. 3–4.
429		193	3·28	
431		562	2·88	
439	3	401	3·58	
443		77	3·20	
445		90	3·78	I. 2–3 : S. 3–4. Very variable.
447		497	3·60	
449		472	3·54	
453	4	129	3·28	
455		662	2·89	I. 2 : S. 3.
457		537	3·17	
461	6	71	2·24	
463		134	1·91	
465		66	2·05	I. 1 : S. 1.
471		79	2·03	Very good.
473		65	2·18	
475		146	2·03	
479	7	162	2·45	
481		187	2·68	
483		313	2·54	I. 1–2 : S. 2–3.
487		148	2·56	
489		128	2·49	
495	8	326	3·26	
497		433	2·89	I. 1–2 : S. 2–3.
499		467	2·30	3–4.
501		136	2·31	Rather variable.
503		107	2·61	
511	10	113	3·21	
513		76	3·54	
515		661	3·64	I. 2–3 : S. 3–4.
517		776	3·55	
519		750	3·93	
527	12	232	2·83	
531	16	72	3·09	I. 2–3 : S. 3.
535	26	580	3·40	I. 3 : S. 3.
537		848	3·58	
539		212	2·92	I. 3 : S. 3–4.
541		278	2·84	
543		481	2·94	
545		370	2·76	I. 2–3 : S. 2–3.
547		848	3·45	
553	Oct. 4	527	3·36	
555		209	3·03	I. 2–3 : S. 2–3.
557		154	3·19	
559		320	3·08	I. 3–4 : S. 3–4.
561		365	3·42	Poor night.
569	5	255	4·39	I. 3–4. : S. 3–4. Too bad to continue.

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The mean values of κ corresponding to different states of the "seeing" were taken from the above tables; as κ is the distance of the artificial double star formed by the images of the satellites, it is to be expected that the better the seeing, the smaller will κ be. The following figures show this:—

	1901.	1902.
I 1 or 1 — 2 : S 2 — 3 at worst : κ =	2″·33	2″·44
I 2 or 2 — 3 : S 2 to 3 — 4 κ =	2·81	3·07
I 3 or 3 — 4 : S 3 — 4 κ =	2·77	3·70

It should be stated with regard to the small value found for the worst "seeing" in 1901, that the mean, viz., 2″·77, depends on only 26 observations, whilst all the other means are derived from twice as many observations.

In the following tables are given the observed distances of each pair of satellites. The sidereal time in the third column is the Cape sidereal time. The next three columns require no explanation. The refraction was treated in the same way as in the case of the standards, viz., a large scale chart was constructed giving the mean refraction to 0″·001 between the hour angles $4\frac{1}{2}$ east and west for a distance of 1000″: the mean refraction was quickly read off the chart for the hour angle of the observation and then multiplied by the meteorological factor to give the true refraction for a distance of 1000″. To find the refraction for the distance, s, of the satellites, this had then only to be multiplied by the fraction $s/1000$. Two small auxiliary tables were constructed to allow for a change in the declination of Jupiter and for a change in position-angle of a pair of satellites. The corrections from these two tables rarely reached 0″·01. This graphical method of computing the correction for refraction was found to be the quickest and by far the least liable to numerical errors.

The κ correction in the next column has already been dealt with.

The scale value correction is the difference between the observed and adopted distance of the standards on the date of observation reduced from the distance of the standards to that of the satellites. By the application of this correction all the observed distances are referred to the adopted distances of the standard stars.

The last column is the sum of the preceding six.

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TABLE IX.

Distances I.–II., 1901.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1901.	h m s	"	"	"	"	"	"	"
31	July 2	16 30 56·7	60·201	− 7·999	− 0·019	+ 0·020	+ 0·079	+ 0·013	52·295
45	8	15 42 32·0	60·201	− 0·321	·000	·025	·030	·012	59·947
63	10	15 47 34·9	140·347	− 9·420	− ·017	·055	·013	·028	131·006
77	11	17 13 16·2	180·374	+ 6·779	+ ·040	·057	·011	·021	187·282
93	12	15 39 1·1	340·842	− 1·283	− ·004	·148	·006	·074	339·783
111	14	15 22 18·2	100·324	− 8·781	+ ·014	·044	·012	·017	91·630
121	16	15 35 27·3	220·480	+ 9·129	+ ·011	·103	·009	·038	229·770
133	18	15 32 57·9	180·374	+ 10·405	+ ·041	·187	·017	·038	190·962
149	22	16 27 3·7	60·201	+ 7·799	+ ·027	·023	·072	·015	68·137
159	27	15 43 22·5	20·049	+ 7·671	− ·002	·012	·084	·005	27·819
185	Aug. 10	16 12 21·2	100·324	+ 0·541	·000	·035	·017	·023	100·940
201	13	20 22 56·8	300·743	+ 11·360	+ ·015	·108	·005	·050	312·281
231	19	20 20 30·7	200·497	− 5·598	− ·028	·067	·014	·039	194·991
247	24	20 31 5·9	120·356	− 4·694	− ·021	·043	·015	·025	115·724
263	26	20 36 23·8	220·480	− 5·000	− ·021	·082	·010	·037	215·588
275	27	20 20 58·4	300·743	+ 12·144	+ ·018	·109	·013	·065	313·092
307	29	21 7 9·7	120·356	− 3·852	− ·010	·050	·023	·027	116·602
321	Sept. 1	20 46 53·3	80·261	− 5·917	− ·012	·028	·032	·015	74·407
337	3	20 31 51·7	280·667	+ 0·514	− ·001	·102	·010	·054	281·346
353	4	20 38 41·2	60·201	+ 2·158	− ·002	·022	·030	·008	62·417
369	9	20 43 10·5	220·480	− 9·006	− ·041	·078	·024	·049	211·584
381	10	20 44 18·8	220·480	+ 11·680	− ·002	·088	·013	·071	232·330
389	14	20 31 58·8	260·622	+ 5·132	+ ·010	·093	·019	·047	265·923
413	16	21 49 56·8	200·497	+ 0·243	− ·001	·107	·011	·034	200·891
421	17	20 35 50·1	160·346	+ 0·526	− ·001	·060	·015	·032	160·978
439	19	20 43 21·4	40·134	+ 3·757	+ ·006	·016	·055	·009	43·977
453	23	20 45 35·0	160·346	− 3·764	− ·018	·057	·024	·033	156·678
457	25	21 12 31·9	80·261	− 0·489	+ ·002	·032	·032	·017	79·855
465	27	20 26 43·5	220·480	+ 0·205	0·000	+ 0·074	+ 0·023	+ 0·052	220·834

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TABLE IX.—*continued.*

DISTANCES I—III., 1901.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1901.	h m s							
33	July 2	16 40 42·0	360·846	+ 0·551	− 0·003	+ 0·126	+ 0·013	+ 0·092	361·625
47	8	15 55 34·8	360·846	+ 0·160	+ ·001	·149	·008	·073	361·237
61	10	15 38 4·3	60·201	+ 1·559	− ·006	·027	·039	·013	61·833
83	11	19 30 19·8	100·324	− 8·831	− ·050	·027	·026	·011	91·507
99	12	16 9 18·9	481·078	− 1·493	− ·004	·181	·005	·106	479·873
119	16	15 26 17·5	300·743	− 10·991	− ·005	·135	·007	·047	289·936
145	22	15 37 13·1	360·886	+ 4·892	+ ·009	·152	·016	·080	366·035
167	27	16 22 35·7	180·374	+ 6·244	+ ·002	·064	·015	·034	186·733
175	29	15 59 43·2	340·842	+ 1·014	− ·004	·129	·006	·060	342·047
187	Aug. 10	16 21 14·1	280·667	− 1·639	− ·001	·094	·007	·063	279·191
203	13	20 32 59·9	220·480	+ 1·331	+ ·001	·082	·009	·036	221·939
217	14	16 2 8·1	400·977	+ 1·810	− ·004	·148	·008	·079	403·018
233	19	20 51 20·8	260·627	− 4·681	− ·019	·100	·011	·051	256·089
249	24	20 41 38·2	280·678	− 1·984	− ·009	·104	·010	·058	278·857
265	26	20 51 5·9	200·407	− 4·634	− ·018	·081	·013	·033	195·882
277	27	20 29 23·4	160·346	+ 0·721	− ·002	·059	·026	·032	161·182
291	28	20 39 33·7	400·977	+ 5·842	+ ·018	·153	·004	·066	407·060
311	29	21 32 38·1	0·000	+ 7·306	+ ·019	·000	·347	·000	7·672
323	Sept. 1	20 56 49·9	260·622	+ 12·159	+ ·010	·108	·006	·055	272·960
339	3	20 41 51·1	140·347	+ 2·524	− ·001	·054	·033	·027	142·984
355	4	20 46 32·0	420·979	+ 1·722	− ·006	·164	·005	·060	422·924
385	10	21 12 21·7	140·315	− 11·104	− ·002	·057	·040	·040	129·346
391	14	20 44 57·9	360·846	− 6·831	− ·021	·132	·014	·064	354·204
411	16	21 34 44·9	20·049	+ 10·806	+ ·005	·011	·055	·000	30·926
423	17	20 47 43·3	120·285	+ 3·703	+ ·010	·048	·023	·026	124·095
443	19	21 11 25·9	80·261	+ 0·519	− ·001	·033	·039	·016	80·867
459	25	21 20 35·9	360·918	− 3·157	− 0·005	+ 0·157	+ 0·016	+ 0·077	358·006

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TABLE IX.—*continued.*

DISTANCES I.—IV., 1901.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1901.	h m s							
29	July 2	16. 20. 41·9	561″·276	− 1″·197	+ 0″·005	+ 0″·210	+ 0″·010	+ 0″·141	560″·445
49	8	16. 6. 51·8	400·977	− 4·339	− ·025	·154	·012	·080	396·859
65	10	15. 59. 43·2	541·215	+ 3·819	− ·001	·210	·006	·116	545·365
85	11	19. 42. 40·3	741·781	− 3·716	− ·014	·221	·005	·084	738·361
101	12	16. 18. 58·7	400·977	− 5·650	− ·027	·145	·008	·087	395·540
113	14	15. 32. 3·2	60·182	− 9·989	+ ·020	·023	·032	·010	50·278
123	16	15. 43. 42·4	320·789	+ 1·035	− ·003	·132	·008	·053	322·014
143	22	15. 26. 58·1	120·285	+ 4·100	+ ·007	·058	·056	·028	124·534
165	27	16. 11. 18·8	721·764	− 4·133	+ ·006	·257	·003	·132	718·029
177	29	16. 10. 44·6	601·372	+ 1·421	− ·008	·219	·004	·106	603·114
189	Aug. 10	16. 29. 5·2	320·789	+ 4·058	+ ·017	·106	·005	·074	325·049
205	13	20. 43. 52·2	481·078	− 4·654	− ·007	·180	·005	·076	476·678
219	14	16. 10. 40·6	661·560	− 8·212	− ·028	·229	·004	·128	653·681
235	19	21. 5. 4·1	280·667	+ 2·274	+ ·008	·119	·013	·056	283·137
251	24	20. 50. 17·2	220·480	− 8·370	− ·048	·079	·012	·044	212·197
267	26	21. 3. 54·8	280·678	− 2·195	− ·008	·120	·007	·047	278·649
279	27	20. 41. 25·4	220·480	− 1·542	+ ·004	·082	·028	·045	219·097
293	28	20. 50. 46·5	601·372	− 2·616	+ ·002	·237	·004	·097	599·096
305	29	20. 57. 28·4	461·054	+ 4·877	+ ·014	·192	·010	·106	466·253
325	Sept. 1	21. 6. 7·0	260·627	− 4·629	− ·006	·105	·007	·052	256·156
341	3	20. 51. 52·8	200·497	− 2·158	+ ·004	·080	·024	·037	198·484
357	4	20. 56. 19·9	140·347	+ 11·941	+ ·010	·062	·028	·022	152·410
371	9	20. 54. 21·6	300·743	+ 10·435	− ·040	·112	·016	·072	311·338
393	14	20. 55. 42·5	420·979	− 10·628	− ·018	·161	·019	·073	410·586
415	16	21. 59. 41·1	541·238	− 5·877	− ·008	·307	·005	·092	535·757
425	17	21. 2. 48·9	340·842	+ 2·867	+ ·013	·139	·010	·070	343·941
447	19	21. 33. 43·6	60·182	− 7·646	− ·024	·029	·080	·011	52·632
463	27	20. 14. 58·9	40·134	+ 3·576	0″·000	+ 0″·012	+ 0″·118	+ 0″·011	43·851

No. 65. Wrong scale division read, causing error of 20″, altered accordingly.

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TABLE IX.—*continued.*

DISTANCES II.—III., 1901.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1901.	h m s							
7	June 24	19 16 54·4	561″·276	− 7″·057	− 0″·027	+ 0″·157	+ 0″·006	+ 0″·093	554″·448
23	29	21 8 3·2	280·667	+ 0·291	− ·001	·106	·015	·068	281·146
37	July 2	17 0 8·0	400·977	+ 0·068	·000	·133	·019	·102	401·299
51	8	16 19 15·3	441·004	− 6·002	− ·040	·162	·008	·088	435·220
67	10	16 10 45·0	80·261	+ 5·500	+ ·010	·032	·031	·018	85·852
79	11	17 24 24·4	260·627	+ 10·602	+ ·050	·082	·008	·031	271·400
95	12	15 49 15·4	140·347	− 5·225	− ·026	·055	·025	·029	135·205
127	16	16 3 59·4	521·193	− 5·002	− ·005	·200	·003	·084	516·473
135	18	15 42 18·1	260·627	+ 11·988	+ ·045	·117	·010	·053	272·840
161	27	15 53 33·3	220·480	+ 4·479	− ·002	·086	·011	·041	225·095
191	Aug. 10	16 38 50·2	180·374	+ 5·070	+ ·021	·059	·012	·042	185·578
207	13	20 56 0·1	541·238	− 9·214	− ·023	·214	·003	·086	532·304
237	19	21 22 46·0	80·261	+ 2·466	+ ·017	·042	·046	·017	82·849
239	23	20 19 7·5	140·347	− 7·210	− ·034	·046	·019	·027	133·195
253	24	20 58 7·7	180·374	− 10·001	− ·048	·071	·015	·036	170·447
269	26	21 19 30·0	20·049	− 1·664	− ·011	·005	·111	·003	18·493
281	27	20 51 45·8	481·078	− 2·198	+ ·005	·186	·017	·099	479·187
301	29	20 37 36·9	120·356	− 3·022	− ·008	·046	·023	·027	117·422
317	31	20 48 57·3	140·347	+ 1·534	− ·002	·054	·030	·035	141·998
327	Sept. 1	21 14 45·9	340·842	+ 5·884	+ ·010	·152	·007	·071	346·966
343	3	21 2 10·0	421·016	+ 9·787	+ ·025	·179	·010	·081	431·098
359	4	21 7 59·8	340·842	+ 11·742	+ ·006	·150	·007	·051	352·798
383	10	20 56 18·9	360·846	+ 5·937	+ ·003	·149	·014	·113	367·062
395	14	21 17 34·6	100·324	− 6·293	− ·018	·040	·076	·017	94·148
405	16	20 52 47·8	240·558	− 2·205	+ ·007	·092	·008	·041	238·501
427	17	21 16 41·9	300·743	− 1·102	− ·001	·134	·011	·062	299·847
437	19	20 33 22·0	40·134	+ 4·925	+ 0·013	+ 0·016	+ 0·054	+ 0·009	45·151

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TABLE IX.—*continued.*

DISTANCES II.—IV., 1901.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1901.	h m s							
5	June 24	18 48 50·0	821·889	− 2·902	− 0·011	+ 0·230	+ 0·003	+ 0·138	819·347
15	25	15 35 15·2	661·560	− 10·039	− ·030	·303	·004	·098	651·896
21	29	20 57 21·7	400·977	+ 4·511	+ ·011	·150	·014	·098	405·761
35	July 2	16 49 9·5	501·131	+ 9·201	+ ·014	·171	·015	·128	510·660
53	8	16 30 17·1	481·078	− 3·633	− ·017	·169	·006	·097	477·700
69	10	16 23 30·5	420·979	− 7·006	− ·018	·149	·005	·089	414·198
81	11	19 16 55·6	561·276	+ 4·807	+ ·023	·163	·005	·065	566·339
97	12	15 58 28·9	741·781	+ 0·476	+ ·001	·299	·003	·164	742·724
109	14	15 11 37·1	40·134	+ 4·947	− ·010	·018	·034	·008	45·131
125	16	15 53 26·3	100·324	+ 1·587	− ·007	·041	·017	·016	101·978
137	18	15 51 20·5	701·694	+ 1·813	·000	·289	·005	·139	703·940
163	27	16 2 51·5	681·608	− 0·040	·000	·257	·004	·126	681·955
193	Aug. 10	16 48 36·5	420·979	− 5·403	+ ·022	·129	·006	·095	415·828
209	13	21 8 45·4	781·825	+ 0·534	− ·002	·335	·003	·126	782·821
227	15	20 30 42·1	180·419	+ 12·663	+ ·070	·065	·013	·042	193·272
241	23	20 32 47·8	340·842	− 3·105	− ·012	·126	·009	·069	337·929
255	24	21 7 37·9	100·324	+ 4·960	+ ·024	·044	·025	·022	105·399
271	26	21 35 20·2	80·261	− 0·404	− ·001	·046	·050	·013	79·965
283	27	21 0 45·8	541·215	− 4·566	− ·008	·219	·013	·111	536·984
309	29	21 16 14·0	340·842	+ 11·522	+ ·023	·160	·009	·081	352·637
329	Sept. 1	21 26 25·2	180·374	− 2·907	+ ·007	·080	·017	·036	177·607
345	3	21 13 42·5	100·324	− 11·342	− ·022	·036	·056	·017	89·069
363	4	21 29 45·5	220·480	+ 10·054	+ ·013	·113	·009	·033	230·702
373	9	21 4 50·3	521 170	+ 0·266	+ ·001	·214	·013	·122	521·786
379	10	20 34 16·1	60·201	+ 1·843	− ·008	·020	·073	·018	62·147
397	14	21 26 53·4	661·538	+ 9·419	+ ·022	·314	·016	·121	671·430
407	16	21 4 3·7	340·842	− 11·014	− ·002	·137	·006	·057	330·026
429	17	21 27 10·5	521·170	− 2·474	− ·002	·243	·011	·107	519·055
441	19	21 0 24·7	100·324	− 9·419	− ·024	·040	·040	·019	90·980
467	27	20 37 44·8	260·627	− 3·608	0·000	+ 0·089	+ 0·019	+ 0·060	257·187

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TABLE IX.—*continued.*

DISTANCES III.—IV., 1901.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1901.	h m s							
3	June 24	17 51 46·4	260″·627	+ 9″·593	+ 0″·033	+ 0″·079	+ 0″·007	+ 0″·046	270″·385
13	25	15 20 48·7	300·743	− 11·337	− ·001	·146	·013	·044	289·578
19	29	20 45 33·4	120·253	+ 3·283	+ ·007	·045	·038	·030	123·656
55	July 8	16 41 38·2	40·134	+ 1·102	+ ·001	·014	·056	·008	41·315
71	10	16 35 31·6	501·131	+ 5·075	+ ·008	·172	·007	·108	506·501
87	11	19 54 0·8	821·944	+ 9·558	+ ·050	·254	·005	·095	831·906
103	12	16 29 11·8	882·050	− 6·633	− ·040	·306	·003	·193	875·879
129	16	16 14 41·7	621·446	− 2·195	+ ·006	·226	·003	·101	619·587
147	22	15 47 13·4	501·131	− 1·307	− ·019	·198	·014	·108	490·125
169	27	16 33 6·9	902·112	+ 3·568	− ·007	·304	·004	·165	906·146
179	29	16 20 55·7	260·627	− 2·466	+ ·011	·092	·012	·046	258·322
195	Aug. 10	16 59 8·1	601·372	+ 1·920	+ ·002	·185	·005	·139	603·623
211	13	21 19 25·9	260·627	− 10·355	− ·019	·113	·011	·040	250·387
221	14	16 19 33·6	260·627	− 7·698	− ·029	·085	·012	·049	253·046
243	23	20 46 21·3	200·497	+ 2·409	+ ·010	·075	·020	·042	203·053
257	24	21 18 46·0	60·201	+ 9·049	+ ·048	·032	·042	·014	69·386
285	27	21 10 49·2	60·201	− 1·880	+ ·003	·028	·106	·012	58·470
295	28	20 59 36·6	200·497	− 4·339	− ·009	·084	·015	·032	196·280
303	29	20 47 9·0	461·054	+ 9·061	+ ·031	·183	·010	·106	470·445
331	Sept. 1	21 37 44·8	521·186	+ 0·296	− ·002	·254	·009	·105	521·848
347	3	21 25 55·7	340·842	+ 8·450	+ ·023	·167	·017	·066	349·565
361	4	21 19 2·2	581·336	− 6·435	− ·014	·263	·004	·082	575·236
399	14	21 38 46·5	761·840	+ 5·774	+ ·012	·384	·012	·139	768·161
409	16	21 17 23·0	561·276	+ 0·536	− ·003	·244	·006	·097	562·156
431	17	21 36 39·0	220·840	− 6·390	− ·031	·104	·020	·045	214·228
445	19	21 22 46·5	140·315	− 13·432	− 0″·028	+ 0″·257	+ 0″·030	+ 0″·025	126·967

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TABLE X.

DISTANCES I.-II., 1902.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1902.	h m s	"	"	"	'	"	"	"
3	June 10	17 30 29·1	20·049	+ 0·301	+ 0·001	+ 0·016	+ 0·215	+ 0·004	20·586
23	16	18 2 18·9	260·622	+ 5·032	+ ·016	·155	·009	+ ·062	265·896
51	24	17 58 12·6	40·134	− 1·260	− ·007	·023	·114	+ ·007	39·011
63	30	18 1 38·9	180·374	+ 10·440	+ ·060	·110	·016	+ ·039	191·039
83	July 3	17 38 36·7	240·558	+ 5·102	+ ·019	·168	·014	+ ·045	245·906
95	7	18 56 21·1	140·315	+ 10·856	+ ·028	·062	·018	+ ·035	151·314
103	8	23 42 48·4	60·182	− 10·480	− ·014	·018	·024	+ ·009	49·749
127	15	17 47 24·8	120·253	− 2·586	− ·006	·073	·020	+ ·014	117·768
143	16	17 50 40·3	80·261	+ 1·426	+ ·008	·048	·053	+ ·021	81·817
159	18	19 12 57·0	260·627	+ 2·185	+ ·008	·098	·025	+ ·066	263·009
163	19	17 48 2·1	60·182	+ 5·879	+ ·048	·040	·068	+ ·012	66·229
181	24	17 38 32·0	360·846	− 3·924	− ·029	·226	·010	+ ·076	357·205
193	26	17 38 10·3	40·134	− 7·263	− ·039	·021	·007	+ ·002	32·562
209	27	17 34 27·9	80·261	+ 0·953	+ ·006	·052	·076	+ ·018	81·366
223	Aug. 3	17 44 14·6	40·134	− 3·538	− ·043	·021	·015	+ ·004	36·593
237	4	23 45 30·7	20·049	− 2·614	− ·013	·008	·071	+ ·002	17·503
245	7	17 48 0·5	300·737	+ 5·578	+ ·038	·218	·027	+ ·012	306·610
265	16	17 56 40·3	100·253	− 4·947	− ·004	·050	·126	+ ·048	95·526
291	18	18 51 55·6	300·743	+ 12·452	+ ·072	·119	·024	+ ·177	313·587
309	20	17 38 23·5	100·324	+ 7·360	+ ·035	·061	·108	+ ·050	107·938
325	21	19 28 25·7	240·558	+ 5·335	+ ·004	·080	·017	+ ·107	246·101
331	22	17 36 23·4	20·049	− 11·986	− ·072	·005	·014	+ ·005	8·015
355	24	23 51 4·6	40·134	− 6·445	− ·019	·013	·061	+ ·015	33·759
371	25	18 42 52·4	340·842	+ 8·854	+ ·058	·136	·009	− ·030	349·869
377	27	17 37 11·7	100·324	− 3·753	− ·017	·055	·029	− ·006	96·632
399	28	17 45 26·6	120·253	+ 1·599	+ ·004	·064	·028	− ·003	121·945
429	Sept. 2	17 53 9·7	200·497	− 7·064	− ·018	·096	·028	− ·005	193·534
445	3	18 20 26·2	100·324	− 9·774	− ·022	·037	·079	− ·007	90·637
475	6	19 10 5·7	160·346	− 14·851	− ·037	·049	·014	− ·020	145·501
479	7	17 29 7·5	160·346	+ 1·749	+ ·003	·093	·018	− ·014	162·195
495	8	17 30 2·7	320·789	+ 5·356	+ ·027	·185	·016	− ·017	326·356
511	10	23 18 21·9	120·253	− 6·934	− ·035	·040	·046	− ·004	113·366
527	12	18 13 24·1	240·558	− 8·851	− ·063	·099	·017	+ ·000	231·760
539	26	23 36 9·4	200·497	+ 10·425	+ ·034	·080	·020	+ ·022	211·078
557	Oct. 4	23 31 33·2	160·346	− 6·014	− 0·015	+ 0·057	+ 0·033	+ 0·002	154·409

No. 3. κ not observed : assumed value 3″·0.
No. 193. Half distance.
No. 223. Double distance.
No. 237. Double distance.
No. 331. Half distance.
No. 355. Double distance.

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TABLE X.—*continued.*

DISTANCES I.—III., 1902.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1902.	h m s	"	"	"	"	"	"	"
5	June 10	17 44 26·3	220·480	+ 5·323	+ 0·015	+ 0·156	+ 0·020	+ 0·038	226·032
25	16	18 13 34·2	180·419	− 4·661	− ·015	·094	·016	+ ·041	175·894
47	21	18 10 19·9	240·558	+ 5·473	+ ·035	·135	·026	+ ·044	246·271
53	24	18 11 10·7	140·347	− 4·140	− ·023	·074	·037	+ ·023	136·318
79	July 1	18 51 45·9	100·324	+ 2·721	+ ·008	·042	·037	+ ·017	103·149
85	3	17 58 0·5	120·253	− 2·175	− ·008	·067	·038	+ ·021	118·196
87	7	17 36 22·2	20·073	− 2·030	− ·005	·013	·046	+ ·004	18·101
105	8	23 54 54·5	140·315	− 6·044	− ·008	·050	·014	+ ·023	134·350
117	14	17 18 0·7	100·253	+ 1·885	+ ·013	·081	·035	+ ·023	102·290
129	15	18 2 17·8	40·134	+ 2·571	+ ·006	·024	·056	+ ·005	40·796
145	16	18 3 21·6	481·078	+ 3·563	+ ·020	·260	·011	+ ·130	485·062
165	19	18 2 38·2	340·810	+ 3·102	+ ·025	·187	·018	+ ·063	344·205
183	24	17 51 44·6	180·419	− 0·150	− ·001	·103	·021	+ ·038	180·430
197	26	18 6 57·7	340·810	+ 1·973	+ ·011	·176	·015	+ ·026	343·011
211	27	17 46 54·0	280·267	+ 3·631	+ ·020	·167	·017	+ ·027	284·562
225	Aug. 3	17 56 3·2	260·627	− 11·502	− ·138	·132	·012	+ ·042	249·173
241	4	0 26 54·4	140·347	+ 0·394	+ ·002	·062	·037	+ ·011	140·853
247	7	17 28 4·0	260·622	+ 8·049	+ ·054	·175	·026	+ ·010	268·936
267	16	18 10 31·7	240·558	− 7·648	− ·005	·110	·052	+ ·120	233·187
273	17	17 33 33·5	180·419	+ 3·721	+ ·013	·111	·041	+ ·102	184·407
293	18	19 2 54·9	441·059	− 5·308	− ·031	·157	·019	+ ·245	436·141
299	19	17 43 8·5	20·073	− 4·574	− ·030	·009	·000	+ ·003	15·471
327	21	19 39 9·3	320·789	− 6·360	− ·005	·100	·026	+ ·136	314·686
333	22	17 46 30·9	300·743	+ 10·470	+ ·063	·164	·012	+ ·128	311·580
345	24	22 27 59·3	200·497	− 0·649	− ·002	·060	·027	+ ·088	200·021
369	25	18 32 44·6	481·078	− 3·505	− ·023	·194	·007	− ·042	477·709
379	27	17 47 45·1	80·261	+ 1·067	+ ·006	·041	·043	− ·005	81·413
397	28	17 34 23·9	380·940	+ 10·089	+ ·023	·225	·010	− ·009	391·478
411	30	23 38 15·6	200·497	+ 6·816	+ ·019	·076	·021	− ·012	207·417
425	Sept. 2	17 33 43·1	20·073	+ 1·122	+ ·003	·012	·204	− ·001	21·413
437	3	17 18 49·1	20·073	− 6·278	− ·014	·009	·359	− ·001	14·148
469	6	18 32 8·7	0·000	+ 6·991	+ ·018	·003	·318	− ·001	7·329
487	7	18 22 45·1	160·346	− 12·964	− ·020	·061	·023	− ·013	147·433
499	8	17 49 23·7	481·078	− 13·933	− ·070	·233	·006	− ·024	467·290
513	10	23 29 20·6	80·261	− 4·033	− ·020	·028	·084	− ·003	76·319
531	16	22 45 24·5	80·261	− 8·179	− ·016	·023	·066	− ·003	72·152
541	26	23 48 30·4	280·667	− 2·524	− ·008	·111	·015	+ ·030	278·291
555	Oct. 4	23 22 46·1	200·497	+ 8·392	+ 0·021	+ 0·075	+ 0·022	+ 0·002	209·009

No. 5. κ not observed : assumed value 3″·0.
Nos. 87, 299, 437, and 469. Double distances.
No. 425. Half distance.

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TABLE X.—*continued.*

DISTANCES I.—IV., 1902.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1902.	h m s							
7	June 10	18. 0. 2·2	461·054	+ 4·005	+ 0·011	+ 0·280	+ 0·010	+ 0·079	465·439
55	24	18. 23. 42·8	20·049	+ 6·285	+ ·034	·014	·370	+ ·004	26·756
65	30	18. 14. 50·8	541·215	− 3·453	− ·020	·280	·006	+ ·109	538·137
71		19. 15. 34·2	541·215	− 4·651	− ·027	·201	·005	+ ·107	536·850
93	July 7	18. 40. 43·0	561·054	+ 8·249	+ ·021	·209	·007	+ ·108	469·648
107	8	0. 11. 16·5	541·238	+ 1·544	+ ·002	·211	·004	+ ·095	543·094
119	14	17. 36. 26·4	681·608	+ 2·296	+ ·015	·461	·005	+ ·152	684·537
131	15	18. 15. 34·8	501·131	+ 1·950	+ ·005	·252	·007	+ ·060	503·405
147	16	18. 14. 36·8	661·560	+ 1·589	+ ·009	·328	·008	+ ·177	663·671
157	18	19. 1. 22·0	240·558	+ 4·170	+ ·015	·095	·021	+ ·060	244·919
185	24	18. 4. 57·9	701·687	+ 3·465	+ ·025	·370	·006	+ ·150	705·703
199	26	18. 17. 1·4	320·789	+ 1·398	+ ·007	·156	·013	+ ·024	322·387
229	Aug. 3	18. 16. 57·9	400·977	− 6·189	− ·074	·183	·011	+ ·066	394·974
249	7	17. 40. 1·1	641·482	− 3·513	− ·024	·378	·010	+ ·024	638·357
275	17	17. 49. 15·4	781·858	− 4·228	− ·015	·419	·016	+ ·429	778·479
295	18	19. 15. 4·1	441·059	+ 3·237	+ ·019	·153	·020	+ ·481	444·969
323	21	19. 18. 9·6	80·261	− 2·256	− ·002	·027	·081	+ ·035	78·146
341	22	19. 6. 18·7	180·374	− 1·318	− ·008	·063	·030	+ ·074	179·215
349	24	22. 57. 7·7	461·054	+ 9·618	+ ·029	·152	·010	+ ·205	471·068
359	25	17. 35. 40·6	781·858	− 11·993	− ·078	·439	·004	− ·068	770·162
373		18. 53. 29·7	761·840	+ 3·854	+ ·025	·283	·005	− ·067	765·940
383	27	18. 19. 46·3	551·215	− 3·288	− ·015	·232	·006	− ·034	538·116
387		19. 0. 26·6	521·186	− 0·140	− ·001	·187	·005	− ·032	521·205
395	28	17. 22. 49·8	220·480	+ 10·678	+ ·025	·144	·015	− ·005	231·337
423	Sept. 2	17. 22. 24·5	761·840	− 3·762	− ·009	·468	·008	− ·021	758·524
447	3	18. 34. 16·8	501·131	− 3·580	− ·008	·195	·013	− ·038	497·713
473	6	18. 58. 22·7	80·261	− 14·507	− ·037	·023	·037	− ·009	65·768
485	7	18. 11. 17·7	0·000	+ 9·646	+ ·015	·004	·082	− ·001	9·746
497	8	17. 39. 43·2	441·004	− 9·179	− ·046	·231	·010	− ·022	431·998
515	10	23. 45. 15·9	641·482	+ 17·951	+ ·090	·257	·010	− ·025	659·765
535	26	23. 9. 21·4	561·276	+ 16·968	+ ·055	·198	·010	+ ·062	578·569
553	Oct. 4	23. 11. 33·9	521·186	+ 5·943	+ 0·014	+ 0·182	+ 0·011	+ 0·006	526·942

No. 7. κ not observed : assumed value 3″·0.
No. 485. Half distance.
No. 553. Scales read wrong, causing error of 20″ ; altered accordingly.

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TABLE X.—*continued.*

DISTANCES II.—III., 1902.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1902.	h m s	"	"	"	"	"	"	"
13	June 11	17 50 8·7	501·131	− 8·383	− 0·027	+ 0·320	+ 0·009	+ 0·110	493·160
29	16	18 39 54·5	100·253	− 9·357	− ·030	·041	·038	+ ·021	90·966
39	19	17 37 29·5	180·419	+ 10·796	+ ·054	·137	·041	+ ·016	191·463
57	24	18 38 27·6	100·253	+ 5·979	+ ·033	·049	·043	+ ·018	106·375
97	July 7	19 10 1·0	120·253	+ 7·152	+ ·019	·049	·018	+ ·030	127·521
109	8	0 25 37·1	100·253	− 5·668	− ·007	·039	·022	+ ·016	94·655
137	15	19 1 51·4	60·182	− 2·721	− ·007	·022	·044	+ ·007	57·527
149	16	18 29 16·0	420·979	− 7·443	− ·041	·187	·014	+ ·110	413·806
167	20	18 10 49·3	160·346	+ 1·985	+ ·010	·084	·024	+ ·018	162·467
187	24	18 20 56·9	541·238	− 9·111	− ·066	·252	·006	+ ·113	532·432
201	26	18 27 23·2	420·979	− 11·767	− ·061	·187	·012	+ ·031	409·381
213	27	17 57 31·6	200·497	− 5·423	− ·030	·107	·029	+ ·041	195·221
227	Aug. 3	18 5 38·8	220·480	+ 2·867	+ ·034	·112	·015	+ ·038	223·546
239	4	0 17 6·4	160·346	− 4·444	− ·022	·065	·028	+ ·012	555·985
251	7	17 55 9·1	581·336	− 2·486	− ·017	·308	·011	+ ·022	579·174
269	16	18 21 56·3	340·842	− 11·866	− ·009	·146	·048	+ ·168	329·329
281	18	17 36 42·4	120·253	+ 2·571	+ ·015	·072	·089	+ ·067	123·067
303	19	18 12 5·3	220·480	+ 6·400	+ ·044	·105	·018	+ ·106	227·153
311	21	17 27 0·4	541·215	+ 4·822	+ ·004	·336	·023	+ ·237	546·637
313		17 41 8·9	541·215	+ 7·463	+ ·006	·302	·021	+ ·239	549·246
317		18 28 22·4	561·276	− 5·257	− ·004	·231	·021	+ ·241	556·508
337	22	18 35 30·3	300·743	− 0·318	− ·002	·119	·014	+ ·125	300·681
343	24	22 15 48·8	260·627	− 7·603	− ·023	·075	·019	+ ·110	253·205
363	25	17 58 48·3	120·253	+ 5·453	+ ·036	·061	·027	− ·011	125·819
381	27	18 0 52·9	20·073	+ 3·887	+ ·018	·011	·127	− ·001	24·115
405	28	18 19 17·9	521·186	− 3·498	− ·008	·223	·007	− ·012	517·898
413	30	23 51 2·7	300·737	− 3·909	− ·011	·115	·016	− ·018	296·930
427	Sept. 2	17 44 9·5	220·480	+ 12·455	+ ·031	·122	·026	− ·006	233·108
443	3	18 8 22·3	80·261	− 3·095	− ·007	·034	·066	− ·006	77·253
457	4	23 27 9·3	541·238	− 4·756	− ·006	·190	·009	− ·025	536·650
463	6	17 46 19·6	120·285	+ 13·375	+ ·034	·068	·014	− ·018	133·758
483	7	17 58 9·4	320·789	− 7·819	− ·012	·148	·010	− ·028	113·088
501	8	17 59 42·4	120·285	+ 15·367	+ ·077	·064	·020	− ·001	135·806
521	10	0 20 56·9	40·134	+ 0·210	+ ·001	·019	·018	− ·001	40·381
543	26	0 1 48·4	481·078	− 0·481	− ·002	·203	·009	+ ·051	480·858
565	Oct. 4	0 18 35·5	0·000	+ 16·614	+ ·042	·008	·005	·000	16·669
569	5	23 8 14·2	240·558	+ 14·282	+ 0·036	+ 0·086	+ 0·038	− 0·019	254·981

Nos. 381, 521. Double distances.
No. 565. Half Distance.

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TABLE X.—*continued.*

DISTANCES II.—IV., 1902.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1902.	h m s	"	"	"	"	"	"	"
15	June 11	18 7 47·7	781·858	+ 0·411	+ 0·001	+ 0·443	+ 0·005	+ 0·173	782·891
31	16	18 51 49·3	320·789	− 7·438	− ·024	·134	·011	+ ·078	313·550
41	19	17 51 59·2	521·186	+ 2·150	+ ·011	·332	·017	+ ·043	523·739
59	24	18 53 16·1	60·182	− 5·894	− ·033	·023	·094	+ ·009	54·381
67	30	18 32 30·4	340·842	− 5·473	− ·032	·157	·008	+ ·067	335·569
73		19 29 52·8	320·789	− 6·535	− ·038	·111	·008	+ ·063	314·398
77	July 1	18 34 26·9	20·049	− 8·926	− ·027	·005	·051	+ ·001	11·153
99	7	19 27 24·4	621·432	+ 2·316	+ ·006	·223	·005	+ ·143	624·125
111	8	0 51 0·1	581·336	− 0·777	− ·001	·277	·004	+ ·099	580·938
135	15	18 45 16·4	380·940	+ 10·811	+ ·025	·166	·009	+ ·047	391·998
151	18	18 0 5·5	20·049	− 11·256	− ·043	·004	·111	+ ·002	8·867
155		18 31 52·5	20·049	− 4·872	− ·019	·007	·023	+ ·003	15·191
155a		18 51 31·2	20·049	− 5·588	− ·021	·010		+ ·007	14·457
169	20	18 22 25·6	120·253	+ 0·772	+ ·004	·058	·039	+ ·014	121·140
189	24	18 35 48·0	340·810	+ 7·463	+ ·054	·152	·009	+ ·074	348·562
203	26	18 38 22·4	380·900	+ 0·744	+ ·004	·163	·014	+ ·029	381·854
231	Aug. 3	18 38 6·9	420·979	+ 8·430	+ ·100	·178	·009	+ ·072	429·768
253	7	18 9 49·0	320·789	+ 4·967	+ ·034	·158	·020	+ ·012	325·980
285	18	18 2 14·1	761·840	+ 3·438	+ ·019	·378	·013	+ ·433	766·121
301	19	17 59 41·7	280·667	− 5·648	− ·038	·136	·025	+ ·127	275·269
305		18 29 29·5	260·622	+ 4·095	+ ·027	·111	·022	+ ·123	265·000
319	21	18 54 5·0	160·346	+ 4·035	+ ·003	·060	·045	+ ·069	164·558
339	22	18 52 56·0	200·497	+ 7·265	+ ·043	·077	·017	+ ·087	207·986
347	24	22 40 49·4	501·124	+ 12·472	+ ·037	·158	·010	+ ·225	514·026
367	25	18 21 27·2	420·979	− 6·011	− ·039	·177	·007	− ·036	515·077
389	27	19 13 56·2	321·446	+ 2·253	+ ·010	·215	·005	− ·039	623·890
401	28	17 56 44·9	100·253	+ 9·387	+ ·021	·054	·040	− ·003	109·752
409	30	23 22 45·1	40·134	− 6·708	− ·018	·012	·155	− ·002	33·573
431	Sept. 2	18 2 24·8	561·276	+ 0·416	+ ·001	·262	·007	− ·014	561·948
439	3	17 32 27·0	400·977	− 0·138	·000	·227	·016	− ·031	401·051
455	4	23 12 12·9	661·538	+ 0·261	·000	·222	·006	− ·030	661·997
465	6	17 58 0·8	80·261	− 15·752	− ·039	·031	·032	− ·009	64·524
471		18 45 0·2	80·261	− 2·160	− ·005	·029	·026	− ·011	78·140
481	7	17 44 29·4	200·452	− 14·765	− ·023	·046	·019	− ·017	185·762
503	8	18 7 36·9	120·324	− 14·186	− ·071	·047	·032	− ·006	106·140
517	10	23 57 14·8	761·840	+ 13·397	+ ·067	·320	·008	− ·029	775·603
545	26	0 12 48·4	360·846	+ 8·124	+ ·026	·168	·010	+ ·039	369·213
561	Oct. 4	23 53 56·3	360·846	+ 4·434	+ 0·011	+ 0·148	+ 0·016	+ 0·004	365·459

No. 77. Half distance. Nos. 151, 155, 409. Double distances.

No. 155*a*. This distance was observed by placing the four images in a diamond form a b b^1 a^1, so that $a\,b^1$ was perpendicular to $a^1\,b$ and the opposite sides of the figure were parallel.

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TABLE X.—*continued.*

DISTANCES III.—IV., 1902.

Rotation No.	Date.	Sidereal Time.	Scales Equivalent.	Micrometer.	Run.	Refraction.	κ	Scale Value.	Observed Distance.
	1902.	h m s	"	"	"	"	"	"	"
9	June 10	18 15 51·3	240·558	− 6·635	− 0·018	+ 0·125	+ 0·019	+ 0·040	134·089
17	11	18 26 40·4	300·737	− 8·826	− ·028	·145	·012	+ ·065	292·105
33	16	19 4 39·4	220·480	+ 6·781	+ ·022	·091	·014	+ ·054	227·442
43	19	18 7 33·5	701·694	+ 3·663	+ ·018	·400	·011	+ ·058	705·844
61	24	19 5 55·8	160·346	+ 4·521	+ ·025	·066	·038	+ ·028	165·024
75	July 1	18 6 13·4	20·073	+ 10·648	+ ·027	·017	·019	+ ·005	30·789
91	7	18 28 51·6	481·078	+ 4·872	+ ·013	·232	·007	+ ·112	486·314
113	8	1 2 57·0	681·608	+ 2·110	+ ·003	·353	·004	+ ·118	684·196
121	14	17 52 33·4	781·858	− 7·508	− ·047	·458	·006	+ ·172	774·939
133	15	17 30 8·8	461·054	− 4·015	− ·009	·210	·008	+ ·055	457·303
173	20	18 53 0·1	40·134	− 6·104	− ·029	·014		+ ·004	34·019
175		19 4 43·7	20·073	+ 11·682	+ ·055	·012		+ ·004	31·881
177		19 23 46·7	60·182	− 3·849	− ·018	·020	·007	+ ·007	56·349
191	24	18 58 10·2	861·980	+ 11·455	+ ·083	·342	·004	+ ·185	874·049
195	26	17 52 20·5	20·073	+ 3·884	+ ·020	·014	·019	+ ·002	24·012
205		18 49 5·1	60·182	+ 1·212	+ ·006	·025	·009	+ ·005	61·439
219	Aug. 3	17 22 7·1	661·560	− 6·148	− ·074	·454	·007	+ ·110	655·909
221		17 33 42·2	661·560	− 7·463	− ·090	·412	·006	+ ·110	654·535
255	7	18 24 50·2	902·076	+ 3·042	+ ·020	·402	·007	+ ·034	905·581
257		18 38 56·8	902·076	+ 3·227	+ ·022	·375	·006	+ ·034	905·740
263	16	17 42 42·2	701·687	+ 6·530	+ ·005	·404	·022	+ ·363	709·011
277	17	18 5 49·6	962·220	+ 2·434	+ ·009	·468	·015	+ ·533	965·679
279		18 36 36·8	962·220	+ 4·681	+ ·016	·396	·015	+ ·533	967·861
283	18	17 48 50·5	902·076	− 10·520	− ·061	·477	·011	+ ·503	892·486
287		18 24 40·7	882·050	+ 3·426	+ ·020	·381	·012	+ ·498	886·387
321	21	19 4 43·7	400·977	− 4·213	− ·003	·140	·022	+ ·173	397·096
335	22	18 0 8·4	501·131	+ 0·279	+ ·002	·243	·010	+ ·208	501·873
353	24	23 37 55·4	260·627	+ 3·042	+ ·009	·079	·015	− ·035	263·755
365	25	18 9 39·4	280·667	+ 7·411	+ ·048	·132	·014	− ·025	288·247
391	27	19 29 4·8	641·482	− 1·998	− ·009	·209	·005	− ·040	639·649
403	28	18 7 1·4	621·432	+ 0·045	·000	·288	·010	− ·014	621·761
415	30	0 6 41·2	320·789	+ 2·266	+ ·007	·135	·021	− ·019	323·199
417	Sept. 1	17 4 6·5	902·076	− 7·758	− ·012	·647	·008	− ·018	894·943
421	2	17 12 40·3	801·875	+ 2·168	+ ·006	·539	·009	− ·022	804·575
449	3	18 42 54·0	461·054	+ 11·285	+ ·025	·178	·013	− ·036	472·519
453	4	22 57 50·8	140·315	− 10·257	− ·013	·041	·042	− ·006	130·222
461	6	17 34 37·6	80·261	− 8·305	− ·021	·038	·036	− ·010	71·999
489	7	18 35 15·8	120·285	+ 8·310	+ ·012	·050	·024	− ·012	128·669
505	8	18 17 52·0	0·000	+ 14·008	+ ·070	·006	·052	− ·001	14·135
519	10	0 8 11·8	761·840	− 12·869	− ·065	·328	·010	− ·028	749·216
537	26	23 23 17·4	842·015	+ 5·162	+ ·017	·305	·008	+ ·091	847·598
547		0 25 29·4	842·015	+ 4·945	+ ·016	·418	·007	+ ·091	847·492
559	Oct. 4	23 41 45·3	320·789	− 0·765	− 0·002	+ 0·123	+ 0·015	+ 0·004	320·164

Nos. 75, 177, 195, 205. Double distances. No. 505. Half distance.
Nos. 173, 175. Images placed in diamond form (v. preceding page.)

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The tables below contain the observations of position-angle. The time given in the third column is the Cape Sidereal time. The instrumental position-angle is the mean of four pointings made in the manner explained above. When the distance exceeded about 500″, the micrometers for reading the position circle were read and the position-angle at each pointing was accordingly measured to single seconds. But when the distance fell short of 500″, the micrometers were not used; the position-angle was read off in both microscopes to the nearest minute.

As in the case of the distances, the mean refraction was taken from a large chart corrected when necessary for change in declination and position-angle by means of two small auxiliary tables and multiplied by the appropriate meteorological factor to give the numbers in the fifth column.

The column headed adjustments contains that part of the correction for instrumental adjustment which varies with the hour-angle, that is, it contains the correction—

$5'' \cos \tau - 30'' \sin \tau$	East hour-angles
$19'' \cos \tau + 30'' \sin \tau$	West ,, ,,

The next column contains the part which does not vary with the hour-angle; see p. 6. This correction is always negative.

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TABLE XI.

Position-Angles I.–II., 1901.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1901.	h m s	° ′ ″	″	″	′ ″	° ′ ″
32	July 2	16 31 14·6	73 15 34	− 16	− 11	−33 34	72 41 33
46	8	15 42 56·0	95 33 4	− 33	− 16	−33 26	94 58 49
64	10	15 48 15·5	85 50 19	− 27	− 16	−33 30	85 16 6
78	11	17 13 24·4	85 0 34	− 10	− 5	−32 50	84 27 29
94	12	15 39 1·2	86 52 0	− 30	− 10	−33 20	86 18 0
112	14	15 22 18·7	86 0 53	− 34	− 18	−33 25	85 26 36
122	16	15 35 17·7	84 23 49	− 30	− 17	−33 36	83 49 26
134	18	15 33 17·2	86 5 15	− 31	− 17	−33 12	85 31 15
150	22	16 26 46·8	79 19 53	− 16	− 11	−33 35	78 45 51
152	23	15 40 56·7	85 36 56	− 27	− 16	−33 13	85 3 0
160	27	15 43 23·6	108 26 17	− 30	− 15	−33 34	107 51 58
186	Aug. 10	16 12 28·5	81 55 0	− 17	− 12	−33 39	81 20 52
202	13	20 22 41·0	87 3 45	+ 21	+ 33	−35 18	86 29 21
232	19	20 20 8·9	86 17 38	+ 21	+ 32	−35 18	85 43 13
248	24	20 31 11·9	83 22 11	+ 24	+ 33	−35 4	82 48 4
264	26	20 35 58·1	86 52 52	+ 26	+ 33	−34 55	86 18 56
276	27	20 20 59·6	88 18 30	+ 21	+ 33	−34 53	87 44 31
308	29	21 7 22·5	88 38 15	+ 34	+ 34	−35 8	88 4 15
322	Sept. 1	20 46 54·8	88 29 0	+ 27	+ 33	−35 10	87 54 50
338	3	20 31 38·1	88 47 4	+ 23	+ 33	−35 1	88 12 59
354	4	20 38 36·9	94 10 0	+ 25	+ 33	−35 0	93 35 58
370	9	20 43 21·0	87 42 0	+ 27	+ 33	−34 56	87 8 4
382	10	20 44 29·3	89 26 8	+ 27	+ 33	−35 8	88 52 0
390	14	20 32 17·8	86 58 30	+ 23	+ 33	−34 58	86 24 28
414	16	21 50 5·8	87 49 45	+ 55	+ 36	−35 4	87 16 12
422	17	20 36 43·7	90 30 26	+ 23	+ 33	−34 56	89 56 26
440	19	20 43 25·6	89 19 15	+ 26	+ 33	−35 4	88 45 10
454	23	20 45 57·1	88 11 30	+ 25	+ 33	−34 52	87 37 36
458	25	21 12 33·3	82 15 30	+ 36	+ 34	−34 54	81 41 46
466	27	20 26 51·5	86 28 45	+ 20	+ 33	−35 3	85 54 35

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TABLE XI.—*continued.*

POSITION-ANGLES I.—III., 1901.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1901.	h m s	° ′ ″	″	″	′ ″	° ′ ″
34	July 2	16 40 41·4	85 33 26	− 16	− 10	−33 34	84 59 26
48	8	15 55 25·2	86 31 53	− 27	− 15	−33 26	85 57 45
62	10	15 38 5·2	71 8 56	− 22	− 17	−33 30	70 34 47
84	11	19 30 42·8	91 43 15	+ 9	+ 26	−34 57	91 8 53
100	12	16 9 9·5	86 53 53	− 21	− 13	−33 20	86 19 59
120	16	15 26 32·6	87 3 26	− 34	− 17	−33 36	86 28 59
146	22	15 37 5·3	88 9 34	− 29	− 16	−33 35	87 35 14
153	23	16 10 8·9	87 39 56	− 22	− 13	−33 13	87 6 8
168	27	16 22 18·9	84 48 30	− 17	− 11	−33 34	84 14 28
176	29	15 50 8·4	88 51 19	− 23	− 14	−33 32	88 17 10
188	Aug. 10	16 21 20·7	86 33 26	− 17	− 10	−33 39	85 59 20
204	13	20 33 12·6	89 2 49	+ 23	+ 33	−35 18	88 28 27
218	14	16 2 39·6	86 52 45	− 21	− 13	−33 33	86 18 38
234	19	20 51 8·0	90 28 26	+ 29	+ 33	−35 18	89 54 10
250	24	20 41 27·3	87 8 49	+ 27	+ 33	−35 4	86 34 45
266	26	20 50 41·2	91 45 26	+ 30	+ 34	−34 55	91 11 35
278	27	20 29 29·8	89 45 49	+ 22	+ 33	−34 53	89 11 51
292	28	20 39 41·7	87 15 11	+ 27	+ 33	−35 2	86 41 9
312	29	21 33 20·8	149 37 53	− 2	+ 35	−35 8	149 3 18
324	Sept. 1	20 57 7·7	87 16 7	+ 30	+ 34	−35 10	86 42 1
340	3	20 41 40·2	88 55 26	+ 25	+ 33	−35 1	88 21 23
356	4	20 46 54·8	87 40 4	+ 27	+ 33	−35 0	87 6 4
386	10	20 11 54·2	89 57 52	+ 35	+ 34	−35 8	89 23 53
392	14	20 44 48·4	88 7 15	+ 28	+ 33	−34 58	87 33 18
412	16	21 33 42·3	61 48 45	+ 51	+ 35	−35 4	61 15 7
424	17	20 47 43·1	89 54 19	+ 27	+ 33	−34 56	89 20 23
444	19	21 11 4·1	82 8 30	+ 37	+ 34	−35 4	81 34 37
460	25	21 20 34·2	88 22 26	+ 37	+ 34	−35 54	87 48 43

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TABLE XI.—*continued.*

POSITION-ANGLES I.—IV., 1901.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1901.	h m s	° ′ ″	″	″	′ ″	° ′ ″
30	July 2	16 20 41·7	86 14 44	− 21	− 13	−33 34	85 40 36
50	8	16 6 48·7	88 24 0	− 23	− 13	−33 26	87 49 58
66	10	16 0 21·8	86 49 19	− 24	− 15	−33 30	86 15 11
86	11	19 42 21·6	86 31 46	+ 11	+ 28	−34 57	85 57 28
102	12	16 19 7·2	85 19 23	− 20	− 12	−33 20	84 45 31
114	14	15 32 1·4	67 39 56	− 21	− 17	−33 25	67 5 53
124	16	15 43 55·4	89 34 11	− 30	− 16	−33 36	88 59 49
144	22	15 27 51·0	80 30 19	− 29	− 17	−33 35	79 55 58
166	27	16 11 51·8	87 55 45	− 21	− 12	−33 34	87 21 38
178	29	16 10 9·1	86 26 23	− 19	− 12	−33 32	85 52 20
190	Aug. 10	16 29 19·0	91 47 30	− 16	− 9	−33 39	91 13 26
206	13	20 44 10·8	88 23 30	+ 27	+ 33	−35 18	87 49 12
220	14	16 10 48·1	87 27 8	− 19	− 12	−33 33	86 53 4
236	19	21 4 18·9	91 19 11	+ 33	+ 34	−35 18	90 45 0
252	24	20 50 26·2	83 14 52	+ 29	+ 33	−35 4	82 40 50
268	26	21 3 51·8	92 11 52	+ 33	+ 34	−34 55	91 38 4
280	27	20 41 3·1	92 22 0	+ 25	+ 33	−34 53	91 48 5
294	28	20 50 38·3	89 8 15	+ 30	+ 33	−35 2	88 34 16
306	29	20 57 7·2	89 14 56	+ 31	+ 34	−35 8	88 40 53
326	Sept. 1	21 6 10·3	84 29 41	+ 35	+ 34	−35 10	83 55 40
342	3	20 52 10·4	93 39 19	+ 28	+ 33	−35 1	93 5 19
358	4	20 56 26·3	94 38 22	+ 30	+ 34	−35 0	94 4 26
372	9	20 54 27·1	85 39 24	+ 31	+ 34	−34 56	85 5 13
394	14	20 55 37·2	89 46 56	+ 29	+ 34	−34 58	89 13 1
416	16	22 0 27·6	88 3 53	+ 62	+ 37	−35 4	87 30 28
426	17	21 3 21·9	86 3 45	+ 33	+ 34	−34 58	85 29 54
448	19	21 34 11·3	105 47 15	+ 32	+ 35	−35 4	105 13 18
464	27	20 14 18·4	68 7 30	+ 20	+ 31	−35 3	67 33 18

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TABLE XI.—*continued.*

POSITION-ANGLES II.—III., 1901.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1901.	h m s					
8	June 24	19 16 38·2	85° 42′ 55″	+ 5″	+ 24″	−35′ 8″	85° 8′ 16″
24	29	21 8 9·4	84 19 38	+ 29	+ 33	−35 15	83 45 25
38	July 2	16 59 50·8	83 58 8	− 14	− 7	−33 34	83 24 13
52	8	16 19 10·8	87 40 30	− 21	− 13	−33 26	87 6 30
68	10	16 11 7·9	95 49 8	− 23	− 13	−33 30	95 15 2
80	11	17 24 39·0	87 2 38	− 9	− 4	−32 50	86 29 35
96	12	15 49 37·4	87 5 11	− 27	− 15	−33 20	86 31 9
128	16	16 4 4·7	85 47 45	− 22	− 13	−33 36	85 13 34
136	18	15 42 18·3	87 44 49	− 30	− 16	−33 12	87 10 51
154	23	16 16 5·6	86 30 41	− 19	− 12	−33 13	85 56 57
162	27	15 53 21·5	87 47 38	− 24	− 14	−33 34	87 13 26
192	Aug. 10	16 39 12·4	88 59 45	− 13	− 8	−33 39	88 25 45
208	13	20 56 10·2	87 49 11	+ 30	+ 34	−35 18	87 14 57
238	19	21 22 5·0	100 6 38	+ 34	+ 35	−35 18	99 32 29
240	23	20 18 58·7	89 21 7	+ 21	+ 32	−35 5	88 46 55
254	24	20 58 15·9	89 48 41	+ 32	+ 34	−35 4	89 14 43
282	27	20 51 48·4	88 40 30	+ 29	+ 34	−34 53	88 6 40
302	29	20 37 52·6	92 22 15	+ 25	+ 33	−35 8	91 48 5
318	31	20 48 44·8	90 14 45	+ 27	+ 33	−34 57	89 40 48
328	Sept. 1	21 14 41·6	87 29 49	+ 38	+ 34	−35 10	86 55 51
344	3	21 1 54·9	89 9 4	+ 33	+ 34	−35 1	88 35 10
360	4	21 7 51·9	86 31 53	+ 36	+ 34	−35 0	85 58 3
384	10	20 56 33·3	89 35 30	+ 31	+ 34	−35 8	89 1 27
396	14	21 17 47·1	91 35 26	+ 38	+ 34	−34 58	91 1 40
406	16	20 53 10·6	84 38 26	+ 31	+ 34	−35 4	84 4 27
428	17	21 16 25·4	90 7 8	+ 38	+ 34	−34 56	89 33 24
438	19	20 33 10·6	75 31 30	+ 24	+ 33	−35 4	74 57 23

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TABLE XI.—*continued.*

POSITION-ANGLES II.—IV., 1901.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1901.	h m s	° ′ ″	″	″	′ ″	° ′ ″
6	June 24	18 53 17·6	85 45 45	+ 1	+ 20	−35 8	85 10 58
16	25	15 34 34·9	85 34 0	− 33	− 17	−33 42	84 59 28
22	29	20 57 24·9	89 16 56	+ 24	+ 33	−35 15	88 42 38
36	July 2	16 49 22·7	87 33 53	− 15	− 9	−33 34	86 59 55
54	8	16 30 21·2	89 10 45	− 18	− 11	−33 26	88 36 50
70	10	16 23 59·3	87 12 45	− 18	− 12	−33 30	86 38 45
82	11	19 16 48·3	87 13 4	+ 6	+ 25	−34 57	86 38 38
98	12	15 58 39·4	86 0 34	− 25	− 14	−33 20	85 26 35
110	14	15 11 35·1	106 44 38	− 47	− 19	−33 25	106 10 7
126	16	15 53 25·1	101 9 8	− 31	− 14	−33 36	100 34 47
138	18	15 51 17·8	87 15 8	− 26	− 15	−33 12	86 41 15
164	27	16 2 49·3	87 9 34	− 22	− 13	−33 34	86 35 25
194	Aug. 10	16 48 42·2	89 31 26	− 12	− 6	−33 39	88 57 29
210	13	21 9 8·0	87 51 0	+ 34	+ 34	−35 18	87 16 50
228	15	20 30 36·9	85 53 4	+ 18	+ 30	−35 36	85 18 16
242	23	20 32 46·8	85 47 4	+ 25	+ 33	−35 5	85 12 59
256	24	21 7 20·7	83 30 30	+ 35	+ 34	−35 4	82 56 35
272	26	21 35 26·5	106 3 38	+ 35	+ 35	−34 55	105 29 53
284	27	21 0 43·3	89 54 11	+ 30	+ 34	−34 53	89 20 22
310	29	21 16 8·1	89 30 8	+ 38	+ 34	−35 8	88 56 12
330	Sept. 1	21 26 28·2	82 46 38	+ 42	+ 35	−35 10	82 12 45
346	3	21 13 43·4	77 36 45	+ 40	+ 34	−35 1	77 2 58
364	4	21 29 35·0	94 0 8	+ 42	+ 35	−35 0	93 26 25
374	9	21 5 25·1	86 27 23	+ 34	+ 34	−34 56	85 53 35
380	10	20 34 29·8	72 17 45	+ 26	+ 33	−35 8	71 39 36
398	14	21 27 13·9	88 34 53	+ 41	+ 35	−34 58	88 4 11
408	16	21 4 20·4	88 14 0	+ 34	+ 34	−35 4	87 40 4
430	17	21 26 50·5	87 26 4	+ 41	+ 35	−34 56	86 52 24
442	19	21 0 41·6	99 1 15	+ 29	+ 34	−35 4	98 27 14
468	27	20 37 29·5	83 18 15	+ 24	+ 33	−35 3	82 44 9

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TABLE XI.—*continued.*

POSITION-ANGLES III.—IV., 1901.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1901.	h m s	° ′ ″	″	″	′ ″	° ′ ″
4	June 24	17 58 26·2	85 45 6	− 5	− 1	−33 58	85 11 2
14	25	15 20 52·8	86 14 25	− 40	− 19	−33 42	85 39 44
20	29	20 45 33·9	100 45 38	+ 21	+ 33	−35 15	100 11 17
40	July 2	17 9 25·7	85 58 38	− 12	− 6	−33 34	85 24 46
56	8	16 42 0·2	104 51 4	− 18	− 10	−33 26	104 17 10
72	10	16 36 1·9	88 39 38	− 17	− 10	−33 30	88 5 41
88	11	19 53 36·8	87 6 38	+ 12	+ 29	−34 57	86 32 22
104	12	16 28 56·4	86 11 23	− 17	− 11	−33 20	85 37 35
130	16	16 14 41·8	88 21 8	− 21	− 13	−33 36	87 46 58
148	22	15 47 19·5	86 6 49	− 25	− 15	−33 35	85 32 34
170	27	16 33 16·1	87 16 38	− 16	− 9	−33 34	86 42 39
180	29	16 20 53·3	83 16 56	− 17	− 11	−33 32	82 42 56
196	Aug. 10	16 58 59·9	89 20 38	− 11	− 5	−33 39	88 46 43
212	13	21 19 46·3	87 54 28	+ 39	+ 34	−35 18	87 20 23
222	14	16 19 32·2	88 26 8	− 18	− 11	−33 33	87 52 6
244	23	20 46 35·0	83 23 45	+ 28	+ 33	−35 5	82 49 41
258	24	21 19 2·3	98 57 56	+ 35	+ 34	−35 4	98 24 1
286	27	21 10 44·6	99 31 4	+ 33	+ 34	−34 53	98 57 18
296	28	20 59 23·7	93 2 19	+ 31	+ 34	−35 2	92 28 22
304	29	20 46 50·7	90 9 45	+ 28	+ 33	−35 8	89 35 38
332	Sept. 1	21 37 48·6	85 53 4	+ 50	+ 35	−35 10	85 19 19
348	3	21 25 59·1	91 56 4	+ 41	+ 35	−35 1	91 22 19
362	4	21 18 55·8	89 29 26	+ 39	+ 34	−35 0	88 55 39
400	14	21 38 53·5	88 58 0	+ 47	+ 36	−34 58	88 24 25
410	16	21 16 50·0	86 42 4	+ 39	+ 34	−35 4	86 8 13
432	17	21 37 4·3	83 41 4	+ 48	+ 35	−34 56	83 7 29
446	19	21 22 54·7	91 32 45	+ 39	+ 34	−35 4	90 58 54

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TABLE XII.

Position-Angles, I.–II., 1902.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1902.	h m s	° ′ ″	″	″	′ ″	° ′ ″
4	June 10	17 30 4·3	73 18 45	− 41	− 24	−32 54	72 44 46
24	16	18 2 25·4	70 25 27	− 29	− 19	−33 2	69 51 37
52	24	17 58 36·2	71 8 42	− 30	− 20	−33 9	70 34 43
64	30	18 0 30·5	70 29 45	− 29	−. 19	−32 57	69 56 0
84	July 3	17 37 55·0	71 5 30	− 35	− 22	−32 58	70 31 35
96	7	18 57 44·2	70 32 23	− 17	− 13	−32 54	69 58 59
104	8	23 42 56·7	71 48 53	+ 28	+ 33	−34 39	71 15 15
128	15	17 46 37·6	71 0 45	− 32	− 20	−33 2	70 26 51
144	16	17 50 0·8	72 30 0	− 29	− 19	−31 1	71 56 11
160	18	19 12 51·2	71 6 53	− 15	− 10	−33 33	70 32 55
164	19	17 48 24·6	71 52 53	− 31	− 20	−31 1	71 19 1
182	24	17 38 24·9	71 34 50	− 32	− 21	−32 46	71 1 11
194	26	17 38 10·3	71 43 0	− 32	− 21	−32 34	71 9 33
210	27	17 33 49·6	70 34 15	− 33	− 21	−32 51	70 0 30
224	Aug. 3	17 44 14·6	67 6 4	− 29	− 19	−33 7	66 32 9
238	4	23 45 30·7	77 37 15	+ 34	+ 34	−34 36	77 3 47
246	7	17 17 32·5	72 2 22	− 36	− 22	−32 48	71 28 36
266	16	17 56 38·8	71 58 45	− 25	− 17	−32 30	71 25 33
292	18	18 52 12·9	72 38 45	− 16	− 11	−32 46	72 5 32
310	20	17 38 12·0	72 39 0	− 28	− 19	−33 1	72 5 12
326	21	19 28 18·4	72 29 15	− 11	− 6	−32 57	71 56 1
332	22	17 36 23·4	74 59 38	− 29	− 19	−32 49	74 26 1
356	24	23 51 4·6	77 28 34	+ 40	+ 34	−34 11	76 55 37
372	25	18 43 3·2	72 50 7	− 16	− 12	−33 3	72 16 36
378	27	17 37 19·4	72 46 45	− 28	− 19	−32 58	72 13 0
400	28	17 45 41·8	72 13 30	− 26	− 17	−32 48	71 39 59
430	Sept. 2	17 53 1·2	72 51 0	− 24	− 17	−33 8	72 17 11
446	3	18 20 33·4	73 6 15	− 18	− 14	−32 58	72 32 45
476	6	19 10 24·8	72 38 15	− 12	− 7	−33 0	72 4 56
480	7	17 30 3·0	73 34 7	− 29	− 19	−33 0	73 0 19
496	8	17 29 53·4	73 11 11	− 29	− 19	−32 58	72 37 25
512	10	23 18 17·9	73 25 45	+ 31	+ 33	−34 22	72 52 27
528	12	18 13 40·9	73 18 23	− 19	− 14	−32 58	72 44 52
540	26	23 36 37·4	73 30 0	+ 38	+ 34	−34 36	72 56 36
558	Oct. 4	23 31 32·2	73 19 45	+ 36	+ 34	−34 45	72 46 10

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TABLE XII.—*continued.*

POSITION-ANGLES I.—III. 1902.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1902.	h m s	° ′ ″	″	″	′ ″	° ′ ″
6	June 10	17 44 26·3	70 44 0	− 34	− 22	−32 54	70 10 10
26	16	18 13 53·5	70 50 4	− 27	− 18	−33 2	70 16 17
48	21	18 10 8·1	70 44 23	− 27	− 18	−33 0	70 10 38
54	24	18 10 47·7	70 53 15	− 27	− 18	−33 9	70 19 21
80	July 1	18 52 9·8	70 56 3	− 18	− 13	−32 52	70 22 40
86	3	17 58 32·5	70 47 45	− 29	− 19	−32 58	70 13 59
88	7	17 36 22·2	69 38 15	− 36	− 22	−32 54	69 4 23
106	8	23 54 56·8	71 1 37	+ 33	+ 34	−34 39	70 28 5
118	14	17 17 51·2	71 16 45	− 42	− 23	−32 57	70 42 43
130	15	18 2 32·8	71 7 45	− 27	− 18	−33 2	70 33 58
146	16	18 3 17·3	71 21 23	− 27	− 18	−33 1	70 47 37
166	19	18 1 59·4	71 28 3	− 27	− 18	−33 1	70 54 17
184	24	17 51 43·4	71 35 8	− 28	− 19	−32 46	71 1 35
198	26	18 6 54·7	71 45 0	− 25	− 17	−32 34	71 11 44
212	27	17 46 36·0	71 45 15	− 29	− 19	−32 51	71 11 36
226	Aug. 3	17 56 14·2	72 0 15	− 27	− 18	−33 7	71 26 23
242	4	0 27 5·6	71 51 45	+ 55	+ 36	−34 36	71 18 40
248	7	17 28 5·5	72 8 34	− 33	− 21	−32 48	71 34 52
268	16	18 10 17·9	72 41 30	− 22	− 16	−32 30	72 8 22
274	17	17 33 54·3	72 38 0	− 29	− 20	−32 47	72 4 24
294	18	19 2 55·7	72 34 34	− 15	− 10	−32 46	72 1 23
300	19	17 43 8·5	76 56 23	− 27	− 18	−32 56	76 22 42
328	21	19 40 30·5	72 40 45	− 10	− 5	−32 57	72 7 33
334	22	17 46 54·4	72 39 37	− 26	− 18	−32 49	72 6 4
346	24	22 28 43·3	72 52 37	+ 15	+ 29	−34 44	72 18 37
370	25	18 32 55·1	72 52 23	− 17	− 13	−33 3	72 18 50
380	27	17 47 52·9	73 18 0	− 26	− 17	−32 58	72 44 19
398	28	17 34 26·6	72 57 30	− 29	− 19	−32 48	72 23 54
412	30	23 37 55·4	73 16 0	+ 36	+ 34	−34 34	72 42 36
426	Sept. 2	17 33 43·1	71 58 37	− 28	− 18	−33 8	71 24 43
438	3	17 18 49·1	65 12 30	− 31	− 21	−32 58	64 38 40
470	6	18 32 8·7	47 53 15	− 17	− 13	−33 0	47 19 45
488	7	18 23 36·5	73 29 19	− 18	− 13	−33 0	72 55 48
500	8	17 49 37·5	73 22 7	− 24	− 17	−32 58	72 48 28
514	10	23 29 32·6	74 13 45	+ 34	+ 34	−34 22	73 40 31
532	16	22 46 3·6	72 49 0	+ 20	+ 32	−34 36	72 15 16
542	26	23 48 34·4	73 36 15	+ 45	+ 34	−34 36	73 2 58
556	Oct. 4	23 23 3·1	73 21 15	+ 33	+ 34	−34 45	72 47 37

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TABLE XII.—*continued.*

POSITION-ANGLES I.—IV., 1902.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1902.	h m s	° ′ ″	″	″	′ ″	° ′ ″
8	June 10	18 0 56·5	71 14 38	− 30	− 20	−32 54	70 40 54
28	16	18 26 7·6	76 13 15	− 24	− 16	−33 2	75 39 33
56	24	18 23 29·5	65 16 23	− 25	− 17	−33 9	64 42 32
66	30	18 15 8·8	71 4 26	− 26	− 17	−32 57	70 30 46
72		19 15 25·7	71 4 14	− 15	− 11	−32 57	70 30 51
94	July 7	18 41 17·8	71 31 56	− 20	− 14	−32 54	70 58 28
108	8	0 11 8·7	71 18 48	+ 40	+ 34	−34 39	70 45 23
120	14	17 37 3·2	71 40 18	− 34	− 21	−32 57	71 6 26
132	15	18 15 9·6	71 43 41	− 24	− 17	−33 2	71 9 58
148	16	18 14 11·1	71 34 8	− 24	− 17	−33 1	71 0 26
158	18	19 1 19·8	71 11 8	− 16	− 12	−33 33	70 37 7
186	24	18 5 0·4	71 55 2	− 26	− 17	−32 46	71 21 33
200	26	18 17 17·6	71 38 45	− 23	− 16	−32 34	71 5 32
230	Aug. 3	18 17 7·4	72 1 35	− 22	− 16	−33 7	71 27 50
250	7	17 39 51·6	72 41 25	− 29	− 19	−32 48	72 7 49
276	17	17 49 52·4	72 54 5	− 27	− 18	−32 47	72 20 33
296	18	19 14 54·8	72 55 26	− 13	− 8	−32 46	72 22 19
324	21	19 18 1·1	75 1 15	− 12	− 6	−32 57	74 28 0
342	22	19 6 14·4	74 10 0	− 13	− 9	−32 49	73 36 49
350	24	22 58 36·3	73 33 0	+ 21	+ 32	−34 44	72 59 9
360	25	17 35 44·6	73 17 23	− 28	− 19	−33 3	72 43 33
374		18 54 10·6	73 16 10	− 15	− 11	−33 3	72 42 41
384	27	18 20 5·2	73 7 52	− 20	− 14	−32 58	72 34 20
388		19 0 11·3	73 7 34	− 13	− 10	−32 58	72 34 13
396	28	17 22 45·8	72 54 45	− 32	− 21	−32 48	72 21 4
424	Sept. 2	17 22 41·0	73 33 1	− 32	− 21	−33 8	72 59 0
448	3	18 33 31·8	73 37 0	− 17	− 13	−32 58	73 3 32
474	6	18 59 8·1	71 11 15	− 13	− 9	−33 0	70 37 53
486	7	18 11 17·7	83 18 52	− 19	− 15	−33 0	82 45 18
498	8	17 39 31·9	74 1 46	− 27	− 18	−32 58	73 28 3
516	10	23 45 18·1	73 50 58	+ 31	+ 33	−34 22	73 17 40
536	26	23 9 34·4	74 12 47	+ 28	+ 33	−34 36	73 39 12
554	Oct. 4	23 11 28·9	74 15 38	+ 29	+ 33	−34 45	73 41 55

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TABLE XII.—*continued.*

POSITION-ANGLES II.—III., 1902.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1902.	h m s	° ′ ″	″	″	′ ″	° ′ ″
14	June 11	17 50 37·7	70 31 23	− 33	− 21	−32 56	69 57 33
30	16	18 39 52·7	69 39 0	− 21	− 14	−33 2	69 5 23
40	19	17 37 31·7	71 0 30	− 37	− 22	−32 52	70 26 39
58	24	18 38 34·9	70 30 15	− 21	− 15	−33 9	69 56 30
98	July 7	19 10 45·1	70 22 23	− 16	− 12	−32 54	69 49 1
110	8	0 25 25·6	70 50 23	+ 48	+ 35	−34 39	70 17 7
138	15	19 2 2·1	70 46 23	− 16	− 12	−33 2	70 12 53
150	16	18 29 31·3	71 7 30	− 21	− 16	−33 1	70 33 52
168	20	18 10 29·3	71 58 0	− 25	− 17	−32 54	71 24 24
188	24	18 21 43·8	71 36 1	− 22	− 16	−32 46	71 2 37
202	26	18 28 5·0	71 45 0	− 21	− 15	−32 34	71 11 50
214	27	17 57 27·8	72 15 53	− 27	− 18	−32 51	71 42 17
228	Aug. 3	18 5 26·8	72 27 0	− 24	− 17	−33 7	71 53 12
240	4	0 17 24·1	71 34 0	+ 49	+ 35	−34 36	71 0 48
252	7	17 54 55·1	72 3 51	− 26	− 17	−32 48	71 30 20
270	16	18 21 15·5	72 27 45	− 20	− 14	−32 30	71 54 41
282	18	17 36 45·6	72 51 15	− 29	− 19	−32 46	72 17 41
304	19	18 12 6·8	72 4 30	− 22	− 16	−32 56	71 30 56
312	21	17 27 1·9	72 34 55	− 31	− 20	−33 1	72 1 3
314		17 40 34·6	72 35 4	− 28	− 18	−33 1	72 1 17
318		18 28 18·1	72 37 42	− 18	− 14	−33 1	72 4 9
338	22	18 35 14·1	72 49 50	− 17	− 13	−32 49	72 16 31
344	24	22 15 27·5	73 14 53	+ 12	+ 29	−34 44	72 40 50
364	25	17 58 35·8	73 4 45	− 23	− 16	−33 3	72 31 3
382	27	18 0 52·9	69 6 22	− 23	− 16	−32 58	68 32 45
406	28	18 19 18·7	72 51 51	− 19	− 14	−32 48	72 18 30
414	30	23 50 58·9	72 57 45	+ 41	+ 34	−34 34	72 24 26
428	Sept. 2	17 44 20·7	72 41 30	− 25	− 17	−33 8	72 7 40
444	3	18 9 12·8	72 4 15	− 21	− 15	−32 58	71 30 41
458	4	23 26 47·3	73 6 34	+ 33	+ 34	−34 31	72 33 10
464	6	17 47 1·8	73 6 15	− 24	− 17	−33 0	72 32 34
484	7	17 57 39·4	73 33 30	− 22	− 16	−33 0	72 59 52
502	8	17 59 39·2	73 44 38	− 22	− 16	−32 58	73 11 2
522	10	0 20 56·9	70 30 23	+ 64	+ 36	−34 22	69 57 41
544	26	0 2 19·4	73 34 50	+ 51	+ 36	−34 36	73 1 41
566	Oct. 4	0 18 35·5	73 17 37	+ 64	+ 36	−34 45	72 44 32
570	5	23 8 16·2	73 53 0	+ 28	+ 33	−34 35	73 19 26

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TABLE XII.—*continued.*

POSITION-ANGLES II.—IV., 1902.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1902.	h m s	° ′ ″	″	″	′ ″	° ′ ″
16	June 11	18 10 18·2	70 53 0	− 27	− 19	−32 56	70 19 18
32	16	18 52 4·0	71 6 30	− 18	− 14	−33 2	70 32 56
42	19	17 50 34·2	71 8 7	− 32	− 20	−32 52	70 34 23
60	24	18 51 59·8	68 4 45	− 18	− 14	−33 9	67 31 4
68	30	18 31 41·9	71 26 41	− 22	− 16	−32 57	70 53 6
74		19 29 55·8	71 26 8	− 13	− 9	−32 57	70 52 49
78	July 1	18 34 26·9	69 43 0	− 22	− 16	−32 52	69 9 30
100	7	19 28 31·1	71 16 45	− 13	− 9	−32 54	70 43 29
112	8	0 46 50·1	71 19 53	+ 62	+ 36	−34 39	70 46 52
136	15	18 45 7·1	71 57 27	− 20	− 14	−33 2	71 23 51
152	18	18 0 5·5	80 10 45	− 27	− 18	−33 33	79 36 27
156		18 31 52·5	66 47 49	− 21	− 16	−33 33	66 13 39
170	20	18 21 59·4	74 3 30	− 22	− 16	−32 54	73 29 58
190	24	18 36 5·2	72 13 26	− 20	− 14	−32 46	71 40 6
204	26	18 38 15·9	71 42 19	− 18	− 14	−32 34	71 9 13
232	Aug. 3	18 38 17·2	71 49 17	− 18	− 14	−33 7	71 15 38
254	7	18 10 8·7	73 19 49	− 23	− 16	−32 48	72 46 22
286	18	18 2 9·6	72 48 25	− 23	− 16	−32 46	72 15 0
302	19	17 59 59·2	73 10 15	− 24	− 16	−32 56	72 36 39
306		18 30 8·5	73 8 4	− 18	− 13	−32 56	72 34 37
320	21	18 54 3·8	71 16 30	− 14	− 11	−32 57	70 43 8
340	22	18 52 34·0	73 44 34	− 14	− 11	−32 49	73 11 20
348	24	22 41 26·7	73 37 45	+ 17	+ 30	−34 44	73 3 48
368	25	18 20 39·0	73 39 30	− 18	− 14	−33 3	73 5 55
390	27	19 14 10·2	73 6 42	− 12	− 6	−32 58	72 33 26
402	28	17 56 58·5	73 30 45	− 23	− 16	−32 48	72 57 18
410	30	23 22 45·1	91 44 38	+ 30	+ 33	−34 34	91 11 7
432	Sept. 2	18 2 2·1	73 46 42	− 22	− 16	−33 8	73 12 56
440	3	17 32 28·2	73 44 30	− 28	− 19	−32 58	73 10 45
456	4	23 12 35·7	73 16 44	+ 26	+ 33	−34 31	72 43 12
466	6	17 57 41·3	73 45 45	− 23	− 16	−33 0	73 12 16
472		18 44 56·9	74 0 15	− 14	− 11	−33 0	73 26 50
482	7	17 45 24·4	74 37 39	− 25	− 17	−33 0	74 3 57
504	8	18 7 44·2	76 36 15	− 21	− 16	−32 58	76 2 40
518	10	23 56 50·1	73 46 23	+ 48	+ 35	−34 22	73 13 24
546	26	0 12 37·4	74 32 27	+ 59	+ 36	−34 36	73 59 26
562	Oct. 4	23 53 56·3	74 41 7	+ 47	+ 35	−34 45	74 7 44

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TABLE XII.—*continued.*

POSITION-ANGLES III.—IV., 1902.

Rotation No.	Date.	Sidereal Time.	Instrumental Position-Angle.	Refraction.	Adjustments.	Index Correction.	Observed Position-Angle.
	1902.	h m s	° ′ ″	″	″	′ ″	° ′ ″
10	June 10	18 16 3·1	71 48 15	− 26	− 17	−32 54	71 14 38
18	11	18 26 51·6	71 32 22	− 25	− 17	−32 56	70 58 44
34	16	19 4 29·1	71 34 52	− 17	− 13	−33 2	71 1 20
44	19	18 7 38·9	71 5 45	− 28	− 19	−32 52	70 32 6
62	24	19 5 41·8	69 52 53	− 16	− 13	−33 9	69 19 15
76	July 1	18 6 13·4	72 8 30	− 27	− 18	−32 52	71 34 53
92	7	18 29 11·3	71 34 41	− 22	− 16	−32 54	71 1 9
114	8	1 3 27·8	71 15 15	+ 76	+ 36	−34 39	70 42 28
122	14	17 52 6·7	71 37 41	− 29	− 19	−32 57	71 3 56
134	15	18 30 48·5	71 46 50	− 22	− 16	−33 2	71 13 10
174	20	18 59 29·2	64 34 15	− 17	− 13	−32 54	64 0 51
178		19 23 46·7	63 3 23	− 13	− 9	−32 54	62 30 7
192	24	18 58 0·7	71 50 39	− 16	− 12	−32 46	71 17 25
196	26	17 52 20·5	74 30 30	− 28	− 19	−32 34	73 57 9
206		18 49 5·1	72 50 37	− 17	− 13	−32 34	72 17 33
220	Aug. 3	17 22 14·9	72 2 49	− 35	− 22	−33 7	71 28 45
222		17 33 3·5	72 1 26	− 31	− 21	−33 7	71 27 27
256	7	18 25 1·7	72 32 2	− 20	− 14	−32 48	71 58 40
258		18 39 10·8	72 31 7	− 18	− 13	−32 48	71 57 48
264	16	17 42 54·3	72 59 25	− 28	− 19	−32 30	72 26 8
278	17	18 5 39·6	72 50 14	− 23	− 16	−32 47	72 16 48
280		18 36 23·4	72 50 3	− 17	− 13	−32 47	72 16 46
284	18	17 48 58·0	72 48 18	− 27	− 18	−32 46	72 14 47
288		18 25 27·4	72 47 41	− 20	− 14	−32 46	72 14 21
322	21	19 4 42·1	73 9 45	− 13	− 9	−32 57	72 36 26
336	22	17 59 46·4	73 14 45	− 23	− 16	−32 49	72 41 17
354	24	23 37 37·1	74 1 0	+ 36	+ 34	−34 11	73 26 59
366	25	18 9 11·6	73 55 41	− 22	− 16	−33 3	73 22 0
392	27	19 28 48·8	73 9 59	− 11	− 5	−32 58	72 36 45
404	28	18 7 29·2	72 57 31	− 22	− 16	−32 48	72 24 5
416	30	0 8 25·5	73 59 30	+ 51	+ 35	−34 34	73 26 22
418	Sept. 1	17 4 4·4	73 29 30	− 36	− 22	−32 57	72 55 35
422	2	17 13 21·0	73 28 30	− 34	− 21	−33 8	72 54 27
450	3	18 43 18·0	73 31 30	− 16	− 11	−32 58	72 58 5
454	4	22 57 14·9	73 51 7	+ 22	+ 33	−34 31	73 17 31
462	6	17 35 15·9	72 29 0	− 25	− 18	−33 0	71 55 17
490	7	18 35 4·1	71 50 45	− 16	− 12	−33 0	71 17 17
506	8	18 17 52·0	63 28 0	− 18	− 14	−32 58	62 54 30
520	10	0 7 24·3	73 52 25	+ 54	+ 36	−34 22	73 19 33
538	26	23 23 27·4	74 1 4	+ 33	+ 34	−34 36	73 27 35
548		0 24 55·4	74 0 1	+ 70	+ 36	−34 36	73 27 11
560	Oct. 4	23 41 23·3	74 48 18	+ 42	+ 34	−34 45	74 14 49

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CHAPTER III.

The Ephemeris.

The theory with which the observations have been compared is that of Souillart, which is practically an extension of Laplace's as given in vol. v. of the *Mécanique Céleste*: the algebraical development of Souillart is to be found in the *Memoirs, R.A.S.*, vol. xlv., the numerical in *Mem. Sav. Étrang*, vol. xxx., No. I. These two memoirs are referred to hereafter as A and B. The numerical theory is based on constants which Damoiseau gives explicitly in his *Introduction des tables Écliptiques* or in the *Memoire Manuscript* used by Souillart or which can be deduced from his formulæ: and though these constants are far from being the best that might have been adopted, yet Souillart's theory agrees with observation sufficiently closely to afford a good basis of comparison. It has also the advantage of being an absolutely self-consistent theory from beginning to end; certain values of the masses and elements of the orbits are adopted and then the numerical values of all the perturbations are found by substituting their adopted values in the algebraical development of the theory. The values of the elements and masses which Souillart adopted are Damoiseau's, but they do not reproduce the same values of some of the most important inequalities as Damoiseau found from observation. Thus, for instance, we have the following values of the co-efficients of the great inequalities in the longitudes of the first three satellites:—

	Souillart.	Damoiseau.	Difference.
I.	1555″	1636″	81″
II.	3693	3867	174
III.	228	262	34

or expressed in seconds of time

	s	s	s
I.	183·6	193·2	9·6
II.	873·1	914·2	41·1
III.	109·1	125·4	16·3

It is unfortunate that Souillart's constants do not lead to values more nearly in agreement with Damoiseau's observed values, for the great inequalities of I. and II. are perhaps the best determined constants of the whole theory. The cause of this disagreement will be pointed out in a later chapter. But in spite of this Souillart's theory has, as pointed out above, the real merit of being perfectly homogeneous.

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Souillart's expressions for the longitudes, radius vectors and latitudes were put into convenient form for practical use by Marth, who published tables for the four satellites in vol. li. of the *Monthly Notices, R.A.S.* These tables have been used throughout the computation of the longitudes and radius vectors, with the exception that for the table giving the inequality in radius vector of the second satellite with argument β_2, another was substituted in which the error pointed out by de Sitter was corrected; the table given in the *Monthly Notices* was calculated with the erroneous arguments $2\beta_2$ and $3\beta_2$: in the new table the correct arguments $3\beta_2$ and $5\beta_2$ were used. Also Marth's tables for the inequalities depending on a_2 and a_4 were expanded tenfold in order to avoid troublesome interpolations.

The mean longitudes have been brought up from Marth's values for 1891 by using his mean motions, *Monthly Notices*, p. 512, which, however, do not agree with Souillart's. Marth's explanation of his method of deriving them is vague: he merely states that he has allowed for the altered constant of the velocity of light, and that some "plausible corrections" have been adopted. An unsuccessful attempt was made to see what these corrections to the mean motions were: the mean longitudes for the epoch 1750, however, were apparently derived as follows. On page 3 of Damoiseau's "Introduction" are found values for the mean longitudes and the mean motions for 1750, January 1, Paris mean midnight: referred to the epoch 1750, January 1, Greenwich mean noon, these are:—

I.	118°·0778
II.	3·1977
III.	35·7522
IV.	341·1234

These angles are reckoned from the moving equinox of the earth; but Marth takes the vernal equinox of Jupiter as his point of zero longitude. The angle between them is denoted by O and is taken equal to the angle $180° + b_4 t + \gamma_4$ of Souillart: this is the mean point of intersection of Jupiter's equator with the fixed plane of reference, viz., the plane of Jupiter's orbit at the epoch January 1, 1850. The adopted values of b_4 and γ_4 are Souillart's, namely:—

$$b_4 = -2''\cdot8726. \qquad \gamma_4 = 314° 46' 16''\cdot56.$$

which, with Le Verrier's constant of precession, give

$$O = 134° 46' 16''\cdot56 - 47''\cdot4374t.$$

where t is reckoned in years from January 1, 1850.

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Hence for January 1, 1750, we have—

$$\begin{aligned} O &= 133^\circ\!\cdot\!4536 \\ l_1 - O &= 344\cdot6242 \\ l_2 - O &= 229\cdot7441 \\ l_3 - O &= 262\cdot2986 \\ l_4 - O &= 207\cdot6698 \end{aligned}$$

These are next corrected for the change in the assumed value of the light equation. Damoiseau following Delambre, *Astronomie*, iii. p. 106, adopted the value $493^s\cdot2$: Marth adopted 498·9: hence Marth's epochs occur $5^s\cdot7$ sooner than Damoiseau's at distance unity or $29^s\cdot6$ at Jupiter's mean distance, viz., 5·2. The above values of l-O require therefore, on this account, the correction

$$\frac{-29\cdot6}{24.60.60}n \text{ or } -0\cdot000343\ n.$$

which gives

$$\begin{aligned} l_1 - O &= 344^\circ\!\cdot\!5546 \\ l_2 - O &= 229\cdot7095 \\ l_3 - O &= 262\cdot2814 \\ l_4 - O &= 207\cdot6624 \end{aligned}$$

These values, however, do not satisfy the well-known relation

$$l_1 - O - 3\,(l_2 - O) + 2\,(l_3 - O) = 180^\circ$$

but if equal and opposite corrections, viz., $-0^\circ\cdot0022$ and $+0^\circ\cdot0022$ are applied to the second and third and the first remains unaltered, the relation is satisfied and we get for 1750, January 1, Greenwich mean noon—

$$\begin{aligned} l_1 - O &= 344^\circ\!\cdot\!5546 \\ l_2 - O &= 229\cdot7073 \\ l_3 - O &= 262\cdot2836 \\ l_4 - O &= 207\cdot6624 \end{aligned}$$

If the value of O for 1750 be taken as $133^\circ\cdot4546$ instead of $133^\circ\cdot4536$, and if these values of the mean longitudes be carried up to 1891 with Marth's mean motions, his values of the longitudes are exactly reproduced. This is probably how he derived the longitudes of the epoch. The mean motions must be assumed to be correct, for it need not be said that to find from observation real corrections to the assumed, *i.e.*, to Marth's values, it would be necessary to discuss a series of observations extending over a much greater period of time than that covered by the present series.

The values of the major axes of the satellites' orbits given by Marth on pp. 516, 517 are not quite correctly associated with the mass of Jupiter, owing to his having neglected all* the non-periodic perturbations of the axes excepting that due to the

* This is not so: as Dr. de Sitter has pointed out to me, Marth has included the second order terms in his expressions for the radius vectors on pp. 514, 515, but has omitted the first order terms. Correction on this account is made in the last chapter, p. 202.

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compression of Jupiter: attention was first drawn to this by Professor Sampson in the *Observatory*, August 1901. The relation between the mean motion n and the semi-major axis a is

$$\frac{\mu}{a^3}(1+f) = n^2 + \text{perturbation terms} = n^2 + \mathrm{P} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i.)}$$

in which μ = sum of masses of Jupiter and the satellite and

$$f = \mathrm{J}b^2/a^2 = b^2(\rho - \tfrac{1}{2}\phi)/a^2$$

b being the equatorial semidiameter of Jupiter, ρ his ellipticity, and ϕ the ratio of centrifugal force to gravity at his equator. By Kepler's third law we have for the earth,

$$1 + \mathrm{M}^1 = (2\pi)^2/\mathrm{T}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii.)}$$

where M^1 is the earth's mass in terms of that of the sun.

Assume for the moment that P is zero and divide (i.) by (ii.); we get

$$\frac{\mu(1+f)}{a^3} = (1+\mathrm{M}^1)\,\mathrm{T}^2\left(\frac{n}{2\pi}\right)^2$$

or writing M for Jupiter's mass, Mm for that of the satellite and t for its period,

$$\mathrm{M}(1+m) = \frac{\mathrm{T}^2}{t^2}\,\frac{a^3(1+\mathrm{M}^1)}{1+f}$$

The reciprocal of the mass of the system is therefore

$$1 \div \mathrm{M}(1+\Sigma m) = \frac{1}{\mathrm{T}^2(1+\mathrm{M}^1)}\,\frac{1+m}{1+\Sigma m}\,\frac{t^2(1+f)}{a^3}.$$

This is the expression used by Marth to connect the major axis and the mass of the system: if Le Verrier's and Souillart's values of the constants are substituted in it, the table on pp. 516, 517 is reproduced. But this relation has been derived on the assumption that P is zero, which is not the case: the correct a to associate with a given mass of the system is the a calculated from this expression plus the perturbative terms. The total perturbation terms are given by Souillart II. pp. 16 and 18, and are in units of the mean distances,

0·0002318 0·0001325 0·0000516 0·0000895.

By far the greater part of this is due to the Compression of Jupiter, which has already been taken into account. This part is denoted by $\frac{1}{3}\frac{(0)}{n}$ and amounts to

0·0002252 0·0000889 0·0000349 0·0000113.

Hence the perturbation terms due to the sun and the other satellites are

0·0000066 0·0000436 0·0000167 0·0000782

and Marth's values of log a″ require corrections of

+4 +19 +7 +34

in units of the sixth decimal place.

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The values of log a'' corresponding to a mass 1/1047·7 adopted throughout this work are thus

2·764534 2·966232 3·169012 3·414255.

According to Marth's table these would correspond not to a mass of 1/1047·7 but to

1/1047·68 1047·56 1047·65 1047·45.

The fundamental plane of reference adopted by Marth is not as stated the plane of Jupiter's equator defined by Souillart. The slight difference between them is, however, immaterial, for the choice of the plane of reference is decided only by considerations of convenience and so long as the plane of reference nearly coincides with Jupiter's equator we attain the desired convenience in computation. It is well to point out that a small error has been made by Marth in reducing to the ecliptic the quantities referring to the moving orbit. At the epoch 1891·75 the longitude of O is in error by +2′ 28″·2. But the quantities referring to the earth's equator are consistent with those referring to the ecliptic. The position of Marth's plane of reference is accordingly defined by longitudes and inclinations referred to the ecliptic, not to the moving orbit of Jupiter. In the present ephemeris the adopted plane of reference is the plane of Jupiter's equator, as defined by Souillart II., equations [72] or in more convenient shape by Tisserand *Méc. Cel.*, iv. p. 61. The periodic terms in Tisserand's expression, which are the nutation of Jupiter's axis, were calculated but were accidentally applied with the wrong sign: as Jupiter's equator was chosen as plane of reference only because of its convenience this mistake is of no consequence. A second small error was made owing to a printer's error [not in the Errata] in Souillart's Tableau LXI. *bis*, p. 160, which has been carried through by Tisserand; the sign of N_3^{IV} is −, not + as printed. The equator of Jupiter relatively to his moving orbit is then defined as follows:—

$$\text{Node} = -\psi^1 = \gamma_4 - 180^\circ + 0''\cdot100\,t + 29''\cdot9 \sin(-G_2) + 58''\cdot3 \sin(-G_3) + 55''\cdot1 \sin(-G_4).$$
$$\text{Inclination} = \omega_1{}^1 = \omega_1 + 0''\cdot0233\,t - 1''\cdot6 \cos(-G_2) - 3''\cdot1 \cos(-G_3) - 3''\cdot0 \cos(-G_4).$$

The periodic terms are the nutation. Le Verrier's precession constant, namely 50″·31, has been used to refer ψ^1 to the earth's equinox. Thus the planes of reference for the observations discussed in this volume are at the mean epochs of the two sets of observations—

	Node.	Inclination.	
1901·61	135° 30′ 35″·2	3° 4′ 6″·6	Referred to Jupiter's moving orbit.
1902·60	135 31 29 ·1	3 4 7 ·0	

The longitudes are reckoned from this node.

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The longitudes of the "périjoves propres" and of the "nœuds propres," *i.e.*, the angles $gt+\beta$ and $bt+\gamma$ are taken directly from Souillart B., pp. 40, 161. The g's and b's include erroneous values of the precession constant, but as only differences of the angles are required, Souillart's values can be used as they stand.

The position of Jupiter's orbit is taken from Le Verrier's tables, *Annales de l'Observatoire*, xii. p. 7.

The obliquity of the ecliptic and the co-ordinates of Jupiter for 1901 and 1902 are from the *Nautical Almanac*. The latter are based on Hill's Tables, which differ slightly from Le Verrier's; as far as concerns the orbit of Jupiter, the difference between the two sets of tables is immaterial, for the orbit is only used as a convenient plane to which we may refer the plane of Jupiter's equator, itself an ill-determined quantity. But we also get different values for the jovicentric co-ordinates of the earth according as we employ one set of tables or the other, that is to say, the mutual distances of the satellites as viewed from the earth are different; so that we may get values of the unknowns varying slightly with the tables we use. The two sets of tables were therefore compared with each other with a result which showed that the differences between them would be quite inappreciable in the computed positions of any of the satellites, and hence that the values of the unknowns would be the same whichever set of tables was employed.

The longitude of the node and the inclination of the adopted plane of reference referred to the movable plane of Jupiter's orbit are :—

		Node Ω.	Inclination.
1901.		° ′ ″	° ′ ″
June	20	135 30 27·04	3 4 6·59
July	20	31·58	6·61
August	19	36·13	6·64
September	18	40·67	6·67
October	18	45·20	6·69
1902.			
May	31	135 31 18·9	3 4 6·9
June	30	23·4	6·9
July	30	27·9	7·0
August	29	32·4	7·0
September	28	36·9	7·0
October	28	41·4	7·0

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The node and inclination of Jupiter's orbit referred to the ecliptic are :—

		Node.	Inclination.
1901.			
June	20	99° 27′ 28″·93	1° 18′ 30″·81
July	20	31·92	30·80
August	19	34·91	30·78
September	18	37·90	30·77
October	18	40·89	30·75
1902.			
May	31	99 28 3·3	1 18 30·7
June	30	6·3	30·7
July	30	9·3	30·6
August	29	12·3	30·6
September	28	15·3	30·6
October	28	18·3	30·6

Hence for Jupiter's equator referred to the ecliptic, we have :—

		Node.	Long. of O.	Inclin.
1901.				
June	20	336°·4496	135°·4861	2°·1528
July	20	336·4507	135·4876	2·1528
August	19	336·4519	135·4891	2·1529
September	18	336·4532	135·4906	2·1529
October	18	336·4546	135·4922	2·1530
1902.				
May	31	336·4637	135·5013	2·1530
June	30	336·4646	135·5026	2·1530
July	30	336·4657	135·5038	2·1530
August	29	336·4670	135·5051	2·1531
September	28	336·4684	135·5063	2·1531
October	28	336·4699	135·5076	2·1531

And for Jupiter's equator referred to the true equator of the earth, we have :—

		Node.	Long. of O.	Inclin.
1901.				
June	20	−2°·0026	135°·3079	25°·4372
July	20	−2·0027	135·3093	25·4376
August	19	−2·0027	135·3104	25·4376
September	18	−2·0025	135·3117	25·4377
October	18	−2·0021	135·3134	25·4382
1902.				
May	31	−2·0016	135·3222	25·4373
June	30	−2·0016	135·3234	25·4374
July	30	−2·0015	135·3247	25·4375
August	29	−2·0014	135·3258	25·4375
September	28	−2·0012	135·3270	25·4374
October	28	−2·0011	135·3284	25·4373

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From these quantities and the co-ordinates of Jupiter in the *Nautical Almanac*, the values of the following angles were next calculated for every alternate Greenwich Mean Noon during the period of observation.

P = the position-angle of the northern end of Jupiter's axis, reckoned in the usual way from the north round through the east.

$L - O + 180^\circ$ = the Jovicentric longitude of the earth reckoned in the plane of Jupiter's equator from O, his vernal equinox.

$\Lambda - O + 180^\circ$ = the Jovicentric longitude of the sun reckoned in the same way.

B = the Jovicentric latitude of the earth.

Since these angles were found from the *apparent* right ascension and declination of Jupiter, *i.e.*, the right ascension and declination as affected by aberration, they are cleared of the effects of aberration: they were therefore interpolated directly for the times for which they were required.

The Light Time (with an assumed Light Equation of $498^s \cdot 9$) and also a correction to the value of log Δ in the *Nautical Almanac* will be found in the same table as these angles. The last correction requires some explanation.

In calculating the co-ordinates of a satellite, we require the distance between the earth at the epoch of observation and the satellite at the moment when the light rays left it. This distance can clearly only be arrived at by succession approximations, but as the distance of the satellites from Jupiter is so small compared to the distance of the earth from Jupiter, there will be only a very small correction to be applied later on if the co-ordinates are calculated with the distance between the earth at the epoch of observation and Jupiter at the moment the light rays left Jupiter. The *Nautical Almanac* gives the logarithm of the distance, Δ, between the positions of the earth and of Jupiter at Greenwich noon: therefore if we interpolate log Δ from the *Nautical Almanac* for the epoch of observation, we shall have to apply a small correction, viz:—

$$\delta_1 (\log \Delta) = 0 \cdot 434\, n_\nu R_\nu \sin (\Lambda - L)$$
$$= [4 \cdot 5603]\, R_\nu \sin (\Lambda - L)$$

where

n_ν = Heliocentric motion (in circular measure) of Jupiter in $498^s \cdot 9$.

R_ν = Radius vector of Jupiter.

$\Lambda - O + 180^\circ$ = Sun's Jovicentric longitude.

$L - O + 180^\circ$ = Earth's Jovicentric longitude.

The corresponding correction to L − O, namely,

$$\delta_1 L = - n_\nu R_\nu \cos (\Lambda - L)$$

amounting to $-0^\circ \cdot 0025$ throughout the period of observation, is not insignificant, but as explained above must not be applied.

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If, on the other hand, we interpolate log Δ for the epoch of observation corrected for aberration, *i.e.*, the epoch of observation antedated by the amount of the "Light-Time," the necessary correction is :—

$$\delta_2 (\log \Delta) = -0{\cdot}434\, n_{III}\, R_{III} \sin JES$$
$$= -[5{\cdot}6344]\, R_{III} \sin JES$$

where

n_{III} = Heliocentric motion (in circular measure) of the earth in $498^s{\cdot}9$.
R_{III} = Radius vector of the earth.
JES = Angular geocentric distance of Jupiter and the sun.
= (L − O) − (Λ − O) + heliocentric longitude of Jupiter − geocentric longitude of the sun.

As before, the corresponding $\delta_2 L$ is not to be applied.

In 1901 log Δ was interpolated for the actual epoch of observation; the proper correction is therefore $\delta_1 (\log\Delta)$. In 1902 log Δ was interpolated for the epoch corrected for aberration, and the proper correction accordingly is $\delta_2 (\log\Delta)$. In the tables below these corrections are given in units of the 6th decimal place.

In the ephemeris for 1901, P, B and L − O were calculated with 5-figure logarithms and by differencing have been kept accurate to three places of decimals of a degree, that is to about 2″: it was thought better, however, in computing the ephemeris for 1902 to do this part of the work with 6-figure logarithms. For this reason these angles for 1901 are expressed in decimals of a degree and for 1902 in minutes and seconds. For actual use, values of all these quantities were interpolated and tabulated for every day at an hour as near as convenient to the mean hour of observation. This was generally 7^h Greenwich mean time. The further interpolation necessary to find the value at the epoch of observation was then but slight.

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TABLE XIII.

EPHEMERIS FOR JUPITER.

Greenwich Mean Noon.		P.	L—O.	B.	Δ—O.	δ_1 (log. Δ).	Light Time.
1901.		°	°	°	°		m.
June	24	354·977	143·495	— 1·864	142·184	0	34·931
	26	355·094	143·240	— 1·864	142·349	0	34·899
	28	355·211	142·986	— 1·865	142·514	0	34·877
	30	355·328	142·730	— 1·866	142·680	0	34·865
July	2	355·446	142·474	— 1·866	142·845	0	34·862
	4	355·563	142·218	— 1·867	143·011	0	34·870
	6	355·680	141·963	— 1·867	143·176	0	34·887
	8	355·796	141·710	— 1·868	143·342	+ 1	34·914
	10	355·911	141·459	— 1·869	143·508	1	34·950
	12	356·025	141·212	— 1·870	143·673	1	34·997
	14	356·138	140·970	— 1·869	143·839	1	35·053
	16	356·248	140·732	— 1·868	144·005	1	35·118
	18	356·356	140·499	— 1·868	144·171	1	35·193
	20	356·461	140·272	— 1·868	144·337	1	35·277
	22	356·563	140·053	— 1·867	144·502	1	35·370
	24	356·661	139·840	— 1·866	144·668	2	35·472
	26	356·755	139·636	— 1·865	144·834	2	35·583
	28	356·845	139·441	— 1·864	145·001	2	35·702
	30	356·932	139·254	— 1·864	145·167	2	35·830
Aug.	1	357·015	139·075	— 1·863	145·333	2	35·965
	3	357·094	138·907	— 1·862	145·499	2	36·109
	5	357·168	138·749	— 1·861	145·665	2	36·260
	7	357·236	138·601	— 1·859	145·832	2	36·418
	9	357·299	138·464	— 1·856	145·998	2	36·584
	11	357·357	138·339	— 1·854	146·164	3	36·756
	13	357·410	138·225	— 1·851	146·331	3	36·936
	15	357·457	138·123	— 1·848	146·497	3	37·122
	17	357·499	138·033	— 1·845	146·664	3	37·313
	19	357·535	137·955	— 1·842	146·831	3	37·511
	21	357·566	137·889	— 1·839	146·997	3	37·714
	23	357·591	137·836	— 1·836	147·164	3	37·923
	25	357·610	137·796	— 1·833	147·331	3	38·137
	27	357·622	137·768	— 1·830	147·497	3	38·355
	29	357·629	137·753	— 1·827	147·664	3	38·578
	31	357·631	137·751	— 1·824	147·831	3	38·804
Sept.	2	357·626	137·760	— 1·819	147·998	3	39·035
	4	357·616	137·782	— 1·815	148·165	3	39·269
	6	357·599	137·818	— 1·811	148·332	3	39·506
	8	357·576	137·866	— 1·807	148·499	4	39·747
	10	357·548	137·925	— 1·803	148·666	4	39·990
	12	357·514	137·998	— 1·799	148·833	4	40·236
	14	357·474	138·084	— 1·795	149·000	4	40·484
	16	357·429	138·181	— 1·790	149·167	4	40·734
	18	357·378	138·291	— 1·784	149·335	4	40·985
	20	357·321	138·413	— 1·778	149·502	4	41·238
	22	357·258	138·546	— 1·773	149·669	4	41·491
	24	357·190	138·691	— 1·767	149·837	4	41·746
	26	357·118	138·848	— 1·762	150·004	4	42·001
	28	357·039	139·015	— 1·756	150·172	4	42·256
	30	356·956	139·194	— 1·749	150·339	+ 4	42·511

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TABLE XIII.—*continued.*

Greenwich Mean Noon.		P.	L—O.	B.	Δ—O.	δ_2 (log. Δ).	Light Time.
1902.		° ′ ″	° ′ ″	° ′ ″	°		m.
May	31	340 15 15	181 41 39	− 0 3 14	171·040	+ 40	38·454
June	2	340 14 41	181 43 26	− 0 2 41	171·214	+ 40	38·210
	4	340 14 22	181 44 26	− 0 2 10	171·387	+ 39	37·972
	6	340 14 16	181 44 40	− 0 1 40	171·560	+ 39	37·736
	8	340 14 22	181 44 7	− 0 1 12	171·734	+ 38	37·504
	10	340 14 40	181 42 49	− 0 0 46	171·908	+ 37	37·278
	12	340 15 12	181 40 45	− 0 0 21	172·081	+ 37	37·055
	14	340 15 56	181 37 57	0 0 0	172·254	+ 36	36·838
	16	340 16 54	181 34 22	+ 0 0 22	172·428	+ 35	36·625
	18	340 18 4	181 30 2	+ 0 0 42	172·602	+ 34	36·418
	20	340 19 26	181 24 58	+ 0 1 0	172·775	+ 33	36·216
	22	340 21 0	181 19 11	+ 0 1 16	172·948	+ 32	36·021
	24	340 22 48	181 12 41	+ 0 1 30	173·122	+ 31	35·832
	26	340 24 48	181 5 28	+ 0 1 44	173·296	+ 29	35·650
	28	340 27 0	180 57 33	+ 0 1 56	173·469	+ 28	35·475
	30	340 29 24	180 48 57	+ 0 2 6	173·643	+ 27	35·306
July	2	340 32 0	180 39 41	+ 0 2 14	173·817	+ 26	35·144
	4	340 34 48	180 29 47	+ 0 2 21	173·991	+ 24	34·990
	6	340 37 47	180 19 17	+ 0 2 26	174·165	+ 23	34·844
	8	340 40 56	180 8 11	+ 0 2 29	174·339	+ 21	34·706
	10	340 44 17	179 56 31	+ 0 2 30	174·513	+ 20	34·576
	12	340 47 48	179 44 19	+ 0 2 29	174·637	+ 18	34·454
	14	340 51 28	179 31 38	+ 0 2 26	174·861	+ 17	34·342
	16	340 55 18	179 18 29	+ 0 2 21	175·035	+ 15	34·239
	18	340 59 16	179 4 54	+ 0 2 15	175·209	+ 14	34·145
	20	341 3 22	178 50 56	+ 0 2 8	175·384	+ 12	33·060
	22	341 7 37	178 36 38	+ 0 2 0	175·558	+ 10	33·983
	24	341 11 59	178 22 0	+ 0 1 51	175·732	+ 9	33·916
	26	341 16 27	178 7 6	+ 0 1 40	175·906	+ 7	33·858
	28	341 21 1	177 51 58	+ 0 1 28	176·080	+ 5	33·810
	30	341 25 40	177 36 38	+ 0 1 14	176·255	+ 4	33·771
Aug.	1	341 30 23	177 21 9	+ 0 1 0	176·430	+ 5	33·742
	3	341 35 10	177 5 34	+ 0 0 44	176·605	+ 3	33·723
	5	341 40 0	176 49 56	+ 0 0 28	176·780	+ 1	33·714
	7	341 44 51	176 34 17	+ 0 0 10	176·955	− 1	33·715
	9	341 49 42	176 18 41	− 0 0 9	177·130	− 3	33·726
	11	341 54 35	176 3 10	− 0 0 27	177·304	− 4	33·747
	13	341 59 51	175 47 47	− 0 0 47	177·479	− 6	33·778
	15	342 4 14	175 32 34	− 0 1 7	177·654	− 8	33·819
	17	342 9 0	175 17 36	− 0 1 28	177·829	− 10	33·870
	19	342 13 42	175 2 53	− 0 1 49	178·004	− 11	33·929
	21	342 18 21	174 48 30	− 0 2 11	178·179	− 13	33·998
	23	342 22 54	174 34 25	− 0 2 32	178·354	− 14	34·076
	25	342 27 21	174 20 44	− 0 2 54	178·529	− 16	34·165
	27	342 31 41	174 7 28	− 0 3 15	178·704	− 18	34·262
	29	342 35 52	173 54 39	− 0 3 37	178·878	− 19	34·367
	31	342 39 56	173 42 20	− 0 3 59	179·053	− 21	34·482
Sept.	2	342 43 50	173 30 32	− 0 4 20	179·229	− 23	34·605
	4	342 47 33	173 19 18	− 0 4 42	179·405	− 24	34·736
	6	342 51 5	173 8 40	− 0 5 2	179·580	− 25	34·876
	8	342 54 26	172 58 40	− 0 5 22	179·756	− 27	35·024

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TABLE XIII.—*continued.*

Greenwich Mean Noon.	P.	L—O.	B.	Λ—O.	δ_2 (log Δ).	Light Time.
1902.	° ′ ″	° ′ ″	° ′ ″	°		m.
Sept. 10	342 57 34	172 49 19	− 0 5 41	179·931	− 28	35·180
12	343 0 30	172 40 38	− 0 6 0	180·106	− 30	35·342
14	343 3 12	172 32 38	− 0 6 18	180·282	− 31	35·513
16	343 5 39	172 25 21	− 0 6 36	180·457	− 32	35·690
18	343 7 52	172 18 47	− 0 6 53	180·632	− 33	35·874
20	343 9 51	172 12 57	− 0 7 10	180·808	− 34	36·063
22	343 11 36	172 7 52	− 0 7 24	180·983	− 35	36·260
24	343 13 5	172 3 33	− 0 7 38	181·159	− 36	36·462
26	343 14 18	172 0 0	− 0 7 50	181·334	− 37	36·670
28	343 15 16	171 57 14	− 0 8 2	181·510	− 37	36·883
30	343 15 57	171 55 15	− 0 8 12	181·686	− 38	37·102
Oct. 2	343 16 22	171 54 4	− 0 8 22	181·862	− 38	37·324
4	343 16 39	171 53 51	− 0 8 29	182·038	− 38	37·550
6	343 16 22	171 54 6	− 0 8 36	182·214	− 39	37·782
8	343 15 58	171 57 18	− 0 8 42	182·390	− 39	38·016

The values of the following angles at Greenwich mean noon on different dates are now tabulated, viz. :—

$g\,t + \beta + 180° - b_4 t - \gamma_4$ (Souillart) or $\omega_1 - O$ (Marth)
$g_1 t + \beta_1 + 180 - b_4 t - \gamma_4$,, $\omega_2 - O$,,
$g_2 t + \beta_2 + 180 - b_4 t - \gamma_4$,, $\omega_3 - O$,,
$g_3 t + \beta_3 + 180 - b_4 t - \gamma_4$,, $\omega_4 - O$,,

and

$b_4 t + \gamma_4 - b\,t - \gamma$ (Souillart) or G_1 (Marth)
$b_4 t + \gamma_4 - b_1 t - \gamma_1$,, G_2 ,,
$b_4 t + \gamma_4 - b_2 t - \gamma_2$,, G_3 ,,
$b_4 t + \gamma_4 - b_3 t - \gamma_3$,, G_4 ,,

	ω_1—O.	ω_2—O.	ω_3—O.	ω_4—O.	G_1.	G_2.	G_3.	G_4.
1901.	°	°	°	°	°	°	°	°
June 20	25·1366	200·0953	209·9955	153·7455	291·2126	59·7914	348·5796	310·6409
July 20	29·6141	201·4022	210·2114	153·8044	295·4453	60·7817	348·7902	310·6979
Aug. 19	34·0915	202·7092	210·4273	153·8632	299·6780	61·7720	349·0008	310·7548
Sept. 18	38·5690	204·0161	210·6432	153·9221	303·9107	62·7623	349·2114	310·8118
Oct. 18	43·0465	205·3231	210·8592	153·9810	308·1433	63·7525	349·4219	310·8687
1902.								
May 31	76·6276	215·1253	212·4786	154·4227	339·8885	71·1797	351·0013	311·2959
June 30	81·1051	216·4323	212·6945	154·4816	344·1211	72·1700	351·2119	311·3528
July 30	85·5826	217·7392	212·9104	154·5405	348·3539	73·1602	351·4224	311·4098
Aug. 29	90·0601	219·0462	213·1264	154·5994	352·5866	74·1505	351·6330	311·4667
Sept. 28	94·5376	220·3531	213·3423	154·6583	356·8193	75·1408	351·8436	311·5237
Oct. 28	99·0150	221·6601	213·5582	154·7172	1·0519	76·1310	352·0541	311·5805

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The longitudes of the nodes and the inclinations of the orbits have been calculated by the same formulæ as Marth used: for convenience of reference they are given here. By Γ and γ are denoted the longitude of the node and the inclination with reference to an adopted fundamental plane, which is not, strictly speaking, the plane defined on p. 58. But the values thus calculated are those adopted for correction by the observations and the corrected nodes and inclinations refer to the above adopted planes of reference.

$$\gamma_1 \sin(O-\Gamma_1) = -33''.035 \sin G_2 - 3''.914 \sin G_3 + 1''.394 \sin G_4 - 0''.915 \sin V - 4''.803 \sin G_1$$
$$\gamma_1 \cos(O-\Gamma_1) = +7''.24 - 33.035 \cos G_2 - 3.914 \cos G_3 + 1.394 \cos G_4 - 0.915 \cos V - 4.803 \cos G_1$$

$$\gamma_2 \sin(O-\Gamma_2) = -1689''.882 \sin G_2 - 102''.264 \sin G_3 - 15''.711 \sin G_4 - 2''.817 \sin V - 0''.372 \sin(V-G_2)$$
$$\gamma_2 \cos(O-\Gamma_2) = 64''.34 - 1689.882 \cos G_2 - 102.264 \cos G_3 - 15.711 \cos G_4 - 2.817 \cos V - 0.372 \cos(V-G_2)$$

$$\gamma_3 \sin(O-\Gamma_3) = +57''.437 \sin G_2 - 644''.173 \sin G_3 - 97''.328 \sin G_4 - 6''.389 \sin V - 0''.368 \sin(V-G_3)$$
$$\gamma_3 \cos(O-\Gamma_3) = 295''.07 + 57.437 \cos G_2 - 644.173 \cos G_3 - 97.328 \cos G_4 - 6.389 \cos V - 0.368 \cos(V-G_3)$$

$$\gamma_4 \sin(O-\Gamma_4) = +1''.546 \sin G_2 + 125''.267 \sin G_3 - 811''.414 \sin G_4 - 13''.694 \sin V - 1''.145 \sin(V-G_4)$$
$$\gamma_4 \cos(O-\Gamma_4) = 1449''.35 + 1.546 \cos G_2 + 125.267 \cos G_3 - 811.414 \cos G_4 - 13.694 \cos V - 1.145 \cos(V-G_4)$$

In this V is written for $2(O-\Lambda)$.

TABLE XIV.

Nodes and Inclinations of Satellites.

		Node (O—Γ).	Inclin. γ.	Node (O—Γ).	Inclin. γ.
		I.		II.	
1901					
June	20	240°.6	0°.0080	237°.91	0°.4692
July	20	241.2	0.0081	238.89	0.4691
August	19	241.7	0.0082	239.87	0.4689
September	18	242.3	0.0083	240.86	0.4687
October	18	242.9	0.0084	241.85	0.4685
1902					
May	31	248.8	0.0090	249.27	0.4670
June	30	249.8	0.0091	250.26	0.4668
July	30	250.8	0.0092	251.26	0.4665
August	29	251.8	0.0093	252.25	0.4663
September	28	252.9	0.0093	253.24	0.4661
October	28	254.0	0.0094	254.23	0.4658
		III.		IV.	
1901					
June	20	146°.71	0°.1237	29°.02	0°.3311
July	20	147.03	0.1239	29.07	0.3306
August	19	147.35	0.1241	29.13	0.3300
September	18	147.66	0.1243	29.19	0.3297
October	18	147.96	0.1245	29.25	0.3294

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TABLE XIV.—*continued.*

	Node (O—Γ).	Inclin. γ.	Node (O—Γ).	Inclin. γ.
	III.		IV.	
1902				
May 31	150°·10	0°·1260	29°·78	0°·3279
June 30	150·36	0·1262	29·85	0·3279
July 30	150·63	0·1263	29·92	0·3278
August 29	150·89	0·1265	29·99	0·3278
September 28	151·13	0·1266	30·06	0·3279
October 28	151·38	0·1267	30·13	0·3279

For convenience of reference, the meaning of the arguments of the inequalities in longitude and radius vector and the inequalities themselves have been taken from Marth's paper and are printed here:—

Let $l_1\ l_2\ l_3\ l_4$ be the mean longitudes reckoned from O, *i.e.*, the angles $l+180°-\beta_4 t-\gamma_4$, $l'+180°-\beta_4 t-\gamma_4$, l'' ..., l'''... of Souillart.

Let $\omega_1\ \omega_2\ \omega_3\ \omega_4$ be the angles $\omega_1-\mathrm{O}$, $\omega_2-\mathrm{O}$, etc., of Marth, or $gt+\beta+180°-\beta_4 t-\gamma$, etc. of Souillart (see p. 65).

Then the arguments in the ephemeris tabulated below represent the following angles :—

I.

$a_1 = 2(l_1-l_2)=4\beta_1$
$\beta_1 = l_2-l_3+180°$
$\gamma_1 = l_1-\omega_1$
$\delta_1 = l_1-\omega_3$
$\epsilon_1 = l_2-2l_3+\omega_1+180°$
$\zeta_1 = l_2-2l_3+\omega_2$
$\eta_1 = l_2-2l_3+\omega_3$
$\theta_1 = l_2-2l_3+\omega_4$

II.

$a_2 = l_1-l_2+180°=2\beta_2$
$\beta_2 = l_2-l_3+180°=\beta_1$
$\gamma_2 = l_2-\omega_2$
$\delta_2 = l_2-\omega_3$
$\epsilon_2 = l_2-\omega_4$
$\zeta_2 = l_2-2l_3+\omega_2=\zeta_1=\xi_3$
$\eta_2 = l_2-2l_3+\omega_3+180°=\eta_3+180°$
$\theta_2 = l_2-2l_3+\omega_4+180°=\theta_3+180°$
$\iota_2 = l_2-2l_3+\omega_1=\epsilon_1+180°$
$\kappa_2 = 2l_1-3l_2+\omega_3$
$\lambda_2 = 2l_1-3l_2+\omega_4$

III.

$a_3 = l_3-\omega_3$
$\beta_3 = l_3-\omega_4$
$\gamma_3 = l_2-l_3+180°=\beta_1$
$\delta_3 = l_3-l_4+180°$
$\epsilon_3 = 2l_2-3l_3+\omega_3+180°$
$\zeta_3 = l_2-2l_3+\omega_2=\xi_2=\xi_1$
$\eta_3 = l_2-2l_3+\omega_3=\eta_2-180°$
$\theta_3 = l_2-2l_3+\omega_4=\theta_2-180°$
$\iota_3 = l_3-\omega_2+180°$

IV.

$a_4 = l_4-\omega_4$
$\beta_4 = l_3-l_4+180°=\delta_3$
$\gamma_4 = l_4-\omega_3+180°$
$\delta_4 = l_4-2l_0+\omega_4$
$\epsilon_4 = 2(l_4-l_0)$
$\zeta_4 = l_3-2l_4+\omega_4$
$\eta_4 = l_2-l_4$
$\theta_4 = l_3-2l_4+\omega+180°$

l_0 in δ_4 and ϵ_4 denotes the heliocentric mean longitude of Jupiter, reckoned from O.

In the ephemeris the arguments are expressed in decimal parts of the circumference of the circle.

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TABLE XV.

MEAN LONGITUDES AND ARGUMENTS OF INEQUALITIES.

First Satellite, 1901 and 1902.

Greenwich Noon.	l_1—O.	S_1.	α_1.	β_1.	γ_1.	δ_1.	ϵ_1.	ζ_1.	η_1.	θ_1.
1901.										
June 20	357°·0884	−0°·0123	0·3122	0·328	0·922	0·409	0·390	0·376	0·404	0·247
30	231·9780	·0123	·9852	·746	·570	·061	·415	·398	·425	·268
July 10	106·8675	·0123	·6582	·165	·218	·713	·440	·420	·446	·289
20	341·7571	·0124	·3312	·583	·867	·365	·464	·441	·466	·309
30	216·6466	·0124	·0042	·001	·515	·017	·489	·463	·487	·330
Aug. 9	91·5362	·0124	·6772	·419	·163	·669	·514	·485	·508	·351
19	326·4257	·0124	·3502	·837	·812	·322	·538	·507	·528	·371
29	201·3153	·0125	·0232	·256	·460	·974	·563	·529	·549	·392
Sept. 8	76·2048	·0125	·6962	·674	·108	·626	·588	·551	·570	·413
18	311·0944	·0125	·3693	·092	·757	·279	·612	·572	·590	·433
28	185·9840	·0126	·0423	·510	·405	·931	·637	·594	·611	·454
Oct. 8	60·8735	−0·0126	0·7153	0·929	0·053	0·583	0·662	0·616	0·632	0·474
1902.										
May 31	0·7782	−0·0127	0·0311	0·258	0·789	0·412	0·242	0·126	0·119	0·958
June 10	235·6678	·0127	·7041	·676	·438	·064	·266	·148	·140	·978
20	110·5573	·0127	·3771	·094	·086	·717	·291	·170	·161	·999
30	345·4469	·0127	·0501	·513	·734	·369	·316	·192	·181	·020
July 10	220·3364	·0128	·7231	·931	·383	·021	·341	·214	·202	·040
20	95·2260	·0128	·3961	·349	·031	·673	·365	·235	·223	·061
30	330·1156	·0128	·0691	·768	·679	·326	·390	·257	·244	·081
Aug. 9	205·0051	·0128	·7421	·186	·327	·978	·415	·279	·264	·102
19	79·8947	·0128	·4151	·604	·976	·630	·439	·300	·285	·123
29	314·7842	·0128	·0881	·022	·624	·282	·464	·322	·306	·143
Sept. 8	189·6738	·0128	·7611	·440	·272	·935	·489	·344	·327	·164
18	64·5634	·0129	·4341	·859	·921	·587	·513	·366	·347	·184
28	299·4529	·0129	·1071	·277	·569	·239	·538	·388	·368	·205
Oct. 8	174·3425	−0·0129	0·7801	0·695	0·217	0·892	0·562	0·409	0·389	0·226

Second Satellite, 1901.

Greenwich Noon.	l_2—O.	S_2.	α_2.	β_2.	γ_2.	δ_2.	ϵ_2.	ζ_2.	η_2	θ_2.	ι_2.	κ_2.	λ_2.
1901.													
June 20	300°·9011	+0°·0073	0·65608	0·328	0·280	0·253	0·409	0·376	0·904	0·747	0·890	0·060	0·903
30	234·6483	73	·49258	·746	·095	·069	·225	·398	·925	·768	·915	·917	·760
July 10	168·3956	73	·32909	·165	·910	·884	·041	·420	·946	·789	·940	·774	·617
20	102·1428	73	·16559	·583	·724	·699	·856	·441	·966	·809	·964	·631	·475
30	35·8901	73	·00209	·001	·539	·515	·672	·463	·987	·830	·989	·488	·332
Aug. 9	329·6374	72	·83860	·419	·354	·330	·488	·485	·008	·851	·014	·345	·189
19	263·3846	72	·67511	·837	·168	·147	·304	·507	·028	·871	·038	·203	·046
29	197·1319	72	·51161	·256	·983	·963	·120	·529	·049	·892	·063	·060	·903
Sept. 8	130·8791	72	·34812	·674	·798	·778	·936	·551	·070	·913	·088	·917	·760
18	64·6264	71	·18463	·092	·613	·594	·752	·572	·090	·933	·112	·775	·617
28	358·3737	71	·02113	·510	·428	·410	·568	·594	·111	·954	·137	·632	·475
Oct. 8	292·1209	+0·0071	0·85764	0·929	0·243	0·226	0·384	0·616	0·132	0·974	0·162	0·489	0·332

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TABLE XV.—*continued.*

Second Satellite, 1902.

Greenwich Noon.	l_2—O.	S_2.	a_2.	β_2.	γ_2.	δ_2.	ϵ_2.	ζ_2.	η_2.	θ_2.	ι_2.	κ_2.	λ_2.
1902.													
May 31	355·1814	+0·0057	0·51555	0·258	0·389	0·396	0·558	0·126	0·619	0·458	0·742	0·635	0·473
June 10	288·9287	56	·35205	·676	·204	·212	·374	·148	·640	·478	·766	·492	·331
20	222·6759	55	·18856	·094	·019	·028	·189	·170	·661	·499	·791	·349	·188
30	156·4232	54	·02507	·513	·833	·844	·005	·192	·681	·520	·816	·206	·045
July 10	90·1704	53	·86157	·931	·648	·659	·821	·214	·702	·540	·841	·064	·902
20	23·9177	52	·69808	·349	·463	·475	637	·235	·723	·561	·865	·921	·759
30	317·6649	51	·53459	·768	·278	·291	·453	·257	·744	·581	·890	·778	·616
Aug. 9	251·4122	50	·37109	·186	·092	·107	·269	·279	·764	·602	·915	·635	·473
19	185·1594	50	·20760	·604	·907	·923	·085	·300	·785	·623	·939	·493	·330
29	118·9067	49	·04410	·022	·722	·738	·901	·322	·806	·643	·964	·350	·187
Sept. 8	52·6540	48	·88061	·440	·537	·554	·717	·344	·827	·664	·989	·207	·044
18	346·4012	47	·71712	·859	·351	·370	·533	·366	·847	·684	·013	·064	·902
28	280·1485	46	·55362	·277	·166	·186	·349	·388	·868	·705	·038	·922	·759
Oct. 8	213·8957	+0·0045	0·39013	0·695	0·981	0·001	0·165	0·409	0·889	0·726	0·063	0·779	0·616

Third Satellite, 1901 and 1902.

Greenwich Noon.	l_3—O.	S_3.	a_3.	β_3.	γ_3.	δ_3.	ϵ_3.	ζ_3.	η_3.	θ_3.	ι_3.
1901.											
June 20	2·8074	+0·0142	0·4245	0·5807	0·3280	0·116	0·732	0·376	0·404	0·247	0·952
30	145·9835	·0142	·8220	·9784	·7463	·915	·171	·398	·425	·268	·349
July 10	289·1596	·0142	·2195	·3761	·1645	·714	·610	·420	·446	·289	·746
20	72·3357	·0142	·6170	·7737	·5828	·512	·049	·441	·466	·309	·142
30	215·5118	·0142	·0145	·1714	·0010	·310	·488	·463	·487	·330	·539
Aug. 9	358·6880	·0142	·4120	·5691	·4193	·108	·927	·485	·508	·351	·935
19	141·8641	·0142	·8095	·9667	·8376	·907	·366	·507	·528	·371	·331
29	285·0402	·0141	·2070	·3644	·2558	·706	·805	·529	·549	·392	·727
Sept. 8	68·2163	·0141	·6045	·7620	·6741	·504	·244	·551	·570	·413	·124
18	211·3924	·0141	·0021	·1596	·0923	·303	·683	·572	·590	·433	·521
28	354·5685	·0141	·3996	·5573	·5106	·102	·122	·594	·611	·454	·917
Oct. 8	137·7446	+0·0141	0·7971	0·9550	0·9288	0·900	0·561	0·616	0·632	0·474	0·314
1902.											
May 31	82·3830	+0·0124	0·6386	0·7999	0·2578	0·665	0·377	0·126	0·119	0·958	0·131
June 10	225·5591	·0123	·0361	·1976	·6760	·463	·816	·148	·140	·978	·528
20	8·7352	·0122	·4336	·5952	·0943	·262	·255	·170	·161	·999	·924
30	151·9113	·0121	·8312	·9929	·5125	·060	·694	·192	·181	·020	·321
July 10	295·0874	·0120	·2287	·3905	·9308	·859	·133	·214	·202	·040	·717
20	78·2635	·0119	·6262	·7882	·3490	·657	·572	·235	·223	·061	·114
30	221·4396	·0118	·0237	·1859	·7673	·456	·011	·257	·244	·081	·510
Aug. 9	4·6157	·0117	·4212	·5835	·1855	·254	·450	·279	·264	·102	·907
19	147·7918	·0116	·8187	·9812	·6038	·053	·889	·300	·285	·123	·303
29	290·9679	·0115	·2162	·3788	·0221	·851	·328	·322	·306	·143	·700
Sept. 8	74·1440	·0114	·6137	·7765	·4403	·650	·767	·344	·327	·164	·096
18	217·3201	·0113	·0112	·1742	·8586	·448	·206	·366	·347	·184	·493
28	0·4962	·0111	·4088	·5718	·2768	·247	·645	·388	·368	·205	·889
Oct. 8	143·6724	+0·0110	0·8063	0·9695	0·6951	0·045	0·084	0·409	0·389	0·226	0·286

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TABLE XV.—*continued.*

Fourth Satellite, 1901 and 1902.

Greenwich Noon.	l_4—O.	S_4.	α_4.	β_4.	γ_4.	δ_4.	ϵ_4.	ζ_4.	η_4.	θ_4.
1901.										
June 20	141°·0462	+0°·0288	0·96473	0·116	0·308	0·000	0·965	0·651	0·444	0·308
30	356·7569	·0288	·56387	·915	·907	·595	·159	·851	·661	·508
July 10	212·4677	·0288	·16301	·714	·506	·190	·353	·050	·878	·707
20	68·1784	·0288	·76215	·512	·105	·784	·546	·250	·094	·907
30	283·8892	·0288	·36129	·310	·704	·379	·740	·449	·311	·106
Aug. 9	139·5999	·0288	·96043	·108	·303	·974	·934	·648	·528	·306
19	355·3107	·0287	·55958	·907	·902	·568	·127	·847	·745	·505
29	211·0214	·0287	·15872	·706	·501	·163	·321	·047	·962	·705
Sept. 8	66·7322	·0286	·75786	·504	·100	·758	·515	·246	·179	·904
18	282·4429	·0286	·35700	·303	·699	·352	·709	·446	·395	·104
28	138·1536	·0285	·95614	·102	·298	·947	·903	·645	·412	·303
Oct. 8	353·8644	+0·0285	0·55528	0·900	0·897	0·542	0·097	0·845	0·629	0·503
1902.										
May 31	23·0667	+0·0249	0·63512	0·665	0·974	0·543	0·178	0·530	0·923	0·191
June 10	238·7774	·0247	·23426	·463	·573	·138	·372	·729	·139	·390
20	94·4882	·0245	·83340	·262	·172	·732	·565	·928	·356	·590
30	310·1989	·0242	·43255	·060	·771	·326	·758	·128	·573	·789
July 10	165·9097	·0240	·03169	·859	·370	·921	·952	·327	·790	·989
20	21·6204	·0237	·63083	·657	·969	·516	·146	·526	·006	·189
30	237·3311	·0235	·22997	·456	·568	·110	·339	·726	·223	·388
Aug. 9	93·0419	·0232	·82911	·254	·167	·704	·532	·925	·440	·588
19	308·7526	·0229	·42826	·053	·766	·299	·726	·125	·657	·787
29	164·4634	·0227	·02740	·851	·365	·894	·920	·324	·873	·987
Sept. 8	20·1741	·0224	·62654	·650	·964	·488	·113	·523	·090	·186
18	235·8848	·0221	·22568	·448	·563	·082	·306	·723	·307	·386
28	91·5966	·0218	·82482	·247	·162	·677	·500	·922	·524	·585
Oct. 8	307·3063	+0·0215	0·42397	0·045	0·761	0·272	0·694	0·121	0·741	0·785

The inequalities in radius vector and longitude are given on pp. 73–5. The inequalities in radius vector are expressed by Souillart in terms of semi-diameters of Jupiter's equator, but Marth expresses them in terms of the satellite's mean distance by means of the values given by Souillart, II. p. 10. These inequalities are all made positive by the addition of constants, the sum of which has been subtracted from the first term depending on the argument α. In his tables Marth tabulates the values of the various inequalities for values of the arguments α, β, etc.

The columns headed S_1 S_2 S_3 S_4 in Table XV contain the sums of all the inequalities in longitudes which are independent of the longitudes of the satellites: they are as follows:—

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S_1	S_2	S_3	S_4	Coefficient of.
+ 2″·206	− 36″·617	− 48″·470	− 115″·012	sin M
− 0·176	− 3·003	− 4·416	− 10·425	sin W
...	− 0·336	− 0·475	− 1·122	sin W_1
− 35·839	− 10·167	− 0·204	...	sin G_2
...	− 5·682	− 8·509	+ 6·187	sin G_3
+ 0·436	− 6·044	− 9·284	− 3·241	sin G_4
− 9·459	− 3·929	− 1·677	...	sin (G_2-G_3)
+ 3·536	− 1·541	− 4·072	− 3·150	sin (G_3-G_4)
− 1·488	− 0·550	− 0·196	...	sin (G_2-G_4)
− 1·320	+ 0·464	+ 2·388	− 2·886	sin (W_3-W_4)

M denotes the mean anomaly of Jupiter, and W and W_1 the values of $5l^{v} - 2l^{iv}$ $- 16^{\circ}\ 38'$ and $l^{iv} - 2l^{v} - 1^{\circ}\ 30'$ in the notation of Le Verrier's Tables.

The mean epochs of the two sets of observations were Greenwich Mean Noon on August 13, 1901 and August 7, 1902: the Julian days being 2415610 and 2415969 respectively. The adopted elements for these epochs are given below: these are the values to be corrected by the observations.

The adopted mass of Jupiter is

$$1 : 1047{\cdot}70.$$

and the logarithms of the corresponding semi-major axes of the satellites' orbits are—

2·764534 2·966232 3·169012 3·414255

These are expressed in seconds of arc at unit distance.

The adopted mean daily motions and periods are—

		d. h. m. s.
203°·4889560	and	1 18 27 33·7
101·3747256	,,	3 13 13 42·0
50·3176104	,,	7 3 42 33·1
21·5710740	,,	16 16 32 11·3

The mean longitudes of the epochs are

	1901.	1902.
ϵ_1	185°·4920	158°·0272
ϵ_2	15·1363	48·6627
ϵ_3	199·9584	263·9805
ϵ_4	225·8842	49·5998

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The values of h and k (see pp. 79, 80) are

	1901.	1902.
h_1	+0°·0010	+0°·0022
k_1	−0·0005	+0·0024
h_2	−0·0107	−0·0144
k_2	−0·0413	−0·0393
h_3	−0·0352	−0·0403
k_3	−0·1707	−0·1683
h_4	+0·3749	+0·3669
k_4	−0·7328	−0·7371

These are referred to the descending nodes of the fundamental planes on Jupiter's actual orbit, as also are the following values of the nodes and inclinations.

	Nodes.		Inclinations.	
	1901.	1902.	1901.	1902.
I.	118°·4	108°·9	0°·0082	0°·0092
II.	120·33	108·48	0·4689	0·4664
III.	212·71	209·30	0·1241	0·1264
IV.	330·88	330·06	0·3301	0·3278

The corresponding values of $p = i \sin(-\Omega)$ and $q = i \cos(-\Omega)$ are—

	1901.	1902.		1901.	1902.
p_1	−0·0072	−0·0087	p_3	+0·0670	+0·0618
q_1	−0·0039	−0·0030	q_3	−0·1044	−0·1103
p_2	−0·4047	−0·4424	p_4	+0·1606	+0·1636
q_2	−0·2368	−0·1479	q_4	+0·2883	+0·2841

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Inequalities in Longitude and Radius Vector.

First Satellite.

Radius Vector.	Longitude.	Period in Days.
$\rho_1 = 1{\cdot}0000096$	v_1 = Mean Longitude	
$-\ 0{\cdot}0037751 \cos \alpha_1$	$+\ 1554''{\cdot}759 \sin \alpha_1$	1·76
$-\ 119 \cos 2\alpha_1$	$+\ 5{\cdot}029 \sin 2\alpha_1$	
$-\ 703$		
	$+\ 12{\cdot}357 \sin 2\beta_1$	7·06
$-\ 173 \cos 2\beta_1$	$-\ 6{\cdot}268 \sin 3\beta_1$	
$+\ 121 \cos 3\beta_1$		
$+\ 294$	$+\ 8{\cdot}796 \sin \gamma_1$	1·77
$-\ 213 \cos \gamma_1$	$+\ 8{\cdot}145 \sin \delta_1$	1·77
$+\ 213$		
	$+\ 11{\cdot}000 \sin \epsilon_1$	400
$-\ 196 \cos \delta_1$		
$+\ 196$	$+\ 23{\cdot}041 \sin \zeta_1$	454
	$+\ 19{\cdot}689 \sin \eta_1$	476
	$+\ 10{\cdot}179 \sin \theta_1$	476

Second Satellite.

$\rho_2 = 1{\cdot}0000421$	v_2 = Mean Longitude	
$-\ 0{\cdot}0090095 \cos \alpha_2$	$+\ 3692''{\cdot}981 \sin \alpha_2$	3·52
$-\ 456 \cos 2\alpha_2$	$+\ 22{\cdot}434 \sin 2\alpha_2$	
$+\ 062 \cos 3\alpha_2$	$+\ 1{\cdot}491 \sin 3\alpha_2$	
$-\ 5060$		
	$+\ 48{\cdot}578 \sin \beta_2$	7·06
$-\ 672 \cos \beta_2$	$-\ 20{\cdot}498 \sin 3\beta_2$	
$+\ 634 \cos 3\beta_2$	$-\ 1{\cdot}505 \sin 5\beta_2$	
$+\ 61 \cos 5\beta_2$		
$+\ 770$	$+\ 45{\cdot}246 \sin \gamma_2$	
$-\ 1096 \cos \gamma_2$	$+\ 79{\cdot}680 \sin \delta_2$	3·55
$+\ 1096$		
	$+\ 42{\cdot}593 \sin \epsilon_2$	
$-\ 1931 \cos \delta_2$		
$+\ 1931$	$+\ 95{\cdot}284 \sin \zeta_2$	454

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Second Satellite—continued.

Radius Vector.	Longitude.	Period in Days.
$\rho_2 = 1{\cdot}0000421$	v_2 = Mean Longitude	
$- 1032 \cos \epsilon_2$		
$+ 1032$	$+ 120''{\cdot}671 \sin \eta_2$	476
	$+ 62{\cdot}966 \sin \theta_2$	476
$- 152 \cos \kappa_2$		
$+ 152$	$+ 6{\cdot}609 \sin \iota_2$	400
$- 79 \cos \lambda_2$	$+ 6{\cdot}947 \sin \kappa_2$	
$+ 79$		3·5
	$+ 3{\cdot}624 \sin \lambda_2$	

Third Satellite.

$\rho_3 = 1{\cdot}0000009$	v_3 = Mean Longitude	
$- 11219 \cos \alpha_3$	$+ 462''{\cdot}880 \sin \alpha_3$	
$- 10 \cos 2\alpha_3$		7·16
$- 13050$	$+ 241{\cdot}905 \sin \beta_3$	
$- 5864 \cos \beta_3$	$+ 228{\cdot}473 \sin \gamma_3$	7·06
$- 8 \cos 2\beta_3$	$- 4{\cdot}084 \sin 2\gamma_3$	
$+ 5872$	$+ 2{\cdot}097 \sin 3\gamma_3$	
	$- 0{\cdot}286 \sin 4\gamma_3$	
$- 5827 \cos \gamma_3$		
$+ 155 \cos 2\gamma_3$	$+ 14{\cdot}620 \sin \delta_3$	12·5
$+ 24 \cos 3\gamma_3$	$+ 49{\cdot}710 \sin 2\delta_3$	
$+ 14 \cos 4\gamma_3$	$- 3{\cdot}503 \sin 3\delta_3$	
$+ 5634$	$+ 0{\cdot}814 \sin 4\delta_3$	
	$- 0{\cdot}258 \sin 5\delta_3$	
$- 221 \cos \delta_3$		
$- 1320 \cos 2\delta_3$	$+ 2{\cdot}012 \sin \epsilon_3$	7·0
$+ 117 \cos 3\delta_3$		
$- 31 \cos 4\delta_3$	$+ 15{\cdot}816 \sin \zeta_3$	454
$+ 11 \cos 5\delta_3$		
$+ 1444$	$+ 23{\cdot}602 \sin \eta_3$	476
$- 50 \cos \epsilon_3$	$+ 12{\cdot}062 \sin \theta_3$	476
$+ 50$		
	$+ 1{\cdot}936 \sin \iota_3$	7·1
$- 50 \cos \iota_3$		
$+ 50$		

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Fourth Satellite.

Radius Vector.	Longitude.	Period in Days.
$\rho_4 = 1{\cdot}0000265$	v_4 = Mean Longitude	
$- 72597 \cos \alpha_4$	$+ 2995''{\cdot}688 \sin \alpha_4$	16·69
$- 264 \cos 2\alpha_4$	$+ 13{\cdot}592 \sin 2\alpha_4$	
$- 3204$		
	$+ 9{\cdot}189 \sin \beta_4$	12·52
$- 1105 \cos \beta_4$	$- 4{\cdot}571 \sin 2\beta_4$	
$+ 201 \cos 2\beta_4$	$+ 0{\cdot}951 \sin 3\beta_4$	
$- 047 \cos 3\beta_4$		
$+ 951$	$+ 59{\cdot}063 \sin \gamma_4$	16·69
$- 1431 \cos \gamma_4$	$+ 22{\cdot}527 \sin \delta$	
$+ 1431$		
	$+ 4{\cdot}208 \sin$	8·4
$- 547 \cos \delta_4$		
$+ 547$	$+ 2{\cdot}535 \sin \zeta_4$	50
$- 148 \cos \epsilon_4$	$+ 1{\cdot}682 \sin \eta_4$	
$+ 148$		
	$+ 1{\cdot}688 \sin \theta_4$	4·5
$- 43 \cos \zeta_4$		
$+ 43$		
$- 84 \cos \eta_4$		
$+ 84$		

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CHAPTER IV.

COMPUTED QUANTITIES.

Co-ordinates and Coefficients. Equations of Condition.

FROM the data in the foregoing chapter the distance and position-angle of each pair of satellites were calculated for the epochs of observation corrected for the aberration of light, that is, antedated by the amount of the light time. The epochs of observation are the Greenwich Mean Times which correspond to the means of the sidereal times of the four pointings in distance and position-angle respectively. In reducing from the Cape sidereal time to Greenwich Mean Time, the adopted longitude of the observatory was $1^h\ 13^m\ 54^s{\cdot}8$ East.

The mean longitude of each satellite and the arguments of its inequalities in longitude and radius vector were then computed for the corrected Greenwich Mean Time; and the inequalities themselves were taken from Marth's tables, with the corrections already noticed. Then from the true longitude and radius vector and the instantaneous node and inclination of the orbit, the co-ordinates of the satellite, as seen from the centre of the earth, were derived by means of the expressions given below: as Marth pointed out, this computation is much simplified by choosing as plane of reference the plane of Jupiter's equator.

Let a = semi-major axis at distance unity.
$a'' = a/\sin 1''$.
ρ = radius vector, in terms of a as unit.
v = true longitude.
Ω = ascending node, ι = inclination } referred to Jupiter's equator.
Δ = jovicentric distance of the earth.
B = " latitude "
$L + 180^\circ$ = " longitude "
Δ' = geocentric distance of the satellite.

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Then

$$\Delta' = \Delta + a\,\rho \cos(v - \mathrm{L}) \cos \mathrm{B}.$$
$$x = \frac{a''\rho}{\Delta'} [\sin(v - \mathrm{L}) - \sin(v - \Omega) \sin \iota \tan \tfrac{1}{2} \iota \cos(\mathrm{L} - \Omega)]$$
$$y = \frac{a''\rho}{\Delta'} [\cos(v - \mathrm{L}) \sin \mathrm{B} + \sin(v - \Omega) \sin \iota \cos \mathrm{B}].$$

x and y are the co-ordinates of the satellite in seconds of arc as seen from the centre of the earth, the origin being at Jupiter's centre and the axes parallel and perpendicular respectively to the plane of Jupiter's equator. The second term in x is insensible for I. and III., and was neglected.

Δ in the above formulæ is the distance between the centre of the earth at the moment of observation and the centre of Jupiter at the moment the light rays left it. In 1901, $\log\Delta$ was interpolated from the Nautical Almanac for the Greenwich Mean Time of observation not corrected for aberration, and the correction to $\log\Delta$ given in the ephemeris for that year was applied: this is the correction denoted by $\delta_1(\log\Delta)$ in the preceding chapter. In 1902, $\log\Delta$ was calculated for the Greenwich Mean Time corrected for aberration, and therefore requires the correction denoted by $\delta_2(\log\Delta)$, which was accordingly applied.

The distance s and the position-angle p of a pair of satellites are given by

$$s \sin(p - \mathrm{P}) = x_2 - x_1,$$
$$s \cos(p - \mathrm{P}) = y_2 - y_1.$$

The angle P was interpolated from the ephemeris for the epoch of observation. The position-angle thus computed for the epoch of observation of distance requires a small correction to reduce it to the epoch of observation of position-angle: as the difference between the two epochs rarely exceeds $0^{m}{\cdot}5$, this small correction is most easily applied to the absolute terms in the equations of condition after the calculation of the coefficients $\frac{\delta p}{\delta v_1}$ and $\frac{\delta p}{\delta v_2}$; thus

$$\delta p = \left(\frac{\delta p}{\delta v_1} n_1 + \frac{\delta p}{\delta v_2} n_2\right) \delta t,$$

where δt is the difference of the epochs and n_1 n_2 are the mean motions in longitude. The corresponding correction to the coefficients of the unknowns is quite insensible.

The quantities that are found by observation are s and p: by comparing with the calculated quantities we get the corrections to the calculated s and p, which in terms of corrections to the co-ordinates x and y are

$$\Delta s = \sin(p - \mathrm{P})\,\Delta(x_2 - x_1) + \cos(p - \mathrm{P})\,\Delta(y_2 - y_1),$$
$$\sin 1'' \, s\Delta p = \cos(p - \mathrm{P})\,\Delta(x_2 - x_1) - \sin(p - \mathrm{P})\,\Delta(y_2 - y_1),$$

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The quantities in x and y which are subject to correction are a'', ρ, v, ι, Ω. Taking them in this order, we have

$$\frac{\delta x}{\delta a} = \frac{x}{a}, \quad \frac{\delta y}{\delta a} = \frac{y}{a},$$
$$\frac{\delta x}{\delta \rho} = \frac{x}{\rho}, \quad \frac{\delta y}{\delta \rho} = \frac{y}{\rho},$$
$$\frac{\delta x}{\delta v} = \mathrm{X} = \frac{a''\rho}{\Delta'} \sin 1^\circ \cos (v - \mathrm{L}),$$
$$\frac{\delta y}{\delta v} = \mathrm{Y} = \frac{a''\rho}{\Delta'} \sin 1^\circ \left\{ - \sin (v - \mathrm{L}) \sin \mathrm{B} + \sin \iota \cos \mathrm{B} \cos (v - \Omega) \right\}.$$

The last two, X and Y, were computed concurrently with the co-ordinates; they are the changes in x and y produced by a change of $+1^\circ$ in longitude. Instead of ι and Ω it is convenient to take two other variables and to follow Marth and put

$$p = \iota \sin (-\Omega),$$
$$q = \iota \cos (-\Omega);$$

thus

$$y = \frac{a''\rho}{\Delta'} \left\{ \cos (v - \mathrm{L}) \sin \mathrm{B} + p \cos \mathrm{B} \cos v + q \cos \mathrm{B} \sin v \right\}$$

and

$$\frac{\delta y}{\delta p} = \frac{a''\rho}{\Delta'} \sin 1^\circ \cos \mathrm{B} \cos v,$$
$$\frac{\delta y}{\delta q} = \frac{a''\rho}{\Delta'} \sin 1^\circ \cos \mathrm{B} \sin v,$$

$\left(\frac{\delta x}{\delta p}$ and $\frac{\delta x}{\delta q}$ are inappreciable and were neglected$\right)$.

These coefficients and $\frac{\delta x}{\delta v}$, $\frac{\delta y}{\delta v}$ were multiplied by sin 1°, that the unknowns Δp, Δq, and Δv might be expressed in degrees instead of in circular measure. Substituting in the expressions for Δs and $s\Delta p$, we find

$$a_1 \frac{\delta s}{\delta a_1} = \rho_1 \frac{\delta s}{\delta \rho_1} = - x_1 \sin (p - \mathrm{P}) - y_1 \cos (p - \mathrm{P}),$$

$$a_2 \frac{\delta s}{\delta a_2} = \rho_2 \frac{\delta s}{\delta \rho_2} = \quad x_2 \sin (p - \mathrm{P}) + y_2 \cos (p - \mathrm{P}),$$

with the check equation, $\rho_1 \frac{\delta s}{\delta \rho_1} + \rho_2 \frac{\delta s}{\delta \rho_2} = s$:

$$s \sin 1'' \rho_1 \frac{\delta p}{\delta \rho_1} = - x_1 \cos (p - \mathrm{P}) + y_1 \sin (p - \mathrm{P}),$$

$$s \sin 1'' \rho_2 \frac{\delta p}{\delta \rho_2} = \quad x_2 \cos (p - \mathrm{P}) - y_2 \sin (p - \mathrm{P}),$$

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with the check equation, $s \sin 1'' \rho_1 \frac{\delta p}{\delta \rho_1} + s \sin 1'' \rho_2 \frac{\delta p}{\delta \rho_2} = 0$:

$$\frac{\delta s}{\delta v_1} = -X_1 \sin (p - P) - Y_1 \cos (p - P),$$

$$\frac{\delta s}{\delta v_2} = X_2 \sin (p - P) + Y_2 \cos (p - P),$$

$$s \sin 1'' \frac{\delta p}{\delta v_1} = -X_1 \cos (p - P) + Y_1 \sin (p - P),$$

$$s \sin 1'' \frac{\delta p}{\delta v_2} = X_2 \cos (p - P) - Y_2 \sin (p - P).$$

Put now $Z = \frac{a''\rho}{\Delta^1} \sin 1^\circ \cos B$, and we get

$$\frac{\delta s}{\delta p_1} = -Z_1 \cos v_1 \cos (p - P), \qquad \frac{\delta s}{\delta p_2} = Z_2 \cos v_2 \cos (p - P),$$

$$\frac{\delta s}{\delta q_1} = -Z_1 \sin v_1 \cos (p - P), \qquad \frac{\delta s}{\delta q_2} = Z_2 \sin v_2 \cos (p - P),$$

$$s \sin 1'' \frac{\delta p}{\delta p_1} = Z_1 \cos v_1 \sin (p - P), \qquad s \sin 1'' \frac{\delta p}{\delta p_2} = -Z_2 \cos v_2 \sin (p - P),$$

$$s \sin 1'' \frac{\delta p}{\delta q_1} = Z_1 \sin v_1 \sin (p - P), \qquad s \sin 1'' \frac{\delta p}{\delta q_2} = -Z_2 \sin v_2 \sin (p - P).$$

The variables we have so far dealt with are a, ρ, v, p and q and the equations of condition would now take the form

$$O - C = a_1 \frac{\delta s}{\delta a_1} \frac{\Delta a_1}{a_1} + a_2 \frac{\delta s}{\delta a_2} \frac{\Delta a_2}{a_2} + \rho_1 \frac{\delta s}{\delta \rho_1} \frac{\Delta \rho_1}{\rho_1} + \rho_2 \frac{\delta s}{\delta \rho_2} \frac{\Delta \rho_2}{\rho_2}$$

(A) . . . .

$$+ \frac{\delta s}{\delta v_1} \Delta v_1 + \frac{\delta s}{\delta v_2} \Delta v_2 + \frac{\delta s}{\delta p_1} \Delta p_1 + \frac{\delta s}{\delta p_2} \Delta p_2 + \frac{\delta s}{\delta q_1} \Delta q_1 + \frac{\delta s}{\delta q_2} \Delta q_2,$$

with an equation of the same form for position-angles. Of these variables a, p, and q are fundamental constants of the theory; but ρ and v are not, and must be expressed in terms of such constants.

The most important terms in the longitude are

$$v = nt + \epsilon + k \sin l - h \cos l + x \sin \psi,$$

in which

$l = nt + \epsilon =$ the mean longitude,

$h = 2e \sin \omega$, $k = 2e \cos \omega$ } e being the eccentricity, ω the perijove.

$x \sin \psi$ is the "Great Inequality" which arises from the near commensurability of the mean motions of the first three satellites. This term represents by far the largest perturbation to which the motions of satellites I. and II. are subject; it is also

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considerable for III., but does not exist in the case of the fourth satellite. Its values for the first three are :—

I. $+ 1555'' \sin 2 (l_1 - l_2)$	Marth $+ 1555'' \sin \alpha_1$,
II. $+ 3693'' \sin (l_1 - l_2 + 180°)$	$+ 3693'' \sin \alpha_2$,
III. $+ 228'' \sin (l_2 - l_3 + 180°)$	$+ 228'' \sin \gamma_3$;

the coefficients are functions of the masses and are constants. The periods are 1·76, 3·52, and 7·06 days respectively, whilst the periodic times of the satellites themselves are 1·77, 3·55, and 7·15 days : the latter are also the periods of the coefficients of h and k in the expression for the longitude, so that though the observations extend over many complete periods of the large inequalities, neither their coefficients nor h and k can be determined by observation with much weight owing to the very close approximation to equality in their periods. h and k are subject to secular variations which are expressed by the series (Souillart—A p. 40, or B p. 24).

$$\tfrac{1}{2} h = \mathrm{M} \sin (gt + \beta) + \mathrm{M}_1 \sin (g_1 t + \beta_1) + \mathrm{M}_2 \sin (g_2 t + \beta_2) + \mathrm{M}_3 \sin (g_3 t + \beta_2),$$
$$\tfrac{1}{2} k = \mathrm{M} \cos (gt + \beta) + \mathrm{M}_1 \cos (g_1 t + \beta_1) + \mathrm{M}_2 \cos (g_2 t + \beta_2) + \mathrm{M}_3 \cos (g_3 t + \beta_3).$$

The angles $gt + \beta$ change slowly with the time (the periods of the four terms are respectively 6·6, 22·6, 136, and 493 years), and it is not possible to separate them out excepting by observations which extend over a very large period of time.

The present series of observations will therefore only give corrections to h and k at the mean epoch of the observations. These terms are those with arguments which Marth denotes by

$$\gamma_1 \ \delta_1, \qquad \gamma_2 \ \delta_2 \ \epsilon_2, \qquad \alpha_3 \ \beta_3 \ \iota_3, \qquad \alpha_4 \ \gamma_4.$$

Of the first two terms in the expression for v, the mean motion, n, must be considered as known since the observations do not cover nearly sufficient time to determine a reliable correction to it.

Thus we have finally :—

$$\Delta v_1 = \Delta \epsilon_1 + \sin l_1 \, \Delta k_1 - \cos l_1 \, \Delta h_1 + \sin \alpha_1 \, \Delta x_1,$$
$$\Delta v_2 = \Delta \epsilon_2 + \sin l_2 \, \Delta k_2 - \cos l_2 \, \Delta h_1 + \sin \alpha_2 \, \Delta x_2,$$
$$\Delta v_3 = \Delta \epsilon_3 + \sin l_3 \, \Delta k_3 - \cos l_3 \, \Delta h_2 + \sin \gamma_3 \, \Delta x_3,$$
$$\Delta v_4 = \Delta \epsilon_4 + \sin l_4 \, \Delta k_4 - \cos l_4 \, \Delta h_3.$$

The expression for the radius vector, in terms of the mean distance as unit, is

$$\rho = 1 - \tfrac{1}{2} \sin 1° \{ k \cos l + h \sin l + x \cos \psi \};$$

and thus

$$\Delta \rho_1 = - \tfrac{1}{2} \sin 1° \{ \cos l_1 \, \Delta k_1 + \sin l_1 \, \Delta h_1 + \cos \alpha_1 \, \Delta x_1 \},$$
$$\Delta \rho_2 = - \tfrac{1}{2} \sin 1° \{ \cos l_2 \, \Delta k_2 + \sin l_2 \, \Delta h_2 + \cos \alpha_2 \, \Delta x_2 \},$$
$$\Delta \rho_3 = - \tfrac{1}{2} \sin 1° \{ \cos l_3 \, \Delta k_3 + \sin l_3 \, \Delta h_3 + \cos \gamma_3 \, \Delta x_3 \},$$
$$\Delta \rho_4 = - \tfrac{1}{2} \sin 1° \{ \cos l_4 \, \Delta k_4 + \sin l_4 \, \Delta h_4 \}.$$

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These were the values of Δv and $\Delta\rho$ which were substituted in the equations of condition (A).

A further substitution was also made for $\Delta a/a$ as follows :

It was seen in the previous chapter that the relation between the semi-major axis and the mass of Jupiter is—

$$\frac{\mu}{a^3}(1+f) = n^2 + \mathrm{P},$$

where μ is the sum of the masses of Jupiter and the satellites and

$$f = \mathrm{J}b^2/a^2.$$

The mean motions were assumed to be known : hence we may put—

$$\mu\,(1 + \mathrm{J}b^2/a^2) = \lambda a^3;$$

and if M is the reciprocal of the mass, we find—

$$\begin{aligned}\frac{\delta\mathrm{M}}{\mathrm{M}} &= -\frac{3\delta a}{a} + \frac{\delta(\mathrm{J}b^2/a^2)}{1+\mathrm{J}b^2/a^2}\\ &= -\frac{3\delta a}{a} + \left[\frac{1}{a^2}\delta(\mathrm{J}b^2) - \frac{2\mathrm{J}b^2}{a^2}\frac{\delta a}{a}\right]\frac{1}{1+\mathrm{J}b^2/a^2}\\ &= -\frac{\delta a}{a}\left[3 + \frac{2\mathrm{J}b^2/a^2}{1+\mathrm{J}b^2/a^2}\right] + \frac{1/a^2}{1+\mathrm{J}b^2/a^2}\delta(\mathrm{J}b^2).\end{aligned}$$

For the first satellite, the second term within the brackets is only 0·0014, for the second it is 0·0005, it has therefore been neglected and we have put—

$$-\frac{\delta a}{a} = \frac{1}{3}\frac{\delta\mathrm{M}}{\mathrm{M}} - \frac{1}{3}\frac{1/a^2}{1+\mathrm{J}b^2/a^2}\delta(\mathrm{J}b^2).$$

It is convenient to take b as unit of length : then if we put $1/a^2 = a$ we get

$$\begin{aligned}\delta s &= \frac{\delta s}{\delta a_1}\delta a_1 + \frac{\delta s}{\delta a_2}\delta a_2 = \frac{\delta s}{\delta\rho_1}\delta\rho_1 + \frac{\delta s}{\delta\rho_2}\delta\rho_2\\ &= -\frac{1}{3}s\frac{\delta\mathrm{M}}{\mathrm{M}} + \frac{1}{3}\left[\frac{a_1}{1+a_1\mathrm{J}}\rho_1\frac{\delta s}{\delta\rho_1} + \frac{a_2}{1+a_2\mathrm{J}}\rho_2\frac{\delta s}{\delta\rho_2}\right]\delta\mathrm{J}.\end{aligned}$$

The numerical values of $a/(1+a\,\mathrm{J})$ are

I. 0·00912.	II. 0·00360.
III. 0·00141.	IV. 0·00046.

In the same way we find

$$\delta p = \frac{1}{3}\left[\frac{a_1}{1+a_1\mathrm{J}}\rho_1\frac{\delta p}{\delta\rho_1} + \frac{a_2}{1+a_2\mathrm{J}}\rho_2\frac{\delta p}{\delta\rho_2}\right]\delta\mathrm{J}$$

the coefficient of δM being zero. In the equations of conditions for 1901 the coefficient $\frac{\delta p}{\delta\mathrm{J}}$ of δJ in position-angle was retained, but it was neglected in 1902, since it rarely amounted to 0·01.

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μ was put $= -\frac{100}{3}\frac{\Delta M}{M}$, and thus our final equations of condition are as follows:

DISTANCE.

$$\frac{s}{100}\mu + \frac{1}{3}\left(\frac{a_1}{1+a_1 J}\rho_1\frac{\delta s}{\delta\rho_1} + \frac{a_2}{1+a_2 J}\rho_2\frac{\delta s}{\delta\rho_2}\right)\Delta J + \frac{\delta s}{\delta p_1}\Delta p_1 + \frac{\delta s}{\delta q_1}\Delta q_1 + \frac{\delta s}{\delta p_2}\Delta p_2 + \frac{\delta s}{\delta q_2}\Delta q_2,$$
$$\left(-\cos l_1\frac{\delta s}{\delta v_1} - \frac{1}{2\rho_1}\sin 1^\circ \sin l_1\rho_1\frac{\delta s}{\delta\rho_1}\right)\Delta h_1 + \left(-\cos l_2\frac{\delta s}{\delta v_2} - \frac{1}{2\rho_2}\sin 1^\circ \sin l_2\rho_2\frac{\delta s}{\delta\rho_2}\right)\Delta h_2,$$
$$\left(+\sin l_1\frac{\delta s}{\delta v_1} - \frac{1}{2\rho_1}\sin 1^\circ \cos l_1\rho_1\frac{\delta s}{\delta\rho_1}\right)\Delta k_1 + \left(\sin l_2\frac{\delta s}{\delta v_2} - \frac{1}{2\rho_2}\sin 1^\circ \cos l_2\rho_2\frac{\delta s}{\delta\rho_2}\right)\Delta k_2,$$
$$\left(+\sin\psi_1\frac{\delta s}{\delta v_1} - \frac{1}{2\rho_1}\sin 1^\circ \cos\psi_1\rho_1\frac{\delta s}{\delta\rho_1}\right)\Delta x_1 + \left(\sin\psi_2\frac{\delta s}{\delta v_2} - \frac{1}{2\rho_2}\sin 1^\circ \cos\psi_2\rho_2\frac{\delta s}{\delta\rho_2}\right)\Delta x_2,$$
$$+\frac{\delta s}{\delta v_1}\Delta\epsilon_1 + \frac{\delta s}{\delta v_2}\Delta\epsilon_2 = O - C.$$

POSITION-ANGLE.

$$s\sin 1''\Big[\frac{1}{3}\left(\frac{a_1}{1+a_1 J}\rho_1\frac{\delta p}{\delta\rho_1} + \frac{a_2}{1+a_2 J}\rho_2\frac{\delta p}{\delta\rho_2}\right)\Delta J + \frac{\delta p}{\delta p_1}\Delta p_1 + \frac{\delta p}{\delta q_1}\Delta q_1 + \frac{\delta p}{\delta p_2}\Delta p_2 + \frac{\delta p}{\delta q_2}\Delta q_2,$$
$$\left(-\cos l_1\frac{\delta p}{\delta v_1} - \frac{1}{2\rho_1}\sin 1^\circ \sin l_1\rho_1\frac{\delta p}{\delta\rho_1}\right)\Delta h_1 + \left(-\cos l_2\frac{\delta p}{\delta v_2} - \frac{1}{2\rho_2}\sin 1^\circ \sin l_2\rho_2\frac{\delta p}{\delta\rho_2}\right)\Delta h_2,$$
$$\left(\sin l_1\frac{\delta p}{\delta v_1} - \frac{1}{2\rho_1}\sin 1^\circ \cos l_1\rho_1\frac{\delta p}{\delta\rho_1}\right)\Delta k_1 + \left(\sin l_2\frac{\delta p}{\delta v_2} - \frac{1}{2\rho_2}\sin 1^\circ \cos l_2\rho_2\frac{\delta p}{\delta\rho_2}\right)\Delta k_2,$$
$$\left(\sin\psi_1\frac{\delta p}{\delta v_1} - \frac{1}{2\rho_1}\sin 1^\circ \cos\psi_1\rho_1\frac{\delta p}{\delta\rho_1}\right)\Delta x_1 + \left(\sin\psi_2\frac{\delta p}{\delta v_2} - \frac{1}{2\rho_2}\sin 1^\circ \cos\psi_2\rho_2\frac{\delta p}{\delta\rho_2}\right)\Delta x_2,$$
$$+\frac{\delta p}{\delta v_1}\Delta\epsilon_1 + \frac{\delta p}{\delta v_2}\Delta\epsilon_2\Big] = (O - C)\, s\sin 1''.$$

For the fourth satellite the last line but one of both equations does not exist.

It was the writer's rule always to take the subscript 1 as referring to that satellite of the pair which had the smaller mean distance: thus in the pair III.–IV., subscript 1 would refer to III.

In the following tables the Greenwich Mean Time of every observation of the satellites and the true longitude and radius vector is given together with its co-ordinates x, y and their rates of change X, Y. The tabular distances and position-angles are not given here but will be found more conveniently placed next to the observed quantities in the tables of the equations of condition. When two satellites were close to one another and the position-angle was changing rapidly, the co-ordinates and thence the distance and position-angle were computed separately for the moment of each pointing: and their mean was taken as the tabular distance and position-angle. The coefficients were computed, as in all other cases, for the mean epoch of the distance observation.

The co-ordinates x were all computed with six-figure logarithms, y with five or four as was necessary, and X and Y with four figures. The distances and position-angles were calculated from the co-ordinates with six figures throughout.

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TABLE XVI.

Co-ordinates of I., 1901.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1901.	h m	°		"	"		
29	July 2	7 51·220	345·9461	1·001298	− 55·450	+ 4·135	− 2·224	− 0·032
31		8 1·440	347·3893	1·001388	− 58·643	+ 4·088	− 2·199	− 0·034
33		8 11·168	348·7630	1·001470	− 61·644	+ 4·042	− 2·175	− 0·035
45	8	6 49·511	117·3584	1·001509	− 57·061	− 4·118	+ 2·205	− 0·032
47		7 2·523	119·1916	1·001399	− 52·982	− 4·174	+ 2·236	− 0·030
49		7 13·774	120·7770	1·001302	− 49·411	− 4·220	+ 2·259	− 0·028
61	10	6 37·161	162·5924	0·998534	+ 49·848	− 4·186	+ 2·247	+ 0·029
63		6 46·646	163·9359	0·998451	+ 52·851	− 4·145	+ 2·225	+ 0·030
65		6 58·755	165·6531	0·998349	+ 56·643	− 4·091	+ 2·197	+ 0·032
77	11	8 8·146	19·5425	1·003300	−117·971	+ 2·362	− 1·275	− 0·067
83		10 24·829	38·7207	1·003645	−135·431	+ 0·965	− 0·526	− 0·077
85		10 37·137	40·4472	1·003663	−136·278	+ 0·831	− 0·455	− 0·078
93	12	6 30·193	208·8356	0·996408	+127·272	− 1·687	+ 0·913	+ 0·091
99		7 0·407	213·1358	0·996323	+130·829	− 1·372	+ 0·744	+ 0·074
101		7 10·043	214·5081	0·996300	+131·812	− 1·268	+ 0·689	+ 0·074
111	14	6 5·604	252·6684	0·996614	+127·703	+ 1·675	− 0·888	+ 0·072
113		6 15·326	254·0513	0·996654	+126·443	+ 1·775	− 0·942	+ 0·072
119	16	6 1·651	299·3178	0·998945	+ 50·161	+ 4·179	− 2·237	+ 0·028
121		6 10·789	300·6119	0·999028	+ 47·257	+ 4·216	− 2·255	+ 0·027
123		6 19·019	301·7768	0·999105	+ 44·621	+ 4·245	− 2·272	+ 0·025
133	18	6 0·367	346·0610	1·002028	− 59·482	+ 4·036	− 2·168	− 0·034
143	22	5 38·479	76·3080	1·003061	−122·864	− 1·990	+ 1·059	− 0·070
145		5 48·700	77·7435	1·003006	−121·295	− 2·089	+ 1·113	− 0·069
149		6 38·406	84·7278	1·002703	−112·594	− 2·551	+ 1·361	− 0·064
152	23	5 48·436	281·8369	0·998200	+ 84·055	+ 3·499	− 1·873	+ 0·047
153		6 17·560	285·9661	0·998444	+ 76·127	+ 3·687	− 1·973	+ 0·043
159	27	5 34·910	13·6765	1·003462	−110·356	+ 2·574	− 1·391	− 0·063
165		6 2·772	17·5858	1·003559	−115·539	+ 2·324	− 1·256	− 0·066
167		6 14·023	19·1642	1·003597	−117·477	+ 2·219	− 1·200	− 0·067
175	29	5 43·222	61·4613	1·003388	−132·538	− 0·943	+ 0·498	− 0·075
177		5 54·214	63·0040	1·003342	−131·712	− 1·058	+ 0·560	− 0·075
185	Aug. 10	5 7·729	338·9689	1·002514	− 46·480	+ 3·996	− 2·160	− 0·026
187		5 16·588	340·2142	1·002574	− 49·160	+ 3·962	− 2·142	− 0·028
189		5 24·415	341·3145	1·002629	− 51·509	+ 3·931	− 2·125	− 0·029
201	13	9 5·559	262·9429	0·998013	+107·292	+ 2·416	− 1·298	+ 0·060
203		9 15·584	264·3650	0·998093	+105·421	+ 2·501	− 1·345	+ 0·059
205		9 26·424	265·9027	0·998185	+103·323	+ 2·591	− 1·394	+ 0·058
217	14	4 41·454	68·4422	1·002678	−122·720	− 1·479	+ 0·791	− 0·069
219		4 49·972	69·6391	1·002620	−121·735	− 1·560	+ 0·836	− 0·068
231	19	8 38·963	39·6027	1·003546	−127·869	+ 0·584	− 0·327	− 0·072
233		9 9·710	43·9174	1·003440	−128·889	+ 0·316	− 0·158	− 0·072
235		9 23·395	45·8381	1·003385	−129·108	+ 0·134	− 0·083	− 0·072
247	24	8 29·342	336·1163	1·002875	− 40·037	+ 3·860	− 2·112	− 0·023
249		8 39·851	337·5930	1·002936	− 43·146	+ 3·826	− 2·093	− 0·024
251		8 48·476	338·8044	1·002987	− 45·672	+ 3·795	− 2·077	− 0·026
263	26	8 26·546	22·3562	1·003730	−114·519	+ 1·720	− 0·950	− 0·064

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TABLE XVI.—*continued.*

CO-ORDINATES OF I., 1901.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1901.	h m	°		″	″		
265	Aug. 26	8 41·206	24·4127	1·003710	−116·391	+ 1·587	− 0·877	− 0·065
267		8 53·985	26·2053	1·003688	−117·898	+ 1·471	− 0·813	− 0·066
275	27	8 7·124	223·3698	0·996698	+125·142	− 0·289	+ 0·168	+ 0·070
277		8 15·517	224·5638	0·996740	+125·319	− 0·206	+ 0·122	+ 0·070
279		8 27·516	226·2702	0·996801	+125·479	− 0·087	+ 0·057	+ 0·070
291	28	8 21·618	68·3456	1·002159	−117·773	− 1·425	+ 0·772	− 0·065
293		8 32·800	69·9192	1·002071	−116·499	− 1·527	+ 0·828	− 0·065
305	29	8 35·436	274·5633	0·999487	+ 85·681	+ 2·914	− 1·593	+ 0·047
307		8 45·096	275·9299	0·999572	+ 83·486	+ 2·979	− 1·628	+ 0·046
311		9 10·510	279·5264	0·999806	+ 77·476	+ 3·140	− 1·716	+ 0·043
321	Sept. 1	8 12·745	161·1857	0·996727	+ 49·155	− 3·590	+ 1·980	+ 0·028
323		8 22·659	162·5960	0·996682	+ 51·928	− 3·551	+ 1·958	+ 0·029
325		8 31·918	164·0133	0·996642	+ 54·684	− 3·508	+ 1·935	+ 0·031
337	3	7 49·662	205·2338	0·996368	+113·507	− 1·476	+ 0·822	+ 0·063
339		7 59·625	206·6518	0·996395	+114·641	− 1·387	+ 0·773	+ 0·064
341		8 9·624	208·0751	0·996425	+115·708	− 1·295	+ 0·723	+ 0·064
353	4	7 52·421	48·7138	1·002861	−123·334	− 0·080	+ 0·035	− 0·068
355		8 0·244	49·8133	1·002812	−123·263	− 0·154	+ 0·076	− 0·068
357		8 10·015	51·1861	1·002751	−123·111	− 0·248	+ 0·128	− 0·068
369	9	7 36·638	344·3513	1·003533	− 54·180	+ 3·418	− 1·901	− 0·030
371		7 47·792	345·9163	1·003568	− 57·133	+ 3·370	− 1·875	− 0·032
381	10	7 33·718	187·3075	0·996225	+ 91·287	− 2·449	+ 1·368	+ 0·052
385		8 1·688	191·2901	0·996250	+ 96·727	− 2·248	+ 1·256	+ 0·053
389	14	7 5·197	277·6384	1·000423	+ 77·495	+ 2·852	− 1·586	+ 0·042
391		7 18·146	279·4665	1·000541	+ 74·565	+ 2·927	− 1·592	+ 0·040
393		7 28·860	280·9788	1·000641	+ 72·084	+ 2·987	− 1·662	+ 0·039
411	16	7 59·380	332·0208	1·003324	− 28·465	+ 3·601	− 2·018	− 0·016
413		8 14·832	334·2043	1·003407	− 32·821	+ 3·565	− 1·996	− 0·018
415		8 24·543	335·5667	1·003447	− 35·532	+ 3·539	− 1·983	− 0·020
421	17	6 56·875	166·4043	0·996305	+ 55·546	− 3·226	+ 1·811	+ 0·030
423		7 8·727	168·0920	0·996281	+ 58·578	− 3·173	+ 1·782	+ 0·032
425		7 23·779	170·2349	0·996256	+ 62·349	− 3·101	+ 1·743	+ 0·034
439	19	6 56·258	213·6432	0·996983	+113·280	− 0·909	+ 0·520	+ 0·062
443		7 24·254	217·6226	0·997148	+115·089	− 0·663	+ 0·381	+ 0·062
447		7 46·487	220·7820	0·997287	+116·132	− 0·464	+ 0·270	+ 0·063
453	23	6 42·248	305·6818	1·002494	+ 26·100	+ 3·503	− 1·980	+ 0·014
457	25	7 1·003	354·9744	1·003762	− 68·390	+ 2·863	− 1·632	− 0·037
459		7 9·047	356·1022	1·003757	− 70·215	+ 2·821	− 1·608	− 0·038
463	27	5 55·494	32·4202	1·002834	−110·267	+ 0·987	− 0·571	− 0·059
465		6 7·207	34·0657	1·002760	−111·149	+ 0·889	− 0·516	− 0·059

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TABLE XVI.—*continued.*

CO-ORDINATES OF II., 1901.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1901.	h m						
5	June 24	10 50·346	31°·6561	0·992500	−203″·111	+ 0″·850	− 1·416	− 0·115
7		11 18·343	33·6571	0·992345	−205·785	+ 0·619	− 1·292	− 0·116
15	25	7 33·379	120·5246	0·995260	− 84·910	− 6·614	+ 3·526	− 0·017
21	29	12 38·921	187·4601	1·005449	+155·840	− 3·478	+ 2·752	+ 0·102
23		12 49·584	188·2023	1·005537	+157·889	− 3·405	+ 2·716	+ 0·102
31	July 2	8 1·440	112·0917	0·994889	−110·785	− 6·465	+ 3·304	− 0·032
35		8 19·603	113·3841	0·995053	−106·530	− 6·506	+ 3·350	− 0·030
37		8 30·548	114·1619	0·995141	−103·930	− 6·527	+ 3·379	− 0·028
45	8	6 49·511	353·6196	0·995403	−116·148	+ 4·654	− 3·247	− 0·085
51		7 26·133	356·2219	0·995072	−124·446	+ 4·427	− 3·154	− 0·089
53		7 37·133	357·0042	0·994977	−126·890	+ 4·356	− 3·124	− 0·090
63	10	6 46·646	197·5801	1·007167	+183·986	− 2·265	+ 2·153	+ 0·112
67		7 9·750	199·1832	1·007302	+187·398	− 2·084	+ 2·063	+ 0·113
69		7 22·473	200·0654	1·007374	+189·213	− 1·984	+ 2·012	+ 0·114
77	11	8 8·146	303·1895	1·002600	+ 68·574	+ 6·788	− 3·662	+ 0·008
79		8 19·251	303·9675	1·002484	+ 65·708	+ 6·793	− 3·677	+ 0·006
81		10 11·463	311·8367	1·001304	+ 36·199	+ 6·773	− 3·799	− 0·010
93	12	6 30·193	38·4157	0·991473	−212·573	− 0·199	− 0·840	− 0·117
95		6 40·420	39·1482	0·991455	−213·163	− 0·285	− 0·793	− 0·117
97		6 49·603	39·8057	0·991434	−213·664	− 0·363	− 0·750	− 0·117
109	14	5 54·947	238·6729	1·008497	+219·302	+ 2·581	− 0·520	+ 0·110
111		6 5·604	239·4107	1·008476	+218·897	+ 2·661	− 0·569	+ 0·110
121	16	6 10·789	83·3292	0·993002	−183·041	− 4·915	+ 2·046	− 0·080
125		6 28·724	84·6090	0·993118	−180·390	− 5·018	+ 2·117	− 0·078
127		6 39·247	85·3599	0·993185	−178·793	− 5·076	+ 2·158	− 0·076
133	18	6 0·367	283·9204	1·004528	+130·840	+ 6·289	− 3·080	+ 0·044
135		6 9·678	284·5701	1·004445	+128·816	+ 6·317	− 3·106	+ 0·043
137		6 18·693	285·1995	1·004361	+126·835	+ 6·344	− 3·130	+ 0·042
149	22	6 38·406	332·0080	0·996882	− 45·057	+ 6·002	− 3·704	− 0·052
152	23	5 48·436	71·2830	0·992479	−200·596	− 3·900	+ 1·370	− 0·094
154		6 23·489	73·7875	0·992668	−196·990	− 4·135	+ 1·522	− 0·092
159	27	5 34·910	116·2967	0·998260	− 84·818	− 6·575	+ 3·451	− 0·017
161		5 45·061	117·0136	0·998368	− 82·345	− 6·582	+ 3·469	− 0·016
163		5 54·339	117·6689	0·998467	− 80·070	− 6·598	+ 3·486	− 0·015
185	Aug. 10	5 7·729	93·5687	0·996542	−147·169	− 5·570	+ 2·585	− 0·057
191		5 34·140	95·4405	0·996805	−142·282	− 5·674	+ 2·668	− 0·053
193		5 43·884	96·1308	0·996900	−140·436	− 5·710	+ 2·698	− 0·052
201	13	9 5·559	54·0174	0·992435	−205·417	− 2·218	+ 0·365	− 0·104
207		9 38·523	56·3727	0·992609	−204·399	− 2·462	+ 0·512	− 0·102
209		9 51·242	57·2815	0·992679	−203·915	− 2·555	+ 0·569	− 0·102
227	15	9 5·241	255·5060	1·005348	+184·867	+ 4·011	− 1·673	+ 0·083
231	19	8 38·963	298·9462	0·998549	+ 66·651	+ 6·257	− 3·380	+ 0·008
237		9 41·044	303·3319	0·997899	+ 51·600	+ 6·268	− 3·455	+ 0·000
239	23	8 21·441	343·6436	0·992822	− 87·694	+ 4·668	− 3·163	− 0·070
241		8 35·075	344·6178	0·992742	− 90·753	+ 4·598	− 3·137	− 0·071

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TABLE XVI.—*continued.*

CO-ORDINATES OF II., 1901.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1901.	h m	°		"	"		
247	Aug. 24	8 29·342	87·0376	0·997155	−155·989	− 4·969	+ 2·223	− 0·063
253		8 56·295	88·9453	0·997432	−151·693	− 5·089	+ 2·313	− 0·060
255		9 5·772	89·6156	0·997527	−150·140	− 5·129	+ 2·344	− 0·059
263	26	8 26·546	287·6853	0·999424	+100·771	+ 5·901	− 3·035	+ 0·028
269		9 9·530	290·7150	0·998993	+ 91·394	+ 5·977	− 3·122	+ 0·023
271		9 25·319	291·8301	0·998804	+ 87·876	+ 5·999	− 3·152	+ 0·021
275	27	8 7·124	28·9259	0·991663	−188·110	+ 0·423	− 1·120	− 0·105
281		8 37·827	31·1233	0·991750	−190·431	+ 0·192	− 0·993	− 0·106
283		8 46·803	31·7659	0·991779	−191·058	+ 0·124	− 0·956	− 0·106
301	29	8 15·632	231·4248	1·006763	+200·150	+ 1·934	− 0·224	+ 0·101
307		8 45·096	233·4713	1·006567	+199·520	+ 2·137	− 0·349	+ 0·099
309		8 54·142	234·1002	1·006508	+199·276	+ 2·200	− 0·387	+ 0·099
317	31	8 18·851	75·8875	0·996347	−173·917	− 4·073	+ 1·623	− 0·076
321	Sept. 1	8 12·745	176·1698	1·008364	+123·614	− 3·596	+ 2·720	+ 0·084
327		8 40·545	178·0940	1·008437	+128·782	− 3·433	+ 2·646	+ 0·086
329		8 52·164	178·8984	1·008463	+130·899	− 3·361	+ 2·615	+ 0·087
337	3	7 49·662	17·2166	0·991475	−167·670	+ 1·584	− 1·728	− 0·099
343		8 19·882	19·3805	0·991528	−171·280	+ 1·369	− 1·615	− 0·100
345		8 31·391	20·2044	0·991552	−172·590	+ 1·287	− 1·572	− 0·100
353	4	7 52·421	119·5660	1·003022	− 61·309	− 5·902	+ 3·252	− 0·006
359		8 21·647	121·6100	1·003309	− 54·652	− 5·911	+ 3·289	− 0·002
363		8 43·348	123·1271	1·003522	− 49·654	− 5·913	+ 3·312	0·000
369	9	7 36·638	263·4334	1·001442	+157·180	+ 4·465	− 1·959	+ 0·064
373		7 58·242	264·9501	1·001218	+154·117	+ 4·559	− 2·031	+ 0·062
379	10	7 23·701	4·8959	0·991385	−139·332	+ 2·677	− 2·271	− 0·088
381		7 33·718	5·6138	0·991388	−140·950	+ 2·613	− 2·241	− 0·088
383		7 45·685	6·4709	0·991392	−142·846	+ 2·537	− 2·204	− 0·089
389	14	7 5·197	49·8124	0·994398	−188·685	− 1·622	+ 0·098	− 0·094
395		7 50·667	53·0485	0·994782	−188·111	− 1·923	+ 0·284	− 0 092
397		7 59·954	53·7091	0·994860	−187·919	− 1·983	+ 0·322	− 0·092
405	16	7 17·844	251·7585	1·002372	+173·416	+ 3·502	− 1·320	+ 0·076
407		7 29·076	252·5457	1·002258	+172·338	+ 3·562	− 1·361	+ 0·075
413		8 14·832	255·7535	1·001787	+167·618	+ 3·796	− 1·526	+ 0·072
421	17	6 56·875	352·5132	0·991409	−105·064	+ 3·588	− 2·692	− 0·073
427		7 37·623	355·4319	0·991378	−112·761	+ 3·369	− 2·595	− 0·076
429		7 48·070	356·1801	0·991376	−114·689	+ 3·311	− 2·569	− 0·077
437	19	6 46·299	194·3276	1·008244	+156·109	− 1·791	+ 1·841	+ 0·092
439		6 56·258	195·0170	1·008212	+157·359	− 1·728	+ 1·807	+ 0·092
441		7 13·265	196·1948	1·008157	+159 442	− 1·620	+ 1·751	+ 0·093
453	23	6 42·248	238·9191	1·003441	+182·363	+ 2·335	− 0·577	+ 0·085
457	25	7 1·003	84·8858	1·000297	−148·280	− 4·165	+ 1·887	− 0·058
465	27	6 7·207	281·9485	0·996837	+109·545	+ 4·888	− 2·537	+ 0·034
467		6 18·196	282·7267	0·996726	+107·549	+ 4 913	− 2·563	+ 0·033

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TABLE XVI.—*continued.*

CO-ORDINATES OF III., 1901.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1901.	h m	°		″	″		
3	June 24	9 53·440	224·9116	0·998077	+346·742	− 1·538	+ 0·907	+ 0·210
7		11 18·343	227·8899	0·998114	+349·033	− 0·910	+ 0·591	+ 0·211
13	25	7 18·977	269·9659	0·999094	+282·094	+ 7·458	− 3·662	+ 0·167
19	29	12 27·149	121·5596	0·999690	−127·216	−11·416	+ 5·716	− 0·073
23		12 49·584	122·3439	0·999660	−122·701	−11·471	+ 5·744	− 0·070
33	July 2	8 11·168	264·0146	0·998982	+299·801	+ 6·592	− 3·218	+ 0·179
37		8 30·548	264·6934	0·999006	+297·605	+ 6·712	− 3·279	+ 0·177
39		8 39·945	265·0226	0·999017	+293·173	+ 6·940	− 3·399	+ 0·175
47	8	7 2·523	203·3280	0·997955	+308·417	− 5·552	+ 2·903	+ 0·188
51		7 26·133	204·1564	0·997956	+310·798	− 5·394	+ 2·826	+ 0·190
55		7 48·453	204·9393	0·997962	+312·994	− 5·440	+ 2·751	+ 0·191
61	10	6 37·161	303·2780	1·000513	+109·549	+11·665	− 5·835	+ 0·062
67		7 9·750	304·4156	1·000555	+102·875	+11·734	− 5·872	+ 0·058
71		7 34·458	305·2782	1·000583	+ 97·792	+11·783	− 5·898	+ 0·055
79	11	8 19·251	357·0239	1·001888	−205·550	+ 9·765	− 4·986	− 0·128
83		10 24·829	1·3953	1·001939	−226·757	+ 9·177	− 4·697	− 0·140
87		10 48·447	2·2177	1·001949	−230·605	+ 9·059	− 4·639	− 0·143
95	12	6 40·420	43·7049	1·001822	−348·297	+ 1·350	− 0·797	− 0·211
99		7 0·407	44·4007	1·001811	−348·814	+ 1·203	− 0·723	− 0·212
103		7 20·232	45·0907	1·001801	−349·276	+ 1·056	− 0·649	− 0·212
119	16	6 1·651	243·8916	0·998640	+339·783	+ 2·985	− 1·390	+ 0·205
127		6 39·247	245·2088	0·998674	+337·873	+ 3·255	− 1·526	+ 0·203
129		6 49·921	245·5828	0·998685	+337·297	+ 3·331	− 1·565	+ 0·203
135	18	6 9·678	344·7608	1·001676	−143·861	+10·958	− 5·563	− 0·091
145	22	5 48·700	185·1249	0·998135	+244·915	− 8·301	+ 4·260	+ 0·151
147		5 58·679	185·4750	0·998133	+246·404	− 8·246	+ 4·234	+ 0·152
153	23	6 17·560	236·6334	0·998501	+343·479	+ 1·618	− 0·706	+ 0·207
154		6 23·489	236·8411	0·998507	+343·331	+ 1·661	− 0·728	+ 0·207
161	27	5 45·061	76·4497	1·001017	−306·886	− 5·588	+ 2·721	− 0·184
167		6 14·023	77·4599	1·000987	−304·055	− 5·771	+ 2·816	− 0·182
169		6 24·513	77·8256	1·000978	−303·008	− 5·839	+ 2·849	− 0·181
175	29	5 43·222	177·1293	0·998205	+209·577	− 9·216	+ 4·715	+ 0·129
179		6 4·372	177·8708	0·998196	+213·062	− 9·119	+ 4·667	+ 0·132
187	Aug. 10	5 16·588	59·9305	1·001592	−328·103	− 2·502	+ 1·170	− 0·197
191		5 34·140	60·5420	1·001580	−327·349	− 2·621	+ 1·231	− 0·196
195		5 54·381	61·2472	1·001564	−326·432	− 2·760	+ 1·301	− 0·196
203	13	9 15·584	219·4781	0·998010	+327·322	− 1·540	+ 0·877	+ 0·197
207		9 38·523	220·2831	0·998020	+327·994	− 1·380	+ 0·796	+ 0·198
211		10 1·887	221·1029	0·998032	+328·613	− 1·218	+ 0·714	+ 0·198
217	14	4 41·454	260·3444	0·998993	+280·287	+ 6·223	− 3·078	+ 0·166
221		4 58·829	260·9529	0·999015	+278·396	+ 6·324	− 3·130	+ 0·165
233	19	9 9·710	160·9492	0·998135	+127·308	−10·187	+ 5·232	+ 0·079
237		9 41·044	162·0482	0·998112	+133·035	−10·098	+ 5·189	+ 0·082
239	23	8 21·441	0·7276	1·002189	−220·781	+ 7·978	− 4·147	− 0·134
243		8 48·594	1·6721	1·002197	−224·650	+ 7·850	− 4·083	− 0·136

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TABLE XVI.—*continued.*

CO-ORDINATES OF III., 1901.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1901.	h m	°		″	″		
249	Aug. 24	8 39·851	51·4635	1·001730	−322·197	− 0·885	+ 0·360	− 0·192
253		8 56·295	52·0461	1·001717	−321·952	− 0·997	+ 0·417	− 0·191
257		9 16·876	52·7627	1·001699	−321·603	− 1·124	+ 0·486	− 0·191
265	26	8 41·206	152·1600	0·998276	+ 79·363	−10·484	+ 5·401	+ 0·050
269		9 9·530	153·1497	0·998256	+ 84·683	−10·431	+ 5·376	+ 0·053
277	27	8 15·517	201·7819	0·997890	+286·500	− 4·585	+ 2·437	+ 0·171
281		8 37·827	202 5647	0·997896	+288·376	− 4·449	+ 2·369	+ 0·172
285		8 56·830	203·2317	0·997902	+289·930	− 4·335	+ 2·310	+ 0·173
291	28	8 21·618	252·4782	0·999013	+289·369	+ 4·690	− 2·325	+ 0·170
295		8 41·609	253·1780	0·999040	+287·720	+ 4·808	− 2·386	+ 0·169
301	29	8 15·632	302·5928	1·000846	+ 83·288	+10·481	− 5·365	+ 0·046
303		8 25·142	302·9247	1·000860	+ 81·509	+10·495	− 5·374	+ 0·045
311		9 10·510	304·5088	1·000924	+ 72·961	+10·561	− 5·412	+ 0·041
317	31	8 18·851	42·9792	1·001744	−315·408	+ 0·712	− 0·460	− 0·187
323	Sept. 1	8 22·659	93·3313	1·000140	−220·254	− 7·732	+ 3·922	− 0·128
327		8 40·545	93·9562	1·000114	−217·774	− 7·810	+ 3·964	− 0·126
331		8 52·214	94·7566	1·000081	−214·557	− 7·911	+ 4·015	− 0·125
339	3	7 59·625	193·4260	0·997913	+257·747	− 5·805	+ 3·074	+ 0·154
343		8 19·882	194·1368	0·997915	+259·903	− 5·695	+ 3·018	+ 0·155
347		8 43·578	194·9682	0·997919	+262·375	− 5·565	+ 2·949	+ 0·156
355	4	8 0·244	243·9381	0·998829	+299·610	+ 3·101	− 1·515	+ 0·176
359		8 21·647	244·6877	0·998853	+298·448	+ 3·232	− 1·582	+ 0·175
361		8 32·656	245·0734	0·998866	+297·832	+ 3·300	− 1·621	+ 0·175
383	10	7 45·685	185·1206	0·998046	+224·185	− 6·842	+ 3·625	+ 0·133
385		8 1·688	185·6826	0·998044	+226·207	− 6·766	+ 3·587	+ 0·135
391	14	7 18·146	25·3756	1·001871	−279·986	+ 3·753	− 2·046	− 0·164
395		7 50·667	26·5077	1·001858	−282·213	+ 3·566	− 1·949	− 0·166
399		8 11·804	27·2435	1·001850	−283·603	+ 3·443	− 1·886	− 0·166
405	16	7 17·844	125·8735	0·999074	− 64·083	− 9·814	+ 5·119	− 0·034
409		7 42·362	126·7314	0·999050	− 59 685	− 9·842	+ 5·135	− 0·032
411		7 59·380	127·3261	0·999033	− 56·629	− 9·860	+ 5·145	− 0·030
423	17	7 8·727	176·0205	0·998108	+183·205	− 7·760	+ 4·127	+ 0·109
427		7 37·623	177·0343	0·998100	+187·346	− 7·648	+ 4·069	+ 0·111
431		7 57·518	177·7322	0·998095	+190·161	− 7 570	+ 4·029	+ 0·113
437	19	6 46·299	276·1416	0·999894	+200·609	+ 7·456	− 3·858	+ 0·114
443		7 24·254	277·4673	0·999940	+195·446	+ 7·604	− 3·938	+ 0·111
445		7 35·567	277·8626	0·999953	+193·885	+ 7·649	− 3·961	+ 0·110
459	25	7 9·047	218·7235	0·998195	+287·668	− 1·500	+ 0·892	+ 0·166

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TABLE XVI.—*continued.*

CO-ORDINATES OF IV., 1901.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1901.	h m	°		″	″		
3	June 24	9 53·440	237·0750	0·999372	+616·526	− 2·285	− 0·684	+ 0·346
5		10 50·346	237·9283	0·999480	+615·976	− 1·990	− 0·845	+ 0·346
13	25	7 18·977	256·3107	1·001782	+570·888	+ 4·423	− 4·227	+ 0·341
15		7 33·379	256·5256	1·001812	+569·993	+ 4·498	− 4·265	+ 0·340
19	29	12 27·149	346·2120	1·006939	−248·445	+19·617	−10·012	− 0·080
21		12 38·921	346·3858	1·006934	−250·206	+19·602	− 9·998	− 0·081
29	July 2	7 51·220	46·1913	1·001950	−616·842	+ 5·659	− 1·177	− 0·335
35		8 19·603	46·6148	1·001899	−617·279	+ 5·516	− 1·097	− 0·336
39		8 39·945	46·9183	1·001862	−617·572	+ 5·412	− 1·039	− 0·336
49	8	7 13·774	176·1776	0·993439	+347·030	−17·963	+ 8·810	+ 0·142
53		7 37·133	176·5325	0·993455	+350·177	−17·912	+ 8·774	+ 0·144
55		7 48·453	176·7044	0·993463	+351·686	−17·888	+ 8·756	+ 0·145
65	10	6 58·755	219·5277	0·997099	+602·056	− 7·441	+ 2·213	+ 0·320
69		7 22·473	219·8850	0·997141	+602·671	− 7·326	+ 2·147	+ 0·321
71		7 34·458	220·0658	0·997162	+603·273	− 7·268	+ 2·113	+ 0·321
81	11	10 11·463	244·0554	1·000062	+602·137	+ 0·896	− 2·383	+ 0·346
85		10 37·137	244·4400	1·000110	+601·235	+ 1·030	− 2·455	+ 0·346
87		10 48·447	244·6094	1·000131	+600·832	+ 1·089	− 2·485	+ 0·346
97	12	6 49·603	262·5552	1·002321	+528·453	+ 7·196	− 5·625	+ 0·324
101		7 10·043	262·8600	1·002356	+526·739	+ 7·295	− 5·675	+ 0·323
103		7 20·232	263·0119	1·002373	+525·877	+ 7·344	− 5·700	+ 0·323
109	14	5 54·947	304·4920	1·006260	+175·882	+17·846	−10·402	+ 0·156
113		6 15·326	304·7934	1·006278	+172·730	+17·894	−10·416	+ 0·155
123	16	6 19·019	347·3939	1·006945	−278·702	+19·094	− 9·672	− 0·099
125		6 28·724	347·5373	1·006941	−280·087	+19·078	− 9·658	− 0·100
129		6 49·921	347·8504	1·006931	−283·107	+19·046	− 9·634	− 0·101
137	18	6 18·693	30·0427	1·003960	−577·422	+10·062	− 3·753	− 0·297
143	22	5 38·479	115·9201	0·994208	−247·024	−15·985	+ 9·636	− 0·190
147		5 58·679	116·2262	0·994184	−244·042	−16·043	+ 9·658	− 0·189
163	27	5 54·339	225·3038	0·997701	+602·155	− 4·795	+ 0·773	+ 0·326
165		6 2·772	225·4304	0·997716	+602·257	− 4·753	+ 0·750	+ 0·326
169		6 24·513	225·7577	0·997753	+602·513	− 4·646	+ 0·689	+ 0·326
177	29	5 54·214	268·4057	1·003082	+470·519	+ 9·328	− 6·670	+ 0·295
179		6 4·372	268·5570	1·003098	+469·510	+ 9·372	− 6·693	+ 0·295
189	Aug. 10	5 24·415	166·2516	0·992902	+272·340	−17·572	+ 8·991	+ 0·097
193		5 43·884	166·5475	0·992914	+275·005	−17·542	+ 8·966	+ 0·099
195		5 54·381	166·7069	0·992917	+276·425	−17·526	+ 8·952	+ 0·100
205	13	9 26·424	235·2209	0·999117	+579·070	− 1·041	− 1·244	+ 0·321
209		9 51·242	235·5934	0·999163	+578·609	− 0·922	− 1·310	+ 0·321
211		10 1·887	235·7532	0·999183	+578·397	− 0·871	− 1·338	+ 0·321
219	14	4 49·972	252·6441	1·001317	+531·512	+ 4·519	− 4·224	+ 0·312
221		4 58·829	252·7766	1·001334	+530·948	+ 4·560	− 4·245	+ 0·311
227	15	9 5·241	277·8996	1·004210	+377·326	+11·711	− 7·791	+ 0·248
235	19	9 23·395	3·3563	1·006131	−412·675	+14·862	− 7·099	− 0·182
241	23	8 35·075	88·5940	0·996808	−428·450	− 8·944	+ 6·448	− 0·266

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TABLE XVI.—*continued.*

CO-ORDINATES OF IV., 1901.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1901.	h m	°		″	″		
243	Aug. 23	8 48·594	88·7978	0·996786	−427·104	− 8·998	+ 6·473	− 0·265
251	24	8 48·476	110·5559	0·994673	−257·570	−13·905	+ 8·728	− 0·187
255		9 5·772	110·8176	0·994653	−255·267	−13·952	+ 8·748	− 0·186
257		9 16·876	110·9857	0·994639	−253·786	−13·984	+ 8·762	− 0·185
267	26	8 53·985	154·3566	0·992875	+159·310	−17·285	+ 9·337	+ 0·033
271		9 25·319	154·8327	0·992876	+163·738	−17·267	+ 9·313	+ 0·035
279	27	8 27·516	175·8291	0·993413	+343·569	−15·357	+ 7·658	+ 0·140
283		8 46·803	176·1220	0·993428	+345·803	−15·313	+ 7·626	+ 0·142
285		8 56·830	176·2745	0·993436	+346·959	−15·292	+ 7·610	+ 0·143
293	28	8 32·800	197·7402	0·994886	+482·193	−11·221	+ 4·862	+ 0·197
295		8 41·609	197·8736	0·994899	+482·845	−11·191	+ 4·842	+ 0·197
303	29	8 25·142	219·3741	0·997066	+551·069	− 5·569	+ 1·416	+ 0·286
305		8 35·436	219·5292	0·997083	+551·291	− 5·525	+ 1·391	+ 0·286
309		8 54·142	219·8110	0·997117	+551·679	− 5·445	+ 1·343	+ 0·287
325	Sept. 1	8 31·918	284·0571	1·004631	+309·476	+12·399	− 8·098	+ 0·210
329		8 52·164	284·3576	1·004662	+307·031	+12·462	− 8·126	+ 0·209
331		9 3·458	284·5253	1·004678	+305·673	+12·497	− 8·141	+ 0·208
341	3	8 9·624	326·3372	1·007103	− 82·819	+17·182	− 9·596	+ 0·010
345		8 31·391	326·6586	1·007109	− 85·904	+17·184	− 9·590	+ 0·008
347		8 43·578	326·8387	1·007112	− 87·619	+17·186	− 9·583	+ 0·007
357	4	8 10·015	347·6088	1·006939	−275·573	+16·149	− 8·391	− 0·099
361		8 32·656	347·9433	1·006928	−278·357	+16·114	− 8·362	− 0·101
363		8 43·348	348·1013	1·006923	−279·665	+16·098	− 8·348	− 0·102
371	9	7 47·792	94·6316	0·996250	−368·597	− 9·765	+ 6·834	− 0·233
373		7 58·242	94·7892	0·996232	−367·503	− 9·799	+ 6·850	− 0·233
379	10	7 23·701	116·0493	0·994216	−199·347	−13·861	+ 8·660	− 0·154
393	14	7 28·860	203·5515	0·995273	+481·646	− 9·315	+ 3·839	+ 0·231
397		7 59·954	204·0215	0·995317	+483·431	− 9·205	+ 3·770	+ 0·232
399		8 11·804	204·2006	0·995335	+484·101	− 9·164	+ 3·744	+ 0·233
407	15	7 29·076	246·8861	1·000407	+502·031	+ 2·273	− 2·964	+ 0·280
409		7 42·362	247·0850	1·000431	+501·441	+ 2·329	− 2·994	+ 0·279
415		8 24·543	247·7161	1·000511	+499·535	+ 2·506	− 3·090	+ 0·279
425	17	7 23·779	268·3025	1·003044	+405·846	+ 7·937	− 5·954	+ 0·246
429		7 48·070	268·6642	1·003087	+403·685	+ 8·026	− 5·998	+ 0·245
431		7 57·518	268·8048	1·003102	+402·843	+ 8·060	− 6·016	+ 0·244
441	19	7 13·265	310·8493	1·006783	+ 69·289	+15·287	− 9·158	+ 0·088
445		7 35·567	311·1791	1·006800	+ 66·271	+15·314	− 9·166	+ 0·086
447		7 46·487	311·3404	1·006810	+ 64·796	+15·348	− 9·168	+ 0·086
463	27	5 55·494	121·5037	0·993713	−152·167	−13·413	+ 8·449	− 0·126
467		6 18·196	121·8479	0·993691	−149·246	−13·454	+ 8·466	− 0·124

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TABLE XVI.—*continued.*

CO-ORDINATES OF I., 1902.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1902.	h m	°		"	"		
3	June 10	10 25·912	323·9825	0·996439	+ 79·233	+ 0·010	− 1·787	0·000
5		10 39·828	325·9630	0·996405	+ 75·647	+ 0·010	− 1·834	0·000
7		10 55·384	328·1767	0·996367	+ 71·531	+ 0·010	− 1·884	0·000
23	16	10 34·712	106·6322	1·002364	−127·931	+ 0·003	+ 0·601	+ 0·001
25		10 45·938	108·2110	1·002438	−126·943	+ 0·004	+ 0·663	+ 0·001
27		10 58·830	110·0366	1·002524	−125·690	+ 0·005	+ 0·732	+ 0·001
47	21	10 23·552	42·5618	0·998655	− 88·240	− 0·054	− 1·758	+ 0·001
51	24	9 59·950	289·0035	0·997244	+128·287	− 0·018	− 0·720	− 0·001
53		10 12·883	290·8413	0·997170	+126·895	− 0·020	− 0·791	− 0·001
55		10 25·385	292·6177	0·997098	+125·425	− 0·023	− 0·859	− 0·001
63	30	9 40·308	67·8792	1·000680	−126·402	− 0·048	− 0·932	+ 0·002
65		9 53·471	69·7362	1·000796	−128·083	− 0·045	− 0·860	+ 0·002
71		10 54·030	78·2757	1·001339	−134·057	− 0·030	− 0·519	+ 0·002
79	July 1	10 26·439	277·1350	0·997564	+136·231	− 0·005	− 0·268	− 0·002
83	3	9 5·774	312·9506	0·996318	+101·581	− 0·071	− 1·619	− 0·002
85		9 25·119	315·7040	0·996282	+ 97·008	− 0·075	− 1·702	− 0·001
87	7	8 48·099	44·9287	0·999519	− 97·942	− 0·091	− 1·726	+ 0·001
93		9 52·274	54·0002	1·000117	−112·378	− 0·080	− 1·435	+ 0·001
95		10 7·865	56·2029	1·000257	−115·472	− 0·074	− 1·358	+ 0·002
103	8	14 49·687	298·9212	0·996536	+121·802	− 0·052	− 1·171	− 0·002
105		15 1·756	300·6390	0·996497	+119·734	− 0·055	− 1·234	− 0·002
107		15 18·079	302·9621	0·996443	+116·767	− 0·059	− 1·317	− 0·002
117	14	8 2·694	22·9007	0·998451	− 55·924	− 0·113	− 2·254	+ 0·001
119		8 21·072	25·5056	0·998608	− 61·750	− 0·111	− 2·208	+ 0·001
127	15	8 28·134	229·1612	0·999812	+107·674	+ 0·082	+ 1·590	− 0·001
129		8 42·978	231·2622	0·999673	+110·930	+ 0·079	+ 1·519	− 0·002
131		8 56·225	233·1331	0·999549	+113·702	+ 0·076	+ 1·455	− 0·002
143	16	8 27·502	73·3652	1·001758	−136·159	− 0·040	− 0·677	+ 0·002
145		8 40·155	75·1465	1·001862	−137·312	− 0·036	− 0·603	+ 0·002
147		8 51·379	76·7266	1·001950	−138·225	− 0·033	− 0·536	+ 0·002
157	18	9 30·233	128·8702	1·003731	−109·160	+ 0·067	+ 1·590	+ 0·002
159		9 41·785	130·4905	1·003741	−106·541	+ 0·069	+ 1·643	+ 0·002
163	19	8 13·212	321·5003	0·996301	+ 85·938	− 0·083	− 1·960	− 0·001
165		8 27·772	323·5732	0·996323	+ 81·821	− 0·086	− 2·013	− 0·001
181	24	7 44·258	254·4369	0·997829	+138·127	+ 0·031	+ 0·596	− 0·002
183		7 57·431	256·3071	0·997731	+139·160	+ 0·028	+ 0·517	− 0·002
185		8 10·616	258·1787	0·997633	+140·044	+ 0·025	+ 0·438	− 0·002
193	26	7 36·089	300·5602	0·996286	+120·100	− 0·041	− 1·335	− 0·001
197		8 4·800	304·6481	0·996263	+114·344	− 0·046	− 1·480	− 0·001
199		8 14·836	306·0770	0·996262	+112·195	− 0·048	− 1·530	− 0·001
209	27	7 28·484	142·9054	1·003617	− 82·339	+ 0·065	+ 2·048	+ 0·001
211		7 40·884	144·6451	1·003583	− 78·732	+ 0·067	+ 2·091	+ 0·001
223	Aug. 3	7 10·818	124·9262	1·003757	−113·563	+ 0·024	+ 1·541	0·000
225		7 22·597	126·5766	1·003751	−110·967	+ 0·025	+ 1·598	0·000
229		7 43·450	129·5031	1·003729	−106·141	+ 0·027	+ 1·694	+ 0·001

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TABLE XVI.—*continued.*

CO-ORDINATES OF I., 1902.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1902.	h m	°		″	″		
237	Aug. 4	13 7·174	19·2241	0·999205	− 54·493	− 0·043	− 2·315	0·000
241		13 48·456	25·0647	0·999587	− 67·741	− 0·042	− 2·207	0·000
245	7	6 28·597	212·5130	0·999756	+ 84·187	+ 0·026	+ 2·025	0·000
247		6 38·961	213·9782	0·999661	+ 87·117	+ 0·026	+ 1·986	0·000
249		6 50·881	215·6641	0·999549	+ 90·421	+ 0·026	+ 1·939	0·000
265	16	6 31·970	244·5038	0·997479	+133·098	− 0·004	+ 0·886	+ 0·001
267		6 45·788	246·4668	0·997385	+134·749	− 0·003	+ 0·806	+ 0·001
273	17	6 4·961	84·6824	1·003365	−143·208	− 0·009	− 0·025	− 0·001
275		6 20·616	86·8800	1·003427	−143·162	− 0·010	+ 0·071	− 0·001
291	18	7 19·150	298·5227	0·996242	+118·661	+ 0·034	− 1·366	+ 0·001
293		7 30·109	300·0829	0·996257	+116·488	+ 0·035	− 1·421	+ 0·001
295		7 42·229	301·8088	0·996278	+113·984	+ 0·036	− 1·482	+ 0·001
299	19	6 6·593	131·5473	1·003511	− 98·310	+ 0·047	+ 1·810	− 0·001
309	20	5 57·889	334·2601	0·997342	+ 50·060	+ 0·062	− 2·320	0·000
323	21	7 33·415	190·5201	1·000512	+ 38·588	− 0·066	+ 2·389	0·000
325		7 43·657	191·9660	1·000399	+ 42·026	− 0·065	+ 2·370	0·000
327		7 54·352	193·4762	1·000318	+ 45·588	− 0·065	+ 2·350	+ 0·001
331	22	5 47·956	19·9640	1·000123	− 60·750	+ 0·067	− 2·243	− 0·001
333		5 58·052	21·3902	1·000215	− 63·937	+ 0·066	− 2·216	− 0·001
341		7 17·628	32·6222	1·000945	− 87·504	+ 0·056	− 1·957	− 0·001
345	24	10 30·801	106·4741	1·003766	−131·717	− 0·044	+ 0·932	− 0·001
349		10 59·859	110·5494	1·003756	−127·578	− 0·049	+ 1·093	− 0·001
355		11 53·661	118·0959	1·003683	−118·219	− 0·060	+ 1·376	− 0·001
359	25	5 35·319	268·0836	0·996383	+140·670	+ 0·016	− 0·162	+ 0·002
369		6 32·229	276·1854	0·996257	+137·950	+ 0·031	− 0·506	+ 0·002
371		6 42·330	277·6234	0·996241	+137·177	+ 0·033	− 0·566	+ 0·002
373		6 52·923	279·1311	0·996231	+136·276	+ 0·035	− 0·629	+ 0·002
377	27	5 28·871	314·4953	0·996763	+ 89·686	+ 0·094	− 1·892	+ 0·001
379		5 39·399	315·9926	0·996812	+ 86·826	+ 0·095	− 1·932	+ 0·001
383		6 11·331	320·5333	0·996979	+ 77·796	+ 0·100	− 2·046	+ 0·001
387		6 51·890	326·2982	0·997217	+ 65·633	+ 0·105	− 2·173	+ 0·001
395	28	5 10·563	154·8444	1·002332	− 46·281	− 0·118	+ 2·327	− 0·001
397		5 22·098	156·4673	1·002249	− 42·481	− 0·119	+ 2·348	0·000
399		5 33·112	158·0171	1·002166	− 38·820	− 0·120	+ 2·367	0·000
411	30	6 17·077	253·7536	0·996689	+137·744	− 0·014	+ 0·424	+ 0·002
423	Sept. 2	4 50·189	89·8269	1·003708	−139·321	− 0·027	+ 0·270	− 0·003
425		5 1·468	91·4090	1·003723	−138·837	− 0·031	+ 0·337	− 0·003
429		5 20·857	94·1287	1·003748	−137·760	− 0·038	+ 0·452	− 0·003
437	3	4 42·613	292·2626	0·996349	+121·677	+ 0·087	− 1·170	+ 0·002
445		5 44·061	301·0081	0·996523	+110·077	+ 0·107	− 1·480	+ 0·002
447		5 57·865	302·9727	0·996574	+107·104	+ 0·111	− 1·546	+ 0·002
469	6	5 43·732	190·8432	0·999726	+ 42·100	− 0·172	+ 2·302	+ 0·001
473		6 9·892	194·5429	0·999483	+ 50·548	− 0·168	+ 2·249	+ 0·001
475		6 21·577	196·1957	0·999378	+ 54·239	− 0·166	+ 2·222	+ 0·001
479	7	4 36·884	25·7146	1·001283	− 74·789	+ 0·156	− 2·035	− 0·001
485		5 18·936	31·6394	1·001645	− 86·456	+ 0·145	− 1·891	− 0·002
487		5 30·361	33·2493	1·001737	− 89·472	+ 0·140	− 1·847	− 0·002

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TABLE XVI.—*continued.*

CO-ORDINATES OF I., 1902.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1902.	h m	°		″	″		
495	Sept. 8	4 33·793	228·0460	0·997418	+112·817	− 0·105	+ 1·374	+ 0·003
497		4 43·442	229·4179	0·997350	+114·663	− 0·101	+ 1·326	+ 0·003
499		4 53·089	230·7868	0·997287	+116·439	− 0·097	+ 1·278	+ 0·003
511	10	10 13·124	323·6200	0·997584	+ 66·797	+ 0·187	− 2·088	+ 0·002
513		10 24·052	325·1713	0·997663	+ 63·535	+ 0·189	− 2·119	+ 0·002
515		10 39·949	327·4283	0·997780	+ 58·707	+ 0·192	− 2·162	+ 0·001
527	12	5 0·984	326·5113	0·997786	+ 60·165	+ 0·201	− 2·139	+ 0·002
531	16	9 16·157	96·1288	1·003746	−131·942	− 0·067	+ 0·563	+ 0·004
535	26	8 59·736	329·1420	0·998535	+ 51·096	+ 0·262	− 2·116	+ 0·002
539		9 26·460	332·9286	0·998770	+ 42·983	+ 0·269	− 2·172	+ 0·001
541		9 38·776	334·6731	0·998878	+ 39·180	+ 0·274	− 2·194	+ 0·002
553	Oct. 4	8 29·602	151·9803	1·000898	− 43·823	− 0·284	+ 2·112	− 0·002
555		8 40·774	153·5564	1·000798	− 40·474	− 0·286	+ 2·131	− 0·002
557		8 49·533	154·7923	1·000717	− 37·827	− 0·288	+ 2·146	− 0·001

CO-ORDINATES OF II., 1902.

3	June 10	10 25·912	333·1615	1·009181	+ 99·764	− 1·118	− 3·200	− 0·021
13	11	10 41·709	74·4152	0·999500	−197·906	− 0·983	− 1·075	+ 0·023
15		10 59·312	75·6558	0·999296	−199·151	− 0·954	− 0·999	+ 0·024
23	16	10 34·712	222·8073	0·995912	+138·311	+ 1·593	+ 2·747	− 0·012
29		11 12·205	225·4669	0·996311	+145·241	+ 1·561	+ 2·634	− 0·013
31		11 24·086	226·3092	0·996440	+147·468	+ 1·551	+ 2·596	− 0·013
39	19	9 58·466	163·5578	0·990778	− 64·467	+ 1·426	+ 3·487	+ 0·018
41		10 12·922	164·5947	0·990800	− 60·843	+ 1·444	+ 3·508	+ 0·017
51	24	9 59·950	310·7946	1·008600	+167·138	− 0·691	− 2·413	− 0·030
57		10 40·093	313·5711	1·008800	+160·288	− 0·775	− 2·553	− 0·029
59		10 54·861	314·5922	1·008839	+157·667	− 0·804	− 2·602	− 0·029
63	30	9 40·308	198·0668	0·993959	+ 64·345	+ 1·892	+ 3·609	0·000
67		10 11·083	200·2588	0·994233	+ 72·239	+ 1·890	+ 3·566	− 0·001
73		11 8·300	204·3317	0·994767	+ 86·631	+ 1·884	+ 3·468	− 0·003
77	July 1	10 9·170	301·1411	1·008268	+190·173	− 0·428	− 1·950	− 0·033
83	3	9 5·774	138·8224	0·990849	−144·697	+ 0·973	+ 2·832	+ 0·029
95	7	10 7·865	189·5838	0·993575	+ 35·876	+ 1·919	+ 3·784	+ 0·005
97		10 21·495	190·5567	0·993687	+ 39·560	+ 1·924	+ 3·774	+ 0·005
99		10 38·839	191·7952	0·993833	+ 44·236	+ 1·929	+ 3·758	+ 0·004
103	8	14 49·687	310·2597	1·009001	+171·207	− 0·752	− 2·523	− 0·032
109		15 32·383	313·2118	1·009079	+163·555	− 0·847	− 2·675	− 0·031
111		15 57·699	314·9616	1·009130	+158·814	− 0·900	− 2·762	− 0·031
127	15	8 28·134	273·5291	1·006594	+225·401	+ 0·487	− 0·286	− 0·034
135		9 25·840	277·5364	1·007030	+223·832	+ 0·351	− 0·560	− 0·034
137		9 42·376	278·6846	1·007149	+223·178	+ 0·312	− 0·639	− 0·034
143	16	8 27·502	13·2557	1·003993	− 54·625	− 1·979	− 3·826	− 0·003
149		9 5·993	15·9401	1·004619	− 64·885	− 1·986	− 3·781	− 0·001

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TABLE XVI.—*continued.*

CO-ORDINATES OF II., 1902.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1902.	h m	°		″	″		
151	July 18	8 29·125	218·0388	0·998588	+141·553	+ 1·847	+ 3·051	− 0·012
155		9 0·822	220·2758	0·998945	+148·335	+ 1·822	+ 2·952	− 0·013
155*a*		9 20·413	221·6576	0·999166	+152·416	+ 1·800	+ 2·890	− 0·014
159		9 41·785	223·1643	0·999407	+156·767	+ 1·780	+ 2·820	− 0·015
163	19	8 13·212	317·2351	1·009158	+151 623	− 0·983	− 2·972	− 0·030
167	20	8 32·044	58·9500	0·997690	−195·654	− 1·479	− 1·960	+ 0·023
169		8 43·618	59·7686	0·997561	−197·214	− 1·460	− 1·911	+ 0·023
181	24	7 44·258	101·5740	0·992030	−219·066	− 0·208	+ 0·900	+ 0·034
187		8 26·557	104·6008	0·991806	−215·979	− 0·106	+ 1·101	+ 0·034
189		8 41·369	105·6611	0·991735	−214·755	− 0·070	+ 1·169	+ 0·034
193	26	7 36·089	304·3852	1·009075	+184·945	− 0·564	− 2·371	+ 0·033
201		8 15·108	307·7790	1·009121	+176·589	− 0·676	− 2·560	+ 0·032
203		8 25·865	308·5355	1·009126	+174·638	− 0·700	− 2·600	+ 0·032
209	27	7 28·484	44·0709	0·999242	−163·950	− 1·746	− 2·752	+ 0·015
213		7 51·483	45·6927	0·998982	−168·307	− 1·720	− 2·670	+ 0·016
223	Aug. 3	7 10·818	32·4499	1·000290	−132·316	− 1·843	− 3·249	+ 0·008
227		7 32·163	33·9527	1·000047	−137·123	− 1·831	− 3·187	+ 0·009
231		8 4·543	36·2335	0·999678	−144·236	− 1·807	− 3·088	+ 0·010
237	4	13 7·174	160·5682	0·992772	− 63·594	+ 1·486	+ 3·789	+ 0·020
239		13 38·683	162·8181	0·992997	− 55·020	+ 1·530	+ 3·832	+ 0·019
245	7	6 28·597	75·2664	0·993607	−222·401	− 1·013	− 0·773	+ 0·027
251		7 5·972	77·9322	0·993318	−224·145	− 0·940	− 0·592	+ 0·028
253		7 20·597	78·9758	0·993205	−224·696	− 0·910	− 0·520	+ 0·028
265	16	6 31·970	269·0788	1·008082	+228·657	+ 0·616	− 0·258	+ 0·029
269		6 57·166	270·8242	1·008204	+228·134	+ 0·565	− 0·379	+ 0·030
281	18	6 4·136	109·3368	0·991004	−205·013	− 0·006	+ 1·608	+ 0·030
285		6 29·595	111·1618	0·990989	−201·958	+ 0·049	+ 1·721	+ 0·030
291		7 19·150	114·7145	0·990987	−195·435	+ 0·156	+ 1·935	+ 0·030
301	19	6 23·101	213·1145	1·001710	+139·998	+ 1·687	+ 3·116	− 0·007
303		6 35·460	213·9809	1·001845	+142·704	+ 1·681	+ 3·079	− 0·007
305		6 52·815	215·1990	1·002034	+146·455	+ 1·671	+ 3·026	− 0·008
309	20	5 57·889	311·3482	1·008429	+157·467	− 0·637	− 2·892	− 0·028
311	21	5 42·567	51·1246	0·995348	−187·550	− 1·455	− 2·180	+ 0·015
313		5 56·670	52·1270	0·995214	−189·678	− 1·440	− 2·121	+ 0·015
317		6 43·766	55·4757	0·994782	−198·357	− 1·383	− 1·923	+ 0·017
319		7 9·405	57·3002	0·994548	−199·713	− 1·350	− 1·812	+ 0·018
325		7 43·657	59·7385	0·994252	−203·880	− 1·305	− 1·662	+ 0·019
331	22	5 47·956	154·4755	0·993479	− 77·381	+ 1·177	+ 3·674	+ 0·021
337		6 46·907	158·6779	0·993948	− 61·755	+ 1·264	+ 3·764.	+ 0·019
339		7 4·287	159·9160	0·994093	− 57·081	+ 1·286	+ 3·787	+ 0·019
343	24	10 18·659	14·5539	1·000657	− 77·746	− 1·663	− 3·699	− 0·003
347		10 43·599	16·3078	1·000377	− 84·177	− 1·667	− 3·655	− 0·003
355		11 53·661	21·2400	0·999592	−101·798	− 1·670	− 3·512	0·000
363	25	5 58·383	98·4428	0·991136	−216·207	− 0·345	+ 0·950	+ 0·028
367		6 20·970	100·0718	0·991097	−214·651	− 0·299	+ 1·057	+ 0·028
371		6 42·330	101·6027	0·991068	−212·938	− 0·255	+ 1·157	+ 0·028
377	27	5 28·871	299·0506	1·008723	+185·703	− 0·235	− 2·266	− 0·028

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TABLE XVI.—*continued.*

CO-ORDINATES OF II., 1902.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1902.	h m						
381	Aug. 27	5 52·494	300°·6851	1·008659	+181″·909	− 0″·281	− 2·251	− 0·028
389		7 5·346	305·7271	1·008418	+169·282	− 0·422	− 2·626	− 0·028
399	28	5 33·112	40·0354	0·996175	−160·848	− 1·526	− 2·707	+ 0·009
401		5 44·386	40·8352	0·996061	−162·978	− 1·519	− 2·667	+ 0·009
405		6 6·875	42·4313	0·995840	−167·132	− 1·503	− 2·586	+ 0·010
409	30	11 1·613	267·1597	1·008526	+224·836	+ 0·659	− 0·233	− 0·026
413		11 29·827	269·1122	1·008616	+224·271	+ 0·608	− 0·367	− 0·026
427	Sept. 2	5 11·880	187·3529	0·999289	+ 53·164	+ 1·502	+ 3·759	+ 0·007
429		5 20·857	187·9860	0·999399	+ 55·548	+ 1·506	+ 3·748	+ 0·007
431		5 30·084	188·6366	0·999491	+ 57·991	+ 1·510	+ 3·738	+ 0·006
439	3	4 56·205	286·5042	1·008881	+205·909	+ 0·151	− 1·533	− 0·027
443		5 32·027	288·9821	1·008830	+201·902	+ 0·084	− 1·688	− 0·027
445		5 44·061	289·8144	1·008812	+200·472	+ 0·062	− 1·739	− 0·027
455	4	10 31·030	50·7897	0·994119	−185·650	− 1·335	− 2·064	+ 0·013
457		10 45·929	51·8509	0·993994	−187·779	− 1·321	− 2·003	+ 0·013
463	6	4 58·041	231·6349	1·005916	+189·018	+ 1·321	+ 2·021	− 0·013
465		5 9·695	232·4456	1·006011	+190·656	+ 1·310	+ 1·974	− 0·013
471		5 56·555	235·7040	1·006381	+196·852	+ 1·266	+ 1·782	− 0·014
475		6 21·577	237·4427	1·006569	+199·899	+ 1·242	+ 1·678	− 0·015
479	7	4 36·884	329·9683	1·005589	+ 86·802	− 0·907	− 3·556	− 0·021
481		4 52·207	331·0351	1·005458	+ 82·977	− 0·929	− 3·583	− 0·021
483		5 5·834	331·9842	1·005339	+ 79·553	− 0·948	− 3·607	− 0·020
495	8	4 33·793	71·4648	0·991988	−213·473	− 0·973	− 0·758	+ 0·019
501		5 3·371	73·5810	0·991849	−214·884	− 0·932	− 0·620	+ 0·020
503		5 11·257	74·1453	0·991812	−215·209	− 0·920	− 0·582	+ 0·020
511	10	10 13·124	298·1455	1·008184	+179·742	− 0·123	− 2·226	− 0·026
517		10 51·897	300·8312	1·008008	+173·530	− 0·192	− 2·370	− 0·025
521		11 15·534	302·4692	1·007893	+169·552	− 0·233	− 2·456	− 0·025
527	12	5 0·984	119·7519	0·991919	−172·033	+ 0·150	+ 2·271	+ 0·025
539	26	9 26·460	117·8157	0·992623	−168·550	+ 0·044	+ 2·123	+ 0·022
543		9 52·038	119·6426	0·992780	−164·598	+ 0·084	+ 2·217	+ 0·022
545		10 3·007	120·4261	0·992847	−162·852	+ 0·102	+ 2·256	+ 0·022
557	Oct. 4	8 49·533	206·4559	1·005353	+116·607	+ 1·229	+ 2·954	0·000
561		9 11·856	208·0106	1·005543	+121·177	+ 1·228	+ 2·898	0·000
565		9 36·440	209·7221	1·005745	+126·105	+ 1·227	+ 2·835	0·000
569	5	8 22·234	304·3860	1·006033	+151·458	− 0·167	− 2·422	− 0·021

CO-ORDINATES OF III., 1902.

5	June 10	10 39·828	248·0151	0·999026	+301·388	+ 0·420	+ 2·309	+ 0·011
9		11 11·160	249·1119	0·999035	+303·897	+ 0·433	+ 2·208	+ 0·011
13	11	10 41·709	298·4559	0·999905	+295·414	+ 0·747	− 2·601	+ 0·001
17		11 18·140	299·7290	0·999934	+292·069	+ 0·749	− 2·714	+ 0·001
25	16	10 45·938	190·0066	0·998849	+ 49·159	− 0·209	+ 5·776	+ 0·012

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TABLE XVI.—*continued.*

Co-ordinates of III., 1902.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1902.	h m	°		"	"		
29	June 16	11 12·205	190·9266	0·998843	+ 54·471	− 0·198	+ 5·762	+ 0·012
33		11 36·887	191·7912	0·998837	+ 59·451	− 0·187	+ 5·748	+ 0·012
39	19	9 58·466	339·4634	1·000794	+126·908	+ 0·493	− 5·488	− 0·009
43		10 28·453	340·5096	1·000812	+121·150	+ 0·485	− 5·529	− 0·009
47	21	10 23·552	80·7527	1·000775	−334·742	− 0·604	− 1·093	+ 0·006
53	24	10 12·883	231·4993	0·998910	+263·291	+ 0·377	+ 3·812	+ 0·010
57		10 40·093	232·4522	0·998918	+266·913	+ 0·386	+ 3·736	+ 0·010
61		11 7·489	233·4117	0·998926	+270·484	+ 0·396	+ 3·658	+ 0·010
75	July 1	9 41·019	222·5913	0·998859	+232·267	+ 0·337	+ 4·521	+ 0·010
79		10 26·439	224·1822	0·998866	+239·405	+ 0·353	+ 4·407	+ 0·010
85	3	9 25·119	322·7582	1·000482	+215·016	+ 0·524	− 4·841	+ 0·008
87	7	8 48·099	162·5079	0·999327	−107·047	− 0·327	+ 5·859	+ 0·010
91		9 40·449	164·3397	0·999301	− 96·244	− 0·307	+ 5·917	+ 0·011
97		10 21·495	165·7761	0·999280	− 87·702	− 0·292	+ 5·957	+ 0·011
105	8	15 1·756	226·0106	0·998890	+253·809	+ 0·398	+ 4·289	+ 0·010
109		15 32·383	227·0831	0·998896	+258·388	+ 0·408	+ 4·205	+ 0·010
113		16 9·614	228·3868	0·998903	+263·834	+ 0·421	+ 4·102	+ 0·010
117	14	8 2·694	153·1667	0·999516	−158·213	− 0·429	+ 5·582	+ 0·010
121		8 37·146	154·3335	0·999492	−151·652	− 0·415	+ 5·599	+ 0·010
129	15	8 42·978	204·9464	0·998846	+154·112	+ 0·160	+ 5·623	+ 0·012
133		9 10·752	205·9193	0·998839	+159·601	+ 0·171	+ 5·576	+ 0·012
137		9 42·376	207·0269	0·998832	+165·771	+ 0·184	+ 5·521	+ 0·012
145	16	8 40·155	255·2796	0·999020	+347·497	+ 0·624	+ 1·511	+ 0·005
149		9 5·993	256·1842	0·999032	+348·846	+ 0·629	+ 1·416	+ 0·005
165	19	8 27·772	45·6956	1·001292	−263·000	− 0·378	− 4·315	+ 0·010
167	20	8 32·044	96·0525	1·000781	−357·797	− 0·701	+ 0·794	− 0·002
173		9 14·110	97·5204	1·000755	−356·498	− 0·703	+ 0·954	− 0·001
175		9 25·805	97·9284	1·000748	−356·088	− 0·704	+ 0·998	− 0·001
177		9 44·803	98·5912	1·000736	−355·402	− 0·705	+ 1·070	− 0·001
183	24	7 57·431	296·3967	0·999634	+319·518	+ 0·707	− 2·973	− 0·002
187		8 26·557	297·4140	0·999659	+316·457	+ 0·705	− 3·073	− 0·002
191		9 3·677	298·7130	0·999691	+312·402	+ 0·701	− 3·199	− 0·003
195	26	7 50·220	36·6349	1·001409	−226·626	− 0·236	− 4·963	− 0·012
197		8 4·800	37·1429	1·001409	−229·143	− 0·242	− 4·927	− 0·012
201		8 25·171	37·8529	1·001410	−232·626	− 0·250	− 4·877	− 0·012
205		8 46·810	38·6070	1·001411	−236·296	− 0·259	− 4·822	− 0·012
211	27	7 40·884	86·4987	1·001016	−363·089	− 0·676	− 0·160	− 0·005
213		7 51·419	86·8684	1·001011	−363·131	− 0·678	− 0·120	− 0·005
219	Aug. 3	6 48·753	76·8969	1·001188	−358·752	− 0·605	− 1·122	− 0·008
221		7 0·307	77·2998	1·001181	−359·188	− 0·608	− 1·077	− 0·008
225		7 22·597	78·0768	1·001168	−359·985	− 0·615	− 0·992	− 0·008
227		7 32·163	78·4104	1·001162	−360·303	− 0·618	− 0·956	− 0·008
239	4	13 38·683	141·4842	0·999564	−210·457	− 0·695	+ 5·168	+ 0·006
241		13 48·456	141·8261	0·999556	−208·680	− 0·693	+ 5·190	+ 0·006
247	7	6 38·961	278·1116	0·999286	+356·430	+ 0·745	− 1·275	+ 0·005

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TABLE XVI.—*continued.*

CO-ORDINATES OF III., 1902.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1902.	h m	°		"	"		
251	Aug. 7	7 5·972	279·0567	0·999311	+355·193	+ 0·749	− 1·377	+ 0·005
255		7 35·576	280·0928	0·999339	+353·726	+ 0·753	− 1·489	+ 0·004
257		7 49·646	280·5850	0·999352	+352·988	+ 0·755	− 1·543	+ 0·004
263	16	6 18·038	10·1300	1·001416	− 92·577	+ 0·398	− 6·138	− 0·014
267		6 45·788	11·0967	1·001425	− 98·510	+ 0·384	− 6·110	− 0·014
269		6 57·166	11·4931	1·001429	−100·935	+ 0·380	− 6·098	− 0·014
273	17	6 4·961	59·8465	1·001326	−327·877	− 0·338	− 2·719	− 0·015
277		6 37·140	60·9679	1·001311	−330·848	− 0·355	− 2·606	− 0·015
279		7 7·842	62·0381	1·001297	−333·565	− 0·371	− 2·498	− 0·014
281	18	6 4·136	110·0500	1·000285	−328·104	− 0·859	+ 2·660	− 0·005
283		6 16·238	110·4728	1·000274	−326·959	− 0·861	+ 2·702	− 0·005
287		6 51·978	111·7210	1·000242	−323·473	− 0·868	+ 2·825	− 0·004
293		7 30·109	113·0528	1·000208	−319·583	− 0·877	+ 2·955	− 0·004
299	19	6 6·593	160·4931	0·999007	− 90·422	− 0·789	+ 6·092	+ 0·008
303		6 35·460	161·5039	0·998985	− 84·229	− 0·780	+ 6·119	+ 0·009
311	21	5 42·567	260·5591	0·999018	+359·474	+ 0·604	+ 0·463	+ 0·013
313		5 56·670	261·0527	0·999028	+359·695	+ 0·610	+ 0·409	+ 0·013
317		6 43·766	262·7016	0·999065	+360·241	+ 0·632	+ 0·228	+ 0·013
321		7 20·020	263·9708	0·999093	+360·454	+ 0·648	+ 0·088	+ 0·012
327		7 54·352	265·1726	0·999119	+360·494	+ 0·662	− 0·044	+ 0·012
333	22	5 58·052	311·4533	1·000288	+247·144	+ 0·964	− 4·593	0·000
335		6 11·640	311·9278	1·000300	+244·953	+ 0·964	− 4·628	0·000
337		6 46·907	313·1597	1·000321	+239·190	+ 0·964	− 4·720	0·000
343	24	10 18·659	60·9391	1·001314	−330·388	− 0·303	− 2·503	− 0·016
345		10 30·801	61·3622	1·001309	−331·431	− 0·310	− 2·461	− 0·016
353		11 40·239	63·7824	1·001278	−337·050	− 0·348	− 2·214	− 0·016
363	25	5 58·383	102·0837	1·000548	−341·940	− 0·851	+ 1·912	− 0·009
365		6 9·205	102·4614	1·000539	−341·198	− 0·855	+ 1·951	− 0·009
369		6 32·229	103·4030	1·000520	−339·572	− 0·863	+ 2·034	− 0·009
379	27	5 39·399	202·1816	0·998535	+168·059	− 0·398	+ 5·496	+ 0·016
381		5 52·494	202·6405	0·998533	+170·578	− 0·391	+ 5·473	+ 0·016
391		7 20·447	205·7228	0·998514	+187·219	− 0·340	+ 5·306	+ 0·017
397	28	5 22·098	252·0341	0·998815	+349·207	+ 0·462	+ 1·291	+ 0·016
403		5 54·631	253·1739	0·998834	+350·619	+ 0·480	+ 1·169	+ 0·016
405		6 6·875	253·6028	0·998841	+351·112	+ 0·486	+ 1·123	+ 0·016
411	30	11 17·077	5·0518	1·001275	− 69·966	+ 0·714	− 6·115	− 0·014
413		11 29·827	5·4962	1·001280	− 72·684	+ 0·708	− 6·103	− 0·014
415		11 45·426	6·0399	1·001286	− 76·009	+ 0·700	− 6·091	− 0·014
417	Sept. 1	4 35·935	91·4430	1·000858	−352·015	− 0·755	+ 0·847	− 0·014
421	2	4 40·481	141·8970	0·999569	−185·342	− 1·104	+ 5·259	+ 0·001
425		5 1·468	142·6300	0·999551	−181·456	− 1·104	+ 5·301	+ 0·001
427		5 11·880	142·9949	0·999542	−179·507	− 1·103	+ 5·320	+ 0·002
437	3	4 42·613	192·3910	0·998574	+114·824	− 0·665	+ 5·821	+ 0·016
443		5 32·027	194·1227	0·998554	+124·859	− 0·637	+ 5·757	+ 0·016
449		6 6·460	195·3296	0·998543	+131·789	− 0·617	+ 5·711	+ 0·018
453	4	10 16·702	254·5614	0·998783	+348·367	+ 0·481	+ 0·933	+ 0·018
457		10 45·929	255·5853	0·998802	+349·271	+ 0·499	+ 0·824	+ 0·018

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TABLE XVI.—*continued.*

CO-ORDINATES OF III., 1902.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1902.	h m	°		"	"		
461	Sept. 6	4 46·373	343·6847	1·000938	+ 57·842	+ 1·069	− 6·071	− 0·008
463		4 58·041	344·0916	1·000947	+ 55·364	+ 1·066	− 6·078	− 0·009
469		5 43·732	345·6853	1·000983	+ 45·641	+ 1·052	− 6·103	− 0·009
483	7	5 5·834	34·5432	1·001645	−233·238	+ 0·328	− 4·601	− 0·020
487		5 30·361	35·3974	1·001649	−237·137	+ 0·312	− 4·539	− 0·020
489		5 42·837	35·8320	1·001651	−239·099	+ 0·304	− 4·507	− 0·020
499	8	4 53·089	84·2663	1·001176	−350·505	− 0·649	+ 0·139	− 0·017
501		5 3·371	84·6249	1·001169	−350·439	− 0·655	+ 0·178	− 0·017
505		5 21·480	85·2563	1·001156	−350·283	− 0·665	+ 0·245	− 0·017
513	10	10 24·052	196·5407	0·998378	+139·915	− 0·697	+ 5·549	+ 0·017
519		11 2·816	197·8995	0·998362	+147·415	− 0·674	+ 5·488	+ 0·018
521		11 15·534	198·3452	0·998357	+150·042	− 0·665	+ 5·466	+ 0·018
531	16	9 16·157	135·9410	0·999696	−203·771	− 1·256	+ 4·814	− 0·003
537	26	9 13·630	279·4349	0·999308	+318·816	+ 0·925	− 1·749	+ 0·017
541		9 38·776	280·3149	0·999334	+317·239	+ 0·940	− 1·834	+ 0·017
543		9 52·038	280·7790	0·999348	+316·378	+ 0·948	− 1·879	+ 0·017
547		10 15·656	281·3738	0·999373	+314·791	+ 0·962	− 1·958	+ 0·016
555	4	8 40·774	320·8098	1·000767	+168·889	+ 1·363	− 4·890	+ 0·003
559		8 59·707	321·4703	1·000786	+165·645	+ 1·364	− 4·923	+ 0·003
565		9 36·440	322·7515	1·000824	+159·293	+ 1·367	− 4·986	+ 0·002
569	5	8 22·234	10·3436	1·001788	−103·306	+ 1·000	− 5·406	− 0·016

CO-ORDINATES OF IV., 1902.

7	June 10	10 55·384	249·4456	1·000816	+536·499	− 3·319	+ 3·834	+ 0·011
9		11 11·160	249·6815	1·000845	+537·439	− 3·316	+ 3·796	+ 0·011
15	11	10 59·312	270·9848	1·003381	+583·497	− 2·869	+ 0·124	+ 0·031
17		11 18·140	271·2650	1·003414	+583·585	− 2·862	+ 0·077	+ 0·031
27	16	10 58·830	17·5222	1·005083	−163·585	+ 2·433	− 9·979	+ 0·041
31		11 24·086	17·8965	1·005050	−167·321	+ 2·448	− 9·961	+ 0·041
33		11 36·887	18·0863	1·005034	−169·220	+ 2·456	− 9·952	+ 0·040
41	19	10 12·922	81·3362	0·997746	−584·827	+ 3·145	− 1·819	− 0·019
43		10 28·453	81·5699	0·997719	−585·241	+ 3·139	− 1·776	− 0·019
55	24	10 25·385	190·6578	0·994259	+ 98·399	− 1·959	+10·292	− 0·046
59		10 54·861	191·1045	0·994291	+103·014	− 1·980	+10·283	− 0·046
61		11 7·489	191·2958	0·994305	+104·990	− 1·989	+10·276	− 0·046
65	30	9 53·471	319·3264	1·006890	+408·871	− 0·951	− 8·078	+ 0·056
67		10 11·083	319·5854	1·006900	+406·789	− 0·936	− 8·112	+ 0·056
71		10 54·030	320·2170	1·006925	+401·646	− 0·901	− 8·191	+ 0·056
73		11 8·300	320·4268	1·006934	+399·961	− 0·889	− 8·217	+ 0·056
75	July 1	9 41·019	340·4024	1·007112	+214·965	+ 0·261	−10·141	+ 0·058
77		10 9·170	340·8184	1·007107	+210·739	+ 0·285	−10·168	+ 0·058
91	7	9 40·449	109·3339	0·994883	−583·302	+ 2·454	+ 3·533	− 0·039
93		9 52·274	109·5128	0·994867	−582·656	+ 2·447	+ 3·567	− 0·039

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TABLE XVI.—*continued.*

CO-ORDINATES OF IV., 1902.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1902.	h m	°		"	"		
99	July 7	10 38·839	110·2176	0·994803	−580·071	+ 2·419	+ 3·693	− 0·040
107	8	15 18·079	136·2929	0·993071	−426·942	+ 1·166	+ 7·775	− 0·054
111		15 57·699	136·8947	0·993048	−422·223	+ 1·134	+ 7·854	− 0·055
113		16 9·614	137·0757	0·993041	−420·796	+ 1·124	+ 7·875	− 0·055
119	14	8 21·072	260·5209	1·001999	+622·132	− 3·309	+ 1·713	+ 0·014
121		8 37·146	260·7607	1·002029	+622·577	− 3·306	+ 1·669	+ 0·015
131	15	8 56·225	282·4735	1·004509	+616·662	− 2·776	− 2·505	+ 0·035
133		9 10·752	282·6891	1·004532	+616·143	− 2·768	− 2·545	+ 0·035
135		9 25·840	282·9131	1·004556	+615·598	− 2·760	− 2·586	+ 0·035
147	16	8 51·379	303·7408	1·006318	+524·176	− 1·860	− 6·282	+ 0·051
151	18	8 29·125	345·9529	1·007182	+144·660	+ 0·594	−10·862	+ 0·060
155		9 0·822	346·4209	1·007169	+139·544	+ 0·622	−10·884	+ 0·060
155A		9 20·413	346·7102	1·007160	+136·382	+ 0·640	−10·896	+ 0·060
157		9 30·233	346·8552	1·007156	+134·790	+ 0·648	−10·902	+ 0·060
169	20	8 43·618	28·8063	1·004144	−319·212	+ 2·781	− 9·649	+ 0·037
173		9 14·110	29·2594	1·004096	−323·586	+ 2·797	− 9·605	+ 0·036
175		9 25·805	29·4331	1·004077	−325·247	+ 2·803	− 9·588	+ 0·037
177		9 44·803	29·7154	1·004048	−327·968	+ 2·813	− 9·557	+ 0·036
185	24	8 10·616	114·7563	0·994300	−566·062	+ 2·241	+ 4·911	− 0·046
189		8 41·369	115·2223	0·994263	−563·720	+ 2·219	+ 4·992	− 0·047
191		9 3·677	115·5603	0·994236	−561·997	+ 2·203	+ 5·049	− 0·047
195	26	7 50·220	158·1763	0·992717	−214·843	− 0·226	+10·361	− 0·061
199		8 14·836	158·5505	0·992721	−210·942	− 0·249	+10·386	− 0·061
203		8 36·128	158·8742	0·992724	−207·559	− 0·269	+10·405	− 0·061
205		8 46·810	159·0366	0·992726	−205·860	− 0·278	+10·416	− 0·061
219	Aug. 3	6 48·753	329·8185	1·007154	+295·843	− 0·130	−10·033	+ 0·064
221		7 0·307	329·9891	1·007155	+294·126	− 0·119	−10·047	+ 0·064
229		7 43·450	330·6260	1·007160	+287·675	− 0·078	−10·105	+ 0·064
231		8 4·543	330·8738	1·007162	+284·509	− 0·059	−10·130	+ 0·064
249	7	6 50·881	55·2598	1·000972	−548·585	+ 3·648	− 5·816	+ 0·006
253		7 20·597	55·7041	1·000916	−551·124	+ 3·650	− 5·740	+ 0·006
255		7 35·576	55·9280	1·000888	−552·391	+ 3·652	− 5·703	+ 0·005
257		7 49·646	56·1383	1·000862	−553·573	+ 3·653	− 5·666	+ 0·005
263	16	6 18·038	250·5481	1·000734	+616·362	− 3·651	+ 2·849	+ 0·016
275	17	6 20·616	272·0668	1·003291	+634·970	− 3·068	− 1·323	+ 0·039
277		6 37·140	272·3127	1·003318	+634·655	− 3·058	− 1·371	+ 0·039
279		7 7·842	272·7696	1·003368	+634·040	− 3·042	− 1·459	+ 0·039
283	18	6 16·238	293·3801	1·005370	+564·641	− 2·040	− 5·293	+ 0·056
285		6 29·595	293·5779	1·005386	+563·596	− 2·029	− 5·328	+ 0·056
287		6 51·978	293·9095	1·005444	+561·826	− 2·010	− 5·384	+ 0·057
295		7 42·229	294·6539	1·005475	+557·782	− 1·970	− 5·512	+ 0·057
301	19	6 23·101	314·7860	1·006722	+414·483	− 0·700	− 8·551	+ 0·066
305		6 52·815	315·2251	1·006742	+410·701	− 0·670	− 8·608	+ 0·066
319	21	7 9·405	358·0136	1·006545	− 36·251	+ 2·135	−11·165	+ 0·056
321		7 20·020	358·1705	1·006537	− 38·010	+ 2·144	−11·165	+ 0·056
323		7 33·415	358·3686	1·006527	− 40·232	+ 2·156	−11·164	+ 0·056
335	22	6 11·640	18·4772	1·005129	−258·071	+ 3·145	−10·203	+ 0·039

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TABLE XVI.—*continued.*

CO-ORDINATES OF IV., 1902.

Rotation No.	Date.	Greenwich Mean Time.	Longitude.	Radius Vector.	x.	y.	X.	Y.
	1902.	h m	°		"	"		
339	Aug. 22	7 4·287	19·2582	1·005063	−266·029	+ 3·174	−10·142	+ 0·038
341		7 17·628	19·4562	1·005046	−268·036	+ 3·183	−10·123	+ 0·038
347	24	10 43·599	65·4458	0·999912	−598·475	+ 3·770	− 3·588	− 0·014
349		10 59·859	65·6895	0·999881	−599·317	+ 3·766	− 3·544	− 0·014
353		11 40·239	66·2946	0·999804	−601·352	+ 3·757	− 3·433	− 0·015
359	25	5 35·319	82·4400	0·997810	−629·908	+ 3·351	− 0·361	− 0·032
365		6 9·205	82·9499	0·997749	−629·995	+ 3·335	− 0·262	− 0·033
367		6 20·970	83·1270	0·997728	−630·007	+ 3·328	− 0·228	− 0·034
373		6 52·923	83·6077	0·997671	−630·026	+ 3·310	− 0·135	− 0·035
383	27	6 11·331	126·5232	0·993582	−460·890	+ 1·020	+ 7·353	− 0·065
387		6 51·890	127·1388	0·993546	−456·272	+ 0·980	+ 7·439	− 0·065
389		7 5·346	127·3430	0·993534	−454·727	+ 0·966	+ 7·466	− 0·065
391		7 20·447	127·5722	0·993520	−452·986	+ 0·952	+ 7·499	− 0·065
395	28	5 10·563	147·4716	0·992773	−277·988	− 0·405	+ 9·723	− 0·067
401		5 44·386	147·9854	0·992766	−272·947	− 0·440	+ 9·767	− 0·067
403		5 54·631	148·1411	0·992764	−271·408	− 0·451	+ 9·777	− 0·067
409	30	11 1·613	196·5270	0·994552	+240·485	− 3·221	+10·000	− 0·038
415		11 45·426	197·1905	0·994610	+247·146	− 3·245	+ 9·949	− 0·037
417	Sept. 1	4 35·935	234·1499	0·998636	+542·447	− 3·920	+ 5·342	+ 0·005
421	2	4 40·481	255·7856	1·001394	+618·471	− 3·543	+ 1·460	+ 0·031
423		4 50·189	255·9305	1·001412	+618·690	− 3·538	+ 1·432	+ 0·031
431		5 30·084	256·5262	1·001488	+619·547	− 3·520	+ 1·320	+ 0·031
439	3	4 56·205	277·4742	1·003966	+606·450	− 2·637	− 2·655	+ 0·052
447		5 57·865	278·3903	1·004065	+603·958	− 2·589	− 2·824	+ 0·053
449		6 6·460	278·5180	1·004079	+603·599	− 2·582	− 2·847	+ 0·053
453	4	10 16·702	303·5747	1·006289	+477·437	− 1·034	− 7·066	+ 0·068
455		10 31·030	303·7867	1·006303	+475·931	− 1·021	− 7·094	+ 0·068
461	6	4 46·373	341·2382	1·007288	+128·735	+ 1·602	−10·674	+ 0·065
465		5 9·695	341·5825	1·007282	+125·042	+ 1·625	−10·686	+ 0·065
471		5 56·555	342·2745	1·007270	+117·600	+ 1·670	−10·713	+ 0·064
473		6 9·892	342·4715	1·007266	+115·476	+ 1·681	−10·721	+ 0·064
481	7	4 52·207	2·6020	1·006431	−103·495	+ 2·861	−10·723	+ 0·050
485		5 18·936	2·9972	1·006405	−107·739	+ 2·880	−10·711	+ 0·049
489		5 42·837	3·3506	1·006382	−111·526	+ 2·896	−10·698	+ 0·049
497	8	4 43·442	23·8164	1·004672	−318·215	+ 3·705	− 9·298	+ 0·028
503		5 11·257	24·2218	1·004630	−321·962	+ 3·716	− 9·258	+ 0·027
505		5 21·480	24·3707	1·004615	−323·336	+ 3·720	− 9·239	+ 0·027
515	10	10 39·949	72·1058	0·998879	−601·923	+ 3·616	− 1·982	− 0·030
517		10 51·897	72·2851	0·998859	−602·247	+ 3·610	− 1·948	− 0·030
519		11 2·816	72·4490	0·998837	−602·534	+ 3·605	− 1·919	− 0·031
535	26	8 59·736	55·7426	1·000921	−528·466	+ 3·959	− 4·548	− 0·017
537		9 13·630	55·9504	1·000896	−529·364	+ 3·956	− 4·514	− 0·017
545		10 3·007	56·6890	1·000805	−532·550	+ 3·942	− 4·393	− 0·018
547		10 15·656	56·8782	1·000782	−533·352	+ 3·938	− 4·362	− 0·018
553	4	8 29·602	229·4754	0·998115	+483·017	− 3·979	+ 5·353	+ 0·010
559		8 59·707	229·9280	0·998169	+485·431	− 3·975	+ 5·288	+ 0·011
561		9 11·856	230·1106	0·998191	+486·395	− 3·973	+ 5·261	+ 0·011

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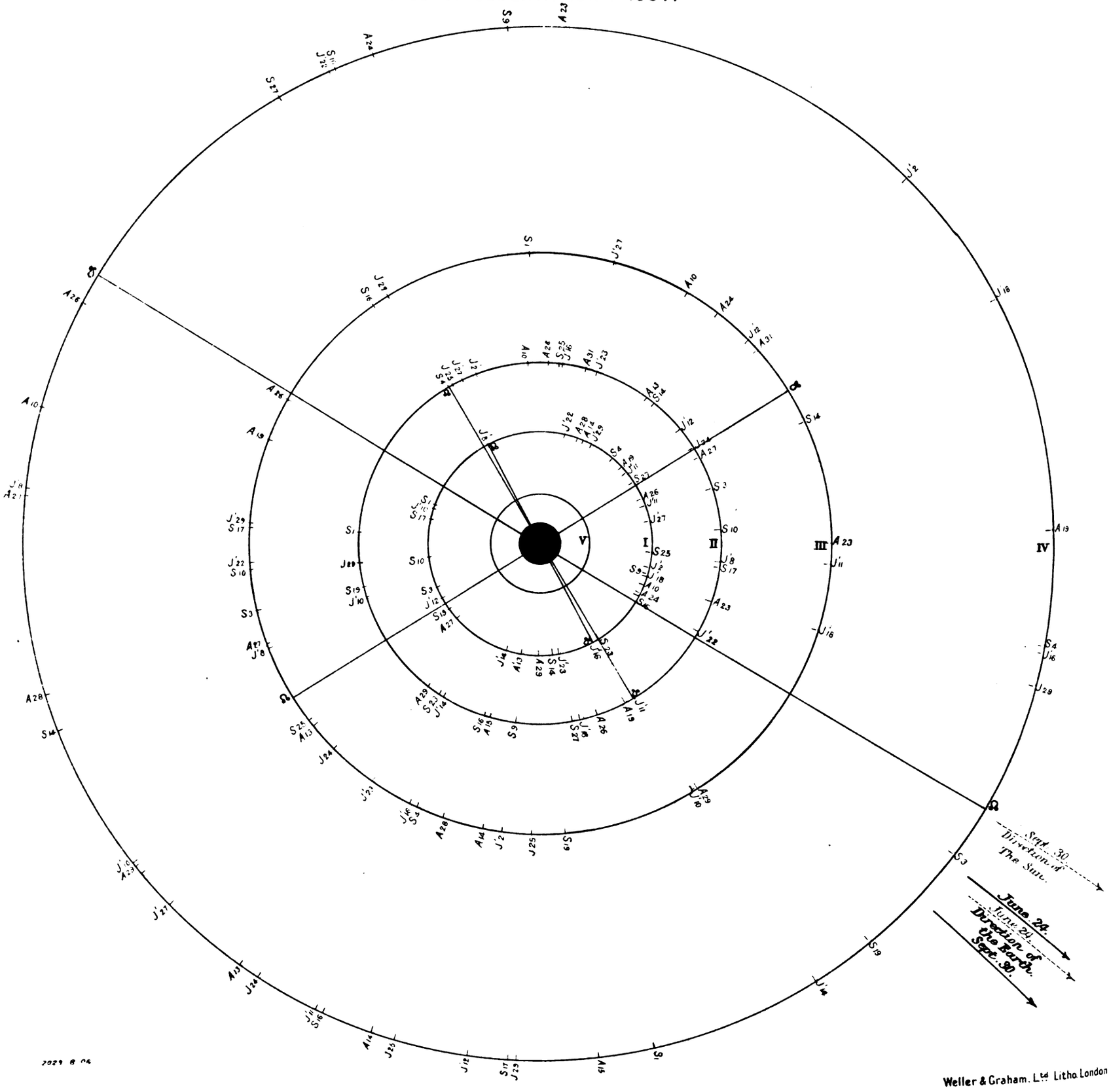
POSITIONS OF THE SATELLITES IN THEIR ORBITS ON THE DATES OF OBSERVATION . 1901.



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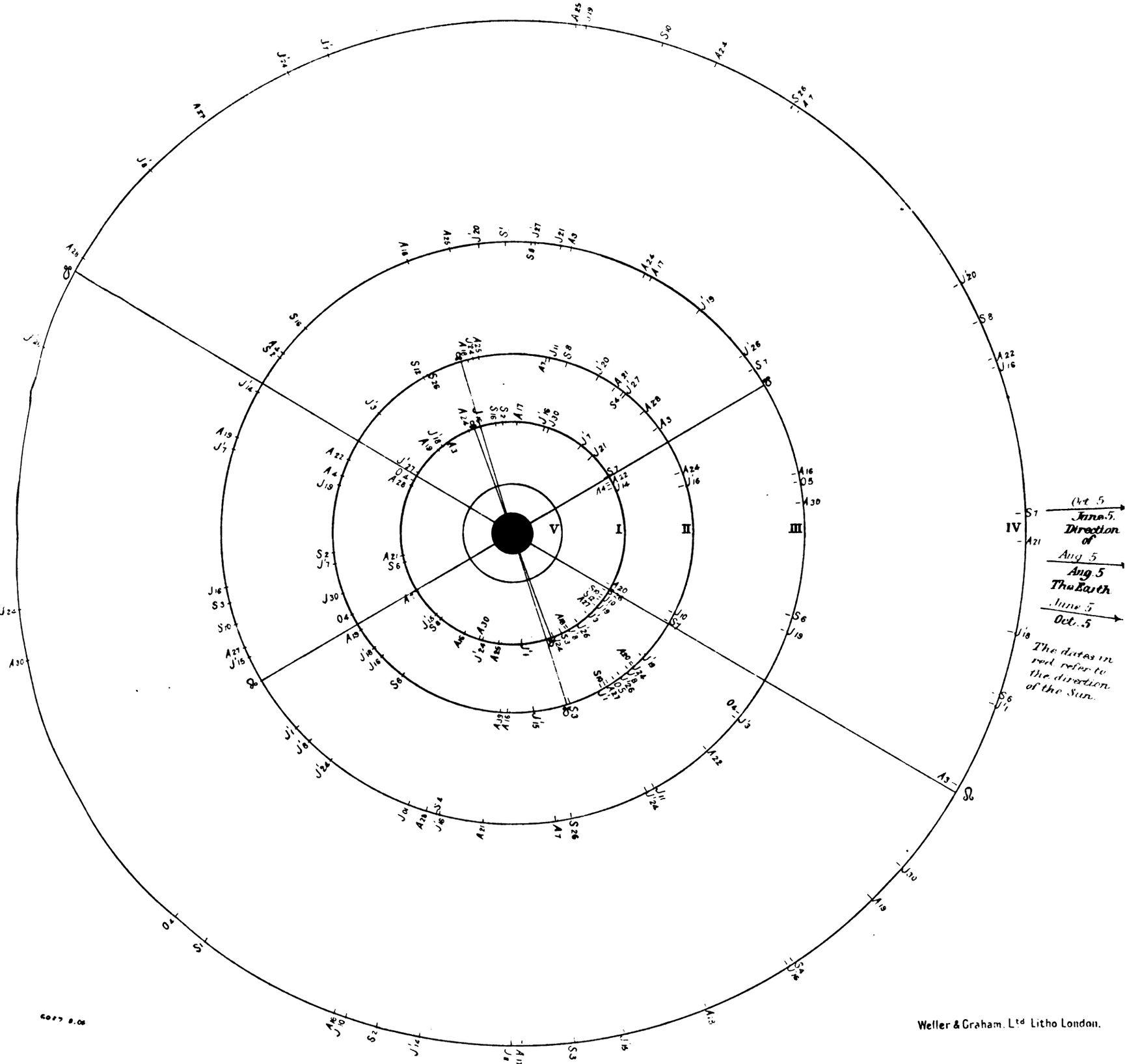
POSITIONS OF THE SATELLITES IN THEIR ORBITS ON THE DATES OF OBSERVATION. 1902.



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It is of interest to see how the observations of each satellite are distributed round its orbit. In the accompanying plates, the orbits of the satellites and the disc of Jupiter are drawn to scale and the line of nodes is marked for each orbit. The position of a satellite is shown for every day of observation and the date is placed close to it: thus J. 10 denotes June 10th, J′. 8 July 8th, A. stands for August, S. and O. for September and October. The directions of the Sun and of the Earth are also given.

It is satisfactory to notice that the gaps in 1901 are filled up in 1902; the observations of the two years combined are very evenly distributed round the entire circumference of the orbits.

In the next set of tables, the rates of change of s and p with respect to ρ_1 and ρ_2 are given for every observation of each pair of satellites. These were calculated from the data in the foregoing tables with the aid of a Brunsviga Calculating Machine, after converting log $\cos(p-\mathrm{P})$ and log $\sin(p-\mathrm{P})$ into natural numbers with five places of decimals. $s \sin 1'' \rho_2 \frac{\delta p}{\delta \rho_2}$ is the same as $s \sin 1'' \rho_1 \frac{\delta p}{\delta \rho_1}$ for I.–II. with its sign changed, and so on: only one need therefore be tabulated, though both were calculated to check each other. $\frac{\delta s}{\delta v_1}$, $\frac{\delta s}{\delta v_2}$, etc., are not tabulated here because they are the coefficients of $\Delta\epsilon_1$, $\Delta\epsilon_2$, etc. in the equations of condition, where they may be found. Thus all the quantities which would be wanted if it were desired to introduce other unknowns, are to be found in the tables contained in this chapter.

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TABLE XVII.

Coefficients, 1901.

	I. and II.				I. and III.		
Rotation No.	$\rho_1 \frac{\delta s}{\delta \rho_1}$.	$\rho_2 \frac{\delta s}{\delta \rho_2}$.	$s \sin 1'' \rho_1 \frac{\delta p}{\delta \rho_1}$.	Rotation No.	$\rho_1 \frac{\delta s}{\delta \rho_1}$.	$\rho_3 \frac{\delta s}{\delta \rho_3}$	$s \sin 1'' \rho_1 \frac{\delta p}{\delta \rho_1}$.
31	− 56·7	+ 109·9	− 15·640	33	+ 61·6	+ 299·8	+ 4·447
45	− 55·9	+ 115·5	+ 12·452	47	+ 53·0	+ 308·4	− 4·376
63	− 52·7	+ 184·0	− 4·902	61	− 47·1	+ 108·9	− 16·840
77	+ 117·8	+ 68·8	+ 5·159	83	− 135·0	+ 226·8	+ 11·168
93	+ 127·3	+ 212·6	+ 1·130	99	+ 130·8	+ 348·8	+ 0·669
111	− 127·7	+ 218·9	+ ·295	119	− 50·2	+ 339·8	+ 4·386
121	+ 47·5	+ 183·1	− 2·341	145	+ 121·3	+ 245·0	− 4·146
133	+ 59·5	+ 130·9	+ 4·740	153	− 76·1	+ 343·5	+ 4·276
149	+ 112·0	− 43·9	+ 11·615	167	− 117·3	+ 303·9	− 7·244
152	+ 84·0	+ 200·6	− 1·314	175	+ 132·5	+ 209·8	− 4·146
159	+ 104·8	− 77·6	− 34·812	187	− 49·1	+ 328·1	− 5·099
185	− 45·9	+ 147·0	− 8·374	203	− 105·4	+ 327·2	+ 4·420
201	+ 107·3	+ 205·4	− 0·826	217	+ 122·7	+ 280·3	+ 0·866
231	+ 127·8	+ 66·8	+ 4·312	233	+ 128·8	+ 127·6	− 4·964
247	− 39·6	+ 156·0	− 6·888	249	− 43·2	+ 322·1	− 4·554
263	+ 114·5	+ 100·9	+ 3·944	265	+ 116·3	+ 79·8	− 5·579
275	+ 125·1	+ 188·1	+ 0·005	277	− 125·2	+ 286·5	+ 3·196
307	− 83·5	+ 199·5	+ 3·587	291	+ 117·8	+ 289·5	+ 0·344
321	− 49·2	+ 123·6	− 3·586	323	+ 51·8	+ 220·3	+ 4·348
337	+ 113·5	+ 167·7	+ 0·241	339	− 114·5	+ 257·7	+ 2·152
353	+ 122·8	− 60·5	− 11·604	355	+ 123·3	+ 299·6	+ 0·795
369	+ 54·2	+ 157·2	+ 3·686	385	− 96·7	+ 226·3	+ 1·125
381	+ 91·3	+ 141·0	+ 0·459	391	+ 74·6	+ 280·0	− 3·101
389	+ 77·5	+ 188·6	− 1·549	423	− 58·6	+ 183·4	− 1·016
413	+ 32·8	+ 167·6	+ 3·603	443	− 114·4	+ 195·2	− 12·438
421	+ 55·6	+ 105·2	+ 0·869	459	+ 70·2	+ 287·6	+ 1·973
439	− 113·3	+ 157·3	+ 1·196				
453	− 26·1	+ 182·3	+ 3·698				
457	− 67·9	+ 148·1	− 8·842				
465	+ 111·1	+ 109·6	+ 2·901				

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TABLE XVII.—*continued.*

COEFFICIENTS, 1901.

I. AND IV.				II. AND III.			
Rotation No.	$\rho_1 \frac{\delta s}{\delta \rho_1}$.	$\rho_4 \frac{\delta s}{\delta \rho_4}$.	$s \sin 1'' \rho_1 \frac{\delta p}{\delta \rho_1}$.	Rotation No.	$\rho_2 \frac{\delta s}{\delta \rho_2}$.	$\rho_3 \frac{\delta s}{\delta \rho_3}$.	$s \sin 1'' \rho_3 \frac{\delta p}{\delta \rho_3}$.
29	− 55·5	+ 616·8	− 3·986	7	+ 205·8	+ 349·0	+ 0·052
49	+ 49·2	+ 347·4	− 5·929	23	+ 157·7	+ 122·9	+ 7·942
65	− 56·6	+ 602·0	− 3·743	37	+ 104·0	+ 297·6	− 3·099
85	+ 136·3	+ 601·2	+ 0·868	51	+ 124·5	+ 310·8	+ 1·620
101	− 131·8	+ 526·8	− 4·125	67	+ 185·2	− 99·9	− 28·178
113	− 120·0	+ 169·0	− 39·895	79	+ 65·8	+ 205·6	− 7·513
123	+ 44·4	+ 279·3	− 6·288	95	− 213·2	+ 348·4	+ 2·865
143	− 122·3	+ 247·2	− 11·786	127	+ 178·9	+ 338·0	− 2·192
165	+ 115·5	+ 602·2	+ 1·185	135	+ 128·7	+ 144·1	− 8·508
177	+ 131·7	+ 470·5	+ 1·214	154	+ 197·0	+ 343·3	− 2·021
189	+ 51·7	+ 272·9	+ 0·509	161	− 82·3	+ 306·9	+ 6·947
205	− 103·3	+ 579·0	+ 3·379	191	− 142·2	+ 327·3	+ 8·021
219	+ 121·7	+ 531·5	− 0·427	207	+ 204·4	+ 328·0	− 2·047
235	− 128·9	+ 412·8	+ 6·561	237	− 49·4	+ 132·4	+ 16·311
251	− 45·2	+ 257·9	− 7·586	239	− 87·8	+ 220·9	− 2·485
267	+ 117·7	+ 160·0	− 6·492	253	− 151·6	+ 321·8	+ 8·732
279	− 125·2	+ 343·7	+ 8·677	281	+ 190·4	+ 288·4	− 1·653
293	+ 116·5	+ 482·3	− 3·413	301	+ 199·5	− 82·5	− 15·849
305	− 85·6	+ 551·3	+ 4·470	317	− 173·7	+ 315·2	+ 9·949
325	− 54·4	+ 309·6	− 6·910	327	+ 128·8	+ 217·9	+ 5·060
341	+ 115·3	+ 84·0	− 9·433	343	+ 171·3	+ 260·0	− 1·437
357	− 122·4	+ 275·7	+ 13·411	359	+ 54·7	+ 298·4	− 4·494
371	− 57·0	+ 368·7	− 5·774	383	+ 142·9	+ 224·3	− 1·113
393	− 71·9	+ 481·7	+ 5·151	395	− 187·7	+ 281·9	+ 12·875
415	+ 35·5	+ 499·5	+ 3·470	405	+ 173·3	+ 64·5	+ 6·210
425	− 62·2	+ 405·9	− 5·102	427	+ 112·8	+ 187·4	− 0·772
447	+ 111·1	− 57·4	− 33·702	437	− 152·4	+ 197·9	− 33·514
463	− 104·0	+ 148·3	− 36·771				

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TABLE XVII.—*continued.*

COEFFICIENTS, 1901.

Rotation No.	II. AND IV. $\rho_2 \frac{\delta s}{\delta \rho_2}$.	$\rho_4 \frac{\delta s}{\delta \rho_4}$.	$s \sin 1'' \rho_2 \frac{\delta p}{\delta \rho_2}$.	Rotation No.	III. AND IV. $\rho_3 \frac{\delta s}{\delta \rho_3}$.	$\rho_4 \frac{\delta s}{\delta \rho_4}$.	$s \sin 1'' \rho_3 \frac{\delta p}{\delta \rho_3}$.
5	+ 203·1	+ 616·0	+ 0·146	3	− 346·7	+ 616·5	− 0·578
15	+ 85·0	+ 570·0	− 5·172	13	− 282·0	+ 570·9	+ 10·423
21	+ 155·8	+ 250·9	− 5·372	19	− 120·3	+ 245·7	+ 42·707
35	− 106·3	+ 617·2	+ 9·011	39	+ 293·2	+ 617·6	− 6·448
53	+ 127·0	+ 350·6	− 1·564	55	− 299·7	+ 340·3	+ 90·678
69	− 189·2	+ 602·9	+ 0·458	71	− 97·3	+ 603·1	+ 15·457
81	− 36·1	+ 602·1	− 7·150	87	+ 230·7	+ 600·8	+ 6·848
97	+ 213·6	+ 528·5	+ 1·812	103	+ 349·3	+ 526·0	+ 3·566
109	+ 206·0	− 160·0	− 75·170	129	+ 337·1	+ 283·5	− 11·871
125	− 174·1	+ 276·7	+ 47·256	147	+ 246·3	+ 244·2	+ 12·162
137	+ 126·8	+ 577·5	− 7·014	169	+ 303·0	+ 602·5	− 5·440
163	+ 80·1	+ 602·1	− 6·386	179	− 211·9	+ 469·0	− 24·421
193	+ 140·2	+ 275·4	− 9·706	195	+ 326·2	+ 276·7	− 10·751
209	+ 203·9	+ 578·6	− 2·130	211	− 328·6	+ 578·4	− 1·665
227	− 184·9	+ 377·5	− 3·380	221	− 278·4	+ 530·9	+ 8·270
241	− 90·5	+ 428·5	− 8·231	243	− 223·3	+ 426·3	− 26·451
255	− 150·0	+ 255·6	− 7·446	257	+ 315·8	− 246·7	− 61·021
271	− 82·2	+ 161·6	+ 31·501	285	− 285·5	+ 343·6	+ 50·446
283	+ 191·0	+ 346·0	− 5·368	295	− 286·3	+ 482·1	+ 28·306
309	− 199·2	+ 551·6	+ 6·522	303	− 81·1	+ 550·9	+ 13·275
329	− 130·1	+ 306·9	− 15·060	331	+ 214·7	+ 305·9	+ 0·505
345	+ 169·5	− 81·4	+ 32·397	347	+ 262·2	+ 88·5	− 11·466
363	− 48·8	+ 279·9	+ 10·615	361	+ 297·6	+ 278·7	− 9·921
373	+ 154·1	+ 367·6	+ 0·317	399	+ 283·6	+ 484·1	− 1·212
379	− 133·6	+ 195·9	− 39·597	409	+ 59·9	+ 501·4	− 8·546
397	+ 187·9	+ 483·4	− 4·005	431	− 189·0	+ 402·3	− 21·488
407	− 172·3	+ 502·0	+ 4·236	445	+ 193·0	− 65·2	− 19·258
429	+ 114·6	+ 403·7	+ 4·355				
441	+ 157·0	− 65·3	− 27·797				
467	+ 107·6	+ 149·9	+ 2·772				

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TABLE XVII. —*continued.*

COEFFICIENTS, 1902.

Rotation No.	I. AND II. $\rho_1 \frac{\delta s}{\delta \rho_1}$.	$\rho_2 \frac{\delta s}{\delta \rho_2}$.	$s \sin 1'' \rho_1 \frac{\delta p}{\delta \rho_1}$.	Rotation No.	I. AND III. $\rho_1 \frac{\delta s}{\delta \rho_1}$.	$\rho_3 \frac{\delta s}{\delta \rho_3}$.	$s \sin 1'' \rho_1 \frac{\delta p}{\delta \rho_1}$.
3	− 79·1	+ 99·6	+ 4·360	5	− 75·6	+ 301·4	− 0·128
23	+ 127·9	+ 138·0	+ 0·768	25	+ 126·9	+ 49·1	− 0·150
51	− 128·3	+ 167·1	+ 2·201	47	− 88·2	+ 334·7	− 0·143
63	+ 126·4	+ 64·3	+ 1·236	53	− 126·9	+ 263·3	− 0·389
83	+ 101·6	+ 144·7	− 0·359	79	− 136·2	+ 239·4	− 0·477
95	+ 115·5	+ 35·9	+ 1·446	85	− 97·0	+ 215·0	− 0·568
103	− 121·8	+ 171·2	+ 1·676	87	− 97·9	+ 107·1	− 2·441
127	− 107·7	+ 225·4	− 0·289	105	− 119·7	+ 253·8	− 0·460
143	+ 136·2	− 54·6	− 3·277	117	− 55·9	+ 158·2	− 0·060
159	+ 106·5	+ 156·8	+ 0·761	129	− 110·9	+ 154·1	− 0·130
163	− 85·9	+ 151·6	+ 1·093	145	+ 137·3	+ 347·5	+ 0·151
181	+ 138·1	+ 219·1	+ 0·061	165	+ 81·8	+ 263·0	+ 0·024
193	− 120·1	+ 184·9	+ 0·926	183	− 139·2	+ 319·5	− 0·495
209	− 82·3	+ 163·9	− 1·893	197	+ 114·3	+ 229·1	+ 0·111
223	− 113·0	+ 131·6	− 11·284	211	− 78·7	+ 363·1	− 0·272
237	− 53·7	+ 62·9	+ 9·049	225	− 111·0	+ 360·0	− 0·310
245	+ 84·2	+ 222·4	+ 0·259	241	− 67·7	+ 208·7	− 0·271
265	− 133·1	+ 228·7	− 0·867	247	− 87·1	+ 356·4	− 0·207
291	+ 118·7	+ 195·4	− 0·080	267	+ 134·7	+ 98·5	− 0·221
309	− 50·1	+ 157·5	+ 0·388	273	− 143·2	+ 327·9	− 0·245
325	+ 42·0	+ 203·9	+ 0·277	293	+ 116·5	+ 319·6	+ 0·208
331	− 60·6	+ 77·3	+ 3·971	299	+ 97·8	− 89·9	− 10·330
355	+ 117·6	− 101·1	− 11·587	327	− 45·6	+ 360·5	− 0·170
371	+ 137·2	+ 212·9	+ 0·079	333	+ 63·9	+ 247·2	+ 0·251
377	− 89·7	+ 185·7	+ 0·402	345	− 131·7	+ 331·4	− 0·131
399	− 38·8	+ 160·8	− 0·326	369	+ 138·0	+ 339·6	+ 0·228
429	+ 137·8	+ 55·5	+ 1·063	379	− 86·8	+ 168·1	+ 0·622
445	− 110·1	+ 200·5	+ 0·162	397	+ 42·5	+ 349·2	− 0·056
475	− 54·2	+ 199·9	− 0·691	411	+ 137·7	+ 70·0	− 0·469
479	+ 74·8	+ 86·8	− 0·336	425	− 138·8	+ 183·3	− 3·310
495	+ 112·8	+ 213·5	+ 0·406	437	+ 121·0	− 114·1	+ 13·157
511	− 66·8	+ 179·8	+ 0·371	469	− 41·9	+ 45·5	− 13·857
527	+ 60·2	+ 172·0	− 0·188	487	− 89·5	+ 237·1	− 0·036
539	+ 43·0	+ 168·6	− 0·223	499	+ 116·4	+ 350·5	+ 0·235
557	+ 37·8	+ 116·6	+ 0·083	513	− 63·5	+ 139·9	+ 0·926
				531	− 132·0	+ 203·8	− 2·116
				541	− 39·2	+ 317·3	+ 0·181
				555	+ 40·5	+ 168·9	+ 0·033

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TABLE XVII.—*continued.*

COEFFICIENTS, 1902.

	I. AND IV.				II. AND III.		
Rotation No.	$\rho_1 \frac{\delta s}{\delta \rho_1}$.	$\rho_4 \frac{\delta s}{\delta \rho_4}$.	$s \sin 1'' \rho_1 \frac{\delta p}{\delta \rho_1}$.	Rotation No.	$\rho_2 \frac{\delta s}{\delta \rho_2}$.	$\rho_3 \frac{\delta s}{\delta \rho_3}$.	$s \sin 1'' \rho_2 \frac{\delta p}{\delta \rho_2}$.
7	− 71·5	+ 536·4	+ 0·522	13	+ 197·9	+ 295·4	− 0·289
27	− 125·4	+ 163·4	+ 8·034	29	+ 145·2	− 54·5	+ 1·252
55	+ 125·1	− 98·0	+ 8·981	39	+ 64·5	+ 126·9	+ 1·112
65	+ 128·1	+ 408·9	− 0·261	57	− 160·3	+ 266·9	− 2·522
71	+ 134·0	+ 401·6	− 0·248	97	+ 39·6	+ 87·7	− 1·235
93	− 112·4	+ 582·6	+ 0·681	109	− 163·6	+ 258·4	− 3·011
107	+ 116·8	+ 426·9	− 0·204	137	+ 223·2	− 165·8	+ 0·185
119	+ 61·7	+ 622·1	− 0·399	149	+ 64·9	+ 348·8	− 1·576
131	− 113·7	+ 616·6	+ 0·722	167	− 195·6	+ 357·8	+ 2·417
147	+ 138·2	+ 524·2	− 0·414	187	+ 216·0	+ 316·5	+ 0·223
157	+ 109·2	+ 134·8	+ 0·326	201	+ 176·6	+ 232·6	+ 0·493
185	+ 140·0	+ 566·1	− 0·465	213	− 168·3	+ 363·1	+ 2·620
199	+ 112·2	+ 211·0	+ 0·117	227	− 137·1	+ 360·3	+ 2·577
229	+ 106·1	+ 287·7	− 0·001	239	− 55·0	+ 210·5	− 2·316
249	+ 90·4	+ 548·6	− 0·539	251	+ 224·1	+ 355·2	− 0·287
275	+ 143·2	+ 635·0	− 0·572	269	+ 228·1	+ 100·9	− 0·437
295	− 114·0	+ 557·8	+ 0·551	281	− 205·0	+ 328·1	− 1·415
323	+ 38·6	+ 40·2	− 1·022	303	+ 142·7	+ 84·2	− 0·133
341	− 87·5	+ 268·1	+ 1·460	311	+ 187·6	+ 359·5	− 0·748
349	− 127·6	+ 599·3	+ 1·082	313	+ 189·7	+ 359·7	− 0·732
359	+ 140·7	+ 629·9	− 0·624	317	+ 196·4	+ 360·2	− 0·674
373	+ 136·3	+ 630·0	− 0·618	337	+ 61·8	+ 239·2	+ 1·202
383	+ 77·8	+ 460·9	− 0·233	343	− 77·7	+ 330·4	+ 2·081
387	+ 65·6	+ 456·3	− 0·215	363	− 216·3	+ 342·0	− 0·526
395	− 46·3	+ 278·0	+ 0·061	381	+ 181·9	− 170·6	+ 2·044
423	+ 139·3	+ 618·7	− 0·672	405	+ 167·1	+ 351·1	− 0·862
447	− 107·1	+ 604·0	+ 0·692	413	+ 224·3	+ 72·7	− 0·684
473	− 50·5	+ 115·5	− 1·605	427	+ 53·2	+ 179·5	− 0·906
485	− 85·8	+ 107·1	+ 10·884	443	+ 201·9	− 124·9	+ 1·806
497	+ 114·7	+ 318·2	− 0·907	457	+ 187·8	+ 349·3	− 0·684
515	+ 58·7	+ 602·0	− 0·497	463	+ 189·0	− 55·4	− 0·960
535	+ 51·1	+ 528·5	− 0·588	483	+ 79·6	+ 233·2	+ 0·623
553	+ 43·8	+ 483·1	− 0·592	501	− 214·9	+ 350·5	+ 1·371
				521	− 169·6	− 150·0	+ 3·984
				543	+ 164·6	+ 316·4	+ 0·380
				565	− 126·1	+ 159·3	+ 0·695
				569	+ 151·5	+ 103·3	− 0·527

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TABLE XVII.—*continued.*

COEFFICIENTS, 1902.

Rotation No.	II. AND IV. $\rho_2 \frac{\delta s}{\delta \rho_2}$.	II. AND IV. $\rho_4 \frac{\delta s}{\delta \rho_4}$.	II. AND IV. $s \sin 1'' \rho_2 \frac{\delta p}{\delta \rho_2}$.	Rotation No.	III. AND IV. $\rho_3 \frac{\delta s}{\delta \rho_3}$.	III. AND IV. $\rho_4 \frac{\delta s}{\delta \rho_4}$.	III. AND IV. $s \sin 1'' \rho_3 \frac{\delta p}{\delta \rho_3}$.
15	+ 199·1	+ 583·4	− 1·442	9	− 303·9	+ 537·3	+ 5·310
31	+ 147·5	+ 167·3	− 1·972	17	− 292·0	+ 583·5	+ 4·367
41	− 60·8	+ 584·8	− 1·246	33	+ 59·4	+ 169·2	− 0·499
59	+ 157·7	− 103·0	+ 4·194	43	+ 121·1	+ 585·2	− 0·940
67	− 72·2	+ 406·8	+ 2·500	61	+ 270·5	− 105·0	+ 3·502
73	− 86·6	+ 400·0	+ 2·650	75	+ 232·2	− 215·0	+ 0·685
77	− 190·1	+ 210·6	− 7·011	91	− 96·3	+ 583·3	+ 0·852
99	+ 44·2	+ 580·0	− 1·965	113	+ 263·8	+ 420·7	− 0·692
111	+ 158·8	+ 422·2	+ 0·344	121	+ 150·6	+ 622·5	− 0·978
135	− 223·8	+ 615·4	+ 2·128	133	− 159·6	+ 616·1	+ 1·199
151	− 130·9	+ 134·2	+ 54·089	173	+ 354·5	− 321·8	+ 36·964
155	+ 147·1	− 138·4	+ 18·343	175	+ 353·8	− 323·1	+ 39·494
169	− 197·1	+ 319·1	+ 8·309	177	+ 352·5	− 325·2	+ 44·457
189	− 214·8	+ 563·6	+ 1·478	191	+ 312·4	+ 562·0	− 1·237
203	+ 174·6	+ 207·6	+ 0·503	195	+ 226·6	− 214·9	− 0·043
231	+ 144·2	+ 284·5	− 1·219	205	+ 236·3	− 205·9	− 0·408
253	− 224·7	+ 551·0	+ 4·049	219	+ 358·7	+ 295·9	− 0·344
285	+ 202·0	+ 563·6	− 0·499	221	+ 359·2	+ 294·2	− 0·339
301	− 140·0	+ 414·5	+ 2·905	255	+ 353·7	+ 552·4	− 1·885
305	− 146·4	+ 410·7	+ 2·969	257	+ 353·0	+ 553·6	− 1·884
319	+ 199·7	− 36·2	+ 2·905	263	+ 92·6	+ 616·4	− 0·131
339	− 57·1	+ 266·0	− 0·770	277	+ 330·8	+ 634·7	− 1·281
347	− 84·2	+ 598·5	+ 2·557	279	+ 333·6	+ 634·1	− 1·291
367	+ 214·7	+ 630·0	+ 2·173	283	+ 327·0	+ 564·7	− 1·294
389	+ 169·3	+ 454·8	+ 0·045	287	+ 323·5	+ 561·9	− 1·285
401	− 163·0	+ 273·0	+ 3·118	321	+ 360·5	+ 38·0	− 2·001
409	− 218·1	+ 234·2	+ 54·827	335	+ 245·0	+ 258·1	− 2·026
431	− 58·0	+ 619·6	+ 2·030	353	− 337·1	+ 601·3	+ 5·578
439	− 205·9	+ 606·5	+ 1·584	365	− 341·2	+ 629·9	+ 5·805
455	+ 185·7	+ 476·0	− 1·246	391	+ 187·2	+ 453·0	− 0·038
465	+ 190·7	− 125·1	− 2·225	403	+ 350·6	+ 271·4	+ 0·045
471	+ 196·9	− 117·6	− 2·269	415	+ 76·0	+ 247·1	− 0·228
481	+ 83·0	+ 103·5	− 0·757	417	+ 352·0	+ 542·5	− 2·001
503	− 215·2	+ 321·9	+ 10·258	421	+ 185·4	+ 618·5	− 1·667
517	+ 173·5	+ 602·3	− 0·658	449	− 131·8	+ 603·6	− 0.069
545	− 162·9	+ 532·6	+ 1·595	453	− 348·4	+ 477·5	+ 4·567
561	− 121·2	+ 486·4	+ 2·950	461	− 57·9	+ 128·8	+ 0·634
				489	+ 239·1	− 111·5	+ 5·160
				505	+ 345·9	− 318·5	+ 55·567
				519	+ 147·4	+ 602·6	− 0·166
				537	+ 318·8	+ 529·4	− 2·064
				547	+ 314·8	+ 533·4	− 2·066
				559	− 165·7	+ 485·4	+ 4·129

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The final coefficients of all the unknowns, except Δp_i and Δq_i, were calculated from the preceding data with the help of the Brunsviga machine; for Δp_i and Δq_i four-place logarithms were used. All the coefficients were found to three places of decimals and were not rounded off to two places till they were written down in the equations of condition. Throughout the computations connected with the calculation of the tabular places and of the coefficients every care was taken to ensure numerical accuracy: every figure from beginning to end was checked and for many of the quantities additional checks were found, more especially for the signs. For instance, in 1901, the coefficients of Δh and Δk have the same sign, *i.e.* that of $-\sin(\boldsymbol{p}-\mathrm{P})$; in 1902, Δk_1 has the sign of $\Delta h_2 \tan l$ or of $\sin(\boldsymbol{p}-\mathrm{P}) \tan l$, and so on. Again, by inspecting the two plates we can check graphically the signs, and to a certain extent also the magnitudes, of the coefficients of $\Delta \epsilon_i$; whilst for Δp_i and Δq_i several checks suggested themselves.

The final equations of condition are given on the following pages.

The small correction mentioned on p. 61 has yet to be applied to the distances and position-angles computed from the formula on p. 77. This is the correction for differential aberration. The co-ordinates from which the distances are derived have been computed for the moment at which the observed rays of light left Jupiter, whereas they should have been computed for the moment at which the rays left the satellite; the co-ordinates tabulated above require, therefore, the correction—

$$\delta x = \frac{\delta x}{\delta v}\delta v = \mathrm{X}\delta v,$$

$$\delta y = \frac{\delta y}{\delta v}v = \mathrm{Y}\delta v,$$

where $\delta v = n\delta t = n\,(\Delta - \Delta')\ 498^s$.

The tabular co-ordinates do not include this.

The corresponding correction to the distance of a pair of satellites is—

$$\delta s = \frac{\delta s}{\delta v_1}\delta v_1 + \frac{\delta s}{\delta v}v_2,$$

with a similar expression for position-angles. This correction is included in the computed distances given on the following pages, but not in the computed position-angles. It was more convenient in the case of the latter to apply the correction to the absolute terms after they had been converted into arc of a great circle; these accordingly include it.

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TABLE XVIII.

EQUATIONS OF CONDITION.

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TABLE XVIII.—Equations of Condition.

Satellites I. and II.—Distances, 1901.

Rotation No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of														O−C.	Residuals.	Rotation No.
					μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	$\Delta \epsilon_1$	$\Delta \epsilon_2$	Δp_1	Δq_1	Δp_2	Δq_2	ΔJ			
	1901																				
31	July 2	53″·200	52″·295	¼	0·53	+2·00	+0·97	−2·19	−2·07	−2·66	−3·21	−2·16	−3·23	+0·47	−0·10	+0·29	−0·70	−0·12	−0″·905	+0″·218	31
45	8	59·733	59·947	1	0·60	+1·45	+1·71	−2·20	−3·09	−1·32	−3·23	+2·19	+3·20	+0·16	−0·32	+0·56	−0·06	−0·09	+0·214	−0·095	45
63	10	131·152	131·006	¼	1·31	−2·02	−1·04	+2·22	+2·53	+0·89	+2·50	−2·22	+2·16	+0·03	−0·01	−0·05	−0·02	+0·18	−0·146	+0·039	63
77	11	186·586	187·282	¼	1·87	−1·54	−0·54	+1·52	+2·55	+2·69	+3·69	+1·28	−3·66	−0·05	−0·02	+0·05	−0·08	+1·32	+0·696	−0·149	77
93	12	339·847	339·783	1	3·40	+1·34	+0·53	−1·35	−1·82	−0·94	−1·98	+0·91	+0·84	+0·01	0·00	+0·01	+0·01	+1·93	−0·064	+0·068	93
111	14	91·200	91·630	1	0·91	−0·80	−1·18	+1·39	+1·35	+1·45	+1·97	+0·89	−0·57	+0·01	+0·02	−0·02	−0·04	−0·37	+0·430	−0·061	111
121	16	230·475	229·770	1	2·30	+1·49	+1·74	−2·28	−1·34	−2·23	−2·52	−2·25	−2·04	+0·05	−0·08	−0·02	−0·15	+1·09	−0·705	+0·041	121
133	18	190·335	190·962	¼	1·90	−1·97	−1·04	+2·12	+1·88	+2·69	+3·24	+2·17	−3·08	−0·03	+0·01	+0·01	−0·04	+1·01	+0·627	−0·044	133
149	22	68·066	68·137	¼	0·68	−0·86	−1·42	+1·64	+3·10	+2·02	+3·56	−1·34	−3·68	−0·03	−0·30	+0·42	−0·22	+0·86	+0·071	+0·073	149
159	27	27·120	27·819	1	0·27	−1·47	−0·58	+1·35	+2·01	+2·65	+3·32	+1·29	+3·26	+0·78	+0·19	+0·56	−1·14	+0·68	+0·699	+0·063	159
185	Aug. 10	101·448	100·940	1	1·01	+1·86	+1·16	−1·87	−1·41	−2·50	−2·87	−2·15	−2·57	+0·20	−0·08	+0·02	−0·34	+0·11	−0·508	−0·044	185
201	13	312·740	312·281	1	3·13	+0·76	+1·41	−1·60	−1·24	−1·36	−1·76	−1·30	−0·36	0·00	−0·03	−0·03	−0·04	+1·72	−0·459	+0·011	201
231	19	194·595	194·991	½	1·95	−0·96	−0·64	+0·94	+2·19	+2·64	+3·23	+0·33	−3·38	−0·05	−0·04	+0·05	−0·09	+1·41	+0·396	−0·085	231
247	24	116·285	115·724	1	1·16	+1·78	+1·18	−1·63	−1·21	−2·30	−2·55	−2·11	−2·21	+0·15	−0·07	−0·01	−0·27	+0·20	−0·561	+0·033	247
263	26	215·325	215·588	1	2·15	−1·26	−0·56	+0·86	+1·80	+2·59	+2·97	+0·95	−3·03	−0·04	−0·02	+0·02	−0·06	+1·41	+0·263	+0·049	263
275	27	313·252	313·092	1	3·13	+0·87	+0·68	−0·88	−1·78	−0·91	−1·32	+0·17	+1·12	0·00	0·00	+0·01	0·00	+1·82	−0·160	+0·093	275
307	29	116·040	116·602	½	1·16	−0·89	−1·55	+1·71	+1·20	+1·30	+1·53	+1·63	−0·35	0·00	−0·02	+0·02	+0·02	−0·04	+0·562	−0·058	307
321	Sept. 1	74·459	74·407	1	0·74	−1·74	−1·04	+1·34	+2·64	+1·26	+1·72	−1·98	+2·72	0·00	0·00	0·00	0·00	0·00	−0·052	−0·041	321
337	3	281·194	281·346	1	2·81	+1·16	+0·55	−0·73	−2·08	−0·90	−1·22	+0·82	+1·73	+0·02	+0·01	+0·04	+0·01	+1·64	+0·152	−0·064	337
353	4	62·289	62·417	1	0·62	−0·78	−0·73	+0·84	+2·01	+2·59	+2·85	−0·04	+3·24	+0·13	+0·15	+0·16	−0·28	+0·90	+0·128	+0·256	353
369	9	211·368	211·584	½	2·11	−1·70	−0·97	+1·10	+1·17	+2·08	+2·17	+1·90	−1·96	−0·01	0·00	0·00	−0·02	+1·06	+0·216	+0·135	369
381	10	232·293	232·330	½	2·32	+1·46	+0·62	−0·72	−2·35	−1·02	−1·17	+1·37	+2·24	+0·04	+0·01	+0·07	+0·01	+1·34	+0·037	+0·111	381
389	14	266·215	265·923	½	2·66	+0·87	+1·49	−1·50	−1·19	−1·16	−1·18	−1·58	−0·10	0·00	−0·04	−0·04	−0·04	+1·39	−0·292	−0·098	389
413	16	200·442	200·891	1	2·00	−1·67	−1·13	+1·11	+1·07	+1·82	+1·81	+2·00	−1·53	0·00	0·00	0·00	0·00	+0·90	+0·449	−0·032	413
421	17	160·760	160·978	1	1·61	+0·06	+0·52	−0·51	−2·55	−1·26	−1·20	+0·18	+2·69	+0·08	−0·02	+0·14	−0·02	+0·89	+0·218	−0·250	421
439	19	44·085	43·977	1	0·44	−0·98	−0·54	+0·49	+2·10	+0·84	+0·73	−0·52	+1·80	−0·03	−0·02	+0·06	+0·02	−0·47	−0·108	+0·105	439
453	23	156·274	156·678	¼	1·56	−1·33	−1·48	+1·34	+1·08	+1·30	+1·18	+1·98	−0·58	+0·01	−0·01	+0·01	+0·02	+0·42	+0·404	−0·064	453
457	25	80·195	79·855	½	0·80	+1·57	+0·73	−0·52	−1·08	−2·00	−1·85	−1·63	−1·88	+0·18	−0·02	−0·02	−0·28	−0·09	−0·340	+0·007	457
465	27	220·720	220·834	½	2·21	−0·97	−0·51	+0·35	+1·50	+2·26	+2·01	+0·52	−2·54	−0·03	−0·02	+0·01	−0·06	+1·41	+0·114	+0·049	465

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TABLE XVIII.—*continued.*

SATELLITES I. AND II.—POSITION-ANGLES, 1901.

Rotation No.	Date.	Computed Position-Angles.	Observed Position-Angles.	Weight.	Coefficients of													O—C.	Residuals.	Rotation No.
					Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	$\Delta \epsilon_1$	$\Delta \epsilon_2$	Δp_1	Δq_1	Δp_2	Δq_2	ΔJ			
	1901																			
32	July 2	254° 1′ 36″	252° 41′ 33″	½	+0·36	+0·22	−0·42	−0·38	−0·59	−0·64	−0·40	−0·69	−2·32	+0·52	−1·41	+3·48	−0·09	−1″·187	+0″·105	32
46	8	274 15 38	274 58 49	1	−0·23	−0·21	+0·31	+0·55	+0·16	+0·53	−0·29	−0·56	+1·10	−2·12	+3·76	−0·42	+0·07	+0·785	−0·130	46
64	10	85 6 31	85 16 6	½	+0·01	−0·04	+0·02	−0·06	+0·06	−0·01	0·00	−0·08	−2·31	+0·67	+3·68	+1·17	−0·03	+0·368	−0·122	64
78	11	84 37 58	84 27 29	½	+0·02	−0·05	+0·02	+0·02	+0·10	+0·08	−0·04	−0·10	+2·28	+0·81	−2·11	+3·22	+0·03	−0·567	−0·124	78
94	12	266 17 45	266 18 0	1	−0·08	+0·06	+0·02	+0·10	−0·07	+0·03	−0·10	−0·12	+2·10	+1·16	+2·98	+2·36	+0·01	+0·022	−0·181	94
112	14	85 32 8	85 26 36	1	+0·03	−0·08	+0·03	−0·06	+0·10	+0·02	+0·08	−0·12	−0·72	−2·29	+1·96	+3·32	0·00	−0·147	−0·051	112
122	16	263 59 33	263 49 26	1	+0·04	+0·11	−0·11	0·00	−0·16	−0·11	−0·12	−0·16	−1·22	+2·06	+0·44	+3·76	−0·01	−0·680	−0·143	122
134	18	85 41 37	85 31 15	¼	+0·02	−0·04	+0·02	−0·02	+0·09	+0·05	−0·01	−0·08	+2·33	−0·58	−0·92	+3·72	+0·03	−0·572	−0·048	134
150	22	79 21 39	78 45 51	¼	−0·08	−0·24	+0·24	+0·32	+0·28	+0·42	−0·23	−0·41	+0·22	+2·36	−3·32	+1·76	+0·06	−0·724	−0·051	150
152	23	265 8 14	265 3 0	½	+0·01	+0·10	−0·08	+0·03	−0·13	−0·08	−0·10	−0·13	−0·49	+2·33	+1·21	+3·56	−0·01	−0·433	−0·188	152
160	27	106 31 22	107 51 58	1	+0·58	+0·17	−0·49	−0·77	−0·91	−1·19	−0·53	−1·15	+2·17	+0·53	+1·57	−3·17	−0·19	+0·654	−0·096	160
186	Aug. 10	261 54 32	261 20 52	1	+0·14	+0·13	−0·18	−0·09	−0·30	−0·30	−0·18	−0·30	−2·14	+0·82	−0·23	+3·62	−0·05	−0·956	+0·131	186
202	13	266 34 19	266 29 21	1	−0·02	+0·08	−0·06	+0·06	−0·09	−0·06	−0·08	−0·11	+0·28	+2·16	+2·12	+2·92	0·00	−0·455	−0·010	202
232	19	85 52 17	85 43 13	½	+0·02	−0·07	+0·06	+0·02	+0·11	+0·11	−0·06	−0·11	+1·74	+1·44	−1·73	+3·12	+0·02	−0·518	−0·200	232
248	24	263 14 58	262 48 4	1	+0·10	+0·11	−0·14	−0·04	−0·24	−0·24	−0·14	−0·23	−2·03	+0·90	+0·18	+3·49	−0·04	−0·902	+0·154	248
264	26	86 30 23	86 18 56	1	+0·03	−0·05	+0·04	0·00	+0·09	+0·09	−0·05	−0·09	+2·05	+0·84	−1·06	+3·34	+0·02	−0·725	0·000	264
276	27	267 45 12	267 44 31	1	−0·05	+0·05	−0·03	+0·10	−0·05	−0·03	−0·07	−0·11	+1·59	+1·50	+3·04	+1·68	0·00	−0·063	+0·001	276
308	29	88 2 43	88 4 15	½	+0·03	−0·04	+0·03	−0·08	+0·06	+0·04	+0·03	−0·10	+0·23	−2·17	+2·08	+2·81	+0·02	+0·052	−0·101	308
322	Sept. 1	87 37 54	87 54 50	1	+0·04	−0·02	+0·01	−0·09	+0·02	+0·01	+0·03	−0·08	−2·04	+0·70	+3·46	−0·23	−0·02	+0·367	−0·012	322
338	3	268 14 33	268 12 59	1	−0·06	+0·03	−0·02	+0·11	−0·03	−0·02	−0·07	−0·12	+1·94	+0·92	+3·24	+1·00	0·00	−0·133	+0·261	338
354	4	92 58 33	93 35 58	1	+0·12	+0·02	−0·04	−0·23	−0·21	−0·24	−0·06	−0·30	+1·41	+1·61	+1·68	−2·96	−0·06	+0·677	−0·080	354
370	9	87 16 23	87 8 4	½	+0·03	−0·02	+0·02	−0·04	+0·07	+0·07	−0·02	−0·07	+2·04	−0·57	+0·38	+3·35	+0·02	−0·511	+0·033	370
382	10	268 47 34	268 52 0	½	−0·08	+0·01	−0·01	+0·14	−0·01	0·00	−0·08	−0·14	+2·08	+0·27	+3·31	+0·32	0·00	+0·302	+0·001	382
390	14	266 30 14	266 24 28	½	0·00	+0·07	−0·07	+0·05	−0·08	−0·08	−0·07	−0·10	−0·28	+2·06	+2·12	+2·52	−0·01	−0·439	+0·035	390
414	16	87 21 10	87 16 12	1	+0·03	−0·02	+0·02	−0·05	+0·06	+0·06	−0·02	−0·07	+1·87	−0·90	+0·81	+3·19	+0·02	−0·290	−0·082	414
422	17	269 49 28	269 56 26	1	−0·04	0·00	0·00	+0·18	+0·03	+0·03	−0·04	−0·19	+1·99	−0·48	+3·22	−0·42	+0·01	+0·341	+0·147	422
440	19	88 24 16	88 45 10	1	+0·07	−0·03	+0·03	−0·12	+0·02	+0·03	+0·07	−0·13	−1·70	−1·13	+3·17	+0·85	+0·01	+0·267	+0·008	440
454	23	87 38 29	87 37 36	¼	+0·03	−0·02	+0·02	−0·07	+0·05	+0·06	0·00	−0·08	+1·18	−1·65	+1·67	+2·77	+0·02	−0·039	−0·056	454
458	25	262 6 48	261 41 46	½	+0·10	+0·09	−0·07	−0·05	−0·23	−0·22	−0·11	−0·22	−2·00	+0·18	+0·28	+3·17	−0·05	−0·579	−0·027	458
466	27	86 1 41	85 54 35	½	+0·03	−0·05	+0·05	−0·01	+0·08	+0·08	−0·05	−0·08	+1·66	+1·12	−0·66	+3·10	+0·02	−0·454	−0·144	466

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TABLE XVIII.—*continued.*

SATELLITES I. AND III.—DISTANCES, 1901.

Rotation No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of											O—C.	Residuals.	Rotation No.
					μ	Δh_1	Δi_1	$\Delta \varpi_1$	Δh_3	Δi_3	$\Delta \varpi_3$	$\Delta \epsilon_1$	$\Delta \epsilon_3$	$\Delta \gamma_1$	$\Delta \eta_1$	$\Delta \gamma_3$	$\Delta \eta_3$	ΔJ		
33	1901 July 2	361·457	361·625	¼	3·61	−2·02	−0·96	+2·21	+2·26	+3·48	−3·83	+2·18	−3·22	−0·02	0·00	0·00	−0·04	+0·99	+0·168	+0·056	33
47	8	361·407	361·237	1	3·61	−1·50	−1·72	+2·25	+3·73	+1·33	−3·74	−2·24	+2·90	0·00	+0·01	+0·02	+0·01	+0·92	−0·170	−0·097	47
61	10	61·764	61·833	¼	0·62	−1·96	−1·03	+2·16	+3·86	+4·20	−5·66	−2·18	−5·62	+0·59	−0·18	+0·86	−1·32	−0·28	+0·069	−0·118	61
83	11	91·704	91·507	¼	0·92	+1·14	+0·59	−1·25	−4·71	−1·87	+4·78	−0·52	+4·66	−0·17	−0·14	+0·55	+0·01	−0·91	−0·197	−0·033	83
99	12	479·648	479·873	1	4·80	+1·25	+0·55	−1·30	−2·64	−1·66	+3·08	+0·74	+0·72	+0·01	+0·01	+0·02	+0·02	+1·68	+0·225	−0·118	99
119	16	289·627	289·936	1	2·90	−1·47	−1·75	+2·27	+2·05	+2·56	−3·26	+2·24	−1·39	0·00	−0·01	+0·01	+0·02	−0·02	+0·309	+0·080	119
145	22	366·257	366·035	¼	3·66	−0·80	−1·31	+1·51	+4·44	+1·75	−4·23	−1·11	+4·26	+0·01	+0·04	+0·10	+0·01	+1·45	−0·222	+0·008	145
167	27	186·750	186·733	1	1·87	+1·47	+0·57	−1·34	−1·99	−3·31	+3·84	−1·20	−2·80	+0·10	+0·03	−0·06	−0·25	−0·64	−0·017	+0·137	167
175	29	342·208	342·047	1	3·42	−0·78	−0·98	+1·25	+4·62	+2·06	−4·37	−0·50	+4·71	+0·03	+0·05	+0·14	−0·01	+1·50	−0·161	−0·039	175
187	Aug. 10	279·303	279·191	1	2·79	+1·87	+1·14	−1·86	−1·90	−2·44	+3·03	−2·14	−1·16	+0·05	−0·02	−0·07	−0·12	+0·02	−0·112	−0·058	187
203	13	221·941	221·939	1	2·22	−0·78	−1·44	+1·63	+2·49	+1·66	−2·54	+1·35	+0·87	0·00	−0·04	+0·08	+0·07	−0·50	−0·002	+0·239	203
217	14	403·076	403·018	1	4·03	−0·71	−1·12	+1·31	+1·88	+3·45	−3·92	−0·79	−3·07	−0·02	−0·04	−0·02	−0·11	+1·51	+0·058	−0·112	217
233	19	256·394	256·089	½	2·56	−0·89	−0·70	+0·96	+4·58	+2·75	−4·15	+0·16	+5·22	+0·07	+0·06	+0·22	−0·08	+1·35	−0·305	+0·049	233
249	24	279·086	278·857	1	2·79	+1·78	+1·15	−1·61	−1·98	−2·02	+2·50	−2·09	−0·36	+0·04	−0·01	−0·06	−0·07	+0·06	−0·229	+0·017	249
265	26	196·115	195·882	1	1·96	−1·21	−0·56	+0·85	+4·45	+3·12	−4·16	+0·87	+5·39	+0·12	+0·06	+0·30	−0·16	+1·17	−0·233	+0·017	265
277	27	161·238	161·182	1	1·61	−0·85	−0·70	+0·89	+3·18	+1·43	−2·16	+0·12	+2·43	−0·04	−0·04	+0·14	+0·06	−0·74	−0·056	−0·056	277
291	28	407·187	407·060	1	4·07	−0·68	−1·09	+1·22	+1·70	+2·98	−3·30	−0·77	−2·32	−0·01	−0·03	−0·02	−0·08	+1·48	−0·127	+0·063	291
311	29	8·696	7·672	½	0·08	+0·47	+0·86	−0·95	−1·81	−2·21	+2·55	−0·93	+2·85	−0·30	+1·84	+2·68	−3·92	+0·30	−1·024	+0·548	311
323	Sept. 1	272·215	272·960	1	2·72	+1·74	+1·01	−1·31	−2·16	−3·79	+4·12	+1·96	−3·92	−0·03	+0·01	0·00	−0·08	+0·78	+0·745	−0·143	323
339	3	143·173	142·984	1	1·43	−1·14	−0·55	+0·72	+3·51	+1·48	−2·01	−0·77	+3·07	−0·06	−0·03	+0·16	+0·04	−0·68	−0·189	−0·085	339
355	4	422·884	422·924	1	4·23	−0·77	−0·75	+0·85	+1·68	+2·52	−2·73	−0·08	−1·51	−0·01	−0·01	−0·02	−0·04	+1·55	+0·040	−0·075	355
385	10	129·557	129·346	½	1·29	−1·39	−0·59	+0·68	+3·76	+1·62	−1·86	−1·25	+3·58	−0·07	−0·01	+0·18	+0·02	−0·56	−0·211	−0·124	385
391	14	354·550	354·204	½	3·54	+0·89	+1·47	−1·48	−2·89	−1·32	+1·36	−1·59	+2·05	0·00	0·00	+0·01	0·00	+1·08	−0·346	−0·018	391
411	16	30·621	30·926	1	0·31	+1·51	+1·05	−1·04	−3·20	−3·38	+3·34	−1·83	−4·63	+0·78	−0·42	+1·37	−1·78	−0·14	+0·305	−0·143	411
423	17	124·709	124·095	1	1·24	−1·64	−0·87	+0·83	+4·00	+1·88	−1·79	−1·78	+4·12	−0·07	+0·02	+0·19	−0·01	−0·28	−0·614	+0·123	423
443	19	80·775	80·867	1	0·81	−0·91	−0·56	+0·52	+2·18	+3·66	−3·55	−0·38	−3·91	+0·17	+0·13	+0·07	−0·53	−0·77	+0·092	−0·180	443
459	25	357·915	358·006	½	3·58	−1·56	−0·72	+0·51	+2·27	+1·40	−1·11	+1·61	+0·89	+0·02	0·00	+0·05	+0·04	+1·05	+0·091	−0·004	459

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TABLE XVIII.—*continued.*

SATELLITES I. AND III.—POSITION-ANGLES, 1901.

Rotation No.	Date.	Computed Position-Angles.	Observed Position-Angles.	Weight.	Coefficients of											O—C.	Residuals.	Rotation No.
					Δh_1	Δk_1	Δx_1	Δh_3	Δk_3	Δx_3	$\Delta \epsilon_1$	$\Delta \epsilon_3$	Δp_1	Δq_1	Δp_3	Δq_3	$\Delta \gamma$		
34	1901 July 2	85 3 53	84 59 26	¼	+0·03	−0·03	0·00	−0·06	+0·20	−0·06	−0·02	−0·20	+2·38	−0·47	+0·64	+6·11	+0·03	−0·467	+0·156	34
48	8	86 2 4	85 57 45	1	+0·02	−0·04	0·00	−0·17	+0·11	+0·06	−0·02	−0·20	−1·18	+2·11	+5·61	+2·42	−0·03	−0·452	+0·114	48
62	10	71 3 38	70 34 47	¼	−0·48	−0·30	+0·56	+0·97	+1·22	−1·53	−0·55	−1·56	−2·22	+0·70	−3·26	+4·96	−0·13	−0·514	+0·120	62
84	11	271 8 7	271 8 53	¼	−0·16	0·00	+0·12	+0·56	+0·08	−0·48	+0·12	−0·56	−1·88	−1·51	+6·11	+0·15	+0·09	+0·022	−0·002	84
100	12	266 21 13	266 19 59	1	−0·06	+0·05	+0·01	+0·16	−0·15	−0·02	−0·08	−0·22	+2·01	+1·31	+4·38	+4·29	0·00	−0·176	−0·098	100
120	16	86 29 57	86 28 59	1	+0·02	−0·04	+0·01	−0·12	+0·16	−0·03	+0·02	−0·20	+1·18	−2·09	+2·68	+5·47	+0·03	−0·080	+0·011	120
146	22	87 32 56	87 35 14	¼	+0·04	−0·04	0·00	−0·22	+0·06	+0·10	−0·05	−0·22	+0·51	+2·34	+6·01	+0·54	−0·03	+0·243	−0·253	146
153	23	87 4 13	87 6 8	½	+0·03	−0·04	+0·01	−0·14	+0·15	−0·02	+0·03	−0·20	+0·65	−2·29	+3·32	+5·04	+0·03	+0·149	−0·142	153
168	27	264 21 39	264 14 28	1	0·00	+0·06	−0·06	0·00	−0·31	+0·24	+0·02	−0·30	−2·24	−0·78	+1·30	+5·86	−0·06	−0·391	+0·202	168
176	29	88 17 7	88 17 10	1	+0·06	−0·04	0·00	−0·24	+0·02	+0·12	−0·06	−0·24	+1·13	+2·08	+5·96	−0·30	−0·03	+0·012	−0·212	176
188	Aug. 10	266 0 31	265 59 20	1	0·00	+0·05	−0·05	+0·07	−0·22	+0·16	−0·02	−0·22	−2·17	+0·78	+2·93	+5·06	−0·04	−0·095	−0·208	188
204	13	88 27 52	88 28 27	1	+0·04	−0·03	+0·01	−0·19	+0·10	−0·02	+0·04	−0·21	−2·27	−2·27	+4·46	+3·67	+0·03	+0·038	−0·072	204
218	14	86 20 38	86 18 38	1	+0·02	−0·08	+0·06	−0·05	+0·22	−0·18	−0·08	−0·22	+0·84	+2·12	+0·97	+5·69	+0·01	−0·226	−0·131	218
234	19	89 53 27	89 54 10	½	+0·09	−0·02	−0·01	−0·29	−0·05	+0·15	−0·08	−0·29	+1·62	+1·56	+5·36	−1·85	−0·04	+0·049	−0·096	234
250	24	266 38 12	266 34 45	1	0·00	+0·04	−0·04	+0·09	−0·18	+0·15	−0·01	−0·20	−2·06	+0·85	+3·51	+4·40	−0·04	−0·280	−0·010	250
266	26	91 8 52	91 11 35	1	+0·13	0·00	−0·03	−0·36	−0·14	+0·22	−0·12	−0·38	+2·01	+0·91	+4·92	−2·60	−0·04	−0·142	−0·140	266
278	27	89 10 49	89 11 51	1	+0·07	−0·03	+0·01	−0·23	+0·06	0·00	+0·07	−0·24	−1·56	−1·54	+5·16	+2·06	+0·02	+0·048	−0·097	278
292	28	86 45 59	86 41 9	1	+0·02	−0·07	+0·06	−0·07	+0·19	−0·17	−0·08	−0·20	+0·81	+2·04	+1·67	+5·30	0·00	−0·570	+0·221	292
312	29	326 31 26	329 3 18	½	−0·81	−1·33	+1·48	+3·08	+3·47	−4·06	+1·44	−4·60	−0·18	+1·12	+1·63	−2·39	−0·52	+0·390	−0·059	312
324	Sept. 1	266 44 49	266 42 1	1	−0·01	+0·04	−0·03	+0·03	−0·19	+0·18	0·00	−0·19	+2·06	−0·64	−0·32	+5·48	+0·03	−0·220	+0·121	324
340	3	89 23 14	89 21 23	1	+0·09	−0·02	+0·01	−0·25	+0·04	0·00	+0·09	−0·25	−1·92	−0·96	+5·30	+1·26	+0·02	−0·077	+0·014	340
356	4	87 10 19	87 6 4	1	+0·04	−0·05	+0·05	−0·09	+0·16	−0·15	−0·07	−0·19	+1·39	+1·64	+2·39	+4·89	+0·01	−0·518	+0·214	356
386	10	89 32 33	89 23 53	½	+0·10	−0·01	0·00	−0·26	+0·02	0·00	+0·10	−0·26	−2·06	−0·41	+5·30	+0·53	+0·01	−0·323	+0·174	386
392	14	267 35 59	267 33 18	½	−0·02	+0·04	−0·04	+0·14	−0·10	+0·10	−0·04	−0·17	−0·34	+2·06	+4·78	+2·27	−0·02	−0·278	−0·015	392
412	16	241 56 11	241 15 7	1	+0·69	+0·52	−0·51	−1·46	+1·69	+1·67	−0·85	−2·23	−1·65	+0·89	−2·88	+3·76	−0·12	−0·372	−0·036	412
424	17	89 30 7	89 20 23	1	+0·10	+0·01	−0·01	−0·26	−0·01	0·00	+0·10	−0·26	−2·01	+0·42	+5·20	−0·36	−0·01	−0·352	+0·254	424
444	19	81 27 53	81 34 37	1	−0·05	−0·10	+0·10	+0·17	+0·50	−0·49	+0·02	−0·51	−1·61	−1·24	−0·67	+5·14	−0·10	+0·155	−0·269	444
460	25	87 49 58	87 48 43	½	+0·06	−0·01	+0·02	−0·15	+0·10	−0·11	−0·06	−0·18	+2·02	−0·14	+3·98	+3·19	+0·02	−0·130	+0·001	460

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TABLE XVIII.—*continued.*

SATELLITES I. AND IV.—DISTANCES, 1901.

Rotation No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of													O—C.	Residuals.	Rotation No.
					μ	Δh_1	Δk_1	Δx_1	Δh_4	Δk_4	$\Delta \epsilon_1$	$\Delta \epsilon_4$	Δp_1	Δq_1	Δp_4	Δq_4	ΔJ			
	1901																			
29	July 2	561″.389	560″.445	½	5·61	+2·04	+1·01	−2·26	−4·73	−2·81	−2·22	+1·18	−0·01	0·00	+0·02	+0·02	−0·22	−0″.944	+0″.069	29
49	8	396·675	396·859	1	3·97	−1·54	−1·71	+2·27	+8·55	+3·69	−2·26	+8·80	−0·04	+0·07	+0·37	−0·02	+0·61	+0·184	+0·062	49
65	10	545·428	545·365	½	5·45	−2·01	−1·01	+2·19	+5·02	+2·72	−2·20	+2·21	−0·01	0·00	+0·05	+0·04	−0·24	−0·063	+0·085	65
85	11	737·512	738·361	½	7·38	−1·12	−0·61	+1·24	+3·60	+4·52	+0·46	−2·46	0·00	0·00	0·00	0·00	+1·52	+0·849	−0·130	85
101	12	395·017	395·540	1	3·95	−1·22	−0·56	+1·28	+3·76	+6·25	−0·69	−5·67	+0·04	+0·03	−0·03	−0·23	−0·96	+0·523	+0·087	101
113	14	49·000	50·278	1	0·49	−0·77	−1·12	+1·33	+6·73	+7·25	+0·87	−9·79	+0·22	+0·76	+2·03	−2·92	−1·02	+1·278	+0·154	113
123	16	323·671	322·014	1	3·24	+1·51	+1·74	−2·30	−8·91	−4·45	−2·27	+9·66	−0·06	+0·09	+0·48	−0·11	+0·53	−1·657	+0·102	123
143	22	124·959	124·534	¼	1·25	+0·79	+1·26	−1·47	−6·19	−7·60	+1·04	−9·56	+0·06	+0·26	+0·52	−1·06	−1·00	−0·425	+0·077	143
165	27	717·833	718·029	1	7·18	−1·50	−0·58	+1·37	+4·23	+3·22	+1·26	+0·75	+0·02	+0·01	+0·07	+0·07	+1·33	+0·196	+0·359	165
177	29	602·315	603·114	1	6·02	−0·77	−1·01	+1·27	+3·81	+6·83	−0·56	−6·66	−0·02	−0·04	0·00	−0·18	+1·42	+0·799	−0·039	177
189	Aug. 10	324·254	325·049	1	3·25	−1·86	−1·12	+1·84	+8·12	+4·49	+2·12	+8·96	+0·14	−0·05	+0·65	−0·16	+0·60	+0·795	−0·031	189
205	13	475·763	476·678	1	4·76	−0·79	−1·46	+1·66	+3·38	+3·96	+1·39	−1·25	0·00	−0·02	+0·04	+0·06	−0·68	+0·915	−0·163	205
219	14	653·275	653·681	1	6·53	−0·71	−1·15	+1·34	+3·08	+5·46	−0·84	−4·22	−0·01	−0·02	−0·03	−0·09	+1·36	+0·406	+0·147	219
235	19	283·957	283·137	½	2·84	+0·86	+0·72	−0·97	−7·30	−3·11	−0·08	+7·08	−0·08	−0·08	+0·52	+0·03	−0·98	−0·820	−0·055	235
251	24	212·642	212·197	1	2·13	+1·78	+1·12	−1·58	−5·24	−7·29	−2·07	−8·68	+0·17	−0·07	+0·29	−0·76	−0·29	−0·445	−0·116	251
267	26	277·829	278·649	1	2·78	−1·18	−0·56	+0·84	+7·78	+5·31	+0·81	+9·31	+0·13	+0·07	+0·59	−0·28	+1·15	+0·820	−0·297	267
279	27	218·614	219·097	1	2·19	−0·82	−0·72	+0·90	+7·27	+3·60	−0·05	+7·53	−0·11	−0·11	+0·68	−0·05	−0·98	+0·483	−0·053	279
293	28	598·768	599·096	1	5·99	−0·68	−1·12	+1·25	+5·89	+2·61	−0·83	+4·86	+0·01	+0·03	+0·15	+0·05	+1·28	+0·328	−0·008	293
305	29	465·636	466·253	½	4·66	−0·86	−1·54	+1·69	+4·10	+2·90	+1·59	+1·39	0·00	−0·04	+0·14	+0·11	−0·53	+0·617	+0·061	305
325	Sept. 1	255·284	256·156	1	2·55	−1·73	−0·99	+1·28	+4·49	+7·23	−1·93	−8·07	+0·13	−0·04	+0·15	−0·59	−0·35	+0·872	−0·236	325
341	3	199·398	198·484	1	1·99	+1·10	+0·56	−0·72	−7·54	−5·92	+0·71	+9·56	+0·18	+0·09	+0·75	−0·50	+1·09	−0·914	−0·008	341
357	4	153·351	152·410	1	1·53	+0·75	+0·77	−0·86	−7·74	−4·12	+0·14	+8·43	−0·14	−0·18	+1·01	−0·22	−0·99	−0·941	−0·007	357
371	9	311·741	311·338	½	3·12	+1·70	+0·94	−1·07	−3·85	−6·49	−1·87	−6·82	+0·09	−0·02	+0·03	−0·40	−0·35	−0·403	−0·048	371
393	14	409·750	410·586	½	4·10	−0·92	−1·52	+1·53	+5·17	+2·40	+1·66	+3·83	+0·01	−0·06	+0·25	+0·11	−0·43	+0·836	−0·112	393
415	16	535·071	535·757	1	5·35	−1·67	−1·11	+1·09	+2·79	+4·55	+1·98	−3·09	0·00	0·00	+0·01	+0·02	+0·55	+0·686	+0·066	415
425	17	343·673	343·941	1	3·44	−1·63	−0·83	+0·79	+3·27	+6·09	−1·74	−5·94	+0·06	−0·01	−0·01	−0·30	−0·38	+0·268	+0·142	425
447	19	53·724	52·632	1	0·54	+0·81	+0·58	−0·54	−6·14	−6·31	+0·24	+8·79	+0·46	+0·39	+1·79	−2·04	+0·99	−1·092	+0·289	447
463	27	44·317	43·851	½	0·44	+0·96	+0·46	−0·31	−5·31	−6·06	−0·56	−7·95	+0·55	+0·35	+1·50	−2·46	−0·88	−0·466	+0·189	463

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TABLE XVIII.—*continued.*

SATELLITES I. AND IV.—POSITION-ANGLES, 1901.

Rotation No.	Date.	Computed Position-Angles.	Observed Position-Angles.	Weight.	Coefficients of												O—C.	Residuals.	Rotation No.
					Δh_1	Δk_1	Δx_1	Δh_4	Δk_4	$\Delta \epsilon_1$	$\Delta \epsilon_4$	Δp_1	Δq_1	Δp_4	Δq_4	ΔJ			
	1901																		
30	July 2	265° 37′ 25″	265° 40′ 36″	½	−0·04	+0·02	+0·02	+0·21	−0·27	+0·04	−0·34	−2·35	+0·59	+ 7·50	+ 7·81	−0·03	+0″·519	−0″·260	30
50	8	87 48 6	87 49 58	1	+0·07	+0·02	−0·06	−0·45	+0·02	+0·05	−0·45	−1·24	+2·08	+10·65	− 0·71	−0·05	+0·215	+0·123	50
66	10	86 16 59	86 15 11	½	+0·05	−0·02	−0·03	−0·24	+0·24	+0·05	−0·33	−2·33	+0·60	+ 8·28	+ 6·83	−0·03	−0·287	+0·176	66
86	11	85 58 57	85 57 28	½	+0·05	−0·06	0·00	−0·16	+0·31	−0·08	−0·35	+1·84	+1·57	+ 4·65	+ 9·71	+0·01	−0·325	+0·062	86
102	12	84 48 14	84 45 31	1	+0·03	−0·06	+0·02	−0·03	+0·45	+0·06	−0·45	−1·98	−1·36	+ 1·34	+10·73	−0·04	−0·313	+0·095	102
114	14	66 57 16	67 5 53	1	−0·23	−0·46	+0·48	+2·30	+2·75	+0·38	−3·57	−0·62	−2·18	− 5·84	+ 8·40	−0·35	+0·128	−0·076	114
124	16	268 53 34	268 59 49	1	−0·09	−0·04	+0·09	+0·54	+0·62	+0·08	−0·54	−1·26	+2·04	+10·54	− 2·36	−0·06	+0·586	−0·094	124
144	22	260 8 45	259 55 58	½	+0·06	+0·21	−0·19	−0·66	−1·09	+0·19	−1·27	−0·56	−2·31	− 4·58	+ 9·43	−0·10	−0·464	+0·020	144
166	27	87 22 41	87 21 38	1	+0·07	−0·03	−0·02	−0·24	+0·23	−0·08	−0·33	+2·26	+0·72	+ 7·39	+ 7·51	+0·01	−0·211	+0·265	166
178	29	85 54 47	85 52 20	1	+0·03	−0·08	+0·05	−0·03	+0·41	−0·08	−0·41	+1·07	+2·11	+ 0·29	+10·57	+0·01	−0·441	−0·026	178
190	Aug. 10	91 8 9	91 13 26	1	+0·16	+0·05	−0·12	−0·67	−0·17	−0·17	−0·69	+2·18	−0·74	+ 9·85	− 2·41	0·00	+0·506	+0·082	190
206	13	87 51 31	87 49 12	1	+0·03	−0·05	+0·03	−0·21	+0·24	+0·05	−0·31	−0·16	−2·27	+ 5·81	+ 8·35	+0·03	−0·322	+0·461	206
220	14	86 54 20	86 53 4	1	+0·03	−0·07	+0·05	−0·11	+0·33	−0·08	−0·35	+0·79	+2·14	+ 3·04	+ 9·73	0·00	−0·239	−0·091	220
236	19	270 30 59	270 45 0	½	−0·09	+0·02	+0·02	+0·55	+0·02	+0·08	−0·55	−1·57	−1·62	+10·08	+ 0·59	+0·06	+1·158	−0·401	236
252	24	262 49 43	262 40 50	1	+0·11	+0·12	−0·14	−0·39	−0·83	−0·15	−0·91	−2·06	+0·80	− 3·43	+ 9·15	−0·07	−0·544	−0·081	252
268	26	91 29 24	91 38 4	1	+0·13	0·00	−0·03	−0·62	−0·24	−0·12	−0·66	+1·98	+0·97	+ 8·76	− 4·20	−0·06	+0·699	−0·217	268
280	27	91 37 42	91 48 5	1	+0·11	0·00	−0·03	−0·67	−0·13	+0·07	−0·68	−1·51	−1·58	+ 9·67	− 0·71	+0·08	+0·660	−0·119	280
294	28	88 33 17	88 34 16	1	+0·05	−0·04	+0·03	−0·25	+0·11	−0·05	−0·28	+0·75	+2·06	+ 9·25	+ 2·96	−0·03	+0·172	+0·013	294
306	29	88 40 3	88 40 53	½	+0·04	−0·02	+0·01	−0·27	+0·16	+0·02	−0·31	+0·17	−2·18	+ 7·49	+ 6·19	+0·04	+0·112	+0·118	306
326	Sept. 1	84 3 15	83 55 40	1	−0·07	−0·08	+0·09	+0·22	+0·68	−0·09	−0·72	−2·07	+0·59	− 2·36	+ 9·43	−0·06	−0·564	+0·099	326
342	3	272 56 11	273 5 19	1	−0·16	−0·01	+0·04	+0·78	+0·42	−0·13	−0·88	+1·88	+1·01	+ 8·04	− 5·36	−0·08	+0·536	+0·058	342
358	4	273 45 7	274 4 26	1	−0·12	−0·03	+0·05	+0·95	+0·32	+0·05	−1·00	−1·34	−1·67	+ 9·39	− 2·06	+0·12	+0·858	−0·146	358
372	9	265 8 26	265 5 13	½	+0·03	+0·06	−0·06	−0·10	−0·51	−0·05	−0·52	−2·06	+0·52	− 0·76	+ 9·35	−0·05	−0·291	−0·116	372
394	14	89 11 12	89 13 1	½	+0·04	0·00	0·00	−0·34	+0·09	−0·01	−0·35	+0·40	−2·04	+ 8·46	+ 3·69	+0·04	+0·217	+0·124	394
416	16	87 31 46	87 30 28	1	+0·03	−0·02	+0·02	−0·13	+0·24	−0·02	−0·27	+1·89	−0·86	+ 3·50	+ 8·55	+0·03	−0·196	+0·152	416
426	17	85 33 11	85 29 54	1	−0·01	−0·05	+0·05	+0·03	+0·44	−0·02	−0·44	−2·02	+0·35	+ 0·27	+ 9·24	−0·04	−0·323	−0·019	426
448	19	284 26 17	285 13 18	1	−0·30	−0·13	+0·12	+1·94	+1·78	−0·14	−2·61	+1·48	+1·28	+ 5·83	− 6·63	−0·29	+0·752	−0·048	448
464	27	248 5 54	247 33 18	½	+0·28	+0·20	−0·16	−1·79	−2·26	−0·13	−2·86	−1·60	−1·02	− 4·38	+ 7·14	−0·32	−0·461	−0·047	464

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TABLE XVIII.—*continued.*

SATELLITES II. AND III.—DISTANCES, 1901.

Rotaiton No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of														O—C.	Residuals.	Rotation No.
					μ	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	$\Delta \epsilon_2$	$\Delta \epsilon_3$	Δp_2	Δq_2	Δp_3	Δq_3	ΔJ			
	1901																				
7	June 24	″554·821	″554·448	½	5·55	−2·09	−0·77	−2·20	+2·66	+1·61	−3·11	+1·29	+0·59	+0·01	+0·01	+0·01	+0·01	+1·23	″−0·373	″+0·144	7
23	29	280·712	281·146	¼	2·81	+2·87	+1·00	+2·96	−3·99	−4·26	+5·65	+2·72	−5·74	−0·11	−0·02	+0·09	−0·15	+0·74	+0·434	+0·099	23
37	July 2	401·756	401·299	¼	4·02	−2·18	−2·74	−3·34	+2·27	+3·51	−3·86	−3·38	−3·27	+0·05	−0·12	−0·02	−0·20	+0·79	−0·457	+0·053	37
51	8	435·355	435·220	1	4·35	−3·09	−1·25	−3·19	+3·68	+1·33	−3·70	+3·15	+2·82	+0·09	−0·01	+0·13	+0·06	+0·89	−0·135	−0·090	51
67	10	85·653	85·852	¼	0·86	+2·43	+0·88	+2·42	−3·98	−4·31	+5·82	+2·02	+5·80	+0·59	+0·20	+0·56	−0·82	+0·53	+0·199	+0·001	67
79	11	271·273	271·400	¼	2·71	+2·58	+2·69	+3·71	−4·88	−2·05	+5·03	−3·68	+4·98	−0·02	+0·04	+0·07	0·00	+0·53	+0·127	−0·051	79
95	12	135·143	135·205	1	1·35	+1·80	+0·95	+1·98	−2·67	−1·64	+3·09	−0·79	+0·80	−0·04	−0·03	+0.05	+0·05	−0·28	+0·062	+0·063	95
127	16	516·734	516·473	1	5·17	−1·37	−2·29	−2·58	+2·03	+2·63	−3·29	−2·16	−1·52	0·00	−0·06	−0·40	−0·09	+1·12	−0·261	−0·038	127
135	18	272·720	272·840	¼	2·73	+1·90	+2·70	+3·26	−5·03	−2·69	+5·42	−3·11	+5·56	−0·02	+0·06	+0·10	−0·03	+0·67	+0·120	−0·043	135
161	27	224·539	225·095	1	2 25	+2·17	+2·80	+3·54	−1·97	−3·27	+3·80	+3·47	−2·72	+0·01	−0·02	+0.01	+0·03	+0·14	+0·556	+0·057	161
191	Aug. 10	185·088	185·578	1	1·85	+1·45	+2·56	+2·94	−1·88	−2·47	+3·06	+2·67	−1·23	+0·01	−0·06	+0·05	+0·08	−0·05	+0·490	−0·027	191
207	13	532·395	532·304	1	5·32	−1·20	−1·43	−1·80	+2·46	+1·68	−2·55	−0·51	+0·80	0·00	−0·01	−0·01	−0·01	+1·20	−0·091	−0·140	207
237	19	83·050	82·849	½	0·83	−2·26	−2·56	−3·18	+4·47	+2·65	−4·03	+3·39	+5·07	+0·39	−0·59	+1·06	−0·34	+0·01	−0·201	−0·078	237
239	23	133·127	133·195	¼	1·33	+2·83	+1·60	+2·36	−4·16	−1·88	+3·02	−3·16	+4·14	−0·08	+0·02	+0·14	0·00	0·00	+0·068	−0·051	239
253	24	170·305	170·447	1	1·70	+1·24	+2·36	+2·61	−1·96	−2·06	+2·53	+2·31	−0·42	0·00	−0·08	+0·08	+0·11	−0·09	+0·142	+0·239	253
269	26	17·751	18·493	1	0·18	+0·76	+0·95	+1·11	−1·77	−1·11	+1·53	−1·16	−2·08	+1·15	−3·04	+4·61	−2·34	+0·11	+0·742	+0·124	269
281	27	478·829	479·187	1	4·79	−1·71	−0·93	−1·32	+3·16	+1·42	−2·15	+0·99	+2·37	+0·03	+0·02	+0·05	+0·02	+1·09	+0·358	−0·551	281
301	29	117·154	117·422	½	1·17	+1·22	+1·24	+1·48	−3·47	−4·14	+4·80	−0·23	+5·36	+0·15	+0·19	+0·21	−0·33	+0·60	+0·268	−0·117	301
317	31	141·570	141·998	1	1·42	+1·05	+1·96	+2·13	−2·21	−1·70	+2·09	+1·62	+0·45	−0·03	−0·11	+0·14	+0·13	−0·18	+0·428	−0·154	317
327	Sept. 1	346·585	346·966	1	3·47	+2·60	+1·21	+1·66	−2·19	−3·81	+4·15	+2·65	−3·96	−0·04	0·00	0·00	−0·07	+0·77	+0·381	+0·118	327
343	3	431·239	431·098	1	4·31	−2·02	−0·89	−1·20	+3·48	+1·47	−2·00	+1·61	+3·02	+0·05	+0·02	+0·09	+0·02	+0·98	−0·141	−0·053	343
359	4	353·226	352·798	1	3·53	−2·09	−2·58	−2·86	+1·67	+2·55	−2·76	−3·29	−1·58	+0·05	−0·08	−0·06	−0·13	+0·62	−0·428	+0·100	359
383	10	367·150	367·062	½	3·67	−2·33	−1·00	−1·16	+3·78	+1·63	−1·88	+2·20	+3·62	+0·08	+0·01	+0·14	+0·01	+0·83	−0·088	−0·081	383
395	14	94·263	94·148	½	0·94	+1·12	+1·24	+1·25	−2·83	−1·33	+1·37	+0·29	+1·94	−0·12	−0·15	+0·28	+0·14	−0·28	−0·115	+0·140	395
405	16	237·881	238·501	1	2·38	+1·05	+1·70	+1·69	−3·46	−3·80	+3·77	−1·31	−5·11	−0·06	−0·18	+0·17	−0·24	+0·72	+0·620	−0·083	405
427	17	300·308	299·847	1	3·00	−2·50	−1·20	−1·14	+3·97	+1·84	−1·75	+2·59	+4·06	+0·12	−0·01	+0·19	−0·01	+0·67	−0·461	+0·235	427
437	19	45·448	45·151	1	0·45	−2·09	−0·81	−0·71	+2·10	+3·56	−3·45	−1·82	−3·75	+0·65	+0·16	+0·11	−1·05	−0·27	−0·297	+0·256	437

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TABLE XVIII.—*continued.*

SATELLITES II. AND III.—POSITION-ANGLES, 1901.

Rotation No.	Date.	Computed Position-Angles.	Observed Position-Angles.	Weight.	Coefficients of												O—C.	Residuals.	Rotation No.
					Δh_2	Δk_2	Δr_2	Δh_3	Δk_3	Δr_3	$\Delta \epsilon_2$	$\Delta \epsilon_3$	Δp_2	Δq_2	Δp_3	Δq_3	ΔJ			
	1901	° ′ ″	° ′ ″																
8	June 24	85 9 53	85 8 16	½	+0·10	−0·07	+0·06	−0·14	+0·16	+0·05	−0·12	−0·21	+3·18	+2·11	+4·10	+4·54	0·00	−″0·265	+″0·010	8
24	29	263 39 26	263 45 25	¼	−0·02	+0·07	+0·03	−0·07	−0·24	+0·18	−0·02	−0·24	+3·83	+0·55	−3·28	+5·18	+0·02	+0·490	−0·136	24
38	July 2	83 34 53	83 24 13	¼	−0·03	−0·14	−0·10	0·00	+0·29	−0·16	−0·14	−0·28	−1·57	+3·49	+0·57	+6·11	−0·01	−1·251	+0·107	38
52	8	87 6 35	87 6 30	1	+0·16	−0·01	+0·12	−0·24	+0·09	+0·13	−0·16	−0·25	+3·82	−0·25	+5·58	+2·50	0·00	−0·012	+0·308	52
68	10	275 12 57	275 15 2	¼	−0·50	−0·09	−0·44	+0·70	+0·60	−0·92	−0·44	−0·89	+3·60	+1·25	+3·42	−5·00	−0·06	+0·077	+0·107	68
80	11	266 37 11	266 29 35	¼	−0·07	+0·01	−0·05	+0·19	−0·06	−0·10	+0·03	−0·18	−2·15	+3·19	+6·13	−0·32	−0·02	−0·598	−0·136	80
96	12	266 44 18	266 31 9	1	−0·11	+0·06	−0·04	+0·18	−0·14	−0·04	+0·13	−0·22	−2·95	−2·40	+4·43	+4·23	+0·01	−0·517	+0·421	96
128	16	85 20 24	85 13 34	1	+0·03	−0·11	−0·06	−0·08	+0·22	−0·10	−0·11	−0·23	+0·31	+3·78	+2·55	+5·53	0·00	−1·025	+0·137	128
136	18	267 20 50	267 10 51	¼	−0·07	+0·01	−0·04	+0·20	−0·02	−0·12	+0·01	−0·19	−0·96	+3·71	+5·89	−1·60	−0·02	−0·792	+0·005	136
154	23	86 0 48	85 56 57	½	+0·04	−0·07	−0·03	−0·09	+0·18	−0·07	−0·08	−0·20	+1·05	+3·61	+3·30	+5·05	0·00	−0·604	−0·113	154
162	27	267 4 0	267 13 26	1	−0·06	+0·03	−0·01	+0·10	−0·15	+0·06	0·00	−0·17	+1·70	−3·34	+1·41	+5·84	+0·02	+0·617	−0·049	162
192	Aug. 10	268 16 56	268 25 45	1	−0·07	+0·01	−0·02	+0·15	−0·12	+0·04	+0·01	−0·18	+0·34	−3·63	+2·87	+5·09	+0·02	+0·477	−0·005	192
208	13	87 18 17	87 14 57	1	+0·07	−0·08	−0·04	−0·14	+0·14	−0·07	−0·10	−0·20	+2·00	+3·00	+4·41	+3·74	0·00	−0·514	−0·005	208
238	19	98 54 30	99 32 29	½	+0·50	+0·48	+0·62	−1·01	−0·47	+0·79	−0·68	−1·10	+1·92	−2·92	+5·30	−1·72	+0·04	+0·854	−0·052	238
240	23	269 1 11	268 46 55	¼	−0·15	−0·02	−0·06	+0·24	−0·03	−0·05	+0·15	−0·24	−3·37	+0·99	+5·66	+0·07	−0·01	−0·552	−0·032	240
254	24	268 58 51	269 14 43	1	−0·08	0·00	−0·02	+0·17	−0·10	+0·04	0·00	−0·18	−0·06	−3·51	+3·46	+4·44	+0·02	+0·788	−0·369	254
282	27	88 10 42	88 6 40	1	+0·11	−0·05	−0·02	−0·18	+0·09	−0·04	−0·12	−0·20	+2·97	+1·79	+5·14	+2·13	0·00	−0·562	+0·417	282
302	29	271 37 2	271 48 5	½	−0·16	−0·02	−0·05	+0·29	+0·20	−0·26	−0·08	−0·33	+2·18	+2·73	+2·98	−4·67	−0·04	+0·383	−0·349	302
318	31	269 34 8	269 40 48	1	−0·09	0·00	−0·02	+0·21	−0·07	+0·03	+0·02	−0·20	−0·84	−3·34	+4·04	+3·76	+0·02	+0·274	+0·044	318
328	Sept. 1	266 54 12	266 55 51	1	−0·06	+0·04	+0·03	+0·03	−0·18	+0·17	−0·05	−0·18	+3·47	−0·12	−0·38	+5·48	+0·01	+0·166	+0·073	328
344	3	88 33 27	88 35 10	1	+0·12	−0·03	−0·01	−0·19	+0·06	−0·03	−0·13	−0·20	+3·20	+1·13	+5·28	+1·33	0·00	+0·211	−0·185	344
360	4	86 7 47	85 58 3	1	−0·01	−0·10	−0·10	−0·06	+0·21	−0·20	−0·09	−0·22	−1·79	+2·91	+2·33	+4·92	−0·01	−1·000	+0·057	360
384	10	89 0 25	89 1 27	½	+0·14	−0·01	0·00	−0·22	+0·03	−0·02	−0·14	−0·23	+3·30	+0·38	+5·31	+0·48	0·00	+0·114	+0·063	384
396	14	270 48 17	271 1 40	½	−0·14	−0·01	−0·01	+0·30	−0·02	+0·02	+0·08	−0·28	−1·98	−2·63	+4·73	+2·36	+0·03	+0·367	−0·181	396
406	16	264 12 38	264 4 27	1	+0·01	+0·16	+0·16	−0·14	−0·29	+0·29	−0·15	−0·32	+1·03	+3·13	−3·06	+4·24	+0·01	−0·560	+0·054	406
428	17	89 29 47	89 33 24	1	+0·17	+0·02	+0·02	−0·26	−0·01	0·00	−0·17	−0·26	+3·24	−0·26	+5·21	−0·27	0·00	+0·309	+0·036	428
438	19	75 36 1	74 57 23	1	−0·35	−0·21	−0·19	+0·38	+0·86	−0·84	−0·28	−0·90	−3·12	−0·80	−0·54	+5·07	−0·07	−0·516	+0·105	438

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TABLE XVIII.—*continued.*

SATELLITES II. AND IV.—DISTANCES, 1901.

Rotation No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of													O—C.	Residuals.	Rotation No.
					μ	Δh_2	Δk_2	Δx_2	Δh_4	Δk_4	$\Delta \epsilon_2$	$\Delta \epsilon_4$	Δp_2	Δq_2	Δp_4	Δq_4	ΔJ			
	1901																			
5	June 24	819·094	819·347	½	8·19	−2·15	−0·76	−2·25	+4·05	+3·63	+1·42	−0·85	+0·01	+0·01	+0·02	+0·03	+1·01	+0·253	+0·056	5
21	29	406·712	405·761	½	4·06	+2·88	+1·02	+2·98	−9 20	−4·44	+2·74	+9·98	+0·22	+0·03	+0·60	−0·15	+0·68	−0·951	+0·057	21
35	July 2	510·884	510·660	¼	5·11	+2·15	+2·73	+3·32	−4·69	−2·84	+3·35	+1·09	+0·04	−0·08	+0·18	+0·18	−0·10	−0·224	+0·053	35
53	8	477·584	477·700	1	4·78	−3·07	−1·23	−3·17	+8·53	+3·66	+3·12	+8·76	+0·18	−0·01	+0·50	−0·03	+0·62	+0·116	+0·303	53
69	10	413·694	414·198	¼	4·14	−2·44	−0·87	−2·43	+4·99	+2·74	−2·01	+2·14	−0·05	−0·02	+0·11	+0·09	−0·40	+0·504	−0·159	69
81	11	565·977	566·339	¼	5·66	−2·81	−2·58	−3·81	+3·61	+4·50	+3·80	−2·39	+0·03	−0·03	+0·05	+0·10	+0·15	+0·362	−0·028	81
97	12	742·147	742·724	1	7·42	−1·78	−0·96	−1·97	+3·75	+6·22	+0·75	−5·62	−0·03	−0·02	−0·01	−0·11	+1·01	+0·577	−0·005	97
109	14	46·040	45·131	1	0·46	+1·25	+1·37	+1·85	−6·68	−7·40	−0·53	+9·86	+0·67	+1·09	+2·04	−2·96	+0·67	−0·909	+0·049	109
125	16	102·578	101·978	1	1·02	+1·30	+2·22	+2·49	−8·64	−4·34	+2·08	+9·36	−0·08	−0·89	+2·48	−0·55	−0·50	−0·600	−0·060	125
137	18	704·262	703·940	¼	7·04	+1·92	+2·70	+3·28	−5·79	−2·40	−3·13	+3·75	0·00	+0·02	+0·05	+0·03	+0·72	−0·322	−0·051	137
163	27	682·236	681·955	1	6·82	−2·19	−2·80	−3·56	+4·24	+3·22	−3·49	+0·77	0·00	−0·01	−0·02	−0·02	+0·56	−0·281	+0·323	163
193	Aug. 10	415·601	415·828	1	4·16	−1·47	−2·58	−2·96	+8·13	+4·47	−2·70	+8·96	−0·01	+0·10	+0·28	−0·07	+0·63	+0·227	−0·130	193
209	13	782·525	782·821	1	7·82	−1·19	−1·46	−1·82	+3·37	+3·99	−0·57	−1·31	0·00	−0·01	−0·01	−0·02	+1·00	+0·296	−0·022	209
227	15	192·605	193·272	1	1·93	−1·16	−2·00	−2·40	+4·23	+7·30	+1·67	−7·78	+0·04	+0·14	+0·06	−0·40	−0·49	+0·667	−0·009	227
241	23	337·966	337·929	¼	3·38	+2·83	+1·57	+2·34	−3·67	−6·48	−3·14	−6·43	+0·14	−0·04	−0·01	−0·40	−0·13	−0·037	−0·040	241
255	24	105·502	105·399	1	1·05	+1·26	+2·36	+2·62	−5·26	−7·29	+2·33	−8·70	0·00	+0·29	+0·29	−0·77	−0·42	−0·103	+0·062	255
271	26	79·343	79·965	½	0·80	−1·83	−2·50	−2·89	+7·44	+5·08	+3·02	+8·89	+0·38	−0·95	+2·58	−1·21	−0·22	+0·622	−0·037	271
283	27	537·074	536·984	½	5·37	−1·69	−0·94	−1·32	+7·38	+3·59	+0·95	+7·62	+0·08	+0·05	+0·28	−0·02	+0·85	−0·090	+0·280	283
309	29	352·486	352·637	½	3·52	−1·18	−1·31	−1·54	+4·08	+2·91	+0·39	+1·34	−0·04	−0·06	+0·16	+0·14	−0·46	+0·151	+0·057	309
329	Sept. 1	176·837	177·607	1	1·77	−2·59	−1·18	−1·63	+4·50	+7·21	−2·61	−8·07	+0·31	−0·01	+0·22	−0·84	−0·33	+0·770	+0·012	329
345	3	88 121	89·069	1	0·88	−1·98	−0·87	−1·16	+7·48	+5·79	+1·56	−9·43	−0·58	−0·21	+1·46	−0·96	−0·57	+0·948	+0·147	345
363	4	231·064	230·702	1	2·31	+2·12	+2·56	+2·84	−7·64	−4·06	+3·30	+8·31	+0·18	−0·27	+0·90	−0·19	−0·05	−0·362	−0·207	363
373	9	521·824	521·786	½	5·22	+1·20	+2·12	+2·21	−3·86	−6·50	−2·03	−6·84	−0·01	−0·09	+0·02	−0·26	+0·72	−0·038	−0·019	373
379	10	62·258	62·147	½	0·62	+2·30	+0·98	+1·14	−5·25	−6·67	−2·21	−8·31	+0·88	+0·08	+1·09	−2·23	−0·39	−0·111	+0·041	379
397	14	671·387	671·430	½	6·71	−1·12	−1·25	−1·27	+5·14	+2·40	−0·32	+3·77	+0·02	+0·03	+0·09	+0·04	+0·90	+0·043	+0·110	397
407	16	329·695	330·026	1	3·30	−1·04	−1·73	−1·72	+2·80	+4·48	+1·36	−2·96	0·00	−0·01	+0·01	+0·03	−0·39	+0·331	+0·008	407
429	17	518·394	519·055	1	5·18	−2·50	−1·18	−1·12	+3·29	+6·12	+2·57	−6·00	−0·03	0·00	0·00	−0·08	+0·60	+0·661	−0·017	429
441	19	91·736	90·980	1	0·92	+2·02	+0·82	+0·72	−6·28	−6·49	+1·70	+9·02	+0·58	+0·17	+1·11	−1·29	+0·54	−0·756	−0·026	441
467	27	257·457	257·187	½	2·57	+1·52	+2·26	+2·01	−5·61	−6·43	−2·55	−8·44	+0·05	−0·22	+0·33	−0·54	+0·46	−0·270	+0·119	467

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TABLE XVIII.—*continued.*

SATELLITES II. AND IV.—POSITION-ANGLES, 1901.

Rotation No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of												O—C.	Residuals.	Rotation No.
					Δh_2	Δk_2	Δx_2	Δh_4	Δk_4	$\Delta \epsilon_2$	$\Delta \epsilon_4$	Δp_2	Δq_2	Δp_4	Δq_4	ΔJ			
	1901																		
6	June 24	85° 12′ 24″	85° 10′ 58″	½	+0·10	−0·06	+0·06	−0·19	+0·29	−0·12	−0·34	+3·25	+2·00	+ 5·72	+ 9·14	0·00	−0″·281	+0″·162	6
22	29	268 33 23	268 42 38	½	−0·26	−0·02	−0·23	+0·64	+0·10	−0·26	−0·65	+3·83	+0·50	+10·58	− 2·56	−0·02	+1·095	−0·090	22
36	July 2	266 49 2	266 59 55	¼	−0·09	−0·02	−0·08	+0·30	−0·21	−0·05	−0·36	+1·52	−3·51	+ 7·43	+ 7·86	+0·03	+1·619	−0·365	36
54	8	88 29 22	88 36 50	1	+0·24	+0·02	+0·20	−0·55	−0·02	−0·24	−0·55	+3·82	−0·20	+10·65	− 0·65	0·00	+1·038	−0·085	54
70	10	86 40 19	86 38 45	¼	+0·13	−0·04	+0·07	−0·27	+0·22	+0·14	−0·35	−3·63	−1·33	+ 8·24	+ 6·88	0·00	−0·191	+0·079	70
82	11	86 35 31	86 38 38	¼	+0·08	−0·01	+0·06	−0·20	+0·26	−0·05	−0·32	+2·57	−2·87	+ 4·71	+ 9·69	+0·02	+0·510	+0·073	82
98	12	85 27 45	85 26 35	1	+0·07	−0·08	0·00	−0·07	+0·38	−0·11	−0·38	+2·92	+2·43	+ 1·40	+10·71	+0·01	−0·255	−0·154	98
110	14	285 31 27	286 10 7	1	−0·53	−0·40	−0·66	+2·39	+2·37	+0·07	−3·30	+1·89	+3·11	+ 5·79	− 8·43	−0·24	+0·512	+0·036	110
126	16	279 51 6	280 34 47	1	−0·37	−0·46	−0·58	+2·22	+0·90	−0·42	−2·37	−0·35	−3·67	+10·27	− 2·27	+0·15	+1·298	+0·080	126
138	18	266 40 28	266 41 15	¼	−0·05	+0·04	−0·01	+0·24	−0·21	−0·02	−0·32	−1·00	+3·70	+ 9·30	+ 5·38	−0·02	+0·160	−0·335	138
164	27	86 39 42	86 35 25	1	+0·04	−0·05	−0·01	−0·19	+0·27	−0·02	−0·32	−1·74	+3·32	+ 7·41	+ 7·49	−0·02	−0·848	+0·141	164
194	Aug. 10	88 58 6	88 57 29	1	+0·09	+0·02	+0·05	−0·36	0·00	+0·02	−0·35	−0·39	+3·62	+ 9·88	− 2·36	−0·03	−0·075	−0·083	194
210	13	87 18 9	87 16 50	1	+0·07	−0·08	−0·04	−0·17	+0·28	−0·10	−0·32	+1·95	+3·03	+ 5·75	+ 8·40	−0·01	−0·295	−0·063	210
228	15	85 10 33	85 18 16	1	+0·01	−0·15	−0·14	+0·10	+0·55	+0·15	−0·56	−0·91	−3·51	− 1·40	+10·09	−0·01	+0·415	−0·306	228
242	23	85 17 54	85 12 57	¼	+0·04	+0·08	+0·09	−0·07	−0·52	−0·06	−0·52	−3·38	+0·93	+ 0·24	+ 9·86	−0·03	−0·487	−0·197	242
256	24	262 48 22	262 56 35	1	+0·06	+0·26	+0·26	−0·39	−0·83	+0·26	−0·92	−0·02	−3·50	− 3·48	+ 9·14	−0·02	+0·251	−0·113	256
272	26	104 40 14	105 29 53	½	+0·60	+0·73	+0·86	−2·38	−1·43	−0·90	−2·76	+1·25	−3·11	+ 8·43	− 3·96	+0·10	+1·172	−0·014	272
284	27	89 16 13	89 20 22	½	+0·14	−0·03	+0·01	−0·36	+0·02	−0·13	−0·36	+2·95	+1·82	+ 9·69	− 0·66	−0·02	+0·647	−0·135	284
310	29	88 52 18	88 56 12	½	+0·10	−0·04	−0·02	−0·28	+0·16	+0·09	−0·32	−2·05	−2·83	+ 7·46	+ 6·23	+0·02	+0·399	−0·012	310
330	Sept. 1	82 29 35	82 12 45	1	−0·14	−0·14	−0·16	+0·35	+0·88	−0·15	−0·94	−3·46	+0·07	− 2·40	+ 9·39	−0·05	−0·866	+0·095	330
346	3	77 13 35	77 2 58	1	−0·27	−0·20	−0·24	+1·29	+1·19	+0·19	−1·74	+3·13	+1·15	− 7·97	+ 5·24	+0·10	−0·273	+0·046	346
364	4	273 4 47	273 26 25	1	−0·25	−0·22	−0·25	+0·86	+0·27	−0·32	−0·90	+1·86	−2·85	+ 9·41	− 1·98	+0·03	+1·444	−0·155	364
374	9	265 58 45	265 53 35	½	0·00	+0·12	+0·12	−0·04	−0·42	−0·12	−0·42	+0·30	+3·36	− 0·78	+ 9·34	0·00	−0·775	−0·107	374
380	10	252 8 4	251 39 36	½	+0·54	+0·30	+0·34	−1·40	−2·04	−0·52	−2·45	−3·20	−0·27	− 3·95	+ 8·08	−0·12	−0·512	−0·231	380
398	14	88 4 57	88 4 11	½	+0·08	−0·05	−0·05	−0·24	+0·14	−0·09	−0·27	+1·95	+2·66	+ 8·44	+ 3·76	−0·01	−0·146	+0·091	398
408	16	87 38 38	87 40 4	1	+0·05	−0·06	−0·06	−0·14	+0·24	+0·07	−0·27	−0·99	−3·15	+ 3·63	+ 8·50	+0·01	+0·138	+0·173	408
430	17	86 52 22	86 52 24	1	+0·06	−0·03	−0·04	−0·05	+0·30	−0·06	−0·30	+3·25	−0·22	+ 0·22	+ 9·25	+0·01	0·000	+0·090	430
442	19	277 57 41	278 27 14	1	−0·47	−0·12	−0·09	+1·22	+1·06	−0·41	−1·60	+3·10	+0·90	+ 5·94	− 6·87	−0·09	+0·796	+0·078	442
468	27	262 58 25	262 44 9	½	+0·07	+0·20	+0·19	−0·37	−0·63	−0·22	−0·73	−0·70	+3·09	− 4·66	+ 7·50	+0·01	−1·075	−0·027	468

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TABLE XVIII.—*continued.*

SATELLITES III. AND IV.—DISTANCES, 1901.

Rotation No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of													O—C.	Residuals.	Rotation No.
					μ	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	Δe_3	Δe_4	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ			
	1901																			
3	June 24	269".784	270".385	½	2·70	−2·78	−1·51	+3·16	+4·09	+3·56	−0·91	−0·68	−0·01	−0·01	+0·02	+0·02	−0·20	+0".601	−0".074	3
13	25	288·813	289·578	½	2·89	−2·45	−3·67	+3·92	+3·75	+5·33	+3·66	−4·22	0·00	−0·06	+0·03	+0·11	−0·14	+0·765	−0·084	13
19	29	125·142	123·656	½	1·25	+3·82	+4·17	−5·46	−8·90	−4·35	+5·56	+9·68	+0·79	−1·29	2·63	−0·64	−0·06	−1·486	+0·103	19
55	July 8	40·636	41·315	1	0·41	−3·43	−1·30	+3·48	+8·08	+3·51	−2·56	+8·29	−1·70	−0·79	+3·27	−0·19	−0·26	+0·679	+0·010	55
71	10	505·849	506·501	¼	5·06	−4·08	−4·34	+5·92	+4·96	+2·76	+5·90	+2·10	+0·13	−0·19	+0·31	+0·26	+0·14	+0·652	−0·155	71
87	11	831·482	831·906	¼	8·31	−4·71	−1·84	+4·76	+3·60	+4·55	+4·64	−2·49	+0·06	0·00	+0·04	+0·09	+0·60	+0·424	−0·045	87
103	12	875·174	875·879	1	8·75	−2·62	−1·68	+3·07	+3·76	+6·27	+0·65	−5·70	−0·03	−0·03	−0·01	−0·08	+0·73	+0·705	+0·007	103
129	16	620·616	619·587	1	6·20	+2·02	+2·65	−3·31	−8·91	−4·40	−1·57	+9·63	+0·06	+0·14	+0·27	−0·06	+0·60	−1·029	−0·142	129
147	22	490·513	490·125	¼	4·90	+4·42	+1·73	−4·20	−6·25	−7·66	+4·24	−9·65	−0·10	−0·01	+0·07	−0·15	+0·46	−0·388	−0·057	147
169	27	905·524	906·146	1	9·05	−1·99	−3·33	+3·86	+4·21	+3·24	−2·85	+0·69	0·00	−0·01	−0·01	−0·01	+0·70	+0·622	+0·038	169
179	29	257·115	258·322	1	2·57	−4·59	−2·02	+4·32	+3·82	+6·81	−4·66	−6·66	+0·43	−0·02	−0·02	−0·76	−0·08	+1·207	−0·238	179
195	Aug. 10	603·026	603·623	1	6·03	−1·87	−2·51	+3·08	+8·13	+4·46	−1·31	+8·95	+0·07	+0·12	+0·24	−0·06	+0·59	+0·597	−0·026	195
211	13	249·785	250·387	1	2·50	−2·42	−1·70	+2·55	+3·36	+4·00	−0·71	−1·34	+0·01	0·00	−0·01	−0·01	−0·20	+0·602	−0·094	211
221	14	252·558	253·046	1	2·53	−1·90	−3·48	+3·95	+3·08	+5·47	+3·13	−4·25	−0·01	−0·04	+0·02	+0·07	−0·15	+0·488	+0·128	221
243	23	203·153	203·053	¼	2·03	+4·13	+1·83	−2·96	−3·68	−6·46	−4·08	−6·43	+0·47	+0·01	−0·02	−0·82	−0·12	−0·100	−0·004	243
257	24	69·014	69·386	1	0·69	−1·88	−2·07	+2·51	+5·18	+7·25	−0·51	+8·64	+0·64	+0·84	+0·66	−1·71	+0·33	+0·372	+0·082	257
285	27	58·065	58·470	½	0·58	−3·04	−1·42	+2·12	+7·21	+3·54	−2·24	+7·45	−0·96	−0·41	+1·83	−0·12	−0·24	+0·405	+0·131	285
295	28	195·779	196·280	1	1·96	−1·69	−3·02	+3·33	+5·85	+2·61	+2·39	+4·81	−0·13	−0·44	+0·76	+0·24	−0·18	+0·501	−0·070	295
303	29	469·845	470·445	½	4·70	−3·50	−4·14	+4·81	+4·10	+2·89	+5·37	+1·40	+0·10	−0·16	+0·26	+0·21	+0·14	+0·600	−0·181	303
331	Sept. 1	520·625	521·848	1	5·21	−2·21	−3·83	+4·17	+4·53	+7·25	−4·01	−8·13	+0·02	−0·21	+0·10	−0·37	+0·44	+1·223	+0·013	331
347	3	350·745	349·565	1	3·51	+3·42	+1·46	−1·97	−7·57	−5·89	+2·93	+9·56	+0·34	+0·09	+0·53	−0·34	+0·41	−1·180	−0·016	347
361	4	576·341	575·236	1	5·76	+1·66	+2·58	−2·79	−7·68	−4·08	−1·62	+8·36	+0·05	+0·11	+0·21	−0·04	+0·55	−1·105	+0·068	361
399	14	767·811	768·161	½	7·68	−2·81	−1·33	+1·37	+5·12	+2·40	+1·88	+3·74	+0·08	+0·04	+0·14	+0·06	+0·62	+0·350	−0·043	399
409	16	561·267	562·156	1	5·61	−3·50	−3·79	+3·75	+2·80	+4·50	−5·13	−2·99	+0·07	−0·09	−0·08	−0·18	+0·31	+0·889	−0·038	409
431	17	213·255	214·228	1	2·13	−3·96	−1·81	+1·71	+3·29	+6·10	−4·03	−5·98	+0·38	−0·02	−0·01	−0·68	−0·08	+0·973	−0·075	431
445	19	127·864	126·967	1	1·28	+2·20	+3·70	−3·59	−6·41	−6·56	−3·96	+9·16	−0·04	+0·31	+0·36	−0·42	+0·24	−0·897	−0·131	445

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TABLE XVIII.—*continued.*

SATELLITES III. AND IV.—POSITION-ANGLES, 1901.

Rotation No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of											O—C.	Residuals.	Rotation No.
					Δl_3	Δk_3	Δx_3	Δl_4	Δk_4	$\Delta \epsilon_3$	$\Delta \epsilon_4$	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ		
	1901	° ′ ″	° ′ ″															
4	June 24	85 9 49	85 11 2	½	+0·15	−0·15	−0·06	−0·19	+0·29	+0·21	−0·34	−4·33	−4·32	+ 5·86	+ 9·05	0·00	+0·081	+0·067	4
14		85 39 33	85 39 44	½	+0·09	−0·20	+0·02	−0·18	+0·35	+0·20	−0·38	0·00	−6·13	+ 2·56	+10·52	+0·01	+0·015	−0·022	14
20	29	279 39 42	280 11 17	½	−1·03	−0·95	+1·37	+2·40	+0·96	−1·35	−2·56	+3·11	−5·06	+10·26	− 2·52	+0·04	+1·138	−0·076	20
40	July 2	265 22 26	265 24 46	¼	−0·07	+0·18	−0·04	+0·18	−0·28	−0·18	−0·33	+0·53	+6·12	+ 7·39	+ 7·90	−0·01	+0·618	−0·393	40
56	8	103 39 8	104 17 10	1	+1·26	+0·28	−1·16	−2·76	−0·97	+1·02	−2·82	−5·28	−2·45	+10·16	− 0·58	+0·09	+0·457	+0·079	56
72	10	88 5 25	88 5 41	¼	+0·21	+0·06	−0·20	−0·40	−0·15	−0·17	−0·40	+3·54	−5·01	+ 8·21	+ 6·90	+0·02	+0·045	+0·105	72
88	11	86 32 50	86 32 22	¼	+0·18	−0·07	−0·10	−0·20	+0·26	−0·19	−0·32	+6·14	+0·24	+ 4·62	+ 9·73	+0·01	−0·117	−0·048	88
104	12	85 38 7	85 37 35	1	+0·12	+0·02	+0·02	−0·08	+0·36	−0·21	−0·36	+4·33	+4·34	+ 1·31	+10·72	0·00	−0·140	−0·364	104
130	16	267 42 58	267 46 58	1	−0·16	+0·10	+0·04	+0·36	−0·03	−0·16	−0·34	+2·52	+5·54	+10·58	− 2·28	−0·01	+0·720	−0·361	130
148	22	265 40 2	265 32 34	¼	−0·08	+0·11	−0·04	−0·06	−0·35	−0·08	−0·34	+6·00	+0·57	− 4·66	+ 9·46	+0·01	−1·063	+0·197	148
170	27	86 44 18	86 42 39	1	+0·08	−0·17	+0·09	−0·20	+0·26	−0·18	−0·32	+1·27	+5·87	+ 7·35	+ 7·55	0·00	−0·431	+0·291	170
180	29	82 46 37	82 42 56	1	−0·20	−0·22	+0·29	+0·18	+0·78	−0·20	−0·78	−5·95	+0·21	+ 0·27	+10·54	−0·02	−0·276	+0·074	180
196	Aug. 10	88 44 21	88 46 43	1	+0·16	−0·10	+0·02	−0·33	+0·02	−0·16	−0·32	+2·81	+5·12	+ 9·89	− 2·34	−0·01	+0·413	−0·099	196
212	13	87 20 32	87 20 23	1	+0·14	−0·14	+0·07	−0·18	+0·27	+0·20	−0·32	−4·35	−3·80	+ 5·73	+ 8·41	0·00	−0·013	+0·185	212
222	14	87 50 21	87 52 6	1	+0·09	−0·13	+0·08	−0·16	+0·24	+0·14	−0·28	−0·91	−5·70	+ 3·02	+ 9·73	+0·01	+0·129	−0·111	222
244	23	262 50 15	262 49 41	¼	+0·21	+0·22	−0·28	−0·22	−0·80	−0·20	−0·80	−5·64	−0·16	+ 0·21	+ 9·83	−0·02	−0·030	−0·119	244
258	24	98 20 31	98 24 1	1	+0·48	+0·24	−0·37	−1·03	−1·15	−0·10	−1·45	+3·35	+4·40	+ 3·45	− 9·00	−0·06	+0·075	+0·136	258
286	27	98 30 1	98 57 18	½	+0·73	+0·17	−0·34	−1·54	−0·55	+0·61	−1·58	−5·02	−2·15	+ 9·53	− 0·62	+0·05	+0·463	+0·037	286
296	28	92 18 53	92 28 22	1	+0·23	+0·10	−0·15	−0·64	−0·06	−0·03	−0·59	−1·60	−5·30	+ 9·21	+ 2·97	+0·03	+0·539	−0·038	296
304	29	89 35 18	89 35 38	½	+0·17	+0·05	−0·09	−0·33	+0·12	−0·14	−0·33	+3·02	−4·66	+ 7·51	+ 6·16	+0·01	+0·044	+0·138	304
332	Sept. 1	85 22 49	85 19 19	1	−0·03	−0·28	+0·28	+0·12	+0·51	−0·28	−0·53	−0·46	+5·47	− 2·44	+ 9·41	0·00	−0·529	−0·038	332
348	3	271 20 18	271 22 19	1	−0·36	−0·01	+0·06	+0·57	+0·25	−0·35	−0·62	+5·25	+1·40	+ 8·10	− 5·29	−0·01	+0·207	+0·305	348
362	4	268 53 14	268 55 39	1	−0·14	+0·09	−0·07	+0·30	−0·03	−0·14	−0·29	+2·30	+4·94	+ 9·45	− 2·02	−0·01	+0·403	−0·082	362
400	14	88 24 21	88 24 25	½	+0·18	−0·08	+0·08	−0·27	+0·13	−0·20	−0·30	+4·71	+2·42	+ 8·42	+ 3·78	0·00	+0·017	+0·060	400
410	16	86 10 38	86 8 13	1	−0·03	−0·16	+0·16	−0·07	+0·34	−0·14	−0·34	−3·13	+4·20	+ 3·60	+ 8·51	−0·01	−0·400	+0·186	410
432	17	83 11 23	83 7 29	1	−0·18	−0·20	+0·19	+0·16	+0·69	−0·18	−0·68	−5·20	+0·21	+ 0·19	+ 9·22	−0·02	−0·234	+0·057	432
446	19	270 46 34	270 58 54	1	−0·18	−0·10	+0·09	+0·43	+0·24	+0·13	−0·46	−0·71	+5·15	+ 6·07	− 6·94	−0·02	+0·456	−0·050	446

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TABLE XVIII.—*continued.*

SATELLITES I. AND II.—DISTANCES, 1902.

Rotation No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of													O—C.	Residuals.	Rotation No.
					μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	$\Delta \epsilon_1$	$\Delta \epsilon_2$	Δp_1	Δq_1	Δp_2	Δq_2	ΔJ			
	1902																			
3	June 10	20".556	20".586	¼	0·21	−1·85	−0·49	+0·12	+3·24	+0·68	+0·36	+1·78	−3·20	+0·10	−0·07	−0·19	+0·07	−0·36	+0".030	+0".142	3
23	16	265·942	265·896	1	2·66	−1·24	−0·26	+0·23	+2·85	−0·93	+1·96	−0·60	+2·75	0·00	−0·01	−0·02	−0·01	+1·66	−0·046	+0·078	23
51	24	38·860	39·011	½	0·39	−1·30	−0·31	+0·35	+2·67	+0·90	+0·48	+0·72	−2·41	+0·01	−0·04	−0·04	+0·05	−0·58	+0·151	+0·016	51
63	30	190·732	191·039	1	1·91	−1·37	+0·44	+1·11	+3·62	−0·53	+2·41	+0·93	+3·61	−0·09	−0·02	−0·04	−0·01	+1·38	+0·307	−0·101	63
83	July 3	246·298	245·906	1	2·46	+1·76	+0·58	−0·54	−2·97	−0·90	−0·98	−1·62	−2·83	0·00	0·00	−0·01	+0·01	+1·45	−0·392	+0·149	83
95	7	151·332	151·314	1	1·51	−1·60	+0·56	+1·42	+3·79	−0·27	+2·55	+1·36	+3·78	−0·02	−0·03	−0·05	0·00	+1·18	−0·018	+0·286	95
103	8	49·413	49·749	1	0·49	−1·50	−0·50	+0·55	+2·76	+0·98	+0·97	+1·17	−2·52	+0·02	−0·03	−0·04	+0·04	−0·50	+0·336	−0·026	103
127	15	117·740	117·768	1	1·18	−1·75	+0·60	+1·64	+1·97	+0·19	+1·23	−1·59	−0·29	0·00	0·00	0·00	−0·01	−0·18	+0·028	−0·154	127
143	16	81·566	81·817	½	0·82	−1·33	+0·30	+1·15	+3·83	−0·47	+3·03	+0·68	−3·82	+0·02	+0·06	−0·09	−0·02	+1·04	+0·251	+0·125	143
159	18	263·303	263·009	½	2·63	−1·77	−0·65	+0·83	+3·01	−0·88	+2·78	−1·64	+2·82	+0·01	−0·01	−0·02	−0·02	+1·53	−0·294	+0·203	159
163	19	65·693	66·229	¼	0·66	−2·00	−0·63	+1·03	+3·07	+1·05	+1·52	+1·96	−2·97	+0·03	−0·02	−0·04	+0·04	−0·24	+0·536	−0·025	163
181	24	357·194	357·205	1	3·57	+1·32	−0·26	−1·18	−2·07	−0·48	−1·30	+0·60	−0·90	0·00	0·00	0·00	0·00	+2·05	+0·011	−0·003	181
193	26	64·851	65·124	1	0·65	−1·58	−0·61	+0·87	+2·66	+1·06	+1·44	+1·34	−2·37	+0·01	−0·02	−0·02	+0·03	−0·43	+0·273	+0·119	193
209	27	81·600	81·366	1	0·82	+2·07	+0·66	−1·25	−2·96	+0·94	−2·92	+2·05	+2·75	−0·04	+0·03	−0·06	−0·06	−0·16	−0·234	+0·153	209
223	Aug. 3	18·819	18·296	1	0·19	+1·68	+0·69	−1·06	−3·32	+0·83	−3·26	+1·53	+3·23	−0·14	+0·20	−0·33	−0·21	−0·46	−0·523	+0·318	223
237	4	9·269	8·752	1	0·09	+2·31	−0·29	−2·13	−3·70	−0·76	−2·78	−2·28	−3·73	−0·39	−0·14	−0·62	+0·22	−0·26	−0·517	−0·031	237
245	7	306·567	306·610	½	3·07	+2·10	−0·48	−2·07	−2·08	+0·28	−1·95	+2·02	+0·77	0·00	0·00	0·00	−0·01	+1·57	+0·043	+0·049	245
265	16	95·559	95·526	½	0·96	−1·43	+0·31	+1·44	+1·98	+0·30	+1·71	−0·89	−0·26	0·00	+0·01	0·00	−0·03	−0·39	−0·033	+0·052	265
291	18	314·116	313·587	1	3·14	+1·56	+0·70	−1·21	−2·37	−1·04	−1·84	−1·36	−1·94	0·00	0·00	0·00	0·00	+1·79	−0·529	+0·197	291
309	20	107·414	107·938	1	1·07	−2·28	−0·62	+1·94	+2·94	+1·25	+2·36	+2·32	−2·89	+0·01	0·00	+0·02	+0·02	+0·11	+0·524	+0·112	309
325	21	245·885	246·101	1	2·46	+2·39	−0·15	−2·32	−2·37	+0·56	−2·43	+2·37	+1·66	−0·01	0·00	0·00	−0·02	+1·12	+0·216	−0·130	325
331	22	16·703	16·030	1	0·17	+2·29	−0·25	−2·25	−3·59	−1·01	−3·10	−2·24	−3·67	−0·15	−0·06	−0·24	+0·11	−0·28	−0·673	+0·129	331
355	24	16·521	16·880	1	0·17	−1·55	−0·72	+1·28	+3·57	−0·51	+3·57	−1·37	−3·50	−0·11	+0·21	−0·36	−0·14	+0·71	+0·359	−0·129	355
371	25	350·124	349·869	1	3·50	+1·27	+0·40	−1·11	−2·07	−0·75	−1·79	−0·57	−1·16	0·00	0·00	0·00	0·00	+2·02	−0·255	+0·024	371
377	27	96·021	96·632	½	0·96	−1·88	−0·80	+1·62	+2·51	+1·19	+2·13	+1·89	−2·27	0·00	0·00	0·00	+0·01	−0·15	+0·611	−0·046	377
399	28	122·008	121·945	1	1·22	+2·32	+0·56	−2·13	−2·97	+0·71	−3·05	+2·37	+2·71	−0·03	+0·01	−0·03	−0·03	+0·22	−0·063	+0·043	399
429	Sept. 2	193·290	193·534	½	1·93	−1·23	−0·36	+1·15	+3·78	+0·02	+3·72	−0·45	+3·75	0·00	−0·02	−0·03	0·00	+1·46	+0·244	−0·019	429
445	3	90·393	90·637	½	0·90	−1·59	−0·78	+1·44	+2·22	+1·04	+2·03	+1·48	−1·74	0·00	0·00	0·00	0·00	−0·28	+0·244	+0·157	445
475	6	145·673	145·501	1	1·46	−2·26	+0·18	+2·27	+2·37	−0·45	+2·41	−2·22	+1·68	+0·02	0·00	−0·02	−0·03	+0·23	−0·172	+0·036	475
479	7	161·597	162·195	1	1·62	−1·74	+0·10	+1·74	+3·47	+1·08	+3·33	+1·62	−3·56	+0·01	0·00	−0·02	+0·01	+1·00	+0·598	+0·052	479
495	8	326·282	326·356	1	3·26	+1·65	−0·37	−1·68	−2·02	+0·14	−2·02	+1·37	+0·76	0·00	0·00	0·00	0·00	+1·80	+0·074	−0·076	495
511	10	112·947	113·366	½	1·13	−2·02	−0·78	+1·97	+2·43	+1·21	+2·34	+2·09	−2·23	0·00	0·00	0·00	0·00	+0·04	+0·419	+0·124	511
527	12	232·209	231·760	1	2·32	+2·11	+0·78	−2·08	−2·43	−1·24	−2·37	−2·19	−2·27	0·00	0·00	0·00	0·00	+1·17	−0·449	−0·040	527
539	26	211·534	211·078	½	2·12	+2·10	+0·67	−2·17	−2·29	−1·21	−2·44	−2·17	−2·12	0·00	0·00	0·00	0·00	+1·00	−0·456	−0·028	539
557	Oct. 4	154·442	154·409	1	1·54	−2·08	−0·61	+2·17	+3·10	−0·37	+2·92	−2·15	+2·95	+0·02	0·00	−0·03	−0·02	+0·77	−0·033	−0·078	557

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TABLE XVIII.—*continued.*

SATELLITES I. AND II.—POSITION-ANGLES, 1902.

Rotation No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of												O—C.	Residuals.	Rotation No.
					Δh_1	Δk_1	Δr_1	Δh_2	Δk_2	Δr_2	$\Delta\epsilon_1$	$\Delta\epsilon_2$	Δp_1	Δq_1	Δp_2	Δq_2			
	1902	° ′ ″	° ′ ″																
4	June 10	73 23 28	72 44 46	¼	+0·10	+0·03	0·00	−0·19	−0·06	0·00	−0·10	+0·20	+1·83	−1·33	−3·40	+1·29	−0″·232	−0″·109	4
24	16	69 56 35	69 51 37	1	0·00	0·00	0·00	+0·02	−0·02	+0·03	0·00	+0·03	−0·66	+2·22	+2·68	+2·48	−0·384	−0·004	24
52	24	71 22 45	70 34 43	½	+0·02	0·00	0·00	−0·06	−0·04	0·00	−0·01	+0·07	+0·77	−2·22	−2·47	+2·86	−0·544	−0·027	52
64	30	69 54 57	69 56 0	1	−0·01	0·00	+0·01	+0·03	−0·02	+0·04	+0·01	+0·04	+0·90	+2·22	+3·59	+1·17	+0·064	−0·106	64
84	July 3	250 48 27	250 31 35	1	0·00	0·00	0·00	+0·03	+0·03	0·00	0·00	+0·04	−1·64	+1·76	−2·86	+2·50	−1·209	+0·325	84
96	7	69 54 44	69 58 59	1	−0·02	0·00	+0·02	+0·04	−0·02	+0·04	+0·02	+0·04	+1·35	+2·02	+3·78	+0·64	+0·179	−0·085	96
104	8	71 30 39	71 15 15	1	+0·02	+0·01	0·00	−0·06	−0·04	0·00	−0·02	+0·07	+1·17	−2·12	−2·53	+2·98	−0·221	−0·394	104
128	15	70 42 13	70 26 51	1	0·00	0·00	0·00	0·00	−0·03	+0·02	0·00	+0·03	−1·61	−1·86	−0·24	+3·94	−0·526	−0·104	128
144	16	72 17 43	71 56 11	½	+0·03	0·00	−0·02	−0·10	0·00	−0·07	−0·01	+0·09	+0·71	+2·37	−3·84	−0·90	−0·508	+0·098	144
160	18	70 37 45	70 32 55	½	−0·01	0·00	0·00	+0·02	−0·03	+0·03	0·00	+0·03	−1·61	+1·89	+2·87	+2·69	−0·371	−0·019	160
164	19	71 49 7	71 19 1	¼	+0·03	0·00	−0·01	−0·06	−0·04	−0·02	−0·03	+0·07	+1·93	−1·54	−2·92	+2·70	−0·576	−0·028	164
182	24	251 10 23	251 1 11	1	0·00	0·00	0·00	0·00	+0·03	−0·02	0·00	+0·03	+0·67	+2·39	−0·79	+3·85	−0·950	−0·015	182
194	26	71 44 53	71 9 33	1	+0·01	0·00	0·00	−0·04	−0·04	0·00	−0·01	+0·05	+1·26	−2·14	−2·26	+3·31	−0·667	+0·021	194
210	27	250 3 9	250 0 30	1	+0·05	−0·01	−0·03	−0·06	+0·04	−0·08	+0·04	+0·08	+2·00	−1·51	+2·85	+2·76	−0·056	−0·061	210
224	Aug. 3	245 54 29	246 32 9	1	+0·17	+0·07	−0·11	−0·33	+0·10	−0·33	+0·15	+0·33	+1·43	−2·05	+3·35	+2·13	+0·205	−0·141	224
238	4	261 9 56	257 3 47	1	−0·39	+0·05	+0·36	+0·63	+0·15	+0·47	+0·38	+0·64	−2·33	−0·81	−3·67	+1·30	−0·662	+0·192	238
246	7	251 33 52	251 28 36	½	0·00	0·00	0·00	0·00	+0·03	−0·02	0·00	+0·03	+2·11	+1·34	+1·01	+3·82	−0·469	−0·161	246
266	16	71 44 58	71 25 33	½	0·00	0·00	0·00	0·00	−0·03	+0·02	0·00	+0·03	−1·07	−2·24	+0·06	+4·00	−0·540	−0·029	266
292	18	252 13 25	252 5 32	1	0·00	0·00	0·00	+0·01	+0·03	0·00	0·00	+0·03	−1·18	+2·18	−1·64	+3·56	−0·719	−0·275	292
310	20	72 38 59	72 5 12	1	+0·01	0·00	−0·01	−0·03	−0·03	−0·02	−0·02	+0·05	+2·23	−1·08	−2·64	+2·99	−1·055	+0·359	310
326	21	252 1 45	251 56 1	1	+0·01	0·00	−0·01	−0·01	+0·02	−0·02	+0·01	+0·03	+2·43	+0·51	+1·98	+3·39	−0·411	−0·049	326
332	22	256 10 5	254 26 1	1	−0·15	+0·02	+0·15	+0·25	+0·09	+0·21	+0·15	+0·26	−2·33	−0·85	−3·52	+1·68	−0·506	−0·060	332
356	24	78 1 57	76 55 37	1	+0·15	+0·07	−0·12	−0·35	+0·04	−0·35	+0·13	+0·34	−1·16	+2·18	−3·65	−1·42	−0·318	+0·039	356
372	25	252 25 8	252 16 36	1	0·00	0·00	0·00	0·00	+0·03	0·00	0·00	+0·03	−0·33	+2·44	−0·78	+3·81	−0·868	−0·103	372
378	27	72 43 58	72 13 0	½	0·00	0·00	0·00	−0·02	−0·03	−0·01	0·00	+0·04	+1·72	−1·75	−1·92	+3·46	−0·865	+0·136	378
400	28	251 54 40	251 39 59	1	+0·03	0·00	−0·02	−0·03	+0·02	−0·04	+0·03	+0·04	+2·28	−0·92	+2·99	+2·51	−0·521	+0·426	400
430	Sept. 2	72 16 48	72 17 11	½	0·00	0·00	0·00	+0·02	−0·01	+0·02	0·00	+0·02	−0·18	+2·44	+3·84	+0·54	+0·021	+0·031	430
446	3	72 47 53	72 32 45	½	0·00	0·00	0·00	−0·01	−0·03	0·00	0·00	+0·03	+1·25	−2·08	−1·32	+3·67	−0·135	−0·346	446
476	6	72 18 18	72 4 56	1	−0·02	0·00	+0·02	+0·02	−0·02	+0·02	−0·02	+0·03	−2·32	−0·67	+2·08	+3·26	−0·567	+0·172	476
480	7	73 15 43	73 0 19	1	+0·01	0·00	−0·01	−0·04	−0·02	−0·03	−0·01	+0·04	+2·18	+1·05	−3·35	+1·93	−0·677	−0·047	480
496	8	252 45 36	252 37 25	1	0·00	0·00	0·00	0·00	+0·02	0·00	0·00	+0·02	+1·61	+1·79	+1·21	+3·61	−0·777	+0·100	496
512	10	73 7 38	72 52 27	½	0·00	0·00	0·00	−0·02	−0·03	−0·02	0·00	+0·03	+1·93	−1·42	−1·81	+3·39	−0·499	−0·116	512
528	12	253 0 3	252 44 52	1	0·00	0·00	0·00	+0·01	+0·02	0·00	0·00	+0·02	−1·99	+1·31	−1·87	+3·27	−1·026	+0·122	528
540	26	253 10 52	252 56 36	½	0·00	0·00	0·00	0·00	+0·02	0·00	0·00	+0·02	−2·04	+1·05	−1·69	+3·21	−0·878	+0·013	540
558	Oct. 4	72 42 43	72 46 10	½	−0·02	−0·01	+0·02	+0·03	−0·01	+0·02	−0·02	+0·03	−2·03	+0·96	+3·21	+1·60	+0·154	−0·180	558

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TABLE XVIII.—*continued.*

SATELLITES I. AND III.—DISTANCES, 1902.

Rotation No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of													O—C.	Residuals.	Rotation No.
					μ	Δh_1	Δk_1	Δx_1	Δh_3	Δk_3	Δx_3	$\Delta \epsilon_1$	$\Delta \epsilon_3$	Δp_1	Δq_1	Δp_3	Δq_3	ΔJ			
	1902	"	"																	
5	June 10	225·738	226·032	¼	2·26	−1·89	−0·47	+0·14	+3·31	−1·15	−2·12	+1·83	+2·31	0·00	0·00	0·00	0·00	−0·27	+0·294	−0·005	5
25	16	176·086	175·894	1	1·76	−1·26	−0·29	+0·21	+5·76	−0·58	−2·75	−0·66	+5·78	0·00	0·00	0·00	0·00	+1·23	−0·192	−0·019	25
47	21	246·500	246·271	½	2·46	+1·82	−0·61	−1·35	−3·06	+0·62	+1·90	−1·76	+1·09	0·00	0·00	0·00	−0·01	−0·33	−0·229	−0·011	47
53	24	136·382	136·318	½	1·36	−1·32	−0·34	+0·33	+4·18	−1·55	−3·35	+0·79	+3·81	0·00	0·00	−0·01	−0·01	−0·79	−0·064	+0·111	53
79	July 1	103·150	103·149	1	1·03	−1·22	−0·11	+0·58	+4·62	−1·56	−3·86	+0·27	+4·41	0·00	0·00	−0·01	−0·01	−0·91	−0·001	−0·009	79
85	3	118·019	118·196	1	1·18	−1·81	−0·58	+0·56	+4·98	+1·45	−1·68	+1·70	−4·84	0·00	0·00	+0·02	−0·02	−0·58	+0·177	+0·127	85
87	7	9·151	9·050	1	0·09	+1·83	−0·60	−1·60	−5·87	−0·86	+2·94	−1·72	−5·86	+0·04	+0·04	+0·15	−0·05	−0·74	−0·101	+0·083	87
105	8	134·048	134·350	1	1·34	−1·53	−0·52	+0·56	+4·58	−1·54	−4·08	+1·23	+4·29	0·00	0·00	−0·01	−0·02	−0·73	+0·302	−0·147	105
117	14	102·334	102·290	1	1·02	+2·27	−0·41	−1·85	−5·60	−1·28	+2·90	−2·25	−5·58	0·00	0·00	+0·02	0·00	−0·29	−0·044	−0·049	117
129	15	43·182	42·796	1	0·43	−1·70	+0·59	+1·61	+5·66	−1·16	−4·76	−1·52	+5·62	0·00	0·00	−0·01	0·00	−0·80	−0·386	+0·059	129
145	16	484·787	485·062	½	4·85	−1·31	+0·27	+1·11	+3·32	−0·68	−2·82	+0·60	+1·51	0·00	0·00	0·00	0·00	+1·73	+0·275	−0·150	145
165	19	344·809	344·205	¼	3·45	+2·04	+0·62	−1·07	−4·66	+1·49	+4·42	−2·01	+4·32	0·00	0·00	0·00	0·00	+1·12	−0·604	+0·084	165
183	24	180·378	180·430	1	1·80	−1·30	+0·22	+1·15	+3·82	+1·43	−2·04	−0·52	−2·97	0·00	0·00	+0·01	−0·02	−0·82	+0·052	−0·113	183
197	26	343·470	343·011	1	3·43	+1·67	+0·65	−0·92	−5·14	+1·38	+4·90	−1·48	+4·93	0·00	0·00	0·00	0·00	+1·36	−0·459	+0·079	197
211	27	284·339	284·562	1	2·84	+2·10	+0·65	−1·28	−3·17	−0·03	+2·51	+2·09	+0·16	0·00	0·00	−0·01	−0·02	−0·21	+0·223	+0·035	211
225	Aug. 3	248·999	249·173	1	2·49	+1·73	+0·71	−1·09	−3·28	+0·33	+2·95	+1·60	+0·99	0·00	0·00	0·00	−0·02	−0·50	+0·174	−0·013	225
241	4	140·989	140·853	1	1·41	+2·25	−0·38	−2·13	−5·21	−1·76	+3·55	−2·21	−5·19	+0·01	0·00	+0·02	−0·02	−0·32	−0·136	+0·035	241
247	7	269·338	268·936	½	2·69	−2·07	+0·49	+2·05	+3·26	+0·84	−2·44	−1·99	−1·28	0·00	0·00	0·00	−0·02	−0·29	−0·402	+0·081	247
267	16	233·228	233·187	½	2·33	+1·40	−0·28	−1·40	−6·16	+0·32	+5·82	+0·81	+6·11	0·00	0·00	+0·01	0·00	+1·37	−0·041	+0·001	267
273	17	184·656	184·407	½	1·85	+1·24	+0·09	−1·12	−3·84	+0·92	+3·90	−0·02	+2·72	0·00	0·00	0·00	0·00	−0·84	−0·249	+0·111	273
293	18	436·072	436·141	1	4·36	+1·59	+0·72	−1·23	−3·73	−1·62	+2·91	−1·42	−2·96	0·00	0·00	0·00	−0·01	+1·51	+0·069	−0·161	293
299	19	7·900	7·736	1	0·08	−1·83	−0·78	+1·45	+5·98	+1·27	−5·16	−1·80	+6·06	−0·17	+0·20	+0·63	−0·22	+0·76	−0·164	−0·233	299
327	21	314·926	314·686	1	3·15	−2·38	+0·18	+2·31	+3·13	+0·32	−2·87	−2·35	−0·04	0·00	0·00	0·00	−0·01	+0·09	−0·240	−0·097	327
333	22	311·093	311·580	1	3·11	−2·27	+0·27	+2·24	+4·64	+2·03	−3·79	+2·22	−4·59	0·00	0·00	+0·01	−0·01	+0·93	+0·487	−0·146	333
345	24	199·698	200·021	1	2·00	+1·36	+0·57	−1·15	−3·71	+0·78	+3·78	+0·93	+2·46	0·00	0·00	0·00	0·00	−0·73	+0·323	−0·284	345
369	25	477·536	477·709	1	4·78	+1·26	+0·37	−1·11	−3·15	−1·29	+2·88	−0·51	−2·03	0·00	0·00	0·00	−0·01	+1·74	+0·173	−0·168	369
379	27	81·191	81·413	½	0·81	−1·91	−0·80	+1·66	+5·64	−0·72	−5·64	+1·93	+5·50	+0·01	−0·01	+0·03	+0·91	−0·56	+0·222	−0·033	379
397	28	391·701	391·278	1	3·92	−2·30	−0·58	+2·10	+3·30	−0·28	−3·28	−2·35	+1·29	0·00	0·00	0·00	+0·00	+0·88	−0·423	+0·092	397
411	30	207·902	207·417	½	2·08	+1·28	−0·07	−1·26	−6·14	−0·08	+6·00	+0·42	+6·12	0·00	0·00	+0·02	0·00	+1·36	−0·485	+0·274	411
425	Sept. 2	42·664	42·826	½	0·43	+1·21	+0·31	−1·14	−5·16	−1·95	+4·76	+0·34	−5·28	0·00	+0·06	+0·12	−0·09	−1·01	+0·162	+0·016	425
437	3	6·932	7·074	½	0·07	+1·42	+0·68	−1·29	−5·87	+0·27	+5·83	−1·16	−5·79	+0·24	−0·10	+0·65	+0·14	+0·94	+0·142	−0·139	437
469	6	3·798	3·664	1	0·04	−2·21	+0·06	+2·20	+5·69	+1·06	−5·52	−2·18	−5·77	+0·77	+0·15	+1·94	−0·49	−0·32	−0·134	−0·048	469
487	7	147·656	147·433	1	1·48	+1·97	−0·35	−2·00	−4·89	+0·95	+4·97	−1·85	+4·54	0·00	0·00	0·00	0·00	−0·48	−0·223	−0·136	487
499	8	466·938	467·290	1	4·67	+1·60	−0·35	−1·63	−3·03	−0·44	+2·97	+1·28	−0·14	0·00	0·00	0·00	0·00	+1·56	+0·352	−0·148	499
513	10	76·353	76·319	½	0·76	−2·05	−0·77	+2·00	+5·67	−0·40	−5·68	+2·12	+5·55	+0·02	−0·02	+0·07	+0·02	−0·38	−0·034	+0·153	513
531	16	71·859	72·152	1	0·72	+1·20	+0·44	−1·21	−4·70	−2·05	+4·72	+0·56	−4·81	0·00	+0·04	+0·07	−0·07	−0·92	+0·293	−0·092	531
541	26	278·064	278·291	½	2·78	−2·12	−0·64	+2·19	+3·05	+1·32	−3·20	+2·19	−1·83	0·00	0·00	0·00	−0·01	+0·09	+0·227	+0·057	541
555	Oct. 4	209·367	209·009	1	2·09	−2·14	−0·65	+2·24	+4·71	+1·97	−5·04	−2·21	−4·89	+0·02	0·00	+0·03	−0·03	+0·61	−0·358	+0·152	555

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TABLE XVIII.—*continued.*

SATELLITES I. AND III.—POSITION-ANGLES, 1902.

Rotation No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of												O—C.	Residuals.	Rotation No.
					Δh_1	Δk_1	Δx_1	Δh_3	Δk_3	Δx_3	$\Delta \epsilon_1$	$\Delta \epsilon_3$	Δp_1	Δq_1	Δp_3	Δq_3			
	1902	° ′ ″	° ′ ″																
6	June 10	70 8 30	70 10 10	¼	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	+1·87	−1·27	+2·15	+5·32	+0″·108	−0″·090	6
26	16	70 21 18	70 16 7	1	0·00	0·00	0·00	−0·02	0·00	+0·01	0·00	−0·02	−0·72	+2·20	+5·74	+1·01	−0·266	+0·015	26
48	21	250 12 52	250 10 38	½	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	−1·72	−1·58	+0·96	+5·87	−0·160	+0·038	48
54	24	70 13 10	70 19 21	½	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	+0·84	−2·20	+3·72	+4·67	+0·245	−0·177	54
80	July 1	70 19 18	70 22 40	1	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	+0·30	−2·38	+4·36	+4·23	+0·101	−0·090	80
86	3	70 16 27	70 13 59	1	0·00	0·00	0·00	+0·03	+0·02	0·00	0·00	−0·03	+1·72	−1·68	−4·88	+3·71	−0·085	+0·144	86
88	7	249 11 1	249 4 23	1	+0·05	−0·02	−0·04	−0·14	−0·02	+0·07	−0·05	−0·14	−1·71	−1·71	−5·84	+1·84	−0·018	+0·103	88
106	8	70 30 21	70 28 5	1	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	+1·24	−2·09	+4·28	+4·43	−0·088	+0·081	106
118	14	250 41 29	250 42 43	1	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	−2·26	−0·96	−5·56	+2·81	+0·037	−0·047	118
130	15	70 47 37	70 33 58	1	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	−1·54	−1·92	+5·65	+2·63	−0·172	+0·171	130
146	16	70 51 19	70 47 37	½	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	+0·63	+2·39	+1·59	+6·05	−0·521	+0·090	146
166	19	250 57 24	250 54 17	¼	0·00	0·00	0·00	−0·01	+0·01	+0·02	0·00	+0·02	−1·99	+1·47	+4·40	+4·51	−0·314	−0·004	166
184	24	70 59 47	71 1 35	1	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	−0·59	−2·41	−2·81	+5·66	+0·094	−0·089	184
198	26	251 15 14	251 11 44	1	0·00	0·00	0·00	+0·01	0·00	0·00	0·00	0·00	−1·41	+2·04	+5·06	+3·83	−0·350	+0·011	198
212	27	251 10 28	251 11 36	1	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	+2·04	−1·45	+3·87	+6·33	+0·094	−0·203	212
226	Aug. 3	251 27 14	251 26 23	1	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	+1·50	−2·02	+1·31	+6·22	−0·050	+0·003	226
242	4	251 23 5	251 18 40	1	+0·01	0·00	−0·01	−0·02	0·00	0·00	−0·01	−0·02	−2·27	−1·06	−4·98	+3·92	−0·180	+0·136	242
248	7	71 36 21	71 34 52	½	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	−2·07	−1·40	−0·89	+6·28	−0·116	−0·006	248
268	16	252 12 59	252 8 22	½	0·00	0·00	0·00	+0·02	0·00	−0·02	0·00	−0·02	+0·99	+2·28	+6·23	+1·22	−0·312	+0·044	268
274	17	252 3 29	252 4 24	½	0·00	0·00	0·00	0·00	−0·01	0·00	0·00	−0·01	−0·23	−2·49	+3·18	+5·48	+0·050	−0·051	274
294	18	252 4 55	252 1 23	1	0·00	0·00	0·00	0·00	−0·01	0·00	0·00	−0·01	−1·24	+2·15	−2·47	+5·81	−0·449	+0·090	294
300	19	78 17 50	76 22 42	1	+0·19	+0·08	−0·15	−0·64	−0·13	+0·56	+0·19	−0·65	−1·65	+1·86	+5·90	−2·09	−0·265	+0·164	300
328	21	72 11 10	72 7 33	1	0·00	0·00	0·00	0·00	+0·01	0·00	0·00	−0·01	−2·41	−0·58	+0·53	+6·27	−0·331	+0·129	328
334	22	72 11 18	72 6 4	1	0·00	0·00	0·00	0·00	+0·01	0·00	0·00	−0·01	+2·31	+0·90	−4·17	+4·72	−0·473	+0·302	334
346	24	252 21 32	252 18 37	1	0·00	0·00	0·00	0·00	−0·01	0·00	0·00	−0·01	+0·70	−2·38	+3·01	+5·52	−0·170	+0·150	346
370	25	252 21 31	252 18 50	1	0·00	0·00	0·00	0·00	−0·01	0·00	0·00	−0·01	−0·26	+2·45	−1·44	+6·10	−0·372	−0·015	370
380	27	72 53 3	72 44 19	½	+0·01	0·00	−0·01	−0·05	+0·01	+0·05	−0·01	−0·05	+1·77	−1·71	+5·77	+2·35	−0·206	+0·165	380
398	28	72 29 10	72 23 54	1	0·00	0·00	0·00	0·00	+0·01	0·00	0·00	−0·01	−2·26	+0·98	+1·92	+5·93	−0·600	+0·282	398
412	30	252 50 55	252 42 36	½	0·00	0·00	0·00	+0·04	0·00	−0·04	0·00	−0·04	+0·68	+2·34	+6·21	+0·55	−0·502	+0·189	412
426	Sept. 2	251 17 44	251 24 43	½	+0·03	+0·01	−0·03	−0·12	−0·05	+0·11	+0·01	−0·13	+0·06	−2·44	−4·88	+3·78	+0·086	+0·009	426
438	3	246 31 11	244 38 0	½	+0·16	+0·08	−0·14	−0·63	+0·02	+0·62	−0·13	−0·62	−2·23	+0·91	−5·97	−1·31	−0·227	+0·143	438
470	6	51 52 18	47 19 45	1	−0·76	+0·03	+0·76	+1·94	+0·38	−1·88	−0·75	−1·98	−2·25	−0·43	−5·64	+1·44	−0·299	+0·294	470
488	7	252 57 10	252 55 48	1	0·00	0·00	0·00	+0·02	−0·01	−0·02	0·00	−0·02	−2·02	−1·33	+5·00	+3·56	−0·059	−0·018	488
500	8	252 50 41	252 48 28	1	0·00	0·00	0·00	0·00	−0·02	0·00	0·00	−0·02	+1·52	+1·86	+0·61	+6·07	−0·301	−0·032	500
514	10	73 38 6	73 40 31	½	+0·02	0·00	−0·02	−0·08	+0·02	+0·08	−0·02	−0·08	+1·96	−1·37	+5·81	+1·72	+0·053	−0·019	514
532	16	252 9 12	252 15 16	1	+0·02	0·00	−0·02	−0·07	−0·04	+0·07	0·00	−0·08	+0·25	−2·36	−4·30	+4·16	+0·126	−0·052	532
542	26	73 6 17	73 2 58	½	0·00	0·00	0·00	0·00	+0·02	0·00	0·00	−0·02	+2·08	−0·98	−1·04	+5·74	−0·268	+0·127	542
556	Oct. 4	72 49 26	72 47 37	1	−0·02	0·00	+0·02	+0·03	+0·03	−0·04	−0·02	−0·04	−2·01	+1·00	−4·43	+3·61	−0·111	−0·081	556

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TABLE XVIII.—*continued.*

SATELLITES I. AND IV.—DISTANCES, 1902.

Rotation No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of												O—C.	Residuals.	Rotation No.
					μ	Δh_1	Δk_1	Δx_1	Δh_4	Δk_4	$\Delta \epsilon_1$	$\Delta \epsilon_4$	Δp_1	Δq_1	Δp_4	Δq_4	ΔJ			
	1902																		
7	June 10	464".973	465".439	¼	4·65	−1·93	−0·46	+0·17	+ 5·76	−1·86	+1·88	+ 3·83	+0·01	0·00	+0·03	+0·07	−0·40	+0".466	+0".093	7
55	24	27·139	26·756	½	0·27	+1·34	+0·36	−0·32	−10·26	+0·96	−0·86	−10·26	+0·06	−0·16	+0·73	+0·14	+1·19	−0·383	−0·146	55
65	30	536·974	538·137	1	5·37	−1·35	+0·41	+1·08	+ 8·43	+2·61	+0·86	− 8·08	0·00	0·00	−0·01	+0·01	+1·36	+1·163	−0·001	65
71	30	535·725	536·850	1	5·36	−1·25	+0·26	+0·90	+ 8·51	+2·60	+0·52	− 8·19	0·00	0·00	−0·01	+0·01	+1·41	+1·125	+0·009	71
93	July 7	470·307	469·648	1	4·70	+1·64	−0·57	−1·46	− 6·02	−1·60	−1·44	− 3·57	0·00	−0·01	−0·02	+0·05	−0·76	−0·659	+0·074	93
107	8	543·757	543·094	1	5·44	+1·58	+0 54	−0·57	− 8·22	−2·63	−1·32	− 7·78	0·00	0·00	−0·02	+0·02	+1·26	−0·663	−0·030	107
119	14	683·878	684·537	1	6·84	−2·23	+0·45	+1·85	+ 5·64	−0·72	+2·21	+ 1·71	+0·01	0·00	0·00	+0·05	+0·85	+0·659	+0·031	119
131	15	502·991	503·405	1	5·03	−1·66	+0·58	+1·57	+ 5·76	+1·36	−1·46	− 2·50	0·00	−0·01	−0·01	+0·06	−0·75	+0·414	−0·026	131
147	16	662·427	663·671	½	6·62	−1·30	+0·24	+1·08	+ 7·25	+2·76	+0·54	− 6·28	0·00	0·00	−0·02	+0·03	+1·50	+1·244	−0·198	147
157	18	244·008	244·919	½	2·44	−1·74	−0 64	+0·81	+10·88	+1·31	−1·59	−10·90	0·00	0·00	+0·03	0·00	+1·06	+0·911	+0·166	157
185	24	706·139	705·703	1	7·06	+1·29	−0·19	−1·11	− 6·59	−2·32	+0·44	− 4·91	0·00	0·00	−0·01	+0·03	+1·54	−0·436	+0·078	185
199	26	323·221	322·387	1	3·23	+1·70	+0·66	−0·93	−10·35	−2·09	−1·53	−10·39	0·00	0·00	0·00	0·00	+1·12	−0·834	+0·046	199
229	Aug. 3	393·878	394·974	1	3·94	−1·79	−0·72	+1·14	+10·02	+2·79	−1·69	−10·10	0·00	0·00	0·00	0·00	+1·10	+1·096	−0·044	229
249	7	638·958	638·357	½	6·39	+2·03	−0·50	−2·02	− 7·22	+2·15	+1·94	+ 5·82	+0·01	0·00	+0·04	+0·05	+1·08	−0·601	+0·106	249
275	17	778·144	778·479	½	7·78	−1·24	−0·14	+1·10	+ 5·55	+1·20	−0·07	− 1·32	0·00	0·00	0·00	+0·04	+1·60	+0·335	+0·139	275
295	18	443·822	444·969	1	4·44	−1·63	−0·74	+1·26	+ 6·67	+3·05	+1·48	− 5·51	0·00	0·00	−0·02	+0·05	−0·78	+1·147	−0·120	295
323	21	78·789	78·146	1	0·79	+2·41	−0·12	−2·32	−11·14	−0·60	+2·39	+11·16	+0·07	+0·01	+0·32	0·00	+0·37	−0·643	−0·315	323
341	22	180·524	179·215	1	1·81	+2·07	−0·40	−2·09	−10·31	+1·28	−1·96	+10·13	−0·04	−0·02	+0·18	+0·06	−0·68	−1·309	−0·088	341
349	24	471·735	471·068	1	4·72	+1·42	+0·63	−1·19	− 6·21	+1·17	+1·09	+ 3·54	0·00	−0·02	+0·04	+0·08	−0·89	−0·667	+0·210	349
359	25	770·583	770·162	1	7·71	+1·23	+0·20	−1·12	− 5·51	−0·29	−0·16	+ 0·36	0·00	+0·01	0·00	+0·05	+1·57	−0·421	−0·060	359
373	25	766·309	765·940	1	7·66	+1·28	+0·43	−1·11	− 5·50	−0·41	−0·63	+ 0·14	0·00	+0·01	0·00	+0·05	+1·53	−0·369	−0·174	373
383	27	538·736	538·116	1	5·39	+2·01	+0·79	−1·75	− 7·65	−3·45	−2·05	− 7·35	0·00	0·00	−0·01	+0·01	+0·92	−0·620	−0·107	383
387	27	521 955	521·205	1	5·22	+2·12	+0·74	−1·87	− 7·71	−3·46	−2·17	− 7·44	0·00	0·00	−0·01	+0·01	+0·81	−0·750	+0·007	387
395	28	231·751	231·337	1	2·32	+2·29	+0·61	−2·07	− 9·51	−3·18	+2·33	− 9·72	0·00	0·00	+0·01	0·00	−0·29	−0·414	+0·208	395
423	Sept. 2	758·013	758·524	½	7·58	−1·21	−0·27	+1·14	+ 5·58	0·00	−0·27	+ 1·43	0·00	+0·01	+0·01	+0·05	+1·56	+0·511	−0·088	423
447	3	496·865	497·713	½	4·97	−1·63	−0·79	+1·48	+ 5·58	+2·10	+1·55	− 2·82	0·00	−0·01	0·00	+0·06	−0·70	+0·848	−0·020	447
473	6	64·996	65·768	1	0·65	−2·29	+0·15	+2·30	+10·53	+2·26	−2·25	−10·72	+0·07	+0·02	+0·30	−0·09	−0·41	+0·772	+0·217	473
485	7	21·433	19·492	1	0·21	+1·99	−0·33	−2·02	−10·66	−0·30	−1·88	+10·63	−0·26	−0·16	+1·39	+0·07	−0·73	−1·941	+0·553	485
497	8	432·866	431·998	1	4·33	+1·62	−0·36	−1·65	− 9·61	+1·32	+1·33	+ 9·30	+0·01	+0·02	+0·09	+0·04	+1·20	−0·868	−0·011	497
515	10	660·636	659·765	½	6·61	+2·09	+0·74	−2·04	− 5·61	+0·35	−2·16	+ 1·98	−0·01	+0·01	+0·02	+0·05	+0·81	−0·871	+0·058	515
535	26	579·568	578·569	1	5·80	+2·04	+0·72	+2·12	− 6·35	+1·25	−2·12	+ 4·55	−0·01	+0·01	+0·04	+0·05	+0·71	−0·999	+0·095	535
553	Oct. 4	526·856	526·942	1	5·27	−2·05	−0·64	−2·14	+ 6·71	−1·23	−2·11	+ 5·35	−0·01	0·00	+0·05	+0·05	+0·62	+0·086	−0·035	553

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TABLE XVIII.—*continued.*

SATELLITES I. AND IV.—POSITION-ANGLES, 1902.

Rotation No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of											O—C.	Residuals.	Rotation No.
					Δh_1	Δk_1	Δx_1	Δh_4	Δk_4	$\Delta \epsilon_1$	$\Delta \epsilon_4$	Δp_1	Δq_1	Δp_4	Δq_4			
	1902	° ′ ″	° ′ ″															
8	June 10	70 39 23	70 40 54	¼	+0·01	0·00	0·00	−0·02	+0·03	−0·01	−0·04	+1·92	−1·19	+ 3·55	+ 9·47	+0·205	+0·039	8
28	16	253 57 10	255 39 33	1	−0·08	−0·02	+0·01	+0·59	−0·12	−0·05	−0·60	+0·79	−2·17	+ 9·90	+ 3·13	+1·131	−0·254	28
56	24	246 17 28	244 42 32	½	+0·09	+0·02	−0·02	−0·76	+0·21	−0·06	−0·78	−0·90	+2·16	−10·23	− 1·92	−0·748	−0·094	56
66	30	70 35 44	70 30 46	1	0·00	0·00	0·00	+0·03	+0·02	0·00	−0·04	+0·83	+2·25	− 8·18	+ 7·02	−0·776	+0·028	66
72	30	70 35 34	70 30 51	1	0·00	0·00	0·00	+0·03	+0·03	0·00	−0·04	+0·49	+2·35	− 8·28	+ 6·90	−0·737	−0·028	72
94	July 7	250 58 27	250 58 28	1	0·00	0·00	0·00	0·00	−0·02	0·00	−0·02	−1·43	−1·97	− 3·60	+10·16	+0·002	−0·174	94
108	8	250 49 44	250 45 23	1	0·00	0·00	0·00	−0·03	−0·02	0·00	−0·04	−1·32	+2·04	− 7·78	+ 7·44	−0·688	−0·051	108
120	14	71 8 12	71 6 26	1	0·00	0·00	0·00	0·00	+0·02	0·00	−0·02	+2·22	+1·06	+ 1·81	+10·84	−0·352	+0·346	120
132	15	71 13 35	71 9 58	1	0·00	0·00	0·00	0·00	+0·02	0·00	−0·02	−1·48	−1·97	− 2·39	+10·79	−0·528	+0·432	132
148	16	71 5 30	71 0 26	½	0·00	0·00	0·00	+0·02	+0·03	0·00	−0·03	+0·57	+2·40	− 6·17	+ 9·23	−0·977	+0·233	148
158	18	70 51 53	70 37 7	½	0·00	0·00	0·00	+0·08	+0·02	0·00	−0·09	−1·56	+1·93	−10·86	+ 2·54	−0·339	−0·388	158
186	24	251 23 40	251 21 33	1	0·00	0·00	0·00	−0·02	−0·03	0·00	−0·03	+0·51	+2·43	− 4·62	+10·02	−0·435	−0·120	186
200	26	251 15 16	251 5 32	1	0·00	0·00	0·00	−0·06	−0·03	0·00	−0·07	−1·46	+2·01	−10·25	+ 4·03	−0·915	+0·033	200
230	Aug. 3	71 36 51	71 27 50	1	0·00	0·00	0·00	+0·05	+0·03	0·00	−0·06	−1·60	+1·94	− 9·83	+ 5·53	−1·032	+0·183	230
250	7	252 5 2	252 7 49	½	−0·01	0·00	+0·01	+0·01	−0·02	−0·01	−0·03	+2·03	+1·46	+ 6·38	+ 9·20	+0·518	−0·169	250
276	17	72 23 8	72 20 33	½	0·00	0·00	0·00	0·00	+0·03	0·00	−0·03	+0·14	+2·50	− 0·40	+11·15	−0·584	+0·205	276
296	18	72 27 39	72 22 19	1	0·00	0·00	0·00	0·00	+0·03	0·00	−0·03	+1·31	−2·11	− 4·67	+10·16	−0·688	+0·489	296
324	21	253 56 0	254 28 0	1	−0·07	0·00	+0·06	+0·26	0·00	−0·07	−0·26	+2·44	+0·45	+11·18	− 0·32	+0·734	+0·027	324
342	22	253 20 53	253 36 49	1	−0·04	0·00	+0·04	+0·13	−0·03	+0·04	−0·14	−2·09	−1·34	+10·52	+ 3·71	+0·837	−0·067	342
350	24	252 53 58	252 59 9	1	−0·01	0·00	+0·01	+0·02	−0·04	0·00	−0·04	+0·87	−2·32	+ 4·55	+10·07	+0·712	−0·287	350
360	25	252 42 45	252 43 33	1	0·00	0·00	0·00	0·00	−0·03	0·00	−0·03	+0·08	+2·46	+ 1·44	+10·90	+0·181	−0·346	360
374	25	252 42 40	252 42 41	1	0·00	0·00	0·00	0·00	−0·04	0·00	−0·04	−0·39	+2·43	+ 1·22	+10·92	+0·004	−0·186	374
384	27	252 38 6	252 34 20	1	0·00	0·00	0·00	−0·03	−0·04	0·00	−0·05	−1·90	+1·56	− 6·49	+ 8·76	−0·590	−0·042	384
388	27	252 38 4	252 34 13	1	0·00	0·00	0·00	−0·03	−0·04	0·00	−0·05	−2·04	+1·36	− 6·58	+ 8·69	−0·585	−0·038	388
396	28	252 30 0	252 21 4	1	0·00	0·00	0·00	−0·07	−0·04	0·00	−0·08	+2·23	−1·05	− 9·16	+ 5·84	−0·601	+0·058	396
424	Sept. 2	73 0 8	72 59 0	½	0·00	0·00	0·00	0·00	+0·04	0·00	−0·04	0·00	+2·45	+ 2·65	+10·57	−0·250	+0·118	424
448	3	73 4 53	73 3 32	½	0·00	0·00	0·00	0·00	+0·04	0·00	−0·04	+1·32	−2·04	− 1·59	+10·79	−0·194	+0·133	448
474	6	71 13 41	70 37 53	1	−0·06	0·00	+0·06	+0·36	+0·10	−0·06	−0·37	−2·34	−0·61	−10·40	+ 3·28	−0·677	+0·021	474
486	7	260 12 52	262 45 18	1	−0·23	+0·04	+0·26	+1·32	+0·02	+0·24	−1·32	−2·04	−1·26	+10·76	+ 0·56	+0·949	+0·019	486
498	8	253 24 59	253 28 3	1	−0·02	0·00	+0·02	+0·05	−0·03	−0·02	−0·05	+1·56	+1·82	+ 9·91	+ 4·37	+0·385	+0·124	498
516	10	253 16 3	253 17 40	½	−0·01	0·00	+0·01	0·00	−0·04	+0·01	−0·04	−2·02	+1·29	+ 3·29	+10·17	+0·311	−0·199	516
536	26	253 36 25	253 39 12	1	−0·01	0·00	+0·01	+0·02	−0·04	+0·01	−0·05	−1·97	+1·18	+ 5·79	+ 8·50	+0·469	−0·252	536
554	Oct. 4	73 40 36	73 41 55	1	+0·01	0·00	−0·01	−0·03	+0·04	+0·01	−0·05	−1·98	+1·05	+ 6·49	+ 7·59	+0·202	+0·074	554

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TABLE XVIII.—*continued.*

SATELLITES II. AND III.—DISTANCES, 1902.

Rotation No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of														O—C.	Residuals.	Rotation No.
					μ	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δe_3	Δe_2	Δe_3	Δp_2	Δq_2	Δp_3	Δq_3	ΔJ			
	1902																				
13	June 11	493″·323	493″·160	1	4·93	−1·94	+0·61	−1·20	+3·50	+1·07	−0·12	+1·08	−2·60	0·00	−0·01	0·00	−0·02	+1·13	−0″·163	+0″·039	13
29	16	90·796	90·966	1	0·91	+2·77	−0·94	+1·93	−5·75	+0·63	+2·79	+2·63	−5·76	−0·05	−0·05	+0·11	+0·02	+0·44	+0·170	+0·067	29
39	19	191·396	191·463	½	1·91	−3·50	−0·45	−1·06	+5·53	+0·90	−1·50	−3·49	−5·49	−0·02	0·00	−0·03	+0·01	+0·41	+0·067	+0·023	39
57	24	106·614	106·375	½	1·07	−2·76	−0·91	−0·52	+4·13	−1·54	−3·32	+2·55	+3·74	−0·03	+0·03	−0·04	−0·05	−0·20	−0·239	+0·087	57
97	July 7	127·291	127·521	1	1·27	+3·78	−0·30	+2·57	−5·96	−0·71	+3·12	+3·77	−5·96	−0·07	−0·01	+0·10	−0·03	+0·27	+0·230	+0·065	97
109	8	94·810	94·655	1	0·95	−2·86	−0·99	−1·02	+4·52	−1·54	−4·04	+2·68	+4·20	−0·04	+0·04	−0·06	−0·06	−0·23	−0·155	−0·038	109
137	15	57·447	57·527	1	0·57	+2·00	+0·36	+1·13	−5·58	+1·22	+4·75	−0·64	−5·52	0·00	0·00	+0·01	0·00	+0·57	+0·080	+0·122	137
149	16	413·712	413·806	½	4·14	−3·78	+0·56	−3·06	+3·30	−0·64	−2·78	+3·78	+1·42	−0·02	0·00	0·00	−0·04	+0·73	+0·094	−0·326	149
167	20	162·158	162·467	1	1·62	+2·46	−0·84	+2·39	−3·19	−0·45	+2·05	−1·96	−0·79	0·00	−0·02	0·00	+0·03	−0·20	+0·309	0·000	167
187	24	532·457	532·432	1	5·32	−2·12	−0·57	−1·28	+3·86	+1·47	−2·06	−1·10	−3·07	0·00	0·00	0·00	0·00	+1·22	−0·025	−0·034	187
201	26	409·204	409·381	1	4·09	+2·77	+1·09	+1·52	−5·09	+1·39	+4·88	−2·56	+4·88	0·00	0·00	0·00	0·00	+0·97	+0·177	−0·124	201
213	27	194·839	195·221	1	1·95	+2·90	−0·94	+2·88	−3·16	−0·05	+2·50	−2·67	+0·12	−0·01	−0·02	0·00	+0·03	−0·09	+0·382	−0·047	213
227	Aug. 3	223·192	223·546	1	2·23	+3·30	−0·85	+3·24	−3·27	+0·31	+2·94	−3·19	+0·96	−0·02	−0·01	0·00	+0·03	+0·02	+0·354	−0·007	227
239	4	155·461	155·985	1	1·55	+3·80	+0·71	+2·88	−5·19	−1·77	+3·53	+3·83	−5·17	−0·05	+0·02	+0·06	−0·07	+0·10	+0·524	−0·182	239
251	7	579·343	579·174	½	5·79	−2·05	+0·19	−1·88	+3·28	+0·88	−2·44	+0·59	−1·38	0·00	−0·01	0·00	−0·02	+1·30	−0·169	0·000	251
269	16	329·046	329·329	½	3·29	+2·01	+0·36	+1·72	−6·15	+0·34	+5·82	−0·38	+6·10	0·00	0·00	0·00	0·00	+0·96	+0·283	−0·187	269
281	18	123·100	123·067	1	1·23	+2·24	+0·92	+1·76	−3·60	−1·51	+2·83	+1·61	−2·66	0·00	+0·03	+0·02	−0·04	−0·28	−0·033	+0·265	281
303	19	226·968	227·153	1	2·27	+3·26	−0·64	+3·28	−6·04	−1·23	+5·24	+3·08	−6·12	−0·04	−0·02	+0·06	−0·02	+0·63	+0·185	+0·203	303
311	21	547·015	546·637	1	5·47	−2·64	+0·71	−2·72	+3·18	+0·07	−2·98	+2·18	+0·46	0·00	−0·01	0·00	−0·02	+1·18	−0·378	+0·080	311
313	21	549·365	549·246	1	5·49	−2·60	+0·69	−2·69	+3·17	+0·09	−2·97	+2·12	+0·41	0·00	−0·01	0·00	−0·02	+1·19	−0·119	−0·163	313
317	21	556·590	556·508	1	5·57	−2·50	+0·65	−2·57	+3·15	+0·18	−2·92	+1·92	+0·23	0·00	−0·01	0·00	−0·02	+1·21	−0·082	−0·195	317
337	22	300·993	300·681	1	3·01	−3·69	−0·91	−3·23	+4·74	+2·03	−3·88	−3·76	−4·72	0·00	0·00	0·00	0·00	+0·55	−0·312	+0·149	337
343	24	252·647	253·205	1	2·53	+3·74	−0·34	+3·69	−3·73	+0·80	+3·81	−3·70	+2·50	−0·02	0·00	+0·02	+0·03	+0·18	+0·558	−0·192	343
363	25	125·639	125·819	1	1·26	+1·90	+0·65	+1·77	−3·32	−1·24	+2·86	+0·95	−1·91	0·00	+0·02	0·00	−0·02	−0·30	+0·180	+0·051	363
381	27	11·377	12·058	½	0·11	+2·51	+1·12	+2·15	−5·62	+0·73	+5·63	−2·25	−5·47	+0·02	−0·03	+0·06	+0·02	+0·41	+0·681	−0·207	381
405	28	518·233	517·898	1	5·18	−2·89	+0·71	−2·97	+3·26	−0·20	−3·22	+2·59	+1·12	−0·01	−0·01	0·00	−0·02	+1·10	−0·335	−0·006	405
413	30	296·950	296·930	¼	2·97	+1·93	+0·41	+1·81	−6·14	−0·06	+5·99	−0·37	+6·10	0·00	0·00	0·00	0·00	+0·91	−0·020	+0·029	413
427	Sept. 2	232·693	233·108	½	2·33	+3·79	+0·04	+3·72	−5·20	−1·94	+4·79	+3·76	−5·32	−0·04	0·00	+0·06	−0·04	+0·44	+0·415	+0·003	427
443	3	77·086	77·253	½	0·77	+2·20	+1·02	+2·01	−5·85	+0·35	+5·83	−1·69	−5·76	+0·01	−0·03	+0·06	+0·01	+0·55	+0·167	+0·145	443
457	4	537·042	536·650	1	5·37	−2·52	+0·59	−2·58	+3·16	−0·03	−3·13	+2·00	+0·82	0·00	−0·01	0·00	−0·02	+1·17	−0·392	+0·107	457
463	6	133·631	133·758	1	1·34	+2·55	−0·53	+2·59	−5·98	−1·18	+5·78	+2·02	+6·08	0·00	0·00	−0·01	0·00	+0·60	+0·127	−0·078	463
483	7	312·788	313·088	1	3·13	+3·52	+1·03	+3·39	−4·94	+0·93	+5·01	−3·61	+4·60	−0·01	0·00	+0·02	+0·01	+0·61	+0·300	+0·021	483
501	8	135·559	135·806	1	1·36	+1·99	−0·08	+1·99	−3·03	−0·46	+2·97	−0·62	−0·18	0·00	0·00	0·00	+0·01	−0·28	+0·247	−0·030	501
521	10	19·549	20·190	½	0·20	+2·57	+1·26	+2·48	−5·60	+0·48	+5·62	−2·46	−5·46	+0·05	−0·07	+0·13	+0·04	+0·40	+0·641	−0·133	521
543	26	480·985	480·858	½	4·81	−2·34	−1·23	−2·49	+3·06	+1·34	−3·22	−2·22	−1·88	0·00	0·00	0·00	−0·01	+1·04	−0·127	−0·049	543
565	Oct. 4	33·186	33·338	¼	0·33	−3·01	+0·41	−2·82	+4·80	+1·93	−5·13	−2·84	−4·99	+0·01	0·00	+0·02	−0·01	−0·23	+0·152	−0·113	565
569	5	25·773	254·981	¼	2·55	+2·47	+1·22	+2·70	−5·48	+0·08	+5·28	−2·42	+5·41	0·00	+0·01	+0·03	0·00	+0·69	+0·208	+0·012	569

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TABLE XVIII.—*continued.*

SATELLITES II. AND III.—POSITION-ANGLES, 1902.

Rotation No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of										O—C.	Residuals.	Rotation No.
					Δl_2	Δk_2	Δz_2	Δh_3	Δk_3	Δx_3	$\Delta \epsilon_2$	$\Delta \epsilon_3$	Δp_2	Δf_{12}	Δl_{13}	Δq_3			
14	1902 June 11	° ′ ″ 70 2 59	° ′ ″ 69 57 33	1	0″00	+0″03	−0″03	0″00	0″00	0″00	+0″03	0″00	+0″97	+3″48	−2″75	+5″07	−0″779	+0″065	14
30	16	249 10 32	249 5 23	1	+0″05	−0″04	+0″05	−0″10	0″00	+0″05	+0″06	−0″10	+2″56	+2″60	−5″73	−1″11	−0″136	−0″026	30
40	19	70 35 47	70 26 39	½	+0″03	+0″02	0″00	−0″02	0″00	+0″01	+0″04	+0″02	−3″51	+1″04	−5″54	+2″08	−0″509	−0″029	40
58	24	69 45 47	69 56 30	½	−0″02	+0″02	−0″02	+0″04	−0″01	−0″03	0″00	+0″03	+2″61	−2″74	+3″64	+4″73	+0″333	+0″159	58
98	July 7	249 40 10	249 49 1	1	+0″06	−0″02	+0″05	−0″09	−0″01	+0″05	+0″06	−0″09	+3″77	+0″70	−5″96	+1″51	+0″328	−0″114	98
110	8	69 56 30	70 17 7	1	−0″02	+0″01	−0″02	+0″05	−0″02	−0″04	0″00	+0″04	+2″68	−2″85	+4″20	+4″52	+0″569	+0″021	110
138	15	250 46 29	250 12 53	1	0″00	−0″03	+0″02	0″00	0″00	0″00	+0″03	0″00	−0″60	+3″90	−5″55	−2″83	−0″561	−0″081	138
150	16	70 34 18	70 33 52	½	−0″02	+0″02	−0″02	+0″02	0″00	−0″01	+0″02	0″00	+3″79	+1″08	+1″49	+6″07	−0″052	−0″013	150
168	20	251 20 37	251 24 24	1	−0″01	−0″02	0″00	+0″02	0″00	−0″02	−0″01	0″00	−2″03	−3″37	−0″66	+6″26	+0″178	+0″052	168
188	24	71 7 31	71 2 37	1	0″00	+0″03	−0″02	0″00	0″00	0″00	+0″03	0″00	−0″99	+3″80	−2″91	+5″60	−0″759	−0″209	188
202	26	251 20 49	251 11 50	1	−0″02	−0″03	0″00	+0″02	0″00	−0″02	+0″04	−0″02	−2″46	+3″17	+5″01	+3″90	−1″069	+0″086	202
214	27	251 37 51	251 42 17	1	−0″02	−0″02	0″00	+0″02	0″00	−0″02	0″00	0″00	−2″77	−2″84	+0″35	+6″32	+0″251	−0″197	214
228	Aug. 3	251 54 36	251 53 12	1	−0″02	−0″02	0″00	+0″02	0″00	−0″03	0″00	−0″01	−3″30	−2″22	+1″28	+6″23	−0″091	−0″021	228
240	4	250 49 45	251 0 48	1	+0″04	0″00	+0″04	−0″06	−0″03	+0″04	+0″04	−0″07	+3″78	−1″17	−3″95	+4″96	+0″501	−0″072	240
252	7	71 35 34	71 30 20	½	0″00	+0″03	−0″02	0″00	0″00	0″00	+0″03	0″00	+0″83	+3″87	−1″00	+6″27	−0″883	+0″023	252
270	16	252 5 23	251 54 41	½	0″00	−0″03	0″00	+0″01	0″00	−0″01	+0″03	−0″01	−0″06	+4″00	+6″22	+1″26	−1″023	+0″130	270
282	18	251 48 9	252 17 41	1	0″00	−0″02	+0″01	−0″02	−0″02	+0″01	−0″02	−0″02	+1″30	−3″70	−2″16	+5″93	+1″057	−0″436	282
304	19	251 37 4	251 30 56	1	+0″03	−0″02	+0″04	−0″05	−0″02	+0″04	+0″04	−0″06	+3″28	+2″21	−5″97	+2″00	−0″407	+0″279	304
312	21	72 5 58	72 1 3	1	0″00	+0″02	−0″02	0″00	+0″01	0″00	+0″02	−0″01	+2″47	+3″06	+1″03	+6″20	−0″782	+0″221	312
314	21	72 6 5	72 1 17	1	0″00	+0″02	−0″02	0″00	+0″01	0″00	+0″02	−0″01	+2″41	+3″10	+0″98	+6″21	−0″769	+0″195	314
318	21	72 6 32	72 4 9	1	0″00	+0″02	−0″02	0″00	+0″01	0″00	+0″02	−0″01	+2″23	+3″24	+0″80	+6″24	−0″386	−0″232	318
338	22	72 24 43	72 16 31	1	+0″02	+0″02	+0″01	−0″01	0″00	+0″01	+0″02	0″00	−3″65	+1″42	−4″31	+4″60	−0″717	−0″007	338
344	24	252 44 36	252 40 50	1	−0″03	−0″01	−0″02	+0″03	−0″02	−0″03	+0″02	−0″03	−3″81	−0″99	+3″05	+5″49	−0″276	−0″101	344
364	25	252 14 3	252 31 3	1	0″00	−0″02	0″00	0″00	−0″02	0″00	−0″02	−0″02	+0″57	−3″85	−1″31	+6″13	+0″621	−0″053	364
382	27	251 58 53	248 32 45	½	+0″01	−0″01	+0″02	−0″04	−0″01	+0″04	0″00	−0″04	−2″02	+3″40	−5″75	−2″40	−0″678	−0″011	382
406	28	72 21 8	72 18 30	1	0″00	+0″02	−0″01	0″00	+0″01	0″00	+0″02	−0″01	+2″88	+2″63	+1″76	+5″98	−0″398	−0″042	406
414	30	252 40 3	252 24 26	¼	0″00	−0″03	0″00	+0″02	0″00	−0″02	+0″03	−0″02	+0″06	+3″93	+6″20	+0″60	−1″349	+0″279	414
428	Sept. 2	252 5 46	252 7 40	½	+0″04	−0″01	+0″04	−0″05	−0″03	+0″04	+0″04	−0″06	+3″84	+0″50	−4″93	+3″72	+0″129	+0″033	428
444	3	252 14 0	251 30 41	½	0″00	−0″04	0″00	−0″04	0″00	+0″04	+0″04	−0″04	−1″27	+3″69	−5″97	−1″50	−0″971	+0″184	444
458	4	72 36 42	72 33 10	1	0″00	+0″02	−0″01	0″00	+0″02	0″00	+0″02	−0″02	+2″38	+3″02	+1″53	+5″96	−0″552	−0″003	458
464	6	252 44 53	252 32 34	1	0″00	−0″02	0″00	0″00	0″00	0″00	+0″02	0″00	+2″40	+3″03	+5″92	−1″69	−0″479	+0″150	464
484	7	253 7 9	252 59 52	1	+0″03	−0″02	−0″03	+0″04	−0″02	−0″04	+0″04	−0″04	−3″41	+1″82	+5″06	+3″48	−0″663	−0″148	484
502	8	253 1 48	253 11 2	1	0″00	−0″02	0″00	+0″01	−0″02	−0″02	−0″02	−0″02	−1″07	−3″65	+0″57	+6″10	+0″365	+0″005	502
522	10	251 42 13	249 57 41	¼	+0″04	0″00	+0″04	−0″11	0″00	+0″11	−0″03	−0″10	−2″06	+3″24	−5″75	−1″91	−0″593	−0″058	522
544	26	73 8 22	73 1 41	¼	0″00	+0″02	0″00	0″00	+0″02	0″00	+0″02	−0″02	−1″80	+3″16	−1″09	+5″73	−0″935	−0″003	544
566	Oct. 4	73 2 0	72 44 32	¼	0″00	+0″01	0″00	+0″01	+0″02	−0″02	−0″01	−0″02	−3″12	−1″78	−4″54	+3″46	−0″169	+0″063	566
570	5	253 32 11	253 19 26	¼	−0″02	−0″02	−0″03	+0″04	−0″01	−0″04	+0″03	−0″04	−2″02	+2″96	+5″61	+1″02	−0″946	+0″067	570

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TABLE XVIII.—*continued.*

SATELLITES II. AND IV.—DISTANCES, 1902.

Rotation No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of													O−C.	Residuals.	Rotation No.
					μ	Δh_2	Δk_2	Δx_2	Δh_4	Δk_4	$\Delta \epsilon_2$	$\Delta \epsilon_4$	Δp_2	Δq_2	Δp_4	Δq_4	ΔJ			
	1902																			
15	June 11	782″.648	782″.891	1	7·83	−3·99	+0·57	−1·16	+ 5·07	−0·14	+1·00	+ 0·12	0·00	+0·01	0·00	+0·02	+0·98	+0″.243	+0″.179	15
31	16	314·787	313·550	1	3·15	+2·74	−0·94	+1·92	− 9·91	+1·78	+2·60	+ 9·96	0·00	+0·01	+0·03	0·00	+0·61	−1·237	+0·170	31
41	19	523·966	523·739	½	5·24	+3·52	+0·43	+1·09	− 5·31	+1·10	+3·51	+ 1·82	+0·01	0·00	0·00	+0·03	+0·05	−0·227	−0·218	41
59	24	54·698	54·381	½	0·55	+2·79	+0·91	+0·53	−10·27	+1·00	−2·60	−10·28	+0·06	−0·06	+0·22	+0·04	+0·52	−0·317	−0·060	59
67	30	334·604	335·569	1	3·35	−3·57	+0·59	−2·44	+ 8·45	+2·60	−3·57	− 8·11	−0·03	−0·01	−0·07	+0·06	−0·07	+0·965	−0·011	67
73	30	313·386	314·398	1	3·13	−3·48	+0·68	−2·47	+ 8·53	+2·60	−3·47	− 8·22	−0·03	−0·01	−0·07	+0·06	−0·13	+1·012	−0·054	73
77	July 1	20·596	22·306	1	0·21	−2·41	−0·84	−0·64	+10·19	+1·60	+1·95	−10·15	−0·07	+0·11	+0·35	−0·12	−0·59	+1·710	−0·539	77
99	7	624·299	624·125	1	6·24	+3·76	−0·34	+2·96	− 6·07	−1·65	+3·76	− 3·69	0·00	0·00	0·00	0·00	+0·42	−0·174	−0·072	99
111	8	581·090	580·938	1	5·81	+2·92	+0·99	+1·07	− 8·28	−2·62	−2·76	− 7·85	0·00	0·00	−0·03	+0·03	+0·77	−0·152	−0·191	111
135	15	391·789	391·998	1	3·92	−1·99	−0·32	−1·14	+ 5·77	+1·39	+0·56	− 2·59	0·00	−0·03	−0·02	+0·09	−0·52	+0·209	+0·301	135
151	18	3·416	4·433	½	0·03	−2·95	+0·80	−2·68	+10·00	+1·30	−2·84	−10·01	−1·14	−0·90	−4·00	+1·00	−0·41	+1·017	−0·122	151
155	18	8·807	7·596	½	0·09	+3·08	−0·85	+2·81	−10·76	−1·33	+2·92	+10·77	−0·41	−0·34	−1·47	+0·36	+0·47	−1·211	+0·001	155
155[a]	18	16·010	14·457	½	0·16	+3·06	−0·86	+2·81	−10·85	−1·30	+2·88	+10·86	−0·21	−0·19	−0·78	+0·19	+0·48	−1·553	+0·261	155[a]
169	20	122·038	121·140	1	1·22	+2·44	−0·83	+2·36	− 9·76	+2·33	−1·91	+ 9·64	−0·07	−0·12	+0·34	+0·19	−0·56	−0·898	+0·096	169
189	24	349·000	348·562	1	3·49	+2·14	+0·60	+1·28	− 6·62	−2·35	+1·17	− 4·99	0·00	−0·02	−0·03	+0·07	−0·52	−0·438	+0·091	189
203	26	382·286	381·854	1	3·82	+2·80	+1·10	+1·54	−10·35	−2·06	−2·60	−10·41	0·00	0·00	−0·01	0·00	+0·73	−0·432	+0·085	203
231	Aug. 3	428·780	429·768	1	4·29	−3·22	+0·87	−3·19	+10·05	+2·78	+3·09	−10·13	−0·01	0·00	+0·04	−0·02	+0·65	+0·988	−0·096	231
253	7	326·432	325·980	½	3·26	+2·03	−0·16	+1·85	− 7·17	+2·14	−0·52	+ 5·74	−0·01	−0·05	+0·09	+0·13	−0·56	−0·452	−0·065	253
285	18	765·595	766·121	1	7·66	−2·28	−0·96	−1·79	+ 6·58	+2·99	−1·72	− 5·33	0·00	0·00	−0·01	+0·03	+0·99	+0·526	+0·139	285
301	19	274·555	275·269	1	2·75	−3·29	+0·62	−3·30	+ 8·56	+3·58	−3·12	− 8·55	−0·03	−0·02	−0·07	+0·07	−0·31	+0·714	+0·142	301
305	19	264·316	265·000	1	2·64	−3·22	+0·65	−3·24	+ 8·60	+3·58	−3·03	− 8·61	−0·03	−0·02	−0·07	+0·07	−0·34	+0·684	+0·182	305
319	21	163·532	164·558	1	1·64	−2·44	+0·61	−2·51	+11·15	+0·64	+1·81	−11·17	−0·04	−0·07	+0·24	0·00	+0·70	+1·026	+0·053	319
339	22	208·888	207·986	1	2·09	+3·72	+0·88	+3·26	−10·32	+1·26	+3·79	+10·14	+0·03	−0·01	+0·10	+0·03	−0·08	−0·902	−0·021	339
347	24	514·322	514·026	1	5·14	+3·71	−0·39	+3·67	− 6·23	+1·18	−3·66	+ 3·59	−0·04	−0·01	+0·05	+0·11	−0·03	−0·296	+0·140	347
367	25	415·365	415·077	1	4·15	+2·05	+0·70	+1·78	− 5·50	−0·36	+1·06	+ 0·23	0·00	−0·03	+0·01	+0·10	−0·48	−0·288	−0·010	367
389	27	624·063	623·890	1	6·24	+2·73	+1·26	+2·32	− 7·73	−3·46	−2·63	− 7·47	0·00	+0·01	−0·01	+0·02	+0·82	−0·173	+0·040	389
401	28	110·045	109·752	1	1·10	+2·94	−0·71	+3·02	− 9·56	−3·13	−2·67	− 9·77	−0·03	−0·02	−0·09	+0·06	−0·46	−0·293	+0·106	401
409	30	16·061	16·786	1	0·16	−1·87	−0·32	−1·76	+ 9·90	−0·69	+0·22	+ 9·71	−0·05	−0·95	+2·51	+0·74	−0·68	+0·725	−0·098	409
431	Sept. 2	561·598	561·948	½	5·62	−3·77	0·00	−3·72	+ 5·56	+0·06	−3·74	+ 1·32	−0·03	0·00	+0·02	+0·09	+0·08	+0·350	−0·143	431
439	3	400·548	401·051	½	4·01	−2·14	−0·96	−1·96	+ 5·54	+2·02	+1·53	− 2·66	0·00	−0·03	0·00	+0·08	−0·46	+0·503	−0·107	439
455	4	661·600	661·997	1	6·62	−2·56	+0·60	−2·62	+ 7·35	+3·66	+2·06	− 7·09	0·00	0·00	0·00	0·00	+0·89	+0·397	+0·254	455
465	6	65·569	64·524	1	0·66	+2·52	−0·52	+2·57	−10·49	−2·33	+1·97	+10·69	+0·01	+0·01	+0·05	−0·02	+0·63	−1·045	−0·056	465
471	6	79·235	78·140	1	0·79	+2·42	−0·48	+2·46	−10·52	−2·27	+1·78	+10·71	+0·01	+0·02	+0·05	−0·02	+0·66	−1·095	−0·016	471
481	7	186·490	185·762	1	1·87	+3·50	+1·06	+3·36	−10·76	−0·34	−3·58	+10·73	−0·07	+0·04	+0·22	+0·01	+0·35	−0·728	−0·083	481
503	8	106·833	106·140	1	1·07	+1·98	−0·06	+1·98	− 9·56	+1·35	−0·58	+ 9·25	−0·05	−0·16	+0·43	+0·19	−0·63	−0·693	−0·129	503
517	10	775·785	775·603	½	7·76	+2·51	+1·25	+2·42	− 5·60	+0·33	−2·37	+ 1·95	0·00	+0·02	+0·02	+0·05	+0·90	−0·182	+0·006	517
545	26	369·709	369·213	1	3·70	+2·36	+1·24	+2·52	− 6·28	+1·21	+2·26	+ 4·39	+0·02	−0·03	+0·06	+0·09	−0·34	−0·496	+0·113	545
561	Oct. 4	365·257	365·459	¼	3·65	−3·06	+0·39	−2·87	+ 6·65	−1·21	−2·90	+ 5·26	−0·05	−0·02	+0·09	+0·11	−0·21	+0·202	−0·019	561

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TABLE XVIII.—*continued.*

SATELLITES II. AND IV.—POSITION-ANGLES, 1902.

Rotation No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of											O—C.	Residuals.	Rotation No.
					Δh_2	Δk_2	Δx_2	Δh_4	Δk_4	$\Delta \epsilon_2$	$\Delta \epsilon_4$	Δp_2	Δq_2	Δp_4	Δq_4			
	1902	° ′ ″	° ′ ″													″	″	
16	June 11	70 23 27	70 19 18	1	0·00	+0·02	−0·02	+0·01	+0·03	+0·02	−0·03	+0·90	+3·50	− 0·18	+10·18	−0·946	+0·299	16
32	16	250 26 59	250 32 56	1	0·00	−0·02	+0·01	−0·02	−0·01	0·00	+0·01	+2·52	+2·64	+ 9·88	+ 3·19	+0·545	−0·172	32
42	19	250 30 12	250 34 23	½	−0·02	−0·02	0·00	0·00	−0·03	−0·03	−0·02	+3·53	−0·97	+ 1·56	+10·25	+0·637	−0·068	42
60	24	249 9 18	247 31 4	½	+0·04	0·00	+0·03	−0·27	+0·01	−0·03	−0·27	−2·66	+2·69	−10·24	− 2·01	−1·571	+0·108	60
68	30	70 58 59	70 53 6	1	+0·03	+0·01	0·00	−0·02	0·00	+0·03	+0·01	−3·55	−1·31	− 8·21	+ 6·99	−0·572	−0·105	68
74	30	71 0 25	70 52 49	1	+0·04	+0·01	+0·01	−0·03	0·00	+0·03	+0·02	−3·45	−1·56	− 8·31	+ 6·87	−0·692	+0·063	74
78	July 1	68 32 11	69 9 30	1	−0·07	0·00	−0·04	+0·41	+0·08	+0·04	−0·41	+1·99	−3·29	−10·21	+ 3·55	+0·223	−0·083	78
100	7	250 42 44	250 43 29	1	0·00	−0·02	0·00	−0·03	−0·03	0·00	−0·04	+3·76	+0·78	− 3·72	+10·11	+0·137	−0·253	100
112	8	250 54 4	250 46 52	1	−0·03	−0·03	0·00	−0·02	−0·02	+0·04	−0·03	−2·76	+2·77	− 7·86	+ 7·36	−1·203	−0·138	112
136	15	71 21 26	71 23 51	1	+0·02	+0·04	0·00	−0·02	+0·02	−0·04	−0·01	+0·52	−3·91	− 2·47	+10·77	+0·274	+0·263	136
152	18	273 19 30	259 36 27	½	+1·18	−0·30	+1·05	−3·96	−0·49	+1·12	+3·96	−2·87	−2·25	−10·05	+ 2·51	−0·804	−0·105	152
156	18	243 11 35	246 13 39	½	+0·42	−0·14	+0·40	−1·53	−0·20	+0·41	+1·54	+2·97	+2·51	+10·74	− 2·59	+0·468	−0·069	156
170	20	253 3 37	253 29 58	1	−0·08	0·00	−0·06	+0·30	−0·08	+0·04	−0·30	−1·98	−3·40	+ 9·76	+ 5·36	+0·936	+0·152	170
190	24	251 35 20	251 40 6	1	−0·02	−0·04	0·00	0·00	−0·02	−0·04	−0·01	+1·06	−3·78	− 4·70	+ 9·98	+0·485	−0·063	190
204	26	251 21 8	251 9 13	1	−0·02	−0·03	0·00	−0·04	−0·02	+0·04	−0·05	−2·50	+3·13	−10·28	+ 3·97	−1·326	−0·196	204
232	Aug. 3	71 21 57	71 15 38	1	−0·01	+0·02	−0·02	+0·10	+0·04	+0·02	−0·10	+3·21	+2·35	− 9·86	+ 5·48	−0·789	−0·034	232
254	7	252 33 37	252 46 22	½	−0·03	−0·03	−0·01	+0·07	−0·04	−0·02	−0·07	−0·76	−3·88	+ 6·31	+ 9·25	+1·211	−0·138	254
286	18	72 21 20	72 15 0	1	+0·02	+0·03	0·00	+0·02	+0·04	+0·04	−0·04	−1·42	+3·66	− 4·47	+10·25	−1·409	+0·236	286
302	19	72 44 13	72 36 39	1	+0·03	+0·01	+0·02	−0·02	+0·01	+0·02	0·00	−3·32	−2·16	− 7·89	+ 7·95	−0·604	+0·124	302
306	19	72 44 50	72 34 37	1	+0·03	+0·01	+0·02	−0·02	+0·01	+0·02	+0·01	−3·24	−2·28	− 7·95	+ 7·89	−0·785	+0·331	306
320	21	71 5 46	70 43 8	1	−0·05	+0·04	−0·06	+0·30	+0·03	+0·06	−0·30	+2·12	+3·31	−11·18	+ 0·39	−1·077	−0·058	320
340	22	252 52 23	253 11 20	1	−0·05	−0·03	−0·04	+0·05	−0·02	−0·05	−0·05	+3·68	−1·34	+10·53	+ 3·68	+1·151	+0·112	340
348	24	253 2 28	253 3 48	1	−0·05	0·00	−0·04	+0·04	−0·04	+0·04	−0·05	−3·78	−1·11	+ 4·59	+10·04	+0·200	−0·106	348
368	25	252 57 57	253 5 55	1	−0·03	−0·03	−0·02	+0·02	−0·03	−0·04	−0·04	+0·68	−3·83	+ 1·32	+10·91	+0·964	−0·174	368
390	27	252 39 58	252 33 26	1	−0·02	−0·03	−0·01	−0·03	−0·04	+0·03	−0·05	−2·31	+3·21	− 6·61	+ 8·66	−1·187	−0·120	390
402	28	253 8 1	252 57 18	1	−0·03	0·00	−0·03	+0·04	0·00	+0·02	+0·03	−2·95	−2·55	− 9·22	+ 5·76	−0·343	−0·107	402
410	30	86 35 35	91 11 7	1	+0·47	+0·11	+0·44	−2·41	+0·19	−0·08	−2·37	−0·19	−3·81	+10·09	+ 2·99	+1·290	−0·011	410
432	Sept. 2	73 15 3	73 12 56	½	+0·04	+0·01	+0·04	−0·03	+0·04	+0·04	−0·04	−3·83	−0·58	+ 2·54	+10·60	−0·346	+0·139	432
440	3	73 10 3	73 10 45	½	+0·02	+0·03	+0·02	−0·01	+0·04	−0·04	−0·03	+1·11	−3·75	− 1·42	+10·81	+0·082	+0·390	440
456	4	72 46 42	72 43 12	1	0·00	+0·02	0·00	+0·05	+0·05	+0·01	−0·07	+2·43	+2·98	− 6·09	+ 9·08	−0·674	−0·112	456
466	6	253 7 58	253 12 6	1	−0·01	−0·02	−0·01	0·00	−0·02	0·00	+0·01	+2·36	+3·07	+10·35	− 3·44	+0·079	+0·276	466
472	6	253 9 2	253 26 50	1	−0·01	−0·02	−0·01	0·00	−0·02	0·00	0·00	+2·18	+3·20	+10·39	− 3·32	+0·410	−0·096	472
482	7	254 2 59	254 3 57	1	−0·09	−0·04	−0·08	+0·17	−0·02	+0·09	−0·17	−3·38	+1·87	+10·86	+ 0·49	+0·048	+0·020	482
504	8	255 24 0	256 2 40	1	−0·09	−0·02	−0·09	+0·38	−0·08	0·00	−0·38	−1·04	−3·65	+ 9·87	+ 4·44	+1·206	+0·034	504
518	10	253 15 6	253 13 24	½	−0·02	−0·03	−0·02	−0·01	+0·02	+0·04	+0·02	−1·97	+3·30	+ 3·25	+10·18	−0·384	−0·189	518
546	26	253 50 14	253 59 26	1	−0·03	−0·03	−0·04	+0·05	−0·05	−0·04	−0·06	+1·84	−3·13	+ 5·64	+ 8·59	+0·990	+0·076	546
562	Oct. 4	74 5 27	74 7 44	¼	+0·05	0·00	+0·05	−0·08	+0·05	+0·04	−0·09	−3·17	−1·69	+ 6·40	+ 7·66	+0·241	+0·077	562

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TABLE XVIII.—*continued.*

SATELLITES III. AND IV.—DISTANCES, 1902.

Rotation No.	Date.	Computed Distance.	Observed Distance.	Weight.	Coefficients of													O—C.	Residuals.	Rotation No.
					μ	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	Δe_3	Δe_4	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ			
	1902	"	"		"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	
9	June 10	233·570	234·089	1	2·34	−3·27	+1·11	+2·07	+ 5·75	−1·84	−2·21	+ 3·80	−0·03	−0·09	+0·06	+0·15	−0·18	+0·519	−0·154	9
17	11	291·538	292·165	1	2·92	−3·55	−1·10	+0·11	+ 5·08	−0·12	+2·71	+ 0·08	+0·04	−0·06	0·00	+0·13	−0·14	+0·567	−0·106	17
33	16	228·673	227·442	1	2·29	+5·73	−0·67	−2·82	− 9·90	+1·79	+5·75	+ 9·95	+0·06	+0·01	+0·11	+0·04	+0·16	−1·231	−0·059	33
43	19	706·396	705·844	½	7·06	+5·34	+0·78	−1·53	− 5·31	+1·08	−5·53	+ 1·78	−0·02	0·00	0·00	+0·04	+0·44	−0·552	+0·084	43
61	24	165·539	165·024	½	1·66	+4·08	−1·52	−3·28	−10·27	+1·02	+3·66	−10·28	−0·05	−0·07	+0·15	+0·03	+0·33	−0·515	−0·009	61
75	July 1	17·254	15·394	1	0·17	+4·70	−1·56	−3·90	−10·18	−1·64	+4·52	+10·14	−0·02	−0·02	−0·04	+0·02	+0·23	−1·860	+0·546	75
91	7	487·045	486·314	1	4·87	+5·92	+0·79	−3·04	− 6·01	−1·58	+5·92	− 3·53	+0·03	0·00	−0·02	+0·06	+0·13	−0·731	+0·125	91
113	8	684·558	684·196	1	6·85	+4·45	−1·53	−3·99	− 8·29	−2·63	+4·10	− 7·88	0·00	0·00	0·00	0·00	+0·57	−0·362	−0·166	113
121	14	774·265	774·939	1	7·73	−5·62	−1·22	+2·94	+ 5·63	−0·71	−5·60	+ 1·67	−0·02	+0·01	0·00	+0·04	+0·50	+0·674	−0·072	121
133	15	456·603	457·303	1	4·57	−5·62	+1·18	+4·76	+ 5·76	+1·37	−5·58	− 2·54	−0·04	−0·02	−0·02	+0·07	+0·06	+0·700	+0·006	133
173	20	33·143	34·019	1	0·33	−3·19	−0·53	+2·00	+ 9·68	−2·35	−0·95	− 9·56	+0·09	−0·66	+1·03	+0·58	+0·35	+0·876	+0·345	173
175	20	31·086	31·881	1	0·31	−3·19	−0·55	+1·99	+ 9·66	−2·35	−0·99	− 9·53	+0·10	−0·70	+1·09	+0·62	+0·35	+0·795	+0·430	175
191	24	874·333	874·049	2	8·74	+3·92	+1·51	−2·08	− 6·65	−2·37	−3·20	− 5·05	0·00	0·00	0·00	+0·02	+0·70	−0·284	−0·003	191
195	26	11·682	12·006	1	0·12	−5·16	+1·38	+4·92	+10·32	+2·11	+4·96	+10·36	0·00	0·00	0·00	0·00	+0·22	+0·324	+0·074	195
205	26	30·335	30·720	1	0·30	−5·05	+1·40	+4·85	+10·38	+2·06	+4·82	+10·42	0·00	0·00	0·00	0·00	+0·24	+0·385	+0·020	205
219	Aug. 3	654·636	655·909	1	6·55	−3·30	+0·39	+3·00	+ 9·96	+2·84	+1·12	−10·03	0·00	0·00	0·00	0·00	+0·64	+1·273	−0·060	219
221	3	653·356	654·535	1	6·54	−3·29	+0·37	+2·99	+ 9·98	+2·83	+1·08	−10·05	0·00	0·00	0·00	0·00	+0·64	+1·179	+0·037	221
255	7	906·097	905·581	½	9·06	+3·30	+0·94	−2·43	− 7·15	+2·13	−1·49	+ 5·70	0·00	+0·02	+0·02	+0·03	+0·75	−0·516	−0·139	255
257	7	906·842	905·740	½	9·07	+3·31	+0·96	−2·43	− 7·14	+2·12	−1·54	+ 5·67	0·00	+0·02	+0·02	+0·03	+0·75	−0·802	+0·062	257
263	16	708·909	709·011	½	7·09	−6·18	+0·27	+5·82	+ 6·03	−0·81	+6·14	+ 2·85	+0·04	0·00	+0·02	+0·06	+0·41	+0·102	+0·091	263
277	17	965·502	965·679	½	9·66	−3·78	+0·89	+3·84	+ 5·56	+1·22	+2·61	− 1·37	0·00	+0·02	0·00	+0·03	+0·76	+0·177	+0·196	277
279	17	967·602	967·861	½	9·68	−3·74	+0·85	+3·79	+ 5·56	+1·26	+2·50	− 1·46	0·00	+0·02	0·00	+0·03	+0·76	+0·259	+0·146	279
283	18	891·645	892·486	1	8·92	−3·62	−1·53	+2·84	+ 6·58	+2·98	−2·70	− 5·29	0·00	0·00	0·00	+0·01	+0·72	+0·841	+0·064	283
287	18	885·343	886·387	1	8·85	−3·67	−1·57	+2·87	+ 6·61	+3·00	−2·82	− 5·38	0·00	0·00	0·00	+0·01	+0·71	+1·044	−0·128	287
321	21	398·423	397·096	1	3·98	+3·14	+0·25	−2·89	−11·15	−0·62	+0·09	+11·16	0·00	+0·02	+0·04	0·00	+0·53	−1·327	+0·005	321
335	22	503·005	501·873	1	5·03	+4·68	+2·03	−3·82	−10·38	+1·21	−4·63	+10·21	−0·02	+0·02	+0·05	+0·02	+0·46	−1·132	+0·022	335
353	24	264·324	263·755	1	2·64	+3·61	−0·69	−3·66	− 6·17	+1·12	−2·21	+ 3·43	−0·04	−0·09	+0·07	+0·16	−0·20	−0·569	+0·054	353
365	25	288·813	288·247	1	2·89	+3·33	+1·26	−2·86	− 5·50	−0·34	+1·95	+ 0·26	+0·02	−0·09	+0·02	+0·16	−0·19	−0·566	+0·034	365
391	27	640·212	639·649	1	6·40	+5·49	−0·83	−5·52	− 7·75	−3·47	+5·31	− 7·50	+0·01	0·00	−0·01	+0·02	+0·47	−0·563	+0·029	391
403	28	622·083	621·761	1	6·22	+3·27	−0·22	−3·23	− 9·57	−3·12	+1·17	− 9·78	0·00	0·00	+0·01	0·00	+0·62	−0·322	−0·210	403
415	30	323·109	323·199	¼	3·23	−6·13	−0·03	+5·99	+10·15	−0·76	+6·09	+ 9·95	+0·08	0·00	+0·13	+0·04	+0·22	+0·090	+0·123	415
417	Sept. 1	894·587	894·943	2	8·94	−3·07	−0·76	+2·87	+ 6·99	−1·45	−0·85	+ 5·34	0·00	+0·02	+0·02	+0·03	+0·74	+0·356	−0·096	417
421	2	803·844	804·575	½	8·04	−5·14	−1·96	+4·73	+ 5·58	−0·01	−5·26	+ 1·46	−0·01	+0·01	0·00	+0·03	+0·55	+0·731	−0·116	421
449	3	471·861	472·519	½	4·72	−5·81	+0·40	+5·80	+ 5·58	+2·11	−5·71	− 2·85	−0·02	0·00	0·00	+0·04	+0·09	+0·658	+0·035	449
453	4	129·108	130·122	1	1·29	−3·18	+0·08	+3·16	+ 7·33	+3·65	−0·93	− 7·07	−0·02	−0·07	−0·07	+0·11	−0·27	+1·014	−0·073	453
461	6	70·908	71·999	1	0·71	−5·96	−1·24	+5·77	+10·48	+2·36	+6·07	−10·68	−0·04	+0·01	+0·08	−0·03	−0·02	+1·091	+0·068	461
489	7	127·613	128·669	1	1·28	−4·87	+0·94	+4·95	+10·74	+0·26	+4·51	−10·70	−0·10	−0·07	+0·22	+0·01	+0·29	+1·056	+0·088	489
505	8	27·324	28·270	1	0·27	−2·99	−0·48	+2·93	+ 9·43	−1·34	−0·24	− 9·11	−0·08	−0·98	+0·16	+0·72	+0·34	+0·946	+0·168	505
519	10	749·927	749·216	½	7·50	+5·62	−0·46	−5·63	− 5·59	+0·32	+5·49	+ 1·92	+0·03	+0·01	+0·02	+0·05	+0·48	−0·711	+0·083	519
537	26	848·194	847·598	1	8·48	+3·03	+1·28	−3·18	− 6·34	+1·24	−1·75	+ 4·51	0·00	+0·02	+0·02	+0·03	+0·69	−0·596	−0·008	537
547	26	848·148	847·492	1	8·48	+3·08	+1·38	−3·24	− 6·26	+1·20	−1·96	+ 4·36	0·00	+0·02	+0·02	+0·03	+0·69	−0·656	+0·069	547
559	Oct. 4	319·836	320·164	¼	3·20	−4·74	−1·95	+5·08	+ 6·67	−1·22	+4·92	+ 5·29	+0·07	−0·06	+0·11	+0·13	−0·01	+0·328	−0·039	559

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TABLE XVIII.—*continued.*

SATELLITES III. AND IV.—POSITION-ANGLES, 1902.

Rotation No.	Date.	Computed Position-Angle.	Observed Position-Angle.	Weight.	Coefficients of											O—C.	Residuals.	Rotation No.
					Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	Δe_3	Δe_4	Δp_3	Δq_3	Δp_4	Δq_4			
	1902	° ′ ″	° ′ ″															
10	June 10	71 9 57	71 14 38	1	+0·06	−0·03	−0·04	−0·07	+0·05	+0·05	−0·07	−2·05	−5·36	+ 3·51	+ 9·49	+0″·317	+0″·046	10
18	11	70 57 37	70 58 44	1	+0·02	−0·05	−0·05	−0·04	+0·03	+0·03	−0·03	+2·86	−5·02	− 0·22	+10·18	+0·095	−0·027	18
34	16	250 56 54	251 1 20	1	−0·08	+0·01	+0·04	+0·07	−0·03	−0·08	−0·07	+5·72	+1·19	+ 9·87	+ 3·22	+0·295	+0·267	34
44	19	250 31 58	250 32 6	½	−0·01	0·00	0·00	0·00	−0·03	+0·01	−0·02	−5·57	+1·97	+ 1·52	+10·26	+0·028	−0·016	44
62	24	249 33 42	249 19 15	½	+0·05	−0·02	−0·04	−0·20	0·00	+0·04	−0·19	+3·56	+4·80	−10·23	− 2·05	−0·694	−0·119	62
76	July 1	250 16 5	251 34 53	1	+0·01	0·00	0·00	−0·10	−0·03	+0·01	+0·10	+4·47	+4·11	+10·19	− 3·62	+0·397	+0·117	76
92	7	250 59 28	251 1 9	1	−0·04	0·00	+0·02	0·00	−0·02	−0·04	−0·02	+5·92	−1·66	− 3·57	+10·17	+0·239	−0·529	92
114	8	250 45 34	250 42 28	1	−0·04	+0·04	+0·05	+0·02	+0·02	−0·05	+0·03	+4·10	+4·61	− 7·89	+ 8·23	−0·617	−0·146	114
122	14	71 5 0	71 3 56	1	+0·03	0·00	−0·02	0·00	+0·02	+0·03	−0·02	−5·57	+2·68	+ 1·76	+10·85	−0·240	+0·233	122
134	15	71 16 15	71 13 10	1	+0·05	−0·01	−0·04	0·00	+0·02	+0·05	−0·02	−5·61	−2·72	− 2·43	+10·77	−0·410	+0·309	134
174	20	64 47 8	64 0 51	1	−0·34	−0·06	+0·21	+1·15	−0·25	−0·11	−1·09	−0·84	+6·20	− 9·66	− 5·43	−0·432	−0·239	174
178	20	63 46 14	62 30 7	1	−0·40	−0·08	+0·25	+1·28	−0·30	−0·14	−1·25	−0·93	+6·17	− 9·60	− 5·48	−0·613	−0·037	178
192	24	251 18 43	251 17 25	1	−0·01	0·00	0·00	−0·03	−0·03	0·00	−0·04	−3·04	+5·55	− 4·76	+ 9·95	−0·330	−0·221	192
196	26	71 14 15	73 57 9	1	0·00	0·00	0·00	+0·06	+0·03	0·00	+0·07	+5·09	+3·79	+10·23	− 4·10	+0·555	−0·013	196
206	26	71 19 25	72 17 33	1	+0·01	0·00	−0·02	+0·05	+0·02	−0·02	+0·05	+4·96	+3·96	+10·29	− 3·94	+0·514	+0·027	206
220	Aug. 3	71 33 21	71 28 45	1	0·00	0·00	0·00	+0·06	+0·03	0·00	−0·07	+1·44	+6·19	− 9·75	+ 5·67	−0·877	−0·018	220
222	3	71 33 18	71 27 27	1	0·00	0·00	0·00	+0·06	+0 03	0·00	−0·07	+1·40	+6·21	− 9·77	+ 5·64	−1·111	+0·213	222
256	7	251 56 38	251 58 40	½	−0·02	0·00	+0·02	0·00	−0·02	0·00	−0·01	−1·11	+6·25	+ 6·28	+ 9·28	+0·536	−0·269	256
258	7	251 56 38	251 57 48	½	−0·02	0·00	+0·02	0·00	−0·02	0·00	−0·01	−1·17	+6·24	+ 6·24	+ 9·30	+0·308	−0·112	258
264	16	72 26 53	72 26 8	½	+0·05	0·00	−0·05	−0·01	+0·03	−0·05	−0·03	+6·25	+1·11	+ 3·71	+10·49	−0·155	+0·173	264
278	17	72 19 16	72 16 48	½	+0·02	−0·01	−0·02	+0·01	+0·04	−0·02	−0·04	+3·07	+5·54	− 0·45	+11·15	−0·693	+0·268	278
280	17	72 19 11	72 16 46	½	+0·02	−0·01	−0·02	+0·01	+0·04	−0·02	−0·04	+2·97	+5·60	− 0·54	+11·15	−0·680	+0·254	280
284	18	72 16 31	72 14 47	1	+0·02	+0·02	−0·01	0·00	−0·02	+0·03	+0·01	−2·21	+5·92	− 4·44	+10·27	−0·450	−0·099	284
288	18	72 16 28	72 14 21	1	+0·02	+0·03	−0·01	0·00	−0·02	+0·03	+0·01	−2·34	+5·86	− 4·53	+10·23	−0·545	−0·007	288
322	21	252 31 57	252 36 26	1	−0·02	+0·01	+0·02	−0·01	−0·02	−0·01	+0·01	+0·66	+6·26	+11·18	− 0·36	+0·520	+0·003	322
336	22	252 36 8	252 41 17	1	−0·03	0·00	+0·02	0·00	−0·02	+0·02	0·00	−4·21	+4·69	+10·58	+ 3·53	+0·755	−0·208	336
354	24	253 19 37	253 27 59	1	−0·07	+0·02	+0·07	+0 07	−0·04	+0·05	−0·07	−2·78	−5·64	+ 4·44	+10·11	+0·642	−0·197	354
366	25	253 17 47	253 22 0	1	−0·05	0·00	+0·05	+0·06	−0·03	−0·02	−0·04	+1·35	−6·12	+ 1·35	+10·91	+0·355	−0·132	366
392	27	252 39 16	252 36 45	1	−0·02	+0·01	+0·03	−0·03	−0·04	−0·03	−0·05	+5·61	+2·70	− 6·64	+ 8·64	−0·469	−0·162	392
404	28	252 29 10	252 24 5	1	0·00	+0·01	0·00	−0·07	−0·04	−0·01	−0·08	+1·80	+5·96	− 9·23	+ 5·74	−0·921	+0·051	404
416	30	73 20 52	73 26 22	¼	+0·09	0·00	−0·09	−0·08	+0·03	−0·09	−0·08	+6·20	+0·66	+10·36	+ 3·21	+0·522	+0·038	416
418	Sept. 1	72 54 28	72 55 35	1	+0·02	−0·01	−0·02	0·00	+0·03	−0·01	−0·02	−0·16	+6·20	+ 6·37	+ 8·81	+0·291	−0·135	418
422	2	72 54 38	72 54 27	½	+0·02	0·00	−0·02	0·00	+0·04	+0·02	−0·04	−4·85	+3·81	+ 2·67	+10·55	−0·043	+0·041	422
450	3	73 0 31	72 58 5	½	+0·04	−0·01	−0·04	0·00	+0·04	+0·04	−0·04	−5·93	−1·63	− 1·62	+10·79	−0·334	+0·178	450
454	4	73 28 40	73 17 31	1	+0·05	−0·02	−0·05	−0·04	0·00	+0·03	+0·02	−1·64	−5·93	− 6·04	+ 9·10	−0·419	+0·167	454
462	6	72 25 35	71 55 17	1	−0·04	−0·02	+0·03	+0·14	+0·05	+0·04	−0·14	+5·91	−1·73	−10·33	+ 3·51	−0·625	−0·051	462
490	7	71 43 20	71 17 17	1	−0·08	0·00	+0·08	+0·27	+0·03	+0·07	−0·27	+4·98	+3·60	−10·85	− 0·64	−0·966	+0·124	490
506	8	63 40 38	62 54 30	1	−0·48	−0·10	+0·47	+1·58	−0·20	−0·06	−1·52	+0·50	+6·02	− 0·97	− 4·41	−0·366	+0·309	506
520	10	253 17 51	253 19 33	½	−0·05	+0·01	+0·05	+0·01	−0·04	−0·05	−0·04	+5·77	+1·86	+ 3·22	+10·20	+0·371	−0·230	520
538	26	253 26 48	253 27 35	1	−0·02	+0·01	+0·01	0·00	−0·04	−0·01	−0·03	−0·96	+5·75	+ 5·76	+ 8·52	+0·193	−0·056	538
548	26	253 26 36	253 27 11	1	−0·02	+0·01	+0·01	0·00	−0·04	0·00	−0·03	−1·17	+5·72	+ 5·61	+ 8·61	+0·144	−0·016	548
560	Oct. 4	74 13 53	74 14 49	¼	+0·08	+0·02	−0·09	−0·09	+0·05	−0·08	−0·10	+4·46	−3·56	+ 6·43	+ 7·64	+0·087	+0·188	560

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CHAPTER V.

Solution of Equations of Condition.

In order to find the values of the unknowns from the equations of condition, the only course is to treat them by the method of least squares. It is especially in cases like the present where several of the unknowns are not well separated from one another, that the method of least squares is of most value: when the unknowns are well separated, other less laborious methods of solution will lead to practically the same results.

Normal equations were therefore formed from the foregoing equations of condition in the usual way.

The equations were multiplied by the square roots of their weights to reduce them all to weight unity. The system of weighting adopted was in general based on the quality of the "seeing" and weights were assigned as follows.

Double weight was given to two equations in 1902, namely numbers 191 and 417. They represent observations of large distances of III–IV, which are important for the determination of the mass: twice the usual number of pointings had therefore been made in each case.

A weight of one half was assigned

(1) Whenever the steadiness of the images S was estimated at 3–4 or 4 and their definition I was not worse than 2–3 (see pp. 23–8).

(2) To the first three nights' observations in 1901: these were the first observations made by the observer with a heliometer.

A weight of one quarter was assigned

(1) Whenever the seeing was such that I and S were both 3–4: when the seeing was worse than this, observation was altogether impossible.

(2) To the observations made on August 23rd, 1901, when it clouded up before observations of the standards could be secured and S was 3–4.

To all other observations weight unity has been assigned.*

For convenience in multiplying the equations by the square roots of the weights, a weight of one-half was taken as 0·49.

It seems quite open to question whether any useful purpose is served by adopting a system of weighting such as the above which is based on the quality of the definition; for experience seems to show that on nights of bad definition the observer, being

* See page 215.

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conscious of the unusual difficulty of making good observations, takes unusual pains to make good pointings. Nights of especially good seeing seem to make observation appear too easy and tempt the observer to relax the concentration of his thought and attention; thus it will often be found that the largest residuals occur on good nights.

In forming the normals it was found convenient to drop the decimal point both in the coefficients and the absolute terms and to perform all the multiplications in full, *i.e.*, to reject no figures in the products. By retaining all the figures the labour was increased but the checking of the work was very much more satisfactory, since all the checks were rigorously applicable up to the last figure. In this, the most laborious part of the whole work, Mr V. A. Löwinger gave me much valued assistance; and I am further indebted to a grant from the Government Grant Committee of the Royal Society for enabling me to employ the services of Mr T. Elliott of Newcastle-on-Tyne, who undertook and successfully carried out the formation of most of the normals for 1902.

The normals are printed on the pages following. The normal for each unknown from every pair of satellites in which it occurs is given with the name of the pair printed opposite it: beneath is the sum of the normals for that unknown, first from the distance equations called S, then from the position-angle equations called P, and lastly from both together. The last are called the combined normals and are denoted by C. The additions were all made and rigorously checked before the two unnecessary figures were rejected.

Before the general solution was started, the normals were separately solved for every pair of satellites, for it was of interest to see how far the values of the unknowns from the different pairs confirmed one another, and it was also hoped that some errors might be detected by an examination of the residuals from these preliminary solutions. The distance normals and the position-angle normals were solved independently; in the first Δp_i and Δq_i were omitted, and in the second they were the only terms retained. In a few cases where the coefficients of Δp_i and Δq_i in the distance normals were large or where the coefficients of Δh_i and Δk_i in the position-angle normals were large, the equation in question was omitted: the reference numbers of the omitted equations are given in the tables on pp. 156–7, which contain the values of the unknowns from these solutions, all (excepting μ and ΔJ) being expressed in decimal parts of a degree. It was not considered worth the labour to determine the weights and therefore the probable errors of the unknowns cannot be stated. The probable error given for each pair of satellites is that of an observation of weight unity; this was derived in the usual way from the sum of the squares of the residuals.

It should be mentioned that a few small errors were discovered by means of these preliminary solutions and that the values of the unknowns have not been corrected for them, since the necessary corrections would in all cases be quite immaterial.

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TABLE XIX.—Normal

		μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	$\Delta \epsilon_1$
μ	I.–II.	+ 76·82	+ 8·33	+ 10·22	− 14·76	− 16·87	− 4·09	− 9·75						+ 2·93
	I.–III.	+ 172·37	− 12·75	− 23·33	+ 23·03				+ 66·44	+ 47·00	− 67·22			− 14·87
	I.–IV.	+ 369·16	− 58·28	− 53·13	+ 70·10							+ 215·73	+ 228·44	− 11·80
	II.–III.	+ 203·51				− 39·60	− 20·04	− 31·76	+ 57·40	+ 16·24	− 33·50			
	II.–IV.	+ 391·10				− 79·20	− 58·09	− 89·16				+ 182·59	+ 182·22	
	III.–IV.	+ 446·41							− 115·36	− 115·87	+ 149·77	+ 116·70	+ 186·70	
	S_1	+1659·36	− 62·70	− 66·24	+ 78·37	− 135·67	− 82·22	− 130·66	+ 8·47	− 52·62	+ 49·05	+ 515·03	+ 597·37	− 23·74
Δh_1	I. II.		+ 36·73	+ 25·85	− 35·59	− 46·55	− 46·94	− 58·83						− 15·27
	I.–III.		+ 35·32	+ 25·54	− 34·54				− 73·66	− 64·08	+ 82·05			− 5·46
	I.–IV.		+ 36·24	+ 27·15	− 36·60							− 150·91	− 137·19	− 8·98
	S_1		+108·29	+ 78·54	−106·72	− 46·55	− 46·94	− 58·83	− 73·66	− 64·08	+ 82·05	− 150·91	− 137·19	− 29·70
	I.–II.		+ 0·50	+ 0·19	− 0·44	− 0·71	− 0·71	− 0·98						− 0·30
	I.–III.		+ 0·93	+ 0·91	− 1·02				− 2·57	− 0·36	+ 3·02			− 1·10
	I.–IV.		+ 0·33	+ 0·20	− 0·20							− 2·10	− 1·80	− 0·11
	P_1		+ 1·76	+ 1·30	− 1·66	− 0·71	− 0·71	− 0·98	− 2·57	− 0·36	+ 3·02	− 2·10	− 1·80	− 1·51
	C_1		+110·05	+ 79·84	− 108·38	− 47·26	− 47·65	− 59·81	− 76·23	− 64·44	+ 85·07	− 153·01	− 138·99	− 31·21
Δk_1	I.–II.			+ 22·60	− 29·12	− 35·37	− 34·72	− 44·24						− 12·69
	I.–III.			+ 22·71	− 29·17				− 57·26	− 49·27	+ 65·39			− 4·47
	I.–IV.			+ 25·13	− 32·07							− 127·09	− 109·46	− 7·48
	S_1			+ 70·45	− 90·36	− 35·37	− 34·72	− 44·24	− 57·26	− 49·27	+ 65·39	− 127·09	− 109·46	− 24·64
	I.–II.			+ 0·18	− 0·26	− 0·27	− 0·38	− 0·50						− 0·12
	I.–III.			+ 1·20	− 1·31				− 2·78	− 1·64	+ 3·74			− 1·33
	I.–IV.			+ 0·31	− 0·30							− 1·63	− 2·13	− 0·17
	P_1			+ 1·70	− 1·87	− 0·27	− 0·38	− 0·50	− 2·78	− 1·64	+ 3·74	− 1·63	− 2·13	− 1·62
	C_1			+ 72·15	− 92·24	− 35·64	− 35·10	− 44·74	− 60·04	− 50·91	+ 69·12	− 128·72	− 111·59	− 26·26
Δx_1	I.–II.				+ 39·88	+ 47·88	+ 47·26	+ 61·52						+ 16·28
	I.–III.				+ 39·34				+ 74·79	+ 65·28	− 88·08			+ 6·72
	I.–IV.				+ 42·89							+ 166·89	+ 141·51	+ 10·34
	S_1				+ 122·11	+ 47·88	+ 47·26	+ 61·51	+ 74·79	+ 65·28	− 88·08	+ 166·89	+ 141·51	+ 33·35
	I.–II.				+ 0·48	+ 0·62	+ 0·73	+ 0·98						+ 0·28
	I.–III.				+ 1·45				+ 3·14	+ 1·97	− 4·16			+ 1·40
	I.–IV.				+ 0·34							+ 1·68	+ 2·11	+ 0·16
	P_1				+ 2·28	+ 0·62	+ 0·73	+ 0·98	+ 3·14	+ 1·97	− 4·16	+ 1·68	+ 2·11	+ 1·84
	C_1				+ 124·38	+ 48·50	+ 47·99	+ 62·50	+ 77·94	+ 67·26	− 92·23	+ 168·57	+ 143·62	+ 35·19
Δh_2	I.–II.					+ 75·97	+ 63·86	+ 82·94						+ 4·61
	II.–III.					+ 73·94	+ 57·40	+ 78·17	− 105·21	− 86·40	+ 114·64			
	II.–IV.					+ 76·32	+ 60·59	+ 82·88				− 216·78	− 193·87	
	S_1					+ 226·23	+ 181·85	+ 244·00	− 105·21	− 86·40	+ 114·64	− 216·78	− 193·87	+ 4·61
	I.–II.					+ 1·14	+ 0·92	+ 1·38						+ 0·24
	II.–III.					+ 0·61	+ 0·30	+ 0·47	− 1·02	− 0·51	+ 0·82			
	II.–IV.					+ 1·27	+ 0·86	+ 1·20				− 4·74	− 3·52	
	P_1					+ 3·01	+ 2·08	+ 3·05	− 1·02	− 0·51	+ 0·82	− 4·74	− 3·52	+ 0·24
	C_1					+ 229·24	+ 183·93	+ 247·05	− 106 23	− 86·91	+ 115·46	− 221·53	− 197·39	+ 4·86

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EQUATIONS, 1901.

$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\Delta\epsilon_4$	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ	n		
− 8·18			+ 1·06	− 0·71	+ 1·12	− 2·26					+ 40·20	− 0·732	I.–II.	μ
	+ 14·17		+ 0·82	− 0·06			+ 2·87	− 3·42			+ 41·32	− 0·756	I.–III.	
		+ 37·81	+ 1·97	+ 0·27					+ 17·73	− 12·11	+ 33·28	+ 22·437	I.–IV.	
+ 32·72	+ 20·87				+ 1·86	− 2·08	+ 4·40	− 2·33			+ 40·70	+ 0·608	II.–III.	
+ 17·10		+ 11·86			+ 3·03	− 1·21			+ 16·79	− 11·44	+ 37·16	+ 10·718	II.–IV.	
	− 56·22	+ 80·14					+ 5·01	− 1·28	+ 13·69	− 10·12	+ 31·46	+ 23·113	III.–IV.	
+ 41·64	− 21·18	+ 129·82	+ 3·85	− 0·50	+ 6·01	− 5·55	+ 12·27	− 7·03	+ 48·22	− 33·67	+ 224·11	+ 55·388	S_1	
− 4·23			+ 0·30	− 1·20	− 0·12	+ 0·15					+ 3·08	− 7·604	I.–II.	Δh_1
	− 35·82		+ 0·98	− 0·27			+ 0·64	− 3·09			− 3·19	+ 2·498	I.–III.	
		+ 13·21	+ 0·16	− 0·02					+ 0·10	− 0·96	− 4·51	− 18·176	I.–IV.	
− 4·23	− 35·82	+ 13·21	+ 1·44	− 1·49	− 0·12	+ 0·15	+ 0·64	− 3·09	+ 0·10	− 0·96	− 4·63	− 23·281	S_1	
− 0·70			− 0·23	+ 0·89	− 0·26	− 0·46					− 0·16	− 0·127	I.–II.	
	+ 0·35		− 0·79	+ 0·30			+ 0·81	+ 3·12			+ 0·14	− 0·427	I.–III.	
		+ 1·05	+ 0·04	− 0·03					+ 1·21	+ 4·22	+ 0·12	− 0·434	I.–IV.	
− 0·70	+ 0·35	+ 1·05	− 0·98	+ 1·16	− 0·26	− 0·46	+ 0·81	+ 3·12	+ 1·21	+ 4·22	+ 0·10	− 0·989	P_1	
− 4·93	− 35·47	+ 14·27	+ 0·46	− 0·33	− 0·38	− 0·31	+ 1·44	+ 0·03	+ 1·31	+ 3·26	− 4·53	− 24·270	C_1	
− 0·27			+ 0·55	− 1·05	+ 0·41	− 0·36					+ 5·12	− 5·928	I.–II.	Δk_1
	− 19·80		+ 0·61	+ 0·34			+ 1·28	− 3·15			− 5·10	+ 0·948	I.–III.	
		+ 17·09	− 0·04	− 0·38					− 1·04	+ 0·80	− 3·64	− 16·070	I.–IV.	
− 0·27	− 19·80	+ 17·09	+ 1·13	− 1·08	+ 0·41	− 0·36	+ 1·28	− 3·15	− 1·04	+ 0·80	− 3·61	− 21·050	S_1	
− 0·19			− 0·57	+ 1·58	+ 0·17	+ 0·84					− 0·07	− 0·382	I.–II.	
	+ 2·04		− 0·78	− 0·42			− 3·06	+ 2·08			+ 0·28	− 0·380	I.–III.	
		+ 1·74	− 0·16	+ 0·40					− 0·39	− 4·89	+ 0·16	− 0·156	I.–IV.	
− 0·19	+ 2·04	+ 1·74	− 1·52	+ 1·57	+ 0·17	+ 0·84	− 3·06	+ 2·08	− 0·39	− 4·89	+ 0·37	− 0·917	P_1	
− 0·46	− 17·76	+ 18·83	− 0·39	+ 0·48	+ 0·58	+ 0·48	− 1·78	− 1·08	− 1·43	− 4·09	− 3·25	− 21·967	C_1	
+ 3·08			− 0·42	+ 1·50	− 0·31	+ 0·02					− 6·64	+ 8·301	I.–II.	Δx_1
	+ 24·78		− 0·52	− 0·47			− 0·90	+ 3·21			+ 6·08	− 1·122	I.–III.	
		− 15·32	+ 0·25	+ 0·56					+ 2·01	− 1·60	+ 5·39	+ 20·503	I.–IV.	
+ 3·08	+ 24·78	− 15·32	− 0·69	+ 1·60	− 0·31	+ 0·02	− 0·90	+ 3·21	+ 2·01	− 1·60	+ 4·83	+ 27·682	S_1	
+ 0·56			+ 0·46	− 1·80	+ 0·00	− 0·21					+ 0·15	+ 0·373	I.–II.	
	− 2·48		+ 0·57	+ 0·45			+ 2·17	− 2·21			− 0·33	+ 0·393	I.–III.	
		− 1·91	+ 0·37	− 0·69					+ 1·14	+ 2·96	− 0·17	+ 0·322	I.–IV.	
+ 0·56	− 2·48	− 1·91	+ 1·40	− 2·03	+ 0·00	− 0·21	+ 2·17	− 2·21	+ 1·14	+ 2·96	− 0·34	+ 1·087	P_1	
+ 3·64	+ 22·30	− 17·22	+ 0·71	− 0·43	− 0·31	− 0·19	+ 1·27	+ 1·00	+ 3·15	+ 1·36	+ 4·48	+ 28·769	C_1	
− 10·31			− 0·12	+ 1·75	− 0·37	− 1·58					− 7·85	+ 7·206	I.–II.	Δh_2
+ 3·66	− 36·39				− 1·89	− 2·10	+ 2·48	+ 0·40			− 10·65	+ 9·757	II.–III.	
+ 0·61		+ 62·32			+ 3·23	+ 1·29			+ 4·51	− 5·82	− 5·12	− 15·851	II.–IV.	
− 6·04	− 36·39	+ 62·32	− 0·12	+ 1·75	+ 0·97	− 2·39	+ 2·48	+ 0·40	+ 4·51	− 5·82	− 23·62	+ 1·112	S_1	
+ 0·67			+ 0·56	− 1·00	+ 1·27	+ 2·20					+ 0·22	− 0·054	I.–II.	
− 0·32	− 0·15				+ 4·30	+ 0·51	+ 3·96	− 2·53			+ 0·05	+ 0·426	II.–III.	
− 0·08		+ 2·40			− 2·62	− 0·74			− 3·60	+ 11·93	+ 0·08	− 1·052	II.–IV.	
+ 0·27	− 0·15	+ 2·40	+ 0·56	− 1·00	+ 2·95	+ 1·97	+ 3·96	− 2·53	− 3·60	+ 11·93	+ 0·35	− 0·681	P_1	
− 5·77	− 36·54	+ 64·72	+ 0·44	+ 0·75	+ 3·92	− 0·42	+ 6·43	− 2·13	+ 0·91	+ 6·11	− 23·27	+ 0·431	C_1	

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TABLE XIX.—Normal

		μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	$\Delta \epsilon_1$
Δk_2	I.-II.						+ 71·37	+ 85·97						+ 28·94
	II.-III.						+ 61·30	+ 75·94	− 85·68	− 78·18	+ 100·45			
	II.-IV.						+ 66·40	+ 81·87				− 195·42	− 174·10	
	S_1						+ 199·08	+ 243·78	− 85·68	− 78·18	+ 100·45	− 195·42	− 174·10	+ 28·94
	I.-II.						+ 1·28	+ 1·58						+ 0·64
	II.-III.						+ 0·35	+ 0·39	− 0·54	− 0·58	+ 0·67			
	II.-IV.						+ 0·94	+ 1·15				− 3·79	− 3·22	
	P_1						+ 2·57	+ 3·13	− 0·54	− 0·58	+ 0·67	− 3·79	− 3·22	+ 0·64
	C_1						+ 201·65	+ 246·91	− 86·23	− 78·76	+ 101·12	− 199·21	− 177·33	+ 29·58
Δx_2	I.-II.							+ 109·08						+ 26·32
	II.-III.							+ 100·34	− 114·24	− 99·23	+ 133·90			
	II.-IV.							+ 108·42				− 257·71	− 221·07	
	S_1							+ 317·83	− 114·24	− 99·23	+ 133·90	− 257·71	− 221·07	+ 26·32
	I.-II.							+ 2·13						+ 0·65
	II.-III.							+ 0·54	− 0·85	− 0·60	+ 0·86			
	II.-IV.							+ 1·54				− 5·14	− 4·04	
	P_1							+ 4·20	− 0·85	− 0·60	+ 0·86	− 5·14	− 4·04	+ 0·65
	C_1							+ 322·03	− 115·08	− 99·83	+ 134·76	− 262·85	− 225·11	+ 26·96
Δh_3	I.-III.								+ 192·20	+ 142·09	− 194·24			− 1·14
	II.-III.								+ 167·94	+ 127·97	− 171·23			
	III.-IV.								+ 166·37	+ 136·56	− 184·20	− 287·43	− 275·03	
	S_1								+ 526·51	+ 406·62	− 549·67	− 287·43	− 275·03	− 1·14
	I.-III.								+ 7·75	+ 3·01	− 9·15			+ 3·30
	II.-III.								+ 1·82	+ 0·76	− 1·33			
	III.-IV.								+ 3·08	+ 1·02	− 2·63	− 6·56	− 2·72	
	P_1								+ 12·66	+ 4·79	− 13·11	− 6·56	− 2·72	+ 3·30
	C_1								+ 539·17	+ 411·41	− 562·78	− 294·00	− 277·75	+ 2·16
Δk_3	I.-III.									+ 141·16	− 174·18			+ 8·07
	II.-III.									+ 126·40	− 155·23			
	III.-IV.									+ 149·85	− 182·18	− 271·94	− 252·82	
	S_1									+ 417·41	− 511·59	− 271·94	− 252·82	+ 8·07
	I.-III.									+ 9·84	− 5·11			+ 0·86
	II.-III.									+ 1·45	− 1·48			
	III.-IV.									+ 0·98	− 1·37	− 2·25	− 1·88	
	P_1									+ 12·27	− 7·95	− 2·25	− 1·88	+ 0·86
	C_1									+ 429·68	− 519·54	− 274·19	− 254·70	+ 8·92
Δx_3	I.-III.										+ 230·16			− 4·37
	II.-III.										+ 202·90			
	III.-IV.										+ 236·44	+ 353·34	+ 326·22	
	S_1										+ 669·50	+ 353·34	+ 326·22	− 4·37
	I.-III.										+ 12·04			− 4·10
	II.-III.										+ 1·80			
	III.-IV.										+ 2·79	+ 5·72	+ 3·00	
	P_1										+ 16·63	+ 5·72	+ 3·00	− 4·10
	C_1										+ 686·13	+ 359·06	+ 329·22	− 8·48

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$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\Delta\epsilon_4$	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ	n		
+ 3·52			+ 0·29	+ 1·72	+ 1·21	− 1·60					+ 2·35	+ 11·793	I.–II.	Δk_2
+ 22·38	− 22·47				− 0·58	− 2·68	+ 4·23	− 0·36			− 7·91	+ 10·041	II.–III.	
+ 12·86		+ 28·26			+ 1·76	+ 0·24			+ 7·11	− 6·30	− 4·44	− 12·245	II.–IV.	
+ 38·75	− 22·47	+ 28·26	+ 0·29	+ 1·72	+ 2·38	− 4·03	+ 4·23	− 0·36	+ 7·11	− 6·30	− 9·99	+ 9·589	S_1	
+ 1·27			+ 0·07	− 2·69	− 1·79	+ 0·81					+ 0·25	+ 0·123	I.–II.	
− 0·23	− 0·30				+ 1·43	− 2·44	+ 0·72	− 2·42			+ 0·04	+ 0·789	II.–III.	
− 0·10		+ 1·79			− 1·95	− 0·67			− 8·20	+ 2·14	+ 0·03	− 0·775	II.–IV.	
+ 0·95	− 0·30	+ 1·79	+ 0·07	− 2·69	− 2·31	− 2·30	+ 0·72	− 2·42	− 8·20	+ 2·14	+ 0·33	+ 0·137	P_1	
+ 39·71	− 22·77	+ 30·05	+ 0·36	− 0·96	+ 0·07	− 6·33	+ 4·95	− 2·79	− 1·09	− 4·16	− 9·67	+ 9·726	C_1	
− 1·96			+ 0·31	+ 2·46	+ 0·67	− 2·19					+ 0·32	+ 13·489	I.–II.	Δx_2
+ 20·22	− 31·93				− 0·57	− 2·91	+ 4·84	− 1·10			− 11·38	+ 12·214	II.–III.	
+ 11·74		+ 35·35			+ 1·96	+ 0·75			+ 7·41	− 7·13	− 7·16	− 16·124	II.–IV.	
+ 30·00	− 31·93	+ 35·35	+ 0·31	+ 2·46	+ 2·06	− 4·35	+ 4·84	− 1·10	+ 7·41	− 7·13	− 18·23	+ 9·579	S_1	
+ 1·37			+ 0·13	− 3·05	− 0·67	+ 2·00					+ 0·34	+ 0·148	I.–II.	
− 0·35	− 0·41				+ 2·47	− 1·89	+ 1·87	− 1·78			+ 0·05	+ 0·760	II.–III.	
− 0·17		+ 2·64			− 1·80	− 0·72			− 7·73	+ 6·27	+ 0·07	− 0·997	II.–IV.	
+ 0·85	− 0·41	+ 2·64	+ 0·13	− 3·05	+ 0·00	− 0·61	+ 1·87	− 1·78	− 7·73	+ 6·27	+ 0·46	− 0·089	P_1	
+ 30·85	− 32·34	+ 37·99	+ 0·44	− 0·59	+ 2·06	− 4·96	+ 6·71	− 2·87	− 0·31	− 0·86	− 17·77	+ 9·490	C_1	
	+ 105·80		− 1·41	+ 0·20			− 1·22	+ 6·97			+ 14·49	− 6·832	I.–III.	Δh_3
+ 16·79	+ 55·75				+ 1·54	+ 5·00	− 6·91	+ 2·53			+ 15·50	− 14·138	II.–III.	
	+ 55·26	+ 113·13					+ 5·12	+ 2·95	− 9·28	+ 6·03	− 1·24	− 43·923	III.–IV.	
+ 16·79	+ 216·80	+ 113·13	− 1·41	+ 0·20	+ 1·54	+ 5·00	− 3·01	+ 12·44	− 9·28	+ 6·03	+ 28·75	− 64·893	S_1	
	− 3·69		+ 1·08	+ 0·06			− 3·66	− 8·30			− 0·63	+ 1·202	I.–III.	
+ 0·75	+ 0·69				− 7·16	− 1·44	− 6·19	+ 2·50			− 0·07	− 0·506	II.–III.	
	+ 2·31	− 3·30					− 8·23	− 4·14	+ 13·55	+ 2·61	+ 0·11	+ 0·165	III.–IV.	
+ 0·75	− 0·69	− 3·30	+ 1·08	+ 0·06	− 7·16	− 1·44	− 18·07	− 9·93	+ 13·55	+ 2·61	− 0·59	+ 0·861	P_1	
+ 17·54	+ 216·11	+ 109·82	− 0·32	+ 0·26	− 5·63	+ 3·56	− 21·08	+ 2·51	+ 4·27	+ 8·64	+ 28·16	− 64·032	C_1	
	+ 47·55		− 1·35	− 0·33			− 3·66	+ 7·43			+ 14·58	− 4·157	I.–III.	Δk_3
− 31·79	+ 33·07				+ 2·00	+ 3·50	− 5·56	− 0·20			+ 5·06	− 14·853	II.–III.	
	+ 18·25	+ 129·57					+ 1·95	+ 1·97	− 2·09	+ 3·02	− 2·55	− 41·217	III.–IV.	
− 31·79	+ 98·87	+ 129·57	− 1·35	− 0·33	+ 2·00	+ 3·50	− 7·27	+ 9·21	− 2·09	+ 3·02	+ 17·09	− 60·226	S_1	
	− 12·28		− 3·38	+ 3·07			− 3·07	+ 6·83			− 1·11	− 0·105	I.–III.	
− 0·08	− 0·39				− 2·70	+ 4·83	+ 0·61	+ 3·16			− 0·11	− 1·564	II.–III.	
	+ 1·06	+ 0·58					+ 0·27	+ 0·07	− 2·34	− 14·61	+ 0·01	+ 0·124	III.–IV.	
− 0·08	− 11·61	+ 0·58	− 3·38	+ 3·07	− 2·70	+ 4·83	− 2·19	+ 10·06	− 2·34	− 14·61	− 1·22	− 1·544	P_1	
− 31·87	+ 87·26	+ 130·15	− 4·73	+ 2·73	− 0·69	+ 8·33	− 9·46	+ 19·27	− 4·43	− 11·59	+ 15·87	− 61·771	C	
	− 74·50		+ 0·90	+ 0·50			+ 2·93	− 7·62			− 21·58	+ 5·467	I.–III.	Δx_3
+ 22·42	− 38·94				− 1·89	− 4·26	+ 7·12	− 1·01			− 10·15	+ 17·548	II.–III.	
	− 34·36	− 138·41					− 5·47	− 3·18	+ 8·58	− 4·91	+ 2·90	+ 52·571	III.–IV.	
+ 22·42	− 147·80	− 138·41	+ 0·90	+ 0·50	− 1·89	− 4·26	+ 4·58	+ 11·81	+ 8·58	− 4·91	− 28·83	+ 75·586	S_1	
	+ 6·13		− 1·43	− 0·27			− 4·40	+ 6·71			+ 0·86	− 1·235	I.–III.	
− 0·22	+ 0·06				+ 4·96	− 3·58	+ 1·58	− 2·63			+ 0·12	+ 1·510	II.–III.	
	− 2·32	+ 1·83					+ 4·98	+ 2·26	− 7·50	+ 10·37	− 0·08	− 0·254	III.–IV.	
− 0·22	+ 3·87	+ 1·83	− 1·43	− 0·27	+ 4·96	− 3·58	+ 2·16	+ 6·33	− 7·50	+ 10·37	+ 0·90	+ 0·020	P_1	
+ 22·20	− 143·93	− 136·58	− 0·53	+ 0·23	+ 3·08	− 7·84	+ 6·74	− 5·49	+ 1·09	+ 5·46	− 27·93	+ 75·606	C_1	

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TABLE XIX.—Normal

		μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	$\Delta \epsilon_1$
Δh_4	I.-IV.											+ 793·10	+ 624·84	+ 33·18
	II.-IV.											+ 727·35	+ 592·39	
	III.-IV.											+ 658·59	+ 519·27	
	S_1											+2179·05	+1736·50	+ 33·18
	I.-IV.											+ 14·68	+ 13·45	+ 0·89
	II.-IV.											+ 19·58	+ 14·36	
	III.-IV.											+ 14·29	+ 5·69	
	P_1											+ 48·55	+ 33·50	+ 0·89
	C_1											+2227·60	+1770·00	+ 34·07
Δk_4	I.-IV.												+ 624·37	+ 20·17
	II.-IV.												+ 605·76	
	III.-IV.												+ 532·06	
	S_1												+1762·19	+ 20·17
	I.-IV.												+ 16·50	+ 0·96
	II.-IV.												+ 14·76	
	III.-IV.												+ 5·12	
	P_1												+ 36·37	+ 0·96
	C_1												+1798·55	+ 21·13
$\Delta \epsilon_1$	I.-II.													+ 45·84
	I.-III.													+ 42·61
	I.-IV.													+ 44·63
	S_1													133·08
	I.-II.													+ 0·55
	I.-III.													+ 1·91
	I.-IV.													+ 0·33
	P_1													+ 2·78
	C_1													+ 135·87
$\Delta \epsilon_2$	I.-II.													
	II.-III.													
	II.-IV.													
	S_1													
	I.-II.													
	II.-III.													
	II.-IV.													
	P_1													
	C_1													
$\Delta \epsilon_3$	I.-III.													
	II.-III.													
	III.-IV.													
	S_1													
	I.-III.													
	II.-III.													
	III.-.IV													
	P_1													
	C_1													

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$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\Delta\epsilon_4$	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ	n		
− 26·80		− 84·33	− 1·33	+ 1·58					− 2·48	+ 3·93	+ 17·99	+ 94·035	I.–IV.	Δh_4
		− 180·82			− 12·23	− 3·40			− 27·34	+ 28·53	+ 11·86	+ 50·559	II.–IV.	
	− 59·36	− 186·37					− 17·28	− 6·46	+ 22·25	− 6·24	− 3·35	+ 86·668	III.–IV.	
− 26·80	− 59·36	− 451·52	− 1·33	+ 1·58	− 12·23	− 3·40	− 17·28	− 6·46	− 7·57	+ 26·22	+ 26·50	+231·263	S_1	
		− 10·47	+ 0·16	− 1·36					− 5·27	− 14·23	− 1·04	+ 3·027	I.–IV.	
+ 0·10		− 12·50			+ 11·22	+ 1·51			+ 22·74	− 37·40	− 0·23	+ 4·815	II.–IV.	
	− 4·98	+ 7·35					+ 20·41	+ 8·75	− 29·33	− 6·49	− 0·23	− 0·151	III.–IV.	
+ 0·10	− 4·98	− 15·62	+ 0·16	− 1·36	+ 11·22	+ 1·51	+ 20·41	+ 8·75	− 11·86	− 58·12	− 1·50	+ 7·691	P_1	
− 26·70	− 64·34	− 467·14	− 1·17	+ 0·22	− 1·01	− 1·89	+ 3·13	+ 2·28	− 19·44	− 31·90	+ 25·00	+238·954	C_1	
		− 193·69	− 2·39	+ 1·40					− 3·87	+ 1·42	+ 12·55	+ 81·669	I.–IV.	Δk_4
+ 3·29		− 227·24			− 12·35	− 8·07			− 24·06	+ 30·95	+ 5·77	+ 49·879	II.–IV.	
	− 79·52	− 251·01					− 1·61	− 2·02	+ 10·29	− 16·76	+ 3·54	+ 79·468	III.–IV.	
+ 3·29	− 79·52	− 671·93	− 2·39	+ 1·40	− 12·35	− 8·07	− 1·61	− 2·02	− 17·64	+ 15·60	+ 21·87	+211·015	S_1	
		− 12·22	+ 2·71	− 0·46					+ 18·07	+ 18·26	− 1·11	+ 2·245	I.–IV.	
+ 0·38		− 10·17			+ 13·49	+ 9·22			+ 29·28	− 12·68	− 0·39	+ 2·379	II.–IV.	
	− 2·14	+ 1·60					− 2·87	+ 0·05	− 1·35	+ 40·67	− 0·04	− 0·863	III.–IV.	
+ 0·38	− 2·14	− 20·79	+ 2·71	− 0·46	+ 13·49	+ 9·22	− 2·87	+ 0·05	+ 45·99	+ 46·25	− 1·55	+ 3·762	P_1	
+ 3·67	− 81·66	− 692·72	+ 0·32	+ 0·94	+ 1·15	+ 1·19	− 4·47	− 1·96	+ 28·36	+ 61·85	+ 20·32	+214·777	C_1	
			+ 0·07	+ 0·27							+ 4·20	+ 10·243	I.–II.	Δe_1
+ 14·89			− 1·68	+ 0·03	+ 1·90	+ 0·68					+ 0·78	+ 5·070	I.–III.	
	− 6·85	+ 37·70	+ 0·15	+ 0·52			− 4·20	+ 6·63	+ 1·94	+ 0·96	+ 1·08	+ 8·203	I.–IV.	
+ 14·89	− 6·85	+ 37·70	− 1·47	+ 0·82	+ 1·90	+ 0·68	− 4·20	+ 6·63	+ 1·94	+ 0·96	+ 6·06	+ 23·516	S_1	
			− 1·53	− 1·50	− 2·41	− 1·26					+ 0·10	+ 0·115	I.–II.	
+ 1·05			+ 0·13	− 1·47							− 0·23	+ 0·725	I.–III.	
	− 1·11						+ 3·77	− 6·30						
		− 0·46	− 1·65	− 2·07					− 4·03	+ 1·22	− 0·04	+ 0·092	I.–IV.	
+ 1·05	− 1·11	− 0·46	− 3·04	− 5·04	− 2·41	− 1·26	+ 3·77	− 6·30	− 4·03	+ 1·22	− 0·16	+ 0·931	P_1	
+ 15·94	− 7·96	+ 37·23	− 4·51	− 4·22	− 0·51	− 0·58	− 0·43	+ 0·33	− 2·09	+ 2·18	+ 5·90	+ 24·448	C_1	
+ 109·73			+ 2·60	+ 1·09	+ 3·99	− 1·45					− 2·38	+ 4·733	I.–II.	Δe_2
+ 100·01	+ 28·48				− 0·97	+ 2·40	− 1·82	+ 5·53			+ 4·32	+ 4·419	II.–III.	
+ 103·99		+ 62·79			− 0·23	− 3·87			+ 15·58	− 2·35	− 2·57	+ 0·713	II.–IV.	
+ 313·73	+ 28·48	+ 62·79	+ 2·60	+ 1·09	+ 2·80	− 2·92	− 1·82	+ 5·53	+ 15·58	− 2·35	− 0·63	+ 9·865	S_1	
+ 2·26			− 2·99	− 1·51	− 7·17	− 1·83					+ 0·23	− 0·802	I.–II.	
+ 0·79	+ 1·36				− 4·31	− 0·63	− 7·06	− 2·59			+ 0·00	− 0·137	II.–III.	
+ 1·31		+ 3·46			− 3·11	+ 0·75			− 19·00	+ 3·51	− 0·05	− 1·649	II.–IV.	
+ 4·35	+ 1·36	+ 3·46	− 2·99	− 1·51	− 14·59	− 1·72	− 7·06	− 2·59	− 19·00	+ 3·51	+ 0·19	− 2·088	P_1	
+ 318·08	+ 29·84	+ 66·25	− 0·39	− 0·42	− 11·79	− 4·63	− 8·88	+ 2·94	− 3·42	+ 1·17	− 0·44	+ 7·777	C_1	
	+ 220·19		− 5·56	+ 4·85			+ 2·03	+ 7·70			+ 8·22	− 13·989	I.–III.	Δe_3
	+ 187·37				− 1·31	+ 6·48	− 4·40	+ 8·35			+ 6·31	− 7·599	II.–III.	
	+ 205·98	+ 36·92					+ 3·76	− 3·43	− 0·98	+ 12·06	− 4·39	− 17·579	III.–IV.	
	+ 613·53	+ 36·92	− 5·56	+ 4·85	− 1·31	+ 6·48	+ 1·40	+ 12·63	− 0·98	+ 12·06	+ 10·13	− 39·168	S_1	
	+ 17·33		+ 6·25	− 5·46			− 12·81	− 19·68			+ 1·55	+ 0·811	I.–III.	
	+ 3·10				− 4·32	+ 1·83	− 18·25	− 13·94			+ 0·02	+ 0·057	II.–III.	
	+ 2·74	− 0·63					− 13·22	− 9·10	− 2·02	− 5·12	+ 0·10	+ 0·010	III.–IV.	
	+ 23·18	− 0·63	+ 6·25	− 5·46	− 4·32	+ 1·83	− 44·28	− 42·72	− 2·02	− 5·12	+ 1·67	+ 0·878	P_1	
	+ 636·71	+ 36·29	+ 0·70	− 0·61	− 5·63	+ 8·30	− 42·88	− 30·09	− 3·00	+ 6·94	+ 11·80	− 38·290	C_1	

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TABLE XIX.—NORMAL

		μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	$\Delta \epsilon_1$
$\Delta \epsilon_4$	I.-IV.													
	II.-IV.													
	III.-IV.													
	S_1													
	I.-IV.													
	II.-IV.													
	III.-IV.													
	P_1													
	C_1													
Δp_1	I.-II.													
	I.-III.													
	I.-IV.													
	S_1													
	I.-II.													
	I.-III.													
	I.-IV.													
	P_1													
	C_1													
Δq_1	I.-II.													
	I.-III.													
	I.-IV.													
	S_1													
	I.-II.													
	I.-III.													
	I.-IV.													
	P_1													
	C_1													
Δp_2	I.-II.													
	II.-III.													
	II.-IV.													
	S_1													
	I.-II.													
	II.-III.													
	II.-IV.													
	P_1													
	C_1													
Δq_2	I.-II.													
	II.-III.													
	II.-IV.													
	S_1													
	I.-II.													
	II.-III.													
	II.-IV.													
	P_1													
	C_1													

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EQUATIONS, 1901.—*continued.*

$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\Delta\epsilon_4$	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ	n		
		+1147·65	− 2·55	− 4·80					+ 28·42	+ 28·40	+ 45·43	− 48·587	I.–IV.	$\Delta\epsilon_4$
		+1093·88			+ 16·05	− 1·73			+ 59·01	− 14·75	+ 28·34	− 50·361	II.–IV.	
		+ 918·73					− 10·03	+ 0·57	+ 70·45	− 12·57	+ 14·55	− 60·100	III.–IV.	
		+3160·26	− 2·55	− 4·80	+ 16·05	− 1·73	− 10·03	+ 0·57	+ 157·88	+ 1·08	+ 88·32	−159·047	S_1	
		+ 30·47	+ 3·99	+ 4·85					− 44·51	− 58·54	+ 2·59	− 3·285	I.–IV.	
		+ 33·51			− 16·94	+ 1·75			− 72·13	− 13·90	+ 0·47	− 7·649	II.–IV.	
		+ 18·12					+ 14·07	+ 0·46	− 87·77	− 18·85	− 0·23	− 3·361	III.–IV.	
		+ 82·10	+ 3·99	+ 4·85	− 16·94	+ 1·75	+ 14·07	+ 0·46	− 204·41	− 91·29	+ 2·81	− 14·295	P_1	
		+3242·36	+ 1·44	+ 0·04	− 0·89	+ 0·02	+ 4·03	+ 1·03	− 46·54	− 90·02	91·15	−173·342	C_1	
			+ 0·80	+ 0·08	+ 0·58	− 1·15					+ 0·73	+ 0·224	I.–II.	Δp_1
			+ 0·82	− 0·58			+ 0·79	− 1·16			− 0·07	+ 0·406	I.–III.	
			+ 0·57	+ 0·48					+ 1·77	− 2·60	+ 0·47	− 0·042	I.–IV.	
			+ 2·20	− 0·03	+ 0·58	− 1·15	+ 0·79	− 1·16	+ 1·77	− 2·60	+ 1·12	+ 0·587	S_1	
			+ 62·95	+ 3·10	+ 17·74	− 4·86					+ 0·15	+ 4·548	I.–II.	
			+ 58·40	+ 9·65			− 10·50	− 16·03			+ 0·72	+ 1·675	I.–III.	
			+ 57·84	+ 11·18					+ 45·57	− 75·89	+ 0·20	+ 3·201	I.–IV.	
			+ 179·20	+ 23·92	+ 17·74	− 4·86	− 10·50	− 16·03	+ 45·57	− 75·89	+ 1·07	+ 9·424	P_1	
			+ 181·39	+ 23·90	+ 18·33	− 6·01	− 9·71	− 17·19	+ 47·34	− 78·49	+ 2·19	+ 10·011	C_1	
				+ 0·21	− 0·09	− 0·14					− 0·02	+ 0·246	I.–II.	Δq_1
				+ 1·89			+ 1·82	− 2·81			+ 0·43	− 1·063	I.–III.	
				+ 0·89					+ 2·36	− 3·44	− 0·02	+ 0·319	I.–IV.	
				+ 2·99	− 0·09	− 0·14	+ 1·82	− 2·81	+ 2·36	− 3·44	+ 0·39	− 0·499	S_1	
				+ 42·77	+ 1·00	+ 20·04					− 0·49	− 6·672	I.–II.	
				+ 48·99			+ 25·25	+ 4·90			− 0·77	− 4·000	I.–III.	
				+ 53·75					+ 36·01	− 17·85	− 0·42	− 1·020	I.–IV.	
				+ 145·51	+ 1·00	+ 20·04	+ 25·25	+ 4·90	+ 36·01	− 17·85	− 1·69	− 11·691	P_1	
				+ 148·50	+ 0·91	+ 19·90	+ 27·08	+ 2·09	+ 38·37	− 21·29	− 1·30	− 12·190	C_1	
					+ 0·75	− 0·79					+ 0·80	+ 0·558	I.–II.	Δp_2
					+ 1·97	− 3·44	+ 5·68	− 3·58			+ 0·33	+ 0·508	II.–III.	
					+ 1·79	+ 0·83			+ 2·33	− 3·68	+ 0·99	− 1·418	II.–IV.	
					+ 4·51	− 3·40	+ 5·68	− 3·58	+ 2·33	− 3·68	+ 2·12	− 0·352	S_1	
					+ 111·72	+ 20·12					− 0·23	+ 8·821	I.–II.	
					+ 117·27	+ 20·07	+ 75·43	+ 21·44			+ 0·26	+ 7·627	II.–III.	
					+ 116·26	+ 25·12			+ 116·47	− 27·79	+ 0·07	+ 16·685	II.–IV.	
					+ 345·25	+ 65·30	+ 75·43	+ 21·44	+ 116·47	− 27·79	+ 0·10	+ 33·133	P_1	
					+ 349·76	+ 61·90	+ 81·11	+ 17·86	+ 118·80	− 31·47	+ 2·22	+ 32·781	C_1	
						+ 1·78					− 1·58	− 0·262	I.–II.	Δq_2
						+ 9·54	− 14·32	+ 7·00			− 0·48	− 2·343	II.–III.	
						+ 2·71			− 1·41	− 2·45	+ 1·32	− 0·907	II.–IV.	
						+ 14·04	− 14·32	+ 7·00	− 1·41	− 2·45	− 0·74	− 3·511	S_1	
						+ 154·78					+ 0·54	− 25·577	I.–II.	
						+ 134·37	− 12·08	− 4·25			− 0·50	− 22·893	II.–III.	
						+ 142·49			+ 26·96	− 13·26	− 1·71	− 21·079	II.–IV.	
						+ 431·63	− 12·08	− 4·25	+ 26·96	− 13·26	− 1·67	− 69·549	P_1	
						+ 445·67	− 26·40	+ 2·75	+ 25·56	− 15·71	− 2·42	− 73·060	C_1	

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TABLE XIX.—Normal

		μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	Δe_1
Δp_3	I.-III.													
	II.-III.													
	III.-IV.													
	S_1													
	I.-III.													
	II.-III.													
	III.-IV.													
	P_1													
	C_1													
Δq_3	I.-III.													
	II.-III.													
	III.-IV.													
	S_1													
	I.-III.													
	II.-III.													
	III.-IV.													
	P_1													
	C_1													
Δp_4	I.-IV.													
	II.-IV.													
	III.-IV.													
	S_1													
	I.-IV.													
	II.-IV.													
	III.-IV.													
	P_1													
	C_1													
Δq_4	I.-IV.													
	II.-IV.													
	III.-IV.													
	S_1													
	I.-IV.													
	II.-IV.													
	III.-IV.													
	P_1													
	C_1													
ΔJ	I.-II.													
	I.-III.													
	I.-IV.													
	II.-III.													
	II.-IV.													
	III.-IV.													
	S_1													
	I.-II.													
	I.-III.													
	I.-IV.													
	II.-III.													
	II.-IV.													
	III.-IV.													
	P_1													
	C_1													

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EQUATIONS, 1901.—*continued.*

$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\Delta\epsilon_4$	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ	n		
							+ 5·93	− 7·92			+ 0·37	− 1·213	I.–III.	Δp_3
							+ 22·10	− 11·20			+ 1·05	+ 3·481	II.–III.	
							+ 4·60	+ 1·64	− 4·86	− 1·78	+ 0·97	− 1·165	III.–IV.	
							+ 32·64	− 17·47	− 4·86	− 1·78	+ 2·39	+ 1·102	S_1	
							+ 334·73	+ 124·46			− 0·35	− 8·035	I.–III.	
							+ 314·43	+ 124·60			+ 0·35	− 0·380	II.–III.	
							+ 273·00	+ 112·72	+ 50·56	− 184·03	− 0·60	+ 3·701	III.–IV.	
							+ 922·17	+ 361·77	+ 50·56	− 184·03	− 0·60	− 4·714	P_1	
							+ 954·81	+ 344·30	45·70	− 185·82	+ 1·79	− 3·611	C_1	
								+ 11·54			− 0·39	+ 1·381	I.–III.	Δq_3
								+ 7·01			− 0·64	− 1·446	II.–III.	
								+ 2·65	− 4·19	− 1·05	+ 0·83	− 0·693	III.–IV.	
								+ 21·21	− 4·19	− 1·05	− 0·20	− 0·758	S_1	
								+ 354·89			− 0·41	− 17·090	I.–III.	
								+ 329·00			+ 0·29	− 14·253	II.–III.	
								+ 382·62	+ 89·73	− 85·31	− 1·29	− 4·286	III.–IV.	
								+1066·51	+ 89·73	− 85·31	− 1·41	− 35·629	P_1	
								+1087·71	+ 85·54	− 86·36	− 1·60	− 36·387	C_1	
									+ 11·99	− 12·74	− 0·61	− 0·784	I.–IV.	Δp_4
									+ 19·10	− 13·70	+ 0·00	− 2·409	II.–IV.	
									+ 17·40	− 2·81	− 0·25	+ 0·154	III.–IV.	
									+ 48·49	− 29·26	− 0·85	− 3·039	S_1	
									+1091·28	− 41·49	+ 1·59	+ 49·937	I.–IV.	
									+ 950·80	− 72·19	− 0·64	+ 60·812	II.–IV.	
									+ 937·86	+ 89·87	+ 1·06	+ 37·223	III.–IV.	
									+2979·95	− 23·82	+ 2·00	+147·972	P_1	
									+3028·44	− 53·07	+ 1·15	+144·933	C_1	
										+ 17·56	+ 2·03	− 0·684	I.–IV.	Δq_4
										+ 16·58	− 1·02	+ 2·115	II.–IV.	
										+ 4·97	− 0·99	− 1·371	III.–IV.	
										+ 39·11	+ 0·02	+ 0·059	S_1	
										+1131·21	− 3·39	− 47·024	I.–IV.	
										+1096·07	+ 1·58	− 39·937	II.–IV.	
										+1096·40	+ 0·48	− 26·596	III.–IV.	
										+3323·68	− 1·32	−113·557	P_1	
										+3362·79	− 1·30	−113·498	C_1	
											+ 24·42	+ 0·317	I.–II.	ΔJ
											+ 20·34	+ 0·040	I.–III.	
											+ 18·79	− 0·112	I.–IV.	
											+ 9·65	− 0·111	II.–III.	
											+ 7·69	− 1·026	II.–IV.	
											+ 3·24	− 0·247	III.–IV.	
											+ 84·13	− 1·139	S_1	
											+ 0·06	− 0·030	I.–II.	
											+ 0·18	− 0·027	I.–III.	
											+ 0·31	− 0·040	I.–IV.	
											+ 0·01	+ 0·120	II.–III.	
											+ 0·12	+ 0·164	II.–IV.	
											+ 0·02	+ 0·079	III.–IV.	
											+ 0·69	+ 0·266	P_1	
											+ 84·82	− 0·873	C_1	

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TABLE XX.—Normal

		μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	$\Delta \epsilon_1$
μ	I.-II.	+ 103·08	+ 13·12	+ 3·10	− 13·58	− 2·73	− 9·97	− 4·38						+ 2·85
	I.-III.	+ 183·72	+ 5·64	+ 4·29	− 3·18				− 28·36	− 7·18	+ 32·84			− 14·79
	I.-IV.	+ 704·56	+ 36·43	+ 17·23	− 35·23							− 152·36	+ 1·28	− 36·26
	II.-III.	+ 321·34				− 36·22	+ 16·10	− 33·33	+ 23·77	+ 21·24	− 1·95			
	II.-IV.	+ 529·23				+ 19·80	+ 37·68	+ 18·51				− 85·94	+ 62·45	
	III.-IV.	+1308·70							+ 6·98	+ 15·62	+ 12·45	− 45·55	+ 24·27	
	S_2	+3150·62	+ 55·19	+ 24·62	− 51·99	− 19·15	+ 43·82	− 19·20	+ 2·39	+ 36·13	+ 43·35	− 283·85	+ 88·00	− 48·20
Δh_1	I.-II.		+ 94·79	+ 11·65	− 78·99	− 144·64	− 12·86	− 118·20						− 5·50
	I.-III.		+ 97·46	+ 8·72	− 80·43				− 240·35	− 19·29	+ 198·46			− 1·84
	I.-IV.		+ 87·00	+ 8·76	− 75·31							− 381·33	− 50·13	− 9·51
	S_2		+ 279·25	+ 29·13	− 234·73	− 144·64	− 12·86	− 118·20	− 240·35	− 19·29	+ 198·46	− 381·33	− 50·13	− 16·84
	I.-II.		+ 0·23	+ 0·00	− 0·20	− 0·41	− 0·05	− 0·33						− 0·13
	I.-III.		+ 0·63	+ 0·00	− 0·62				− 1·66	− 0·32	+ 1·59			+ 0·59
	I.-IV.		+ 0·07	− 0·01	− 0·07							− 0·43	+ 0·01	− 0·05
	P_2		+ 0·94	− 0·01	− 0·90	− 0·41	− 0·05	− 0·33	− 1·66	− 0·32	+ 1·59	− 0·43	+ 0·01	+ 0·42
	C_2		+ 280·19	+ 29·12	− 235·62	− 145·05	− 12·91	− 118·53	− 242·01	− 19·61	+ 200·05	− 381·76	− 50·12	− 16·42
Δk_1	I.-II.			+ 7·88	− 7·45	− 17·80	− 3·72	− 15·00						− 3·68
	I.-III.			+ 7·69	− 4·80				− 22·81	− 0·60	+ 22·80			+ 2·61
	I.-IV.			+ 7·55	− 6·12							− 35·54	− 10·01	− 3·14
	S_2			+ 23·12	− 18·36	− 17·80	− 3·72	− 15·00	− 22·81	− 0·60	+ 22·80	− 35·54	− 10·01	− 4·21
	I.-II.			+ 0·01	+ 0·01	− 0·01	+ 0·02	− 0·02						+ 0·04
	I.-III.			+ 0·01	+ 0·01				− 0·02	+ 0·00	+ 0·01			− 0·01
	I.-IV.			+ 0·00	+ 0·01							+ 0·04	+ 0·00	+ 0·01
	P_2			+ 0·03	+ 0·02	− 0·01	+ 0·02	− 0·02	− 0·02	+ 0·00	+ 0·01	+ 0·04	+ 0·00	+ 0·04
	C_2			+ 23·14	− 18·34	− 17·82	− 3·70	− 15·02	− 22·82	− 0·60	+ 22·82	− 35·50	− 10·01	− 4·17
Δx_1	I.-II.				+ 70·65	+ 116·87	+ 10·82	+ 100·59						+ 2·11
	I.-III.				+ 71·39				+ 194·27	+ 21·76	− 165·96			− 2·44
	I.-IV.				+ 68·27							+ 323·38	+ 38·36	+ 5·32
	S_2				+ 210·31	+ 116·87	+ 10·82	+ 100·59	+ 194·27	+ 21·76	− 165·96	+ 323·38	+ 38·36	+ 4·99
	I.-II.				+ 0·18	+ 0·35	+ 0·05	+ 0·29						+ 0·13
	I.-III.				+ 0·61				+ 1·62	+ 0·31	− 1·56			− 0·59
	I.-IV.				+ 0·08							+ 0·40	+ 0·01	+ 0·06
	P_2				+ 0·87	+ 0·35	+ 0·05	+ 0·29	+ 1·62	+ 0·31	− 1·56			− 0·41
	C_2				+ 211·18	+ 117·21	+ 10·86	+ 100·87	+ 195·89	+ 22·07	− 167·52	+ 323·78	+ 38·37	+ 4·58
Δh_2	I.-II.					+ 238·75	+ 17·34	+ 190·89						+ 17·16
	II.-III.					+ 238·09	+ 1·80	+ 201·32	− 359·06	− 21·02	+ 293·06			
	II.-IV.					+ 273·76	− 0·87	+ 219·34				− 775·70	− 105·88	
	S_2					+ 750·60	+ 18·28	+ 611·55	− 359·06	− 21·02	+ 293·06	− 775·70	− 105·88	+ 17·16
	I.-II.					+ 0·72	+ 0·08	+ 0·60						+ 0·19
	II.-III.					+ 0·02	+ 0·00	+ 0·01	− 0·02	+ 0·00	+ 0·01			
	II.-IV.					+ 1·03	− 0·14	+ 0·93				− 3·85	− 0·21	
	P_2					+ 1·76	− 0·06	+ 1·54	− 0·02	+ 0·00	+ 0·01	− 3·85	− 0·21	+ 0·19
	C_2					+ 752·36	+ 18·21	+ 613·09	− 359·08	− 21·02	+ 293·08	− 779·54	− 106·09	+ 17·35

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$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\Delta\epsilon_4$	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ	n		
+ 0·80			− 0·27	− 0·07	− 0·76	− 0·22					+ 53·01	− 1·902	I.–II.	μ
	+ 10·61		+ 0·10	+ 0·05			+ 0·43	− 0·61			+ 32·45	+ 0·190	I.–III.	
		− 255·25	+ 0·00	+ 0·10					+ 1·88	+ 4·53	+ 96·89	− 7·379	I.–IV.	
+ 30·12	− 17·35				− 0·63	− 0·42	+ 0·60	− 0·86			+ 64·74	− 0·942	II.–III.	
− 38·08		− 273·34			− 0·98	− 1·15			+ 1·94	+ 4·91	+ 33·67	+ 4·211	II.–IV.	
	− 93·09	− 154·66					− 0·03	− 0·61	+ 3·06	+ 6·54	+ 98·46	+ 7·266	III.–IV.	
− 7·16	− 99·83	− 683·24	− 0·16	+ 0·09	− 2·37	− 1·79	+ 0·99	− 2·08	+ 6·88	+ 15·98	+ 379·23	+ 1·444	S_2	
− 1·23			− 1·59	− 0·15	− 1·62	+ 0·31					+ 8·60	− 12·035	I.–II.	Δh_1
	− 28·52		− 1·20	− 0·58			− 4·66	+ 1·42			+ 2·34	+ 1·425	I.–III.	
		+ 80·90	− 0·57	− 0·42					+ 3·89	+ 0·70	+ 5·98	− 37·018	I.–IV.	
− 1·23	− 28·52	+ 80·90	− 3·36	− 1·15	− 1·62	+ 0·31	− 4·66	+ 1·42	+ 3·89	+ 0·70	+ 16·92	− 47·628	S_2	
− 0·17			+ 1·71	+ 0·17	+ 1·73	− 0·29						+ 0·259	I.–II.	
	+ 1·32		+ 1·19	+ 0·53			+ 4·62	− 1·37				+ 0·162	I.–III.	
		+ 0·37	+ 0·39	+ 0·57					− 4·56	− 1·07		− 0·407	I.–IV.	
− 0·17	+ 1·32	+ 0·37	+ 3·29	+ 1·27	+ 1·73	− 0·29	+ 4·62	− 1·37	− 4·56	− 1·07		+ 0·014	P_2	
− 1·40	− 27·20	+ 81·27	− 0·07	+ 0·12	+ 0·12	+ 0·03	− 0·04	+ 0·05	− 0·68	− 0·37	+ 16·92	− 47·614	C_2	
+ 10·62			− 0·01	+ 0·11	+ 0·23	− 0·26					+ 2·26	− 2·743	I.–II.	Δk_1
	− 4·23		+ 0·21	− 0·17			− 0·29	+ 0·18			+ 0·98	+ 0·578	I.–III.	
		− 31·02	+ 0·10	+ 0·04					− 0·43	+ 0·03	+ 3·94	− 2·693	I.–IV.	
+ 10·62	− 4·23	− 31·02	+ 0·30	− 0·02	+ 0·23	− 0·26	− 0·29	+ 0·18	− 0·43	+ 0·03	+ 7·18	− 4·859	S_2	
+ 0·08			− 0·13	− 0·07	− 0·37	+ 0·15						− 0·055	I.–II.	
	− 0·14		− 0·26	+ 0·19			+ 0·13	− 0·19				− 0·039	I.–III.	
		− 0·05	− 0·10	+ 0·01					+ 0·16	− 0·05		+ 0·010	I.–IV.	
+ 0·08	− 0·14	− 0·05	− 0·49	+ 0·13	− 0·37	+ 0·15	+ 0·13	− 0·19	+ 0·16	− 0·05		− 0·084	P_2	
+ 10·70	− 4·36	− 31·06	− 0·19	+ 0·11	− 0·13	− 0·11	− 0·15	− 0·01	− 0·27	− 0·02	+ 7·18	− 4·942	C_2	
+ 5·37			+ 1·35	+ 0·34	+ 1·53	− 0·52					− 7·93	+ 9·654	I.–II.	Δx_1
	+ 14·95		+ 1·28	+ 0·51			+ 4·48	− 1·32			− 0·42	− 1·935	I.–III.	
		− 90·79	+ 0·60	+ 0·34					− 3·56	− 0·65	− 4·57	+ 31·757	I.–IV.	
+ 5·37	+ 14·95	− 90·79	+ 3·24	+ 1·20	+ 1·53	− 0·52	+ 4·48	− 1·32	− 3·56	− 0·65	− 12·92	39·476	S_2	
+ 0·19			− 1·42	− 0·35	− 1·67	+ 0·53						− 0·265	I.–II.	
	− 1·35		− 1·29	− 0·47			− 4·49	+ 1·29				− 0·174	I.–III.	
		− 0·39	− 0·55	− 0·39					+ 3·74	+ 0·85		+ 0·323	I.–IV.	
+ 0·19	− 1·35	− 0·39	− 3·26	− 1·21	− 1·67	+ 0·53	− 4·49	+ 1·29	+ 3·74	+ 0·85		− 0·115	P_2	
+ 5·56	+ 13·59	− 91·18	− 0·03	− 0·01	− 0·13	+ 0·02	− 0·01	− 0·03	+ 0·18	+ 0·20	− 12·92	+ 39·361	C_2	
+ 8·49			+ 2·30	+ 0·29	+ 2·07	− 0·55					− 0·10	+ 20·380	I.–II.	Δh_2
− 27·80					− 0·70	− 0·33	+ 1·97	+ 0·55			− 11·02	+ 20·503	II.–III.	
+ 30·65	− 42·56				+ 1·13	+ 1·68			+ 0·46	+ 0·71	+ 5·11	− 60·304	II.–IV.	
		+ 350·68												
+ 11·35	− 42·56	+ 350·68	+ 2·30	+ 0·29	+ 2·50	+ 0·80	+ 1·97	+ 0·55	+ 0·46	+ 0·71	− 6·01	− 19·421	S_2	
+ 0·21			− 2·91	− 0·12	− 2·03	+ 0·44						− 0·372	I.–II.	
+ 0·01	− 0·02				+ 0·62	+ 0·55	− 1·62	− 0·19				− 0·019	II.–III.	
+ 0·67		+ 1·59			− 1·34	− 2·09			− 1·77	+ 0·52		− 0·111	II.–IV.	
+ 0·90	− 0·02	+ 1·59	− 2·91	− 0·12	− 2·76	− 1·11	− 1·62	− 0·19	− 1·77	+ 0·52		− 0·503	P_2	
+ 12·25	− 42·58	+ 352·28	− 0·61	+ 0·17	− 0·26	− 0·31	+ 0·34	+ 0·36	− 1·30	− 0·20	− 6·01	− 19·925	C_2	

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TABLE XX.—Normal

		μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	$\Delta \epsilon_1$
Δk_2	I.-II.						+ 18·93	+ 9·06						+ 31·46
	II.-III.						+ 16·82	− 4·00	− 11·86	+ 0·66	+ 14·46			
	II.-IV.						+ 18·98	− 5·33				+ 1·44	+ 1·90	
	S_2						+ 54·74	− 0·26	− 11·86	+ 0·66	+ 14·46	+ 1·44	+ 1·90	+ 31·46
	I.-II.						+ 0·06	+ 0·04						+ 0·10
	II.-III.						+ 0·01	− 0·01	+ 0·01	+ 0·00	+ 0·00			
	II.-IV.						+ 0·08	− 0·13				+ 0·42	+ 0·12	
	P_2						+ 0·16	− 0·10	+ 0·01	+ 0·00	+ 0·00	+ 0·42	+ 0·12	+ 0·10
	C_2						+ 54·90	− 0·37	− 11·85	+ 0·67	+ 14·46	+ 1·86	+ 2·03	+ 31·56
Δx_2	I.-II.							+ 163·98						+ 0·90
	II.-III.							+ 180·86	− 297·52	− 22·01	+ 251·72			
	II.-IV.							+ 192·50				− 637·17	− 87·77	
	S_2							+ 537·34	− 297·52	− 22·01	+ 251·72	− 637·17	− 87·77	+ 0·90
	I.-II.							+ 0·52						+ 0·11
	II.-III.							+ 0·02	− 0·02	+ 0·00	+ 0·02			
	II.-IV.							+ 0·85				− 3·53	− 0·19	
	P_2							+ 1·38	− 0·02	+ 0·00	+ 0·02	− 3·53	− 0·19	+ 0·11
	C_2							+ 538·72	− 297·54	− 22·01	+ 251·73	− 640·69	− 87·97	+ 1·02
Δh_3	I.-III.								+ 654·31	+ 41·81	− 536·01			+ 17·67
	II.-III.								+ 588·47	+ 30·35	− 479·61			
	III.-IV.								+ 685·73	+ 23·51	− 554·51	−1222·09	− 103·17	
	S_2								+1928·51	+ 95·67	−1570·13	−1222·09	− 103·17	+ 17·67
	I.-III.								+ 4·41	+ 0·82	− 4·22			− 1·53
	II.-III.								+ 0·04	+ 0·00	− 0·03			
	III.-IV.								+ 0·55	+ 0·09	− 0·44	− 1·72	+ 0·32	
	P_2								+ 5·00	+ 0·92	− 4·69	− 1·72	+ 0·32	− 1·53
	C_2								+1933·52	+ 96·59	−1574·82	−1223·81	− 102·85	+ 16·14
Δk_3	I.-III.									+ 45·59	− 27·54			+ 0·40
	II.-III.									+ 32·38	− 16·23			
	III.-IV.									+ 45·71	− 7·40	− 37·49	+ 10·68	
	S_2									+ 123·68	− 51·17	− 37·49	+ 10·68	+ 0·40
	I.-III.									+ 0·17	− 0·79			− 0·31
	II.-III.									+ 0·01	− 0·00			
	III.-IV.									+ 0·03	− 0·07	− 0·32	+ 0·05	
	P_2									+ 0·20	− 0·86	− 0·32	+ 0·05	− 0·31
	C_2									+ 123·88	− 52·03	− 37·81	+ 10·73	+ 0·09
Δx_3	I.-III.										+ 466·04			− 0·10
	II.-III.										+ 419·14			
	III.-IV.										+ 482·83	+1012·42	+ 99·01	
	S_2										+1368·01	+1012·42	+ 99·01	− 0·10
	I.-III.										+ 4·06			+ 1·47
	II.-III.										+ 0·03			
	II.-IV.										+ 0·37	+ 1·35	− 0·24	
	P_2										+ 4·45	+ 1·35	− 0·24	+ 1·47
	C_2										+1372·46	+1013·78	+ 98·77	+ 1·38

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$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\Delta\epsilon_4$	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ	n		
+ 0·49			+ 0·43	+ 0·20	+ 0·53	− 0·28					− 11·15	+ 5·153	I.–II.	Δk_2
+ 6·40	+ 4·31				+ 0·17	+ 0·00	+ 0·14	− 0·15			+ 5·01	+ 0·084	II.–III.	
− 15·18		− 71·98			− 0·13	+ 0·09			− 1·60	+ 0·22	+ 3·25	+ 2·755	II.–IV.	
− 8·29	+ 4·31	− 71·98	+ 0·43	+ 0·20	+ 0·57	− 0·19	+ 0·14	− 0·15	− 1·60	+ 0·22	− 2·88	+ 7·993	S_2	
+ 0·16			− 0·58	+ 0·22	− 0·44	+ 0·34						− 0·155	I.–II.	
+ 0·00	+ 0·01				+ 0·28	+ 0·30	+ 0·16	+ 0·41				− 0·043	II.–III.	
− 0·20		− 0·94			+ 0·19	− 0·18			− 0·40	− 0·37		+ 0·010	II.–IV.	
− 0·05	+ 0·01	− 0·94	− 0·58	+ 0·22	+ 0·03	+ 0·46	+ 0·16	+ 0·41	− 0·40	− 0·37		− 0·188	P_2	
− 8·34	+ 4·32	− 72·92	− 0·15	+ 0·42	+ 0·60	+ 0·27	+ 0·30	+ 0·26	− 2·01	− 0·14	− 2·88	+ 7·805	C_2	
+ 1·27			+ 1·84	+ 0·38	+ 1·81	− 0·49					+ 2·55	+ 16·146	I.–II.	Δx_2
− 31·33	− 26·25				− 0·66	− 0·20	+ 1·46	+ 0·48			− 11·86	+ 18·365	II.–III.	
+ 29·04		+ 341·22			+ 0·80	+ 1·80			+ 0·70	− 0·77	+ 2·72	− 49·925	II.–IV.	
− 1·01	− 26·25	+ 341·22	+ 1·84	+ 0·38	+ 1·95	+ 1·11	+ 1·46	+ 0·48	+ 0·70	− 0·77	− 6·58	− 15·414	S_2	
+ 0·12			− 2·20	− 0·40	− 1·84	+ 0·49						− 0·278	I.–II.	
+ 0·00	− 0·02				+ 0·48	− 0·08	− 1·75	− 0·82				+ 0·106	II.–III.	
+ 0·61		+ 1·41			− 1·07	− 1·96			− 0·85	+ 0·20		+ 0·008	II.–IV.	
+ 0·73	− 0·02	+ 1·41	− 2·20	− 0·40	− 2·44	− 1·56	− 1·75	− 0·82	− 0·85	+ 0·20		− 0·164	P_2	
− 0·28	− 26·27	+ 342·63	− 0·37	− 0·02	− 0·49	− 0·45	− 0·29	− 0·34	− 0·15	− 0·57	− 6·58	− 15·579	C_2	
	+ 124·90		+ 2·55	+ 1·69			+ 11·62	− 3·88			− 10·97	− 1·823	I.–III.	Δk_3
+ 15·63	+ 79·78				+ 1·03	+ 0·70	− 3·40	− 0·66			+ 3·88	− 30·143	II.–III.	
	+ 50·83	+ 231·18					+ 0·71	+ 7·76	− 7·64	− 5·99	− 0·44	−114·276	III.–IV.	
+ 15·63	+ 255·51	+ 231·18	+ 2·55	+ 1·69	+ 1·03	+ 0·70	+ 8·93	+ 3·22	− 7·64	− 5·99	− 7·54	−146·243	S_2	
	− 3·20		− 2·51	− 1·65			− 11·88	+ 3·87				− 0·361	I.–III.	
− 0·01	+ 0·03				− 1·17	− 0·88	+ 3·30	+ 0·71				− 0·007	II.–III.	
	+ 0·13	+ 1·63					− 1·20	− 8·11	+ 7·51	+ 6·13		+ 0·494	III.–IV.	
− 0·01	− 3·04	+ 1·63	− 2·51	− 1·65	− 1·17	− 0·88	− 9·79	− 3·54	+ 7·51	+ 6·13		+ 0·127	P_2	
+ 15·61	+ 252·47	+ 232·80	+ 0·05	+ 0·04	− 0·14	− 0·18	− 0·86	− 0·31	− 0·13	+ 0·14	− 7·54	−146·116	C_2	
	− 1·01		+ 0·61	+ 0·24			+ 2·65	− 0·56			+ 4·51	− 1·596	I.–III.	Δk_3
− 52·60	− 7·99				+ 0·28	− 0·23	− 0·02	+ 0·44			+ 8·15	− 1·029	II.–III.	
	− 23·24	+ 66·98					− 0·19	+ 1·18	− 1·15	− 0·75	+ 0·79	− 5·853	III.–IV.	
− 52·60	− 32·25	+ 66·98	+ 0·61	+ 0·24	+ 0·28	− 0·23	+ 2·43	+ 1·07	− 1·15	− 0·75	+ 13·46	− 8·478	S_2	
	− 0·66		− 0·65	− 0·24			− 2·75	+ 0·57				− 0·095	I.–III.	
+ 0·00	+ 0·01				− 0·07	+ 0·42	+ 0·21	− 0·49				− 0·080	II.–III.	
	+ 0·02	+ 0·32					+ 0·11	− 0·40	+ 1·50	+ 1·24		+ 0·100	III.–IV.	
+ 0·00	− 0·64	+ 0·32	− 0·65	− 0·24	− 0·07	+ 0·42	− 2·43	− 0·31	+ 1·50	+ 1·24		− 0·076	P_2	
− 52·61	− 32·89	+ 67·31	− 0·04	− 0·01	+ 0·21	+ 0·18	+ 0·00	+ 0·75	+ 0·35	+ 0·50	+ 13·46	− 8·553	C_2	
	− 72·80		− 2·71	− 1·63			− 11·27	+ 3·72			+ 12·12	+ 2·572	I.–III.	Δx_3
− 43·07	− 51·48				− 0·61	− 0·55	+ 2·51	+ 0·69			+ 1·60	+ 25·886	II.–III.	
	− 19·10	− 187·76					− 0·75	− 5·81	+ 5·40	+ 4·26	+ 2·53	+ 92·423	III.–IV.	
− 43·07	− 143·38	− 187·76	− 2·71	− 1·63	− 0·61	− 0·55	− 9·51	− 1·41	+ 5·40	+ 4·26	+ 16·25	+120·881	S_2	
	+ 3·15		+ 2·75	+ 1·55			+ 11·51	− 3·71				+ 0·368	I.–III.	
+ 0·01	− 0·02				+ 0·83	+ 0·77	− 2·53	− 0·96				− 0·012	II.–III.	
	− 0·09	− 1·28					+ 1·01	+ 6·36	− 5·44	− 4·84		− 0·390	III.–IV.	
+ 0·01	+ 3·04	− 1·28	+ 2·75	+ 1·55	+ 0·83	+ 0·77	+ 9·99	+ 1·69	− 5·44	− 4·84		− 0·034	P_2	
− 43·07	− 140·35	− 189·04	+ 0·04	− 0·08	+ 0·22	+ 0·21	+ 0·48	+ 0·28	− 0·04	− 0·58	+ 16·25	+120·847	C_2	

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TABLE XX.—NORMAL

		μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	$\Delta \epsilon_1$
Δh_4	I.-IV.											+1778·05	+ 245·68	+ 28·24
	II.-IV.											+2429·58	+ 309·59	
	III.-IV.											+2449·13	+ 182·65	
	S_2											+6656·77	+ 737·91	+ 28·24
	I.-IV.											+ 2·61	− 0·08	+ 0·27
	II.-IV.											+ 15·22	+ 0·62	
	III.-IV.											+ 5·63	− 0·97	
	P_2											+ 23·45	− 0·44	+ 0·27
	C_2											+6680·22	+ 737·47	+ 28·51
Δk_4	I.-IV.												+ 110·93	+ 11·42
	II.-IV.												+ 149·68	
	III.-IV.												+ 149·04	
	S_2												+ 409·67	+ 11·42
	I.-IV.												+ 0·07	+ 0·00
	II.-IV.												+ 0·21	
	III.-IV.												+ 0·22	
	P_2												+ 0·51	+ 0·00
	C_2												+ 410·18	+ 11·42
$\Delta \epsilon_1$	I.-II.													+ 81·35
	I.-III.													+ 80·18
	I.-IV.													+ 72·19
	S_2													+ 233·72
	I.-II.													+ 0·21
	I.-III.													+ 0·61
	I.-IV.													+ 0·07
	P_2													+ 0·90
	C_2													+ 234·62
$\Delta \epsilon_2$	I.-II.													
	II.-III.													
	II.-IV.													
	S_2													
	I.-II.													
	II.-III.													
	II.-IV.													
	P_2													
	C_2													
$\Delta \epsilon_3$	I.-III.													
	II.-III.													
	III.-IV.													
	S_2													
	I.-III.													
	II.-III.													
	III.-IV.													
	P_2													
	C_2													

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$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\Delta\epsilon_4$	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ	n		
− 44·06	+ 4·83	− 467·96 −1156·00 − 685·12	+ 2·83	+ 2·52	− 3·67	− 8·36	− 0·61	− 23·44	− 21·75 + 8·58 + 22·48	− 3·86 + 3·83 + 18·11	− 20·27 − 23·45 + 2·47	+173·725 +197·745 +229·203	I.–IV. II.–IV. III.–IV.	Δh_4
− 44·06	+ 4·83	−2309·08	+ 2·83	+ 2·52	− 3·67	− 8·36	− 0·61	− 23·44	+ 9·32	+ 18·08	− 41·25	+600·673	S_2	
− 2·22	− 0·39	− 2·02 − 3·62 − 5·34	− 2·28	− 3·88	+ 4·94	+ 8·71	+ 0·81	+ 25·20	+ 25·39 − 9·77 − 27·91	+ 4·52 − 4·09 − 19·81		+ 2·337 − 1·009 − 2·110	I.–IV. II.–IV. III.–IV.	
− 2·22	− 0·39	− 10·98	− 2·28	− 3·88	+ 4·94	+ 8·71	+ 0·81	+ 25·20	− 12·28	− 19·38		− 0·782	P_2	
− 46·28	+ 4·44	−2320·06	+ 0·55	− 1·36	+ 1·27	+ 0·36	+ 0·20	+ 1·75	− 2·96	− 1·29	− 41·25	+599·891	C_2	
− 12·25	− 19·18	+ 24·17 − 91·51 + 57·25	+ 0·18	− 0·01	− 0·94	− 0·33	− 0·64	+ 4·41	+ 0·85 − 0·88 − 4·90	+ 0·16 + 1·32 − 3·59	− 4·06 − 6·06 − 0·37	+ 24·400 + 23·230 + 17·472	I.–IV. II.–IV. III.–IV.	Δk_4
− 12·25	− 19·18	− 10·09	+ 0·18	− 0·01	− 0·94	− 0·33	− 0·64	+ 4·41	− 4·94	− 2·10	− 10·48	+ 65·101	S_2	
− 0·32	+ 0·09	− 0·06 − 1·53 + 0·94	− 0·22	+ 0·23	+ 0·81	+ 0·33	+ 0·76	− 5·19	− 3·48 − 1·29 + 4·13	− 1·62 − 0·47 + 3·55		− 0·369 − 0·046 + 0·212	I.–IV. II.–IV. III.–IV.	
− 0·32	+ 0·09	− 0·65	− 0·22	+ 0·23	+ 0·81	+ 0·33	+ 0·76	− 5·19	− 0·65	+ 1·46		− 0·203	P_2	
− 12·57	− 19·09	− 10·74	− 0·04	+ 0·22	− 0·12	+ 0·00	+ 0·13	− 0·78	− 5·58	− 0·65	− 10·48	+ 64·898	C_2	
+ 23·85	+ 27·96	+ 63·04	+ 0·92 − 1·62 + 0·67	+ 0·41 − 0·70 + 0·36	+ 1·58	− 0·78	− 5·95	+ 1·46	− 3·11	− 0·18	− 3·01 − 6·12 − 4·66	+ 9·821 + 9·843 + 11·333	I.–II. I.–III. I.–IV.	Δe_1
+ 23·85	+ 27·96	+ 63·04	− 0·03	+ 0·08	+ 1·58	− 0·78	− 5·95	+ 1·46	− 3·11	− 0·18	− 13·79	+ 30·997	S_2	
+ 0·37	+ 1·41	− 0·23	− 1·07 + 1·64 − 0·73	− 0·36 + 0·70 − 0·31	− 1·32	+ 0·82	+ 6·04	− 1·60	+ 2·52	+ 0·04		− 0·291 + 0·195 + 0·211	I.–II. I.–III. I.–IV.	
+ 0·37	+ 1·41	− 0·23	− 0·16	+ 0·03	− 1·32	+ 0·82	+ 6·04	− 1·60	+ 2·52	+ 0·04		+ 0·116	P_2	
+ 24·22	+ 29·36	+ 62·81	− 0·19	+ 0·10	+ 0·26	+ 0·04	+ 0·09	− 0·14	− 0·59	− 0·14	− 13·79	+ 31·113	C_2	
+ 201·35 + 191·44 + 211·95	− 45·69	+ 197·26	+ 1·16	+ 0·65	+ 3·15 − 0·85 + 1·71	− 2·31 + 0·19 + 0·79	+ 0·59	− 1·55	+ 3·92	− 2·61	+ 6·08 + 8·36 + 4·37	− 2·002 − 5·895 − 12·632	I.–II. II.–III. II.–IV.	Δe_2
+ 604·73	− 45·69	+ 197·26	+ 1·16	+ 0·65	+ 4·00	− 1·34	+ 0·59	− 1·55	+ 3·92	− 2·61	+ 18·81	− 20·529	S_2	
+ 0·75 + 0·03 + 0·74	− 0·02	+ 2·61	− 1·29	− 0·68	− 3·77 + 0·59 − 2·44	+ 3·98 + 1·68 + 0·81	− 0·80	+ 1·49	− 6·22	+ 2·12		− 1·068 − 0·323 − 0·936	I.–II. II.–III. II.–IV.	
+ 1·52	− 0·02	+ 2·61	− 1·29	− 0·68	− 5·62	+ 6·48	− 0·80	+ 1·49	− 6·22	+ 2·12		− 2·327	P_2	
+ 606·25	− 45·71	+ 199·86	− 0·13	− 0·03	− 1·62	+ 5·14	− 0·21	− 0·06	− 2·30	− 0·49	+ 18·81	− 22·856	C_2	
	+ 549·62 + 448·03 + 502·37	− 23·75	− 6·47	− 0·02	+ 0·78	+ 0·85	− 11·06 − 2·99 + 0·84	+ 2·36 + 0·38 + 1·06	− 0·06	− 1·60	+ 12·36 − 7·49 − 3·30	− 9·683 − 6·496 − 15·777	I.–III. II.–III. III.–IV.	Δe_3
	+1500·03	− 23·75	− 6·47	− 0·02	+ 0·78	+ 0·85	− 13·21	+ 3·80	− 0·06	− 1·60	+ 1·58	− 31·956	S_2	
	+ 4·57 + 0·04 + 0·08	+ 0·36	+ 6·37	− 0·04	− 0·66	− 0·97	+ 10·25 + 2·07 − 1·97	− 3·03 − 1·41 − 2·70	+ 0·31	+ 2·47		+ 0·883 + 0·139 + 0·026	I.–III. II.–III. III.–IV.	
	+ 4·69	+ 0·36	+ 6·37	− 0·04	− 0·66	− 0·97	+ 10·34	− 7·15	+ 0·31	+ 2·47		+ 1·047	P_2	
	+1504·72	− 23·39	− 0·10	− 0·06	+ 0·12	− 0·12	− 2·87	− 3·35	+ 0·25	+ 0·87	+ 1·58	− 30·908	C_2	

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TABLE XX.—NORMAL

		μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	$\Delta \epsilon_1$
$\Delta \epsilon_4$	I.-IV.													
	II.-IV.													
	III.-IV													
	S_2													
	I.-IV.													
	II.-IV.													
	III.-IV.													
	P_2													
	C_2													
Δp_1	I.-II.													
	I.-III.													
	I.-IV.													
	S_2													
	I.-II.													
	I.-III.													
	I.-IV.													
	P_2													
	C_2													
Δq_1	I.-II.													
	I.-III.													
	I.-IV.													
	S_2													
	I.-II.													
	I.-III.													
	I.-IV.													
	P_2													
	C_2													
Δp_2	I.-II.													
	II.-III.													
	II.-IV.													
	S_2													
	I.-II.													
	II.-III.													
	II.-IV.													
	P_2													
	C_2													
Δq_2	I.-II.													
	II.-III.													
	II.-IV.													
	S_2													
	I.-II.													
	II.-III.													
	II.-IV.													
	P_2													
	C_2													

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$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\Delta\epsilon_4$	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ	n		
		+1443·68	− 3·37	+ 0·95					+ 15·40	+ 1·71	− 64·78	− 77·390	I.–IV.	$\Delta\epsilon_4$
		+2153·59			+ 2·53	− 9·05			+ 41·01	+ 7·36	− 18·04	−143·761	II.–IV.	
		+1987·64					+ 0·99	+ 23·51	− 22·02	− 16·44	− 16·56	−125·672	III.–IV.	
		+5584·90	− 3·37	− 0·95	+ 2·53	− 9·05	+ 0·99	+ 23·51	+ 34·40	− 7·37	− 99·38	−346·823	S_2	
		+ 2·68	+ 3·39	+ 1·42					− 13·76	− 11·05		− 1·386	I.–IV.	
		+ 15·06			− 2·90	+ 8·23			− 33·91	− 14·24		− 4·455	II.–IV.	
		+ 5·24					− 0·90	− 25·13	+ 31·59	+ 11·64		+ 2·427	III.–IV.	
		+ 22·97	+ 3·39	+ 1·42	− 2·90	+ 8·23	− 0·90	− 25·13	− 16·09	− 13·65		− 3·414	P_2	
		+5607·88	+ 0·02	+ 0·47	− 0·37	− 0·82	+ 0·10	− 1·61	+ 18·31	− 21·02	− 99·38	−350·237	C_2	
			+ 0·22	+ 0·01	+ 0·36	− 0·05					− 0·01	+ 0·339	I.–II.	Δp_1
			+ 0·65	+ 0·07			+ 1·47	− 0·33			− 0·29	− 0·070	I.–III.	
			+ 0·08	+ 0·04					− 0·30	− 0·02	+ 0·25	+ 0·567	I.–IV.	
			+ 0·96	+ 0·12	+ 0·36	− 0·05	+ 1·47	− 0·33	− 0·30	− 0·02		+ 0·836	S_2	
			+ 78·72	− 13·97	+ 25·78	+ 20·36						+ 0·910	I.–II.	
			+ 80·72	− 9·13			+ 37·12	− 19·05				+ 3·688	I.–III.	
			+ 70·51	− 8·66					+ 64·79	− 46·30		+ 3·684	I.–IV.	
			+ 229·95	− 31·76	+ 25·78	+ 20·36	+ 37·12	− 19·05	+ 64·79	− 46·30		+ 8·282	P_2	
			+ 230·90	− 31·64	+ 26·15	+ 20·31	+ 38·60	− 19·38	+ 64·48	− 46·32	− 0·05	+ 9·118	C_2	
				+ 0·12	− 0·04	− 0·12					+ 0·08	+ 0·056	I.–II.	Δq_1
				+ 0·07			+ 0·40	− 0·13			− 0·03	− 0·049	I.–III.	
				+ 0·04					− 0·27	− 0·03	+ 0·15	+ 0·347	I.–IV.	
				+ 0·23	− 0·04	− 0·12	+ 0·40	− 0·13	− 0·27	− 0·03	+ 0·20	+ 0·354	S_2	
				+ 87·11	+ 15·72	+ 1·02						− 6·170	I.–II.	
				+ 99·10			+ 19·33	− 71·96				− 6·251	I.–III.	
				+ 96·16					− 119·15	+ 136·85		− 15·087	I.–IV.	
				+ 282·38	+ 15·72	+ 1·02	+ 19·33	− 71·96	− 119·15	+ 136·85		− 27·507	P_2	
				+ 282·60	+ 15·68	+ 0·91	+ 19·72	− 72·09	− 119·43	+ 136·82	+ 0·20	− 27·153	C_2	
					+ 0·71	− 0·04					− 0·09	+ 0·497	I.–II.	Δp_2
					+ 0·02	+ 0·00	− 0·01	+ 0·01			− 0·08	− 0·060	II.–III.	
					+ 0·78	+ 0·65			+ 2·42	− 0·72	+ 0·24	− 0·372	II.–IV.	
					+ 1·50	+ 0·61	− 0·01	+ 0·01	+ 2·42	− 0·72	+ 0·06	+ 0·065	S_2	
					+ 194·56	− 14·09						+ 20·330	I.–II.	
					+ 197·13	+ 36·92	− 36·66	+ 27·84				+ 8·597	II.–III.	
					+ 212·17	+ 36·30			+ 161·25	− 117·59		+ 25·949	II.–IV.	
					+ 603·85	+ 59·13	− 36·66	+ 27·84	+ 161·25	− 117·59		+ 54·876	P_2	
					+ 605·36	+ 59·74	− 36·68	+ 27·84	+ 163·67	− 118·31	+ 0·06	+ 54·941	C_2	
						+ 0·14					− 0·21	− 0·066	I.–II.	Δq_2
						+ 0·01	− 0·01	− 0·01			− 0·16	− 0·046	II.–III.	
						+ 1·44			− 0·38	− 1·31	+ 0·91	− 0·522	II.–IV.	
						+ 1·59	− 0·01	− 0·01	− 0·38	− 1·31	+ 0·54	− 0·633	S_2	
						+ 224·82						− 41·619	I.–II.	
						+ 249·94	− 37·40	+ 10·08				− 46·903	II.–III.	
						+ 267·38			− 65·56	− 161·42		− 46·879	II.–IV.	
						+ 742·15	− 37·40	+ 10·08	− 65·56	− 161·42		−135·401	P_2	
						+ 743·74	− 37·42	+ 10·07	− 65·94	− 162·73	+ 0·54	−136·035	C_2	

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TABLE XX.—NORMAL

		μ	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	$\Delta \epsilon_1$
Δp_3	I.–III.													
	II.–III.													
	III.–IV.													
	S_2													
	I.–III.													
	II.–III.													
	III.–IV.													
	P_2													
	C_2													
Δq_3	I.–III.													
	II.–III.													
	III.–IV.													
	S_2													
	I.–II.													
	II.–III.													
	III.–IV.													
	P_2													
	C_2													
Δp_4	I.–IV.													
	II.–IV.													
	III.–IV.													
	S_2													
	I.–IV.													
	II.–IV.													
	III.–IV.													
	P_2													
	C_2													
Δq_4	I.–IV.													
	II.–IV.													
	III.–IV.													
	S_2													
	I.–IV.													
	II.–IV.													
	III.–IV.													
	P_2													
	C_2													
ΔJ	I.–II.													
	I.–III.													
	I.–IV.													
	II.–III.													
	II.–IV.													
	III.–IV.													
	C_2													

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EQUATIONS, 1902.—*contd.*

$\Delta\epsilon_2$	$\Delta\epsilon_3$	$\Delta\epsilon_4$	Δp_1	Δq_1	Δq_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4	ΔJ	n		
							+ 4·40	− 1·06			− 0·06	− 0·312	I.–III.	Δp_3
							+ 0·05	+ 0·00			+ 0·21	+ 0·191	II.–III.	
							+ 0·05	− 0·04	+ 0·17	+ 0·06	+ 0·02	− 0·146	III.–IV.	
							+ 4·51	− 1·10	+ 0·17	+ 0·06	+ 0·17	− 0·267	S_2	
							+ 555·06	+ 24·20				− 4·092	I.–III.	
							+ 441·85	+ 31·07				− 3·972	II.–III.	
							+ 493·07	+ 71·91	− 36·89	− 86·44		− 5·695	III.–IV.	
							+1489·97	+ 127·18	− 36·89	− 86·44		− 13·759	P_2	
							+1494·48	+ 126·08	− 36·72	− 86·39	+ 0·17	− 14·026	C_2	
								+ 0·31			+ 0·21	+ 0·084	I.–III.	Δq_3
								+ 0·02			− 0·14	+ 0·062	II.–III.	
								+ 1·93	− 1·62	− 1·58	− 0·66	− 2·204	III.–IV.	
								+ 2·27	− 1·62	− 1·58	− 0·59	− 2·058	S_2	
								+ 616·66				− 20·472	I.–III.	
								+ 697·31				− 19·631	II.–III.	
								+ 855·19	− 75·97	+ 209·98		− 31·750	III.–IV.	
								+2169·16	− 75·97	+ 209·98		− 71·853	P_2	
								+2171·43	− 77·59	+ 208·40	− 0·59	− 73·911	C_2	
									+ 2·43	+ 0·14	− 0·60	− 3·203	I.–IV.	Δp_4
									+ 16·11	− 0·33	− 1·66	+ 0·895	II.–IV.	
									+ 2·38	+ 1·42	+ 1·01	+ 1·936	III.–IV.	
									+ 20·93	+ 1·23	− 1·25	− 0·373	S_2	
									+1548·61	− 292·33		+123·904	I.–IV.	
									+2071·94	− 462·30		+162·963	II.–IV.	
									+1846·76	− 122·86		+119·885	III.–IV.	
									+5467·31	− 877·49		+406·751	P_2	
									+5488·24	− 876·26	− 1·25	+406·379	C_2	
										+ 0·06	+ 0·37	+ 0·249	I.–IV.	Δq_4
										+ 1·29	− 0·96	− 0·384	II.–IV.	
										+ 1·37	+ 0·81	+ 1·640	III.–IV.	
										+ 2·73	+ 0·22	+ 1·505	S_2	
										+1783·29		− 43·815	I.–IV.	
										+1728·66		− 25·638	II.–IV.	
										+2261·14		− 27·464	III.–IV.	
										+5773·10		− 96·918	P_2	
										+5775·82	+ 0·22	− 95·412	C_2	
											+ 33·31	− 0·940	I.–II.	ΔJ
											+ 24·96	− 0·753	I.–III.	
											+ 29·50	+ 0·970	I.–IV.	
											+ 15·88	− 0·734	II.–III.	
											+ 10·91	− 2·147	II.–IV.	
											+ 9·01	+ 0·686	III.–IV.	
											+ 123·56	− 2·918	C_2	

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TABLE XXI.

VALUES OF UNKNOWNS FROM PRELIMINARY SOLUTIONS, 1901.

1901.	I.–II.	I.–III.	I.–IV.	II.–III.	II.–IV.	III.–IV.
μ	+0 ·0735	+0 ·0055	−0 ·0027	+0 ·0104	+0 ·0013	+0 ·0016
Δh_1	+0°·0172	−0°·0541	+0°·1350			
Δk_1	+0 ·1026	−0 ·1746	−0 ·1618			
Δx_1	+0 ·0920	−0 ·2326	−0 ·0530			
Δh_2	−0 ·0612			+0°·0218	+0°·0305	
Δk_2	−0 ·0725			−0 ·1055	+0 ·0906	
Δx_2	+0 ·1932			+0 ·3014	+0 ·0267	
Δh_3		−0 ·0080		+0 ·1166		−0°·0455
Δk_3		−0 ·1768		−0 ·4401		−0 ·0537
Δx_3		−0 ·1510		−0 ·3043		+0 ·0222
Δh_4			+0 ·0839		+0 ·0563	+0 ·0616
Δk_4			+0 ·0445		+0 ·0656	+0 ·0042
$\Delta\epsilon_1$	+0 ·1680	+0 ·1410	+0 ·1262			
$\Delta\epsilon_2$	+0 ·0210			−0 ·0811	+0 ·0284	
$\Delta\epsilon_3$		−0 ·0625		−0 ·0006		−0 ·0407
$\Delta\epsilon_4$			−0 ·0411		+0 ·0170	−0 ·0346
ΔJ	−0 ·1488	+0 ·0084	−0 ·0100	−0 ·0098	−0 ·0568	0 ·0000
No. of Equations	29	27	28	26	29	26
$[vv]$	0″·217	0″·180	0″·416	0″·368	0″·288	0″·141
p.e.	±0″·071	±0″·079	±0″·104	±0″·100	±0″·089	±0″·060
Reference No. of Equations omitted	none	none	123	269	none	19
Δp_1	+0°·0340	+0°·0283	−0°·0206			
Δq_1	−0 ·0826	−0 ·0841	−0 ·0613			
Δp_2	+0 ·1044			+0°·1181	+0°·1235	
Δq_2	−0 ·1670			−0 ·1912	−0 ·1824	
Δp_3		−0 ·0028		−0 ·0185		−0°·0103
Δq_3		−0 ·0424		−0 ·0465		−0 ·0284
Δp_4			+0 ·0461		+0 ·0516	+0 ·0447
Δq_4			−0 ·0431		−0 ·0321	−0 ·0334
No. of Equations	30	26	27	27	30	27
$[vv]$	0″·414	0″·568	1″·069	0″·866	0″·767	0″·911
p.e.	±0″·085	±0 ·108	±0 ·146	±0 ·131	±0 ·116	±0 ·134
Reference No. of Equations omitted	none	312,412	30	270	none	none

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TABLE XXII.

VALUES OF UNKNOWNS FROM PRELIMINARY SOLUTIONS, 1902.

1902.	I.–II.	I.–III.	I.–IV.	II.–III.	II.–IV.	III.–IV.
μ	−0 ·1147	+0 ·0148	−0 ·0032	+0 ·0008	+0 ·0056	+0 ·0203
Δh_1	−0°·0598	−0°·1901	+0°·1137			
Δk_1	−0 ·0366	+0 ·1529	−0 ·0364			
Δx_1	−0 ·1072	−0 ·2866	+0 ·0914			
Δh_2	−0 ·0060			−0°·0497	+0°·0253	
Δk_2	+0 ·2601			+0 ·0742	+0 ·0446	
Δx_2	+0 ·1035			+0 ·1780	+0 ·1190	
Δh_3		−0 ·0622		−0 ·0317		−0°·0542
Δk_3		+0 ·0170		+0 ·0049		−0 ·0191
Δx_3		−0 ·0990		−0 ·0530		−0 ·0266
Δh_4			+0 ·0908		+0 ·1061	+0 ·0615
Δk_4			+0 ·0286		−0 ·0025	+0 ·0341
$\Delta\epsilon_1$	+0 ·0350	+0 ·1311	+0 ·1528			
$\Delta\epsilon_2$	−0 ·0172			−0 ·0241	−0 ·0247	
$\Delta\epsilon_3$		−0 ·0249		−0 ·0116		−0 ·0284
$\Delta\epsilon_4$			−0 ·0305		−0 ·0291	−0 ·0390
ΔJ	+0 ·2322	+0 ·0210	+0 ·0633	+0 ·0435	−0 ·0801	−0 ·2894
No. of Equations	35	38	32	37	38	43
$[vv]$	0″·417	0″·431	0″·558	0″·513	0″·871	0″·724
p.e.	±0″·085	±0″·083	±0″·104	±0″·091	±0″·115	±0″·097
Δp_1	+0°·0114	+0°·0272	−0°·0295			
Δq_1	−0 ·0844	−0 ·0908	−0 ·0534			
Δp_2	+0 ·0968			+0°·0820	+0°·0996	
Δq_2	−0 ·1797			−0 ·2012	−0 ·1783	
Δp_3		−0 ·0042		−0 ·0173		−0°·0036
Δq_3		−0 ·0428		−0 ·0277		−0 ·0297
Δp_4			+0 ·0754		+0 ·0631	+0 ·0632
Δq_4			−0 ·0089		−0 ·0073	−0 ·0061
No. of Equations	35	38	33	37	34	42
$[vv]$	1″·014	0″·553	1″·231	0″·624	0″·941	1″·368
p.e.	±0″·122	±0·085	±0 ·137	±0 ·092	±0 ·118	±0 ·126

The only equations omitted in the preliminary solutions for 1902 were Nos. 152, 155, and 410 in Position-angles II.–IV.

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It will be noticed that the values of both μ and ΔJ differ considerably: the explanation of this is as follows. The co-efficient of $\frac{1}{3}\Delta J$ is—

$$\frac{a_1}{1+a_1 J}\rho_1\frac{\partial s}{\partial \rho_1}+\frac{a_2}{1+a_2 J}\rho_2\frac{\partial s}{\partial \rho_2}.$$

This may be written—

$$\frac{a_1}{1+a_1 J}\left(s-\rho_2\frac{\partial s}{\partial \rho_2}\right)+\frac{a_2}{1+a_2 J}\rho_2\frac{\partial s}{\partial \rho_2}$$

or

$$\tfrac{1}{2}\left(\frac{a_1}{1+a_1 J}+\frac{a_2}{1+a_2 J}\right)s+\tfrac{1}{2}\left(\frac{a_1}{1+a_1 J}-\frac{a_2}{1+a_2 J}\right)s-\left(\frac{a_1}{1+a_1 J}-\frac{a_2}{1+a_2 J}\right)\rho_2\frac{\partial s}{\partial \rho_2}$$

or

$$\tfrac{1}{2}\left(\frac{a_1}{1+a_1 J}+\frac{a_2}{1+a_2 J}\right)s+\left(\frac{a_1}{1+a_1 J}-\frac{a_2}{1+a_2 J}\right)\left(\tfrac{1}{2}s-\rho_2\frac{\partial s}{\partial \rho_2}\right).$$

The second term is small, especially for I. and II., where $\frac{1}{2}\ s-\rho_2\frac{\delta s}{\delta \rho_2}$ is small: if it is neglected, the coefficients of $-\frac{1}{3}\delta M/M$ and of

$$\tfrac{1}{6}\left(\frac{a_1}{1-a_1 J}+\frac{a_2}{1+a_2 J}\right)\Delta J$$

are both s, and hence in the solutions of the normals μ and ΔJ cannot be separated: a linear function of them is determined, namely—

$$f(\mu_1 J)=\mu+\frac{100}{6}\left(\frac{a_1}{1+a_1 J}+\frac{a_2}{1+a_2 J}\right)\Delta J.$$

This is tabulated below and if we take

$$f(\mu_1 J)=-\frac{100}{3}\ \frac{\partial M}{M}$$

we get the values of the mass given in the same table.

TABLE XXIII.

VALUES OF THE MASS FROM THE SEVERAL PAIRS OF SATELLITES.

		I.-II.	I.-III.	I.-IV.	II.-III.	II.-IV.	III.-IV.	Weighted Mean.
$f(\mu_1 J)$	1901	−0·0211	+0·0099	−0·0075	+0·0079	−0·0102	+0·0016	
	1902	+0·0330	+0·0258	+0·0271	+0·0117	−0·0106	−0·0069	
Reciprocal of mass	1901	1048·36	1047·39	1047·94	1047·45	1048·02	1047·65	1047·78
	1902	1046·66	1046·89	1046·85	1047·33	1048·03	1047·92	1047·43
Weighted Mean		1047·45	1047·14	1047·32	1047·38	1048·03	1047·82	1047·58

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The various determinations of the mass are as accordant as can be expected. It is not easy to arrive at a very reliable estimate of the probable errors owing to the intricate manner in which the unknowns are involved in one another, but inspection of the normals will show that the weights of the determinations from I.–II. and I.–III. are very much less than those of the others. The weight from I.–IV. is surprisingly large owing to the number of big distances that happen to have been observed. If, however, we assign as weights simply the coefficient of μ in the normal for μ, we get the weighted means of the table, which agree with one another sufficiently well.

The disagreement of the various determinations of the unknowns Δh and Δk may be explained in the same way as in the case of μ and ΔJ.

If the eccentricities and inclinations are supposed to be zero and if $B = 0$ and $p - P = \pm 90^\circ$ the formulæ on page 79 become—

$$\frac{\partial s}{\partial v_1} = K_1 \cos (l_1 - L)$$
$$\frac{\partial s}{\partial \rho_1} = \frac{K_1}{\sin 1^\circ} \sin (l_1 - L)$$

where

$$K_1 = \frac{a_1'' \rho_1}{\Delta_1'} \sin 1^\circ \sin (p - P).$$

Hence

$$\frac{\partial s}{\partial h_1} = K_1 [\cos l_1 \cos (l_1 - L) + \tfrac{1}{2} \sin l_1 \sin (l_1 - L)]$$
$$\frac{\partial s}{\partial k_1} = - K_1 [\sin l_1 \cos (l_1 - L) - \tfrac{1}{2} \cos l_1 \sin (l_1 - L)]$$
$$\frac{\partial s}{\partial x_1} = - K_1 [\sin a_1 \cos (l_1 - L) - \tfrac{1}{2} \cos a_1 \sin (l_1 - L)].$$

The same formulæ hold for the other satellites with the necessary change of suffixes: for satellite IV. the last equation is non-existent. Now let—

$$c = l_1 - 2 l_2 = l_2 - 2 l_3 + 180^\circ$$

then

$$a_1 = 2 (l_1 - l_2) = l_1 + c$$
$$a_2 = l_1 - l_2 + 180^\circ = l_2 + c + 180$$
$$\gamma_2 = l_2 - l_3 + 180^\circ = l_3 + c.$$

Thus we get

$$\frac{\partial s}{\partial h} = \tfrac{1}{4} K [3 \cos L + \cos (2 l - L)]$$
$$\frac{\partial s}{\partial k} = - \tfrac{1}{4} K [3 \sin L + \sin (2 l - L)]$$
$$\frac{\partial s}{\partial x} = \mp \tfrac{1}{4} K [3 \sin (L + c) + \sin (2 l + c - L)]$$

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where the lower sign in the last formula is to be taken for satellite II. Neglecting the second terms within the brackets, terms of comparatively short periods, we have—

$$\frac{\partial s}{\partial h} = \tfrac{3}{4}\,\mathrm{K}\cos \mathrm{L},$$

$$\frac{\partial s}{\partial k} = -\tfrac{3}{4}\,\mathrm{K}\sin \mathrm{L},$$

$$\frac{\partial s}{\partial x} = \mp\tfrac{3}{4}\,\mathrm{K}\sin(\mathrm{L}+c).$$

Now L changes very slowly; its changes during the two periods of observation were 5° and 10° respectively, whilst in 1901 c varied from 115° to 190°, and in 1902 from 18° to 106°. If there were no variation at all, it would be possible to determine only a linear function of Δh, Δk and Δx, viz.,

$$\mathrm{F} = \cos \mathrm{L}\,\Delta h - \sin \mathrm{L}\,\Delta k \mp \sin(\mathrm{L}+c)\,\Delta x.$$

On the supposition of no variation of L and c, the functions F were computed with their mean values, *i.e.*

1901	L = 138°·2	c = 155°·2
1902	L = 176·6	c = 60·7

and are tabulated below. For the sake of comparison, the same functions have been calculated with the final values of the unknowns Δh, Δk and Δx and are tabulated with the others.

TABLE XXIV.

	1901.	1902.		1901.	1902.
I.– II.	+0·0033	−0·0303	I.–III.	−0·0147	−0·0222
F_1 I.–III.	−0·0567	−0·0605	F_3 II.–III.	−0·0726	−0·0133
I.–IV.	−0·0414	−0·0344	III.–IV.	+0·0810	+0·0331
General Solution	+0·0469	−0·0069	General Solution	+0·0287	−0·0012
I.– II.	−0·0833	−0·0964	I.–IV.	−0·0922	−0·0923
F_2 II.–III.	−0·2225	−0·1047	F_4 II.–IV.	−0·0857	−0·1035
II.–IV.	−0·1076	−0·1281	III.–IV.	−0·0487	−0·0591
General Solution	−0·0644	−0·0827	General Solution	−0·0737	−0·0855

Here again it is not easy to arrive at a good approximation of the probable errors of these quantities: the following, however, are the best estimates that it is possible to give.

Probable error of F_1 = ± 0·05
" F_2 ± 0·03
" F_3 ± 0·02
" F_4 ± 0·01

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These apply to the values found in the preliminary solutions, but the probable errors of the values from the general solution can be rigorously found by the method explained on page 162: their values have been roughly found in this way, and are

	1901.	1902.
Probable error of F_1	$\pm$ 0·0207	$\pm$ 0·0169
„ F_2	$\pm$ 0·0129	$\pm$ 0·0139
„ F_3	$\pm$ 0·0086	$\pm$ 0·0090
„ F_4	$\pm$ 0·0059	$\pm$ 0·0036

The agreement of the different determinations is as good as can be expected: the discordances do not appear to be entirely due to accidental error, since the determinations of the two years more or less confirm one another: thus F_3 from III. – IV. differs from the other two values considerably and in the same direction in both years: so also F_4.

Turning now to the position-angle unknowns, we see that the agreement of the various determinations is satisfactory excepting in the case of Δp_1 and Δq_1. It is significant that in both years the values of Δp_1 and Δq_1 from I.–IV. differ widely and in the same direction from the other two values: this will be referred to further on. On the whole the observations of position-angle are not so good as those of distance. It was the writer's impression in 1901 that of the two the observation of position-angle was the easier; but the preliminary solutions do not confirm this, and as these solutions were made before re-commencing observations in 1902, the greatest possible care was taken with the measurement of position-angle in 1902. But the results of this year are no more satisfactory than those of 1901. Before discussing this further, however, it will be best to proceed to the solutions of the combined normals and see if they throw any light upon the question.

After the revision of the equations of condition and the normals had been made by means of the preliminary solutions, the solution of the final combined normals was undertaken. As the normals for the two years were treated somewhat differently, it will be convenient to take them separately.

Normals for 1901.

The solution was not made in quite the usual form, and it is therefore necessary to explain the method in outline.

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Let the normals be

$$\begin{aligned}
(aa)\,x + (ab)\,y + (ac)\,z + \ldots\ldots &= (an).\\
(ba)\,x + (bb)\,y + (bc)\,z + \ldots\ldots &= (bn).\\
(ca)\,x + (cb)\,y + (cc)\,z + \ldots\ldots &= (cn).\\
\cdot \quad \cdot \quad \cdot \quad \cdot &\\
\cdot \quad \cdot \quad \cdot \quad \cdot &
\end{aligned}$$

and let Δ denote the determinant

$$\begin{vmatrix}
(aa) & (ab) & (ac) & . \\
(ba) & (bb) & (bc) & . \\
(ca) & (cb) & (cc) & . \\
\cdot & \cdot & \cdot & \cdot
\end{vmatrix}$$

and let $\Delta_{(aa)}$, $\Delta_{(ab)}$, etc., denote the first minors of (aa), (ab), etc.: then by the elementary properties of determinants it follows that the solution of the normals is—

$$x = \frac{\Delta_{(aa)}}{\Delta}(an) + \frac{\Delta_{(ab)}}{\Delta}(bn) + \frac{\Delta_{(ac)}}{\Delta}(cn) + \ldots$$

$$y = \frac{\Delta_{(ba)}}{\Delta}(an) + \frac{\Delta_{(bb)}}{\Delta}(bn) + \frac{\Delta_{(bc)}}{\Delta}(cn) + \ldots$$

Since $\Delta_{(ik)}\Delta = {}_{(ki)}$, this solution also has the form of normal equations. The weight of x is $\Delta/\Delta_{(aa)}$, of y it is $\Delta/\Delta_{(bb)}$, and so on. The complete solution then of n normals is equivalent to the valuation of a symmetrical determinant of the nth order and of all its first minors. In this form of solution the substitution of the numerical values of the absolute terms is not made till the coefficients $\Delta_{(ik)}/\Delta$ have all been evaluated, and though so simple algebraically, the application of the above method to an actual numerical example is practically no shorter than the usual method of solving the normals and finding the weights, but it has the great advantage of making it an easy matter to find the corrections to the unknowns corresponding to a change in any of the absolute terms; thus if the absolute term in the normal for z were in error by the amount α, the corresponding corrections to the unknowns would be

$$\partial x = -\frac{\Delta_{(ac)}}{\Delta}\alpha, \qquad \partial y = -\frac{\Delta_{(bc)}}{\Delta}\alpha, \qquad \partial z = -\frac{\Delta_{(cc)}}{\Delta}\alpha, \text{ etc.}$$

This form of solution has other advantages also; for instance, it is easy to find the weight and thence the probable error of any function of the unknowns. For suppose the function be

$$F = Ax + By + Cz + \ldots;$$

substitute A for (an), B for (bn), etc., in the solution of the normals; let x^1, y^1, z^1 . . . be the resulting values of x, y, z . . .; then if r_F and r be the probable errors of the function F and of an observation of weight unity, we have—

$$r_F^2 = (Ax^1 + By^1 + Cz^1 + \ldots)\, r^2.$$

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This method of solution is on the whole slightly easier to control than the usual one : numerous checks can be applied throughout the work and, what is most important, there are checks for both rows and columns of numbers. Any one who has made a solution of a large number of normals will at once appreciate the importance of this : it is in fact the only way to detect compensating errors.

More need not here be said. Dr de Sitter in his paper on the "Satellites of Jupiter" has explained it very fully, and, as he there remarks, the method deserves to be better known than it is.

The combined normals for 1901, *i.e.* those called C_1, were solved in this form, and the solution is printed below. The unknown ΔJ was taken over to the right hand side of the normals and all the other unknowns are expressed in terms of it. The capital letters M, H_1, etc., denote the absolute terms in the normals for μ, Δh, etc., minus the term depending on ΔJ : their values can be taken from the pages on which the normals are printed ; thus, for instance—

$$K_2 = 9''{\cdot}726 + 9{\cdot}67\ \Delta J.$$

As the solution has the form of normals, *i.e.* is symmetrical about the leading diagonal, it will be understood that the coefficient of any unknown x in the equation for another unknown y is the same as the coefficient of y in the equation for x ; for instance the expression for $\Delta\epsilon_2$ is obtained by starting at the top of the column E_2 and following it down to the last number in it, and then going along that line to the end on the right. The unit in this table is one in the sixth decimal place.

The numerical values of M, H_1, etc., were next substituted in the above symbolical solution and thus the values of the unknowns were derived. The unknowns were then substituted in the normals and found to satisfy them all within a few thousandths of a second of arc. These, therefore, are the values of the unknowns from the combined normals of 1901 : their weights are the reciprocals of the coefficients in the diagonal of the solution which is printed on the next two pages.

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TABLE XXV.—SOLUTION

	M	H_1	K_1	X_1	H_2	K_2	X_2	H_3	K_3	X_3	H_4	K_4
$\mu =$	+ 860	− 45	+ 1418	− 723	+ 1094	− 1540	+ 1280	− 504	+ 1	− 947	+ 405	− 366
$\Delta h_1 =$		+ 90643	+ 33793	+ 78475	+ 17492	+ 25977	− 17289	+ 11046	+ 12245	+ 7749	+ 871	+ 6533
$\Delta k_1 =$			+304491	+234166	+ 23956	+ 38834	− 36490	+ 16591	+ 16054	+ 16991	+ 408	+ 5366
$\Delta x_1 =$				+255072	+ 13434	+ 47292	− 49806	+ 15445	+ 20585	+ 30206	− 4427	+ 4541
$\Delta h_2 =$					+ 40311	+ 15234	− 31943	+ 6751	+ 7125	+ 3023	+ 1766	+ 2896
$\Delta k_2 =$						+113784	− 94787	+ 10898	+ 16728	+ 19066	− 2387	+ 5330
$\Delta x_2 =$							+103500	− 9537	− 16822	− 22872	+ 3634	− 3188
$\Delta h_3 =$								+ 21285	+ 8026	+ 20313	− 127	+ 2206
$\Delta k_3 =$									+ 34968	+ 31922	− 414	+ 1300
$\Delta x_3 =$										+ 45611	− 2534	+ 609
$\Delta h_4 =$											+ 2957	− 1878
$\Delta k_4 =$												+ 4017
$\Delta e_1 =$												
$\Delta e_2 =$												
$\Delta e_3 =$												
$\Delta e_4 =$												
$\Delta p_1 =$												
$\Delta q_1 =$												
$\Delta p_2 =$												
$\Delta q_2 =$												
$\Delta p_3 =$												
$\Delta q_3 =$												
$\Delta p_4 =$												
$\Delta q_4 =$												

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OF NORMALS, 1901.

E_1	E_2	E_3	E_4	P_1	Q_1	P_2	Q_2	P_3	Q_3	P_4	Q_4	
+ 576	+ 101	+ 123	− 121	+ 9	+ 13	− 22	− 6	− 16	+ 14	− 6	+ 16	$=\mu$
+ 2549	− 963	+ 1604	+ 382	− 200	+ 136	+ 155	+ 2	+ 107	− 176	− 154	− 191	$=\Delta h_1$
+ 3474	− 3400	+ 125	− 397	+ 397	− 635	− 403	− 47	+ 667	− 241	− 176	+ 160	$=\Delta k_1$
− 2621	− 3346	+ 927	− 106	− 37	− 386	− 68	+ 48	+ 527	− 334	− 238	− 13	$=\Delta x_1$
+ 5564	+ 1942	+ 417	− 386	+ 252	− 272	− 128	− 167	+ 150	− 111	− 47	− 88	$=\Delta h_2$
− 5406	− 4923	− 144	+ 444	+ 222	− 350	+ 108	+ 436	+ 313	− 236	− 61	− 12	$=\Delta k_2$
+ 2503	+ 1908	+ 99	+ 87	− 330	+ 488	− 47	− 214	− 345	+ 275	+ 64	+ 52	$=\Delta x_2$
+ 1063	− 1885	− 2547	+ 150	+ 302	− 326	− 43	+ 153	+ 344	− 241	− 52	− 78	$=\Delta h_3$
− 456	+ 490	+ 379	− 260	+ 1054	− 944	− 202	+ 19	+ 582	− 614	+ 6	+ 49	$=\Delta k_3$
− 1576	− 1234	− 1570	− 128	+ 1008	− 977	− 170	+ 182	+ 692	− 635	− 25	− 8	$=\Delta x_3$
+ 655	+ 404	+ 12	− 176	+ 38	+ 16	− 6	− 2	− 19	+ 7	+ 33	+ 61	$=\Delta h_4$
+ 391	− 546	+ 84	+ 398	− 22	− 29	− 6	− 13	+ 46	− 19	− 44	− 88	$=\Delta k_4$
+ 9625	+ 124	+ 191	− 181	+ 194	+ 192	− 42	− 55	+ 8	+ 13	− 10	− 16	$=\Delta e_1$
	+ 3915	+ 97	− 162	+ 21	− 23	+ 86	− 28	+ 11	− 28	+ 10	+ 8	$=\Delta e_2$
		+ 2132	− 19	− 11	+ 20	+ 6	− 51	+ 32	+ 41	+ 1	+ 5	$=\Delta e_3$
			+ 381	− 17	+ 5	+ 3	+ 3	− 2	+ 4	+ 2	− 1	$=\Delta e_4$
				+ 5806	− 964	− 337	+ 181	+ 140	+ 56	− 67	+ 137	$=\Delta p_1$
					+ 7031	+ 183	− 360	− 266	+ 72	− 76	+ 7	$=\Delta q_1$
						+ 3065	− 447	− 294	+ 49	− 110	+ 2	$=\Delta p_2$
							+ 2339	+ 131	− 36	− 1	+ 15	$=\Delta q_2$
								+ 1247	− 390	+ 4	+ 58	$=\Delta p_3$
									+ 1055	− 27	+ 7	$=\Delta q_3$
										+ 338	+ 3	$=\Delta p_4$
											+ 307	$=\Delta q_4$

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Normals for 1902.

In 1902 it happened that the earth lay very nearly in the plane of Jupiter's equator and consequently very nearly also in the plane of the orbits of the satellites, * whilst in 1901 the earth was more than 1° below these planes. A necessary result of this was that in 1902 the distance and the position-angle unknowns were well separated, very much better than in 1901, as examination of the normals will show. It was determined, therefore, to solve the two sets of normals, S_2 and P_2, independently, and by a process of approximation to find those values of the unknowns which would satisfy the combined normals, C_2, it being assumed that those are the values desired. The procedure was as follows :—

In the eight normals for the unknowns Δp, Δq from the position-angle equations, *i.e.* the normals P_2, all the other unknowns were taken over to the right hand side of the equations and the eight normals were solved in the symbolical form explained above. These normals are as follows :—

Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4.	
+229·95	− 31·76	+ 25·78	+ 20·36	+ 37·12	− 19·05	+ 64·79	− 46·30	= P_1
	+282·38	+ 15·72	+ 1·02	+ 19·33	− 71·96	− 119·15	+ 136·85	= Q_1
		+603·85	+ 59·13	− 36·66	+ 27·84	+ 161·25	− 117·59	= P_2
			+742·15	− 37·40	+ 10·08	− 65·56	− 161·42	= Q_2
				+1489·97	+ 127·18	− 36·89	− 86·44	= P_3
					+2169·16	− 75·97	+ 209·98	= Q_3
						+5467·31	− 877·49	= P_4
							+5773·10	= Q_4

and their solution is,

P_1	Q_1	P_2	Q_2	P_3	Q_3	P_4	Q_4	
+ 4492	+ 524	− 194	− 118	− 132	+ 66	− 36	+ 6	= Δp_1
	+ 3721	− 163	− 25	− 82	+ 145	+ 67	− 84	= Δq_1
		+ 1704	− 125	+ 49	− 36	− 49	+ 28	= Δp_2
			+ 1374	+ 39	− 12	+ 28	+ 41	= Δq_2
				+ 683	− 46	+ 5	+ 16	= Δp_3
					+ 472	+ 6	− 21	= Δq_3
						+ 191	+ 27	= Δp_4
							+ 182	= Δq_4

The unit is one in the sixth decimal place.

The values of P_1, Q_1, etc., are as follows :—

* On three occasions the occultation of one satellite by another was observed.

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TABLE XXVI.

VALUES OF P_1, Q_1, ETC.

	Δh_1	Δk_1	Δx_1	Δh_2	Δk_2	Δx_2	Δh_3	Δk_3	Δx_3	Δh_4	Δk_4	$\Delta \epsilon_1$	$\Delta \epsilon_2$	$\Delta \epsilon_3$	$\Delta \epsilon_4$	
$P_1 =$	−3·29	+0·49	+3·26	+2·91	+0·58	+2·20	+2·51	+0·65	−2·75	+ 2·28	+0·22	+0·16	+1·29	− 6·37	− 3·39	+ 8″·282
$Q_1 =$	−1·27	−0·13	+1·21	+0·12	−0·22	+0·40	+1·65	+0·24	−1·55	+ 3·88	−0·23	−0·03	+0·68	+ 0·04	− 1·42	− 27·507
$P_2 =$	−1·73	+0·37	+1·67	+2·76	−0·03	+2·44	+1·17	+0·07	−0·83	− 4·94	−0·81	+1·32	+5·62	+ 0·66	+ 2·90	+ 54·876
$Q_2 =$	+0·29	−0·15	−0·53	+1·11	−0·46	+1·56	+0·88	−0·42	−0·77	− 8·71	−0·33	−0·82	−6·48	+ 0·97	− 8·23	−135·401
$P_3 =$	−4·62	−0·13	+4·49	+1·62	−0·16	+1·75	+9·79	+2·43	−9·99	− 0·81	−0·76	−6·04	+0·80	−10·34	+ 0·90	− 13·759
$Q_3 =$	+1·37	+0·19	−1·29	+0·19	−0·41	+0·82	+3·54	+0·31	−1·69	−25·20	+5·19	+1·60	−1·49	+ 7·15	+25·13	− 71·853
$P_4 =$	+4·56	−0·16	−3·74	+1·77	+0·40	+0·85	−7·51	−1·50	+5·44	+12·28	+0·65	−2·52	+6·22	− 0·31	+16·09	+406·751
$Q_4 =$	+1·07	+0·05	−0·85	−0·52	+0·37	−0·20	−6·13	−1·24	+4·84	+19·38	−1·46	−0·04	−2·12	− 2·47	+13·65	− 96·918

TABLE XXVII.

SOLUTION OF NORMALS S_2.

	M	H_1	K_1	X_1	H_2	K_2	X_2	H_3	K_3	X_3	H_4	K_4	E_1	E_2	E_3	E_4	
μ	+358	+ 854	− 496	+ 830	+ 356	− 340	− 304	+ 46	− 165	− 41	+ 90	− 174	+ 109	− 45	+ 17	+ 70	$= \mu$
Δh_1		+132587	−41043	+116150	+33881	− 5156	−19302	+20857	− 9015	+10769	+ 6350	+ 140	+ 2416	− 1373	−866	+1075	$= \Delta h_1$
Δk_1			+65655	− 39726	− 8876	+ 7152	+ 9821	− 6961	+ 2943	− 7621	− 295	+ 744	− 854	− 746	+198	+ 217	$= \Delta k_1$
Δx_1				+124018	+25677	−10593	−24430	+15388	−10810	+13715	+ 2027	− 31	+ 3417	− 1452	−402	+ 754	$= \Delta x_1$
Δh_2					+32145	− 7335	−28888	+ 9252	− 3952	+ 5320	+ 2727	− 178	− 485	− 960	−362	+ 579	$= \Delta h_2$
Δk_2						+25252	+11199	− 1250	+ 1670	− 3376	+ 870	− 1	− 3117	+ 477	− 79	+ 297	$= \Delta k_2$
Δx_2							+36434	− 5586	+ 4856	− 7365	+ 178	+ 456	+ 96	+ 952	+ 49	− 399	$= \Delta x_2$
Δh_3								+13783	− 4279	+12376	+ 1720	− 394	− 619	− 29	−859	+ 270	$= \Delta h_3$
Δk_3									+10929	− 4504	− 97	− 20	− 129	+ 984	+464	− 253	$= \Delta k_3$
Δx_3										+15364	− 188	− 485	− 405	+ 183	−652	+ 66	$= \Delta x_3$
Δh_4											+ 1214	− 404	− 178	− 96	−135	+ 209	$= \Delta h_4$
Δk_4												+ 3318	− 45	+ 72	+ 60	− 199	$= \Delta k_4$
$\Delta \epsilon_1$													+ 4965	− 248	− 24	− 94	$= \Delta \epsilon_1$
$\Delta \epsilon_2$														+ 1848	+ 75	− 112	$= \Delta \epsilon_2$
$\Delta \epsilon_3$															+744	− 19	$= \Delta \epsilon_3$
$\Delta \epsilon_4$																+ 262	$= \Delta \epsilon_4$

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The first approximations to the values of Δp_i and Δq_i were found by neglecting all the other unknowns in the expressions for P_1, Q_1, etc., that is, by retaining only the numerical absolute terms and substituting them in the symbolical solution for Δp, Δq, etc. The result of this substitution is given in the first column of the table below (Table XXVIII.).

The distance equations, S_2, were next solved in the symbolical form, after transferring Δp_i and Δq_i and ΔJ to the right hand sides. The solution is given in Table XXVII., the unit being as before one in the sixth decimal place.

The expressions for M, H_1, K_1, etc., may be taken from the pages containing the normals and are therefore not reproduced here. We have for instance from p. 149 :—

$$K_2 = 7''{\cdot}993 - 0{\cdot}43\Delta p_1 - 0{\cdot}20\Delta q_1 - 0{\cdot}57\Delta p_2 + 0{\cdot}19\Delta q_2 - 0{\cdot}14\Delta p_3 + 0{\cdot}15\Delta q_3 + 1{\cdot}60\Delta p_4 - 0{\cdot}22\Delta q_4 + 2{\cdot}88\Delta J.$$

The previously found first approximations to Δp_1, Δq_1, etc., were substituted in these expressions, and the resulting values of M, H_1, K_1, etc., were in turn substituted in the above equations for μ, Δh_1, Δk_1, etc. This provides us with first approximations to μ, Δh_1, Δk_1, etc. (Column 1 of the next table).

The values of the omitted terms in P_1, Q_1, etc., were then formed by substituting in them these first approximations to Δh_1, Δk_1, etc., and from the expressions for Δp_1, Δq_1, etc. (page 166) were determined the corresponding corrections to Δp_1, Δq_1, etc. This gives a second approximation to Δp_1, Δq_1, etc. (Column 2).

From what has been already said it will be clear how corrections to Δh_1, Δk_1, corresponding to the corrections to Δp_1, Δq_1, etc., were determined (Column 2). This process of approximation was continued one step further, after which no further improvement could be made (Column 3). The fourth column is the sum of the preceding three and contains those values of the unknowns which satisfy both the distance and the position-angle normals, *i.e.* both S_2 and P_2 but not the sum of the two, *i.e.* they do not satisfy the combined normals C_2, and it is such values that we seek.

Now the expressions for the unknowns are very nearly the same as they would have been had they been derived from the combined normals, C_2; let us suppose they had been so derived. If, then, the substitution of the unknowns in the normals, C_2, results in absolute terms $(an) - \alpha$, $(bn) - \beta$, etc., instead of (an), (bn) etc., or, in other words, fails to satisfy them by the quantities α, β, etc., we may take α, β, etc., as new absolute terms and find the corresponding corrections to the unknowns: the corrected unknowns

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will then satisfy the normals. This was the course adopted in the present case. The values in Column 4 were substituted in the combined normals and the "residuals" were used as absolute terms in the expressions on page 162 to give the corrections to the unknowns necessary to make them satisfy the combined normals. These corrections are tabulated in Column 5. Column 6 is the sum of Columns 4 and 5, and contains the values of the unknowns which satisfy the combined normals.

TABLE XXVIII.

APPROXIMATIONS TO UNKNOWNS.

	1 First Approximation from Solutions.	2 Second Approximation.	3 Third Approximation.	4 Values Substituted in Combined Normals.	5 Correction to satisfy Combined Normals.	6 Final Values.
μ	+0·002339	−0·000699	0·000000	+0·001640	+0·000054	+0·001694
Δk_1	−0·021964	−0·017014	+0·000005	−0·038973	−0·004785	−0·043758
Δk_1	−0·007280	+0·000755	+0·000002	−0·006523	−0·002185	−0·008708
Δx_1	−0·048043	−0·009744	+0·000001	−0·057786	−0·002866	−0·060652
Δh_2	−0·029071	−0·008811	+0·000002	−0·037880	−0·003363	−0·041243
Δk_2	+0·038017	−0·000865	0·000000	+0·037152	−0·002723	+0·034429
Δx_2	+0·138766	+0·004257	0·000000	+0·143023	+0·001700	+0·144723
Δh_3	−0·016620	−0·003876	0·000000	−0·020496	−0·001577	−0·022073
Δk_3	−0·017837	+0·001094	0·000000	−0·016743	+0·000281	−0·016462
Δx_3	−0·028035	−0·000641	−0·000001	−0·028677	−0·000228	−0·028905
Δh_4	+0·088034	−0·002952	+0·000001	+0·085083	−0·000632	+0·084451
Δk_4	+0·020197	+0·003225	0·000000	+0·023422	−0·000811	+0·022611
$\Delta \epsilon_1$	+0·132749	+0·000198	0·000000	+0·132947	+0·000887	+0·133834
$\Delta \epsilon_2$	−0·024736	−0·000839	0·000000	−0·025575	+0·000183	−0·025392
$\Delta \epsilon_3$	−0·023616	+0·000305	0·000000	−0·023311	+0·000527	−0·022784
$\Delta \epsilon_4$	−0·032815	−0·001388	0·000000	−0·034203	−0·000081	−0·034284
Δp_1	+0·009760	+0·003040	−0·000017	+0·012783	+0·000881	+0·013664
Δq_1	−0·077365	+0·001382	−0·000017	−0·076000	+0·000516	−0·075484
Δp_2	+0·092734	−0·000563	−0·000005	+0·092166	+0·000011	+0·092177
Δq_2	−0·185609	−0·000356	+0·000041	−0·185924	−0·000115	−0·186039
Δp_3	−0·006849	−0·000340	−0·000003	−0·007192	+0·000043	−0·007149
Δq_3	−0·032536	−0·001178	+0·000019	−0·033695	+0·000311	−0·033384
Δp_4	+0·066003	+0·000067	−0·000017	+0·066053	−0·000240	+0·065813
Δq_4	−0·007063	+0·000254	−0·000010	−0·006819	−0·000167	−0·006986

The next table contains the final values of the unknowns from the equations of condition for the two years, together with their weights and probable errors. With regard to the weights for 1902, it must be pointed out that they are not perfectly rigorous; they are the weights from two solutions, one of the distance normals, S_2, the other of the position-angle normals, P_2, whereas the rigorous weights would be those

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from the solution of the combined normals, C_2; but an examination of the normals will show that the weights given cannot differ materially from the weights which would be found from the solution of the combined normals.

TABLE XXIX.

Values of Unknowns.

Unknown.	Value, 1901.	Probable Error.	Correction to Unknown for Error of Jb_2.	Weight.	Value, 1902.	Probable Error.	Correction to Unknown for Error of Jb_2.	Weight.
μ	+0·00069	±0·00295	− 0·1605 ΔJ	1163	+0·00169	±0·00202	− 0·1225 ΔJ	2795
Δh_1	+0°·0756	0°·0303	+ 0·0170 ΔJ	11	−0°·0438	0°·0378	− 0·1759 ΔJ	8
Δk_1	−0·0843	0·0581	− 0·2956 ΔJ	3	−0·0087	0·0276	− 0·0198 ΔJ	15
Δx_1	+0·0512	0·0503	+ 0·1325 ΔJ	4	−0·0606	0·0378	− 0·1573 ΔJ	8
Δh_2	−0·0188	0·0201	+ 0·0442 ΔJ	25	−0·0412	0·0193	− 0·1144 ΔJ	31
Δk_2	+0·0260	0·0335	+ 0·0488 ΔJ	9	+0·0344	0·0169	+ 0·1665 ΔJ	40
Δx_2	+0·0665	0·0318	− 0·2315 ΔJ	10	+0·1447	0·0205	+ 0·1219 ΔJ	27
Δh_3	+0·0088	0·0147	+ 0·0473 ΔJ	47	−0·0221	0·0125	− 0·0588 ΔJ	73
Δk_3	−0·0789	0·0187	+ 0·1553 ΔJ	29	−0·0165	0·0112	− 0·0893 ΔJ	91
Δx_3	−0·0189	0·0214	+ 0·2959 ΔJ	22	−0·0289	0·0133	− 0·0710 ΔJ	65
Δh_4	+0·06942	0·00547	− 0·0667 ΔJ	338	+0·08445	0·00372	− 0·0095 ΔJ	824
Δk_4	+0·03301	0·00638	+ 0·0305 ΔJ	249	+0·02261	0·00616	+ 0·0612 ΔJ	301
$\Delta \epsilon_1$	+0·13587	0·00987	− 0·1064 ΔJ	104	+0·13383	0·00754	+ 0·0163 ΔJ	201
$\Delta \epsilon_2$	+0·01954	0·00630	+ 0·0349 ΔJ	255	−0·02539	0·00460	− 0·0402 ΔJ	541
$\Delta \epsilon_3$	−0·03803	0·00465	− 0·0186 ΔJ	469	−0·02278	0·00292	− 0·0123 ΔJ	1344
$\Delta \epsilon_4$	−0·03640	0·00196	− 0·0159 ΔJ	2625	−0·03428	0·00173	+ 0·0027 ΔJ	3811
Δp_1	+0·01696	0·00767	Negligible	172	+0·01366	0·00717	Negligible.	223
Δq_1	−0·06946	0·00845	,,	142	−0·07548	0·00653	,,	269
Δp_2	+0·11134	0·00557	,,	326	+0·09218	0·00442	,,	587
Δq_2	−0·17697	0·00486	,,	428	−0·18604	0·00397	,,	728
Δp_3	−0·01484	0·00355	,,	802	−0·00715	0·00279	,,	1465
Δq_3	−0·03602	0·00327	,,	948	−0·03338	0·00232	,,	2120
Δp_4	+0·04563	0·00185	,,	2955	+0·06581	0·00147	,,	5236
Δq_4	−0·03592	±0·00176	,,	3253	−0·00699	±0·00144	,,	5493

	1901.	1902.
Sum of squares of residuals	6″·971	10″·528
Number of equations of condition	337	444
Probable error of one observation	±0″·1006	±0″·1068

The sums of the squares of the residuals were calculated from the expression

$$[pvv] = [pnn] - [pan]\,x - [pbn]\,y - , \text{ etc.},$$

which serves as a valuable check on the sum of the squares of the actual residuals.

These values of the unknowns were substituted in the weighted equations of condition and the residuals, which are printed with the equations, were found. The sums

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of the squares of the residuals from every pair of satellites are tabulated below. This table shows clearly the superior accuracy of the observations of distance, which was referred to on page 161. To make the quantities for the various pairs comparable with one another, the same table contains the squares of the mean errors for the various pairs—

$$\epsilon^2 = \frac{[pvv]}{n - m},$$

where n is the number of equations and m is the number of unknowns in the equations for the pair under consideration.

TABLE XXX.

		I.–II.	I.–III.	I.–IV.	II.–III.	II.–IV.	III.–IV.	Total.
1901.	Distances . . .	0".319	0".569	0".586	0".678	0".440	0".249	2".840
[*pvv*]	Position-angles . .	0·385	0·617	0·774	0·898	0·678	0·833	4·184
								7·024
1902.	Distances . . .	0·534	0·558	0·774	0·588	0·913	0·966	4·334
[*pvv*]	Position-angles . .	1·037	0·640	1·384	0·786	0·993	1·382	6·224
								10·558
1901.	Distances . . .	0·0189	0·0406	0·0366	0·0484	0·0259	0·0178	
ϵ^2	Position-angles . .	0·0214	0·0386	0·0455	0·0599	0·0376	0·0520	
1902.	Distances . . .	0·0243	0·0223	0·0387	0·0245	0·0351	0·0322	
ϵ^2	Position-angles . .	0·0451	0·0246	0·0629	0·0314	0·0382	0·0446	

The square of the mean error and the probable error of one observation of weight unity are—

In distance . . . $\begin{cases} 1901.\ \epsilon_s^2 = 0''{\cdot}0200,\ r_s = \pm 0''{\cdot}095; \\ 1902.\ \epsilon_s^2 = 0\ {\cdot}0219,\ r_s = \pm 0\ {\cdot}100. \end{cases}$

In position-angle . . $\begin{cases} 1901.\ \epsilon_p^2 = 0\ {\cdot}0292,\ r_p = \pm 0\ {\cdot}115; \\ 1902.\ \epsilon_{p2} = 0\ {\cdot}0313,\ r_p = \pm 0\ {\cdot}119. \end{cases}$

It is clear from this that even in 1902, when special pains were taken in making observations of position-angle, they still are worse than the observations of distance. The residuals were accordingly tabulated for every day of observation and it appeared that in the case of satellite IV. large residuals occurred on the same day; also, that the same change in the y co-ordinate of IV. would cause these large residuals practically to vanish. Now the expression for y is

$$y = \frac{a''\rho}{\Delta'}\left\{\cos(v - l)\sin B + \sin\lambda\cos B\right\},$$

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λ being the latitude of the satellite; if the earth is in the plane of reference, we have $B = 0$ and

$$s \sin 1'' \, \Delta p = (- z_2 \Delta\lambda_2 + z_1 \Delta\lambda_1) \sin (p - P),$$

where z is put for $\frac{a''\rho}{\Delta'} \sin 1''$ and $\Delta\lambda$ is expressed in seconds of arc. In the preliminary solutions the residuals from the pairs involving IV. were larger than in the other three pairs and an attempt was therefore made to see if a systematic correction to the latitude of IV. would account for these large residuals. Sin $(p - P)$ was put $= \pm 1$ and $\frac{a''\rho}{\Delta'} \sin 1''$ was assumed to be a constant; hence we have

$$- z_4 \Delta\lambda_4 = \text{residual} \times \text{cosec} (p - P).$$

The daily mean was taken for the three pairs I.–IV., II.–IV. and III.–IV. and was plotted with the date as abscissa. For both years of observation there were indications of a period of about eight days in the graph of residuals; this is half the period of IV. To test this period further, the daily mean was plotted with twice the longitude of IV. as abscissa—that is to say, all the observations were brought within one period. The annexed diagram shows the result of this.

It appears that the daily means, v, are distributed about curves whose analytical expressions are approximately—

$$1901. \quad v = 0''{\cdot}050 + 0''{\cdot}150 \sin (2 l_4 + 10^\circ);$$
$$1902. \quad v = 0 \cdot 100 + 0 \cdot 150 \sin (2 l_4 - 100).$$

The individual residuals for I.–V., II.–IV. III.–IV. were then corrected for this systematic run, and the sum of the squares of the corrected residuals found. The results were as follows—

Sum of Squares of Residuals.		I.–IV.	II.–IV.	III.–IV.	Sum.
1901	Before correction . .	0''·774	0''·678	0''·833	2''·285
	After ,, . .	0·454	0·614	0·640	1·708
1902	Before correction . .	1·384	0·993	1·382	3·759
	After ,, . .	0·493	0·390	0·772	1·655

Thus the sum of the squares of the residuals in position-angle for the case of the fourth satellite is reduced by 25 per cent in 1901 and by 56 per cent in 1902. It will be noticed that it is the residuals of the pair I.–IV. that are most improved, and that in the preliminary solutions the values of the unknowns from this pair differed widely from those given by the other pairs. The absolute terms of I.–IV. for 1901 were

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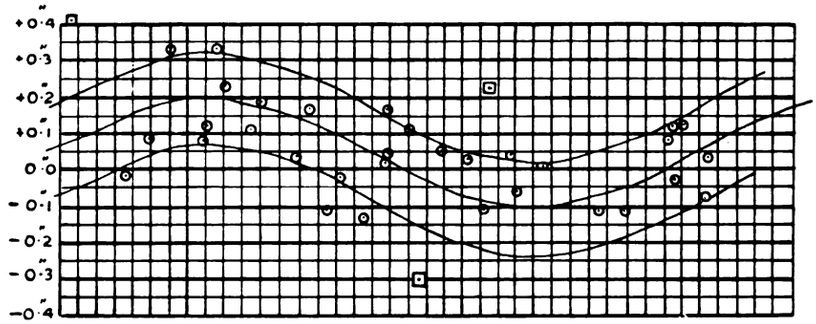
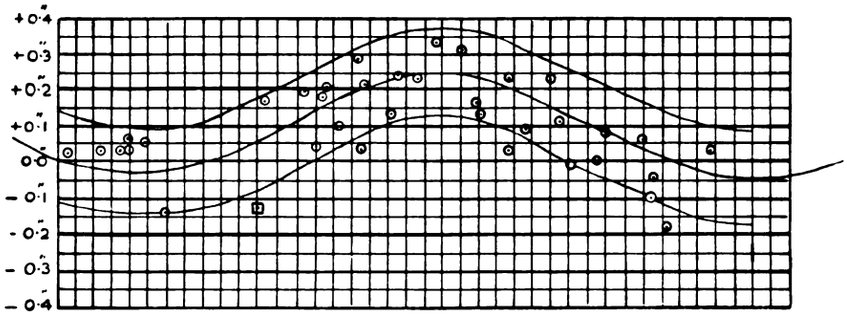
[*To face page* 172.

GRAPH OF MEAN RESIDUALS IN POSITION-ANGLE.

1901



1902



The abscissa are twice the longitude of IV.

Points enclosed in squares depend on only one observation.

The two outside curves are parallel to the middle one, which is the graph of the mean residual. They are drawn at a distance of ±0″·150 from it, so that any point outside the curved band represents a corrected residual exceeding ±0″·150.

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accordingly corrected for a systematic error, which was graphically derived from the residuals in the preliminary solution, viz.—

$$0''\cdot105 + 0''\cdot210 \sin (2v + 25^\circ).$$

This was the expression found on a first inspection of the residuals for 1901, and the preliminary solution was made again with the new absolute terms: the sum of the squares of the residuals was reduced from $1''\cdot07$ to $0''\cdot63$, and the values of the unknowns were altered as follows—

	Preliminary Solution.		Final Solution.
	Old Values.	New Values.	Uncorrected.
Δp_1	−0·0206	+0·0066	+0·0170
Δq_1	−0·0613	−0·0618	−0·0695
Δp_4	+0·0461	+0·0572	+0·0456
Δq_4	−0·0431	−0·0301	−0·0359

Thus the application of this systematic correction brings the preliminary solution of I.–IV. much better into line with the others.

The evidence that has now been given demonstrates pretty conclusively the reality of the existence of a systematic error, but no explanation of it can be found. It was naturally at first attributed to some error in the method of reduction or in the reductions themselves, but careful re-examination proves that this is not the case and the fact remains unexplained. For this reason the final values of the unknowns have not been corrected by taking account of it in the absolute terms: the only effect of the periodic part is to increase the probable error of the unknowns, but the non-periodic part will have some effect on them. Until some explanation, however, or some further confirmation of the inequality—at present purely empirical—has been found, it seems best to disregard it: if it is confirmed by the photographs which were taken in 1902 and are now being reduced by the writer, then the unknowns should perhaps be corrected for it; but until the photographs are reduced it seems best to leave the matter as it now stands.

It remains to examine the residuals in distance. The only systematic error here could be one depending on the measured distance: the residuals were accordingly arranged in order of distance and the mean of every five consecutive residuals was assumed to be the residual for their mean distance. Since the correction for "crossing through" or the κ correction has been applied, it was not to be expected that any systematic run would show itself, and the following table bears this out.

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TABLE XXXI.

RESIDUALS ARRANGED IN ORDER OF DISTANCE MEASURED—1901.

Distance.	Residual.	Distance	Residual.	Distance.	Residual.
30	+0".117	196	−0".031	370	−0".005
47	+0.173	208	−0.018	403	+0.024
59	+0.050	220	+0.068	420	−0.106
71	+0.075	236	−0.046	465	−0.014
84	−0.030	256	−0.035	493	−0.166
94	−0.005	271	−0.082	522	−0.040
111	−0.030	280	−0.061	546	+0.129
126	+0.010	295	+0.032	581	−0.009
137	−0.039	320	+0.037	646	+0.082
160	−0.028	342	+0.050	734	+0.026
186	−0.016	353	+0.024	855	+0.007

1902.

Distance.	Residual.	Distance.	Residual.	Distance.	Residual.
6	−0".108	162	−0".017	407	−0".109
12	−0.036	182	−0.038	440	−0.076
18	+0.146	194	−0.086	469	+0.052
23	+0.003	212	−0.074	485	−0.041
31	+0.126	229	+0.060	507	+0.022
42	+0.098	238	+0.033	528	−0.054
62	+0.075	250	−0.067	542	+0.010
72	+0.044	265	+0.120	565	−0.081
81	−0.017	283	+0.032	618	−0.065
94	+0.038	306	+0.007	654	−0.028
105	+0.002	315	+0.050	689	+0.056
116	+0.041	325	−0.032	750	+0.010
125	+0.113	343	+0.061	776	+0.038
134	−0.043	362	+0.029	854	−0.037
148	+0.028	390	+0.090	922	+0.040

If these residuals are plotted, they appear to be uniformly distributed about the zero line. The distance observations may therefore be taken as free from systematic error.

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CHAPTER VI.

THE STANDARD STARS AND THE SYSTEMATIC DIFFERENCE BETWEEN LONG AND SHORT DISTANCES OBSERVED WITH THE HELIOMETER.

THE positions of the standard stars are required to determine the absolute scale-value and the zero of position-angle. Preliminary values of their distance and position-angle were adopted in the reductions, and we must now inquire whether those values are in need of appreciable corrections.

Let us first consider what degree of accuracy we wish to attain. The scale-value, *i.e.* the value in arc of one scale division of the heliometer, was found from measurement of the known distance of the standards and the mass of Jupiter is the only quantity that is affected by an error in the adopted scale-value; for the corresponding error in any absolute term of the equations of condition is proportional to s, the distance observed, and the coefficient of the correction to the mass is also s. Hence an error in the distance of the standards affects only the mass. Now it is desirable that the error in the mass due to an incorrect scale-value should not exceed its probable error as derived from the inter-agreement of the observations: this may be put at ± 6 in 100,000. The corresponding error of scale is one third of this, or ± 2 in 100,000: hence taking the distance of the standards as $5000''$, we get $\pm 0''{\cdot}1$ as the limit of error of their distance. If the error of their position-angle expressed in arc of a great circle were the same, the error in the zero of position-angle would be $\pm 4''$. The only way of determining the positions of the standards is to collect observations made with the meridian circles, and it need hardly be said that to attain the desired accuracy a large number of meridian observations is necessary; at the same time what are required are not absolute right ascensions and declinations but their differences.

The four stars used as standards in this investigation are on Sir D. Gill's List of Zodiacal Stars and have recently been observed at several observatories.

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Standard Stars, 1901.

The recent observations of these stars are as follows :—

STANDARDS, 1901.—RIGHT ASCENSIONS.

Place of Observation.	24 Sagittarii. Epoch.	R.A. 1901.	No. of Obs.	26 Sagittarii. Epoch.	R.A. 1901.	No. of Obs.	Δα	Remarks.
		h m s			h m s		m s	
Poulkova	1895·5	18 27 50·58	4	1896·5	18 35 49·15	2	7 58·57	Poulk. Obs. II., vol. ix. part iii.
Washington	1902·6	50·60	7	1902·6	49·35	7	58·75	Astr. Journ., xxiii. p. 56. 6 in. T. C.
Lick	1902·5	50·59	2	1902·5	49·30	2	58·71	Lick Obs. Bull., 43.
Cape	1901·5	50·64	4	1901·5	49·40	4	58·76	Cape M.S.
Heidelberg	1903·0	50·61	2	1903·0	49·32	2	58·71	Communicated by Dr. Courvoisier.
Cambridge	1901·2	50·59	4	1901·0	49·31	6	58·72	Communicated by Mr. Hartley.
Greenwich	1900·6	50·58	5	1899·6	49·43	4	58·85	Communicated by Mr. Dyson.
Simple Mean		50·599			49·323		58·724	
Weighted Mean		50·598			49·336		58·738	

STANDARDS, 1901.—DECLINATIONS.

Place of Observation.	24 Sagittarii. Epoch.	Dec. 1901.	No. of Obs.	26 Sagittarii. Epoch.	Dec. 1901.	No. of Obs.	Δδ	Remarks.
Poulkova	1895·5	−24° 6′ 21″·20	4	1896·5	−23° 55′ 34″·85	2	10′ 46″·35	Poulk. Obs. II., vol. ix. part iii.
Washington	1902·6	22·43	7	1902·6	32·83	7	49·60	Astr. Journ., xxiii. p. 56. 6 in. T. C.
Lick	1902·5	22·82	2	1902·5	32·84	2	49·98	Lick. Obs. Bull., 43.
Cape	1901·5	21·81	4	1901·5	32·05	4	49·76	Cape M.S.
Heidelberg	1903·0	22·06	2	1903·0	32·46	2	49·60	Communicated by Dr. Courvoisier.
Charkow	1897·3	21·67	3	1899·5	33·07	4	48·60	Communicated by Dr. L. Struve.
Cambridge	1901·2	23·08	4	1901·0	33·54	6	49·54	Communicated by Mr. Hartley.
Greenwich	1900·6	22·42	5	1899·6	33·45	4	48·97	Communicated by Mr. Dyson.
Simple Mean		22·19			33·14		49·05	
Weighted Mean		22·20			33·09		49·11	
Rejecting Poulkova Simple Mean		22·33			33·04		49·29	
Weighted Mean		22·35			32·96		49·39	

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These observations were combined by taking (i.) simple means and (ii.) weighted means, the observations being weighted proportionally to the number of observations. Since the number of observations does not vary much, it is immaterial whether in taking the means we adopt as the weight the number itself or its square root. It is difficult to know how to treat these results, for they are far from satisfactory. I have adopted the mean of the simple and weighted means and conclude that the recent observations give

$$\Delta\alpha = 7^m\ 58^s{\cdot}731.$$
$$\Delta\delta = 10'\ 49''{\cdot}34.$$

Besides the recent observations I have also utilised the places of these stars from published catalogues. Mr. W. J. Thackery of the Royal Observatory, Greenwich, kindly communicated these places to me: they are contained in the following table, p. 178.

Newcomb's systematic corrections have been applied to reduce to the system of his Fundamental Catalogue. The precessions are those of Struve-Peters. Weights were assigned by means of Newcomb's tables, pp. 238–40 of his Fundamental Catalogue, and equations were solved to give the places for 1900 and the proper motions: the latter are given in the table. After applying precession and proper motion the following values of the co-ordinates were found for 1901 :—

	R. A. 1901.	Dec. 1901.
24 Sagittarii . . .	18 27 50·631	−24 6 22·11.
26 Sagittarii . . .	18 35 49·363	−23 55 32·02.

Thus the catalogues give :—

$$\Delta\alpha = 7^m\ 58^s{\cdot}732.$$
$$\Delta\delta = 10'\ 50''{\cdot}09.$$

This value of $\Delta\alpha$ only differs $0^s{\cdot}001$ from that found from the recent observations, but the values of $\Delta\delta$ differ by $0''{\cdot}75$. It was decided to take means giving weight unity to the values from the catalogues and weight 3 to those from the recent observations. Hence the final values are :—

$$\Delta\alpha = 7^m\ 58^s{\cdot}731.$$
$$\Delta\delta = 10'\ 49''{\cdot}53.$$

and the corresponding values of the distance and position-angle (1901) are :—

$$s = 6591''{\cdot}364.$$
$$p = 84^\circ\ 20'\ 40''.$$

In the reduction of the observations the preliminary values adopted were :—

$$s = 6591''{\cdot}100.$$
$$p = 84^\circ\ 20'\ 36''.$$

The difference in position-angle is insignificant, but the difference in distance involves a not inappreciable correction to the mass of Jupiter.

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TABLE XXXII.

Meridian Places of Standards, 1901.

	Name of Catalogue.	R.A. from Catalogue.	Epoch.	No. of Observations.	Correction for Precession.	Systematic Correction.	Adopted R.A. 1900.	Seconds Corrected by Proper Motion.	N.P.D. from Catalogue.	Epoch.	No. of Observations.	Correction for Precession.	Systematic Correction.	Adopted N.P.D. 1900.	Seconds Corrected by Proper Motion.
		h m s			m s	s	h m s	s	° ′ ″			′ ″	″	° ′ ″	″
24 Sagittarii	Bradley	18 18 55·36	1755·4	14	+8 51·80	−·08	18 27 47·08	46·96	114 11 20·0	1755·2	5	−4 55·80	−·5	114 6 23·7	24·3
	Madras 1835	18 23 48·77	1835·2	2	+3 58·36	−·11	18 27 47·02	46·97	114 8 51·47	1833·4	3	−2 26·44	−·35	114 6 24·68	24·94
	Cape 1860	18 25 20·27	1857·3	1	+2 26·68	+·03	18 27 46·98	46·94	114 7 56·45	1857·3	1	−1 32·76	·00	114 6 23·69	23·86
	Cape 1880	18 26 33·62	1878·6	3	+1 13·33	+·06	18 27 47·01	46·99	114 7 12·17	1878·6	3	−0 47·44	−·10	114 6 24·63	24·71
	Greenwich 1880	18 26 33·62	1879·8	6	+1 13·33	+·04	18 27 46·99	46·97	114 7 12·65	1879·8	6	−0 47·44	−·78	114 6 24·43	24·51
	Cape 1890	18 27 10·26	1891·7	3	+0 36·67	+·02	18 27 46·95	46·94	114 6 48·56	1891·7	3	−0 23·98	+·04	114 6 24·62	24·65
	Greenwich 1890	18 27 10·27	1890·9	6	+0 36·67	+·05	18 27 46·99	46·98	114 6 49·20	1890·9	6	−0 23·98	−·66	114 6 24·56	24·60
	Greenwich 1890	18 27 46·88	1898·6	3	...	+·05	18 27 46·93	46·93	114 6 24·90	1898·6	3	...	−·66	114 6 24·24	24·24
	1900 Adopted Precession +3·6664 Adopted Proper Motion −0·00084								1900 Adopted Precession −2·425 Adopted Proper Motion +0·0040						
26 Sagittarii	Bradley	18 26 54·72	1754·3	7	+8 50·78	−·08	18 35 45·42	45·72	114 2 9·1	1754·6	3	−6 36·4	−·5	113 55 32·2	35·8
	Madras 1835	18 31 47·78	1836·2	9	+3 57·87	−·11	18 35 45·54	45·67	113 58 45·35	1838·0	9	−3 11·49	−·35	113 55 33·5	35·1
	Cape 1840	18 32 6·04	1840·5	2	+3 39·58	+·01	18 35 45·63	45·75	113 58 31·00	1835·4	3	−2 57·54	−·33	113 55 33·13	34·76
	Cape 1880	18 34 32·36	1878·6	3	+1 13·18	+·06	18 35 45·60	45·64	113 56 36·03	1878·6	3	−1 1·30	−·10	113 55 34·63	35·17
	Greenwich 1880	18 34 32·42	1880·6	5	+1 13·18	+·04	18 35 45·64	45·68	113 56 36·52	1879·8	6	−1 1·30	−·78	113 55 34·44	34·95
	Greenwich 1890	18 35 9·05	1891·8	5	+0 36·59	+·05	18 35 45·69	45·71	113 56 6·66	1891·8	5	−0 30·91	−·66	113 55 34·10	34·31
	Greenwich 1900	18 35 45·71	1899·6	4	...	+·05	18 35 45·76	45·76	113 55 36·55	1899·6	4	...	−·66	113 55 35·89	35·90
	1900 Adopted Precession +3·6586 Adopted Proper Motion +0·00205								1900 Adopted Precession −3·117 Adopted Proper Motion +0·0252						

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Standard Stars, 1902.

The recent observations of these stars are collected in the following table.

STANDARDS, 1902.—RIGHT ASCENSIONS.

Place of Observation.	21 Capricorni. Epoch.	R.A. 1902.	No. of Obs.	23 Capricorni. Epoch.	R.A. 1902.	No. of Obs.	Δα	Remarks.
		h m s			h m s		m s	
Poulkova	1895·7	20 55 20·83	5	1895·6	21 0 26·31	5	5 5·48	Poulk. Obs. II., vol. ix. part iii.
Washington	1902 6	20·88	6	1902·6	26·36	6	5·48	Astro. Journ., xxiii. p. 56. 6 in. T. C.
Lick	1902·7	20·88	2	1902·7	26·35	2	5·47	Lick Obs. Bull., 43.
Cape	1901·9	20·91	10	1902·0	26·34	26	5·43	Cape M.S.
Heidelberg	1903·6	20·88	2	1903·6	26·32	2	5·44	Communicated by Dr. Courvoisier.
Cambridge	1902·5	20·86	12		26·36	N.A.	5·50	Communicated by Mr. Hartley.
Washington	1900·7	20·88	2	1900·7	26·36	A.E.	5·48	Washington Obs.
Greenwich	1901·9	20·83	5	1901·3	26·36	66	5·53	Communicated by Mr. Dyson.
Simple Mean		20·869			26·345		5·476	
Weighted Mean		20·870			26·349		5·479	

STANDARDS, 1902.—DECLINATIONS.

Place of Observation.	21 Capricorni. Epoch.	Dec. 1902.	No. of Obs.	23 Capricorni. Epoch.	Dec. 1902.	No. of Obs.	Δδ	Remarks.
		° ′ ″			° ′ ″		′ ″	
Poulkova	1895·7	−17 54 47·38	5	1895·6	−17 37 20·92	5	17 26·46	Poulk. Obs. II., vol. ix. part iii.
Washington	1902·6	47·20	7	1902·6	20·40	7	26·80	Astr. Journ., xxiii. p. 56. 6 in. T. C.
Lick	1902·7	47·34	2	1902·7	20·38	2	26·96	Lick. Obs. Bull., 43.
Cape	1901·9	47·56	10	1901·9	20·52	26	27·04	Cape M.S.
Heidelberg	1903·6	48·09	2	1903·6	21·11	2	26·98	Communicated by Dr. Courvoisier.
Charkow	1896·5	47·42	5	1896·7	20·58	5	26·84	Communicated by Dr. L. Struve.
Cambridge	1902·5	47·80	12		20·97	N.A.	26·83	Communicated by Mr. Hartley.
Washington	1900·7	47·64	2		20·97	A.E.	26·67	Washington Obs.
Greenich	1901·9	48·20	5	1901·3	20·51	32	27·69	Communicated by Mr. Dyson.
Simple Mean		47·63			20·71		26·92	
Weighted Mean		47·61			20·73		26·88	

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At Cambridge and Washington 23 Capricorni was regularly used as a clock-star, and it seemed best to adopt the place given in the ephemeris which was used at the Observatory. The weight assigned to these was 30; the weights for the other observations were equal to the number of observations when it did not exceed 30. When it exceeded 30, a maximum weight of 30 was assigned.

The following discussion of the places of these stars was made by Mr. Löwinger. As the stars are close together the differences of right ascension and declination can be determined free from systematic error, provided only those catalogues are used which contain both stars. The following table shows the values of $\Delta\alpha$ and $\Delta\delta$ from the catalogues which contain both stars:—

	$\Delta\alpha$ (1900).	$\Delta\delta$ (1900).	Residuals.	
			$\Delta\alpha$	$\Delta\delta$
	m s	′ ″	s	″
1755 Bradley	5 5·47	17 25·26	+0·007	−0·11
1800 Piazzi	·41	22·75	− ·053	−2·62
1845 Paris	·41	25·13	− ·053	−0·24
1860 Paris	·47	24·81	+ ·007	−0·56
1875 Paris	·48	25·27	+ ·017	−0·10
1875 C.G.A.	·55	24·52	+ ·087	−0·85
1880 Greenwich	·38	25·73	− ·083	+0·36
1890 Greenwich	·48	26·38	+ ·017	+1·01
1890 Radcliffe	·47	26·26	+ ·007	+0·89
1900 Cape	·49	26·64	+ ·027	+1·27
1902 Washington	·48	26·30	+ ·017	+0·93
Simple Mean	5 5·463	17 25·37		

These were brought up to 1900 using Auwers' proper motions, viz.:—

21 Capricorni	$-0^s\cdot0041$	$+0''\cdot012$
23 (θ) Capricorni	$+0^s\cdot0040$	$-0''\cdot054$

The right ascensions appear to be fairly satisfactory and were accepted as they stand, but there can be no doubt from the appearance of the residuals in declination that there is an error in the adopted proper motions; the small residual in the case of Bradley is due to the fact that Auwers used his position for determining the proper motion which has been here adopted. To find a correction to it, Bradley's $\Delta\delta$ was omitted, and from the remaining ten values a correction of $+0''\cdot0427$ per annum was found; this gave the following values for 1900.

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	Δδ (1900).	Residual.
1755	(17′ 31″·55)	(+4″·96)
1800	27·02	+0·43
1845	27·48	+0·89
1860	26·52	−0·07
1875	26·34	−0·25
1875	25·59	−1·00
1880	26·58	−0·01
1890	26·81	+0·22
1890	26·69	+0·10
1900	26·64	+0·05
1902	26·21	−0·38
Mean	17 26·59	

The representation of all except the Bradley observations is much improved and the correction of 0″·0427 has been adopted. Thus the following values were found :—

$$\Delta\alpha = 5^m\ 5^s{\cdot}463 + [-0^s{\cdot}0120 + 0^s{\cdot}0081]\,(t - 1900)$$
$$\Delta\delta = 17'\ 26''{\cdot}59 + [+0''{\cdot}318 - 0''{\cdot}023]\ (t - 1900)$$

Collecting results, we get :—

	Δα (1902).	Δδ (1902).
Recent observations . . .	5m 5s·478	17′ 26″·90
Catalogues	5 5·455	17 27 ·18

If, as before, we give weight unity to the values from the catalogues and weight three to those from the recent observations, we find the following final values :—

$$\Delta\alpha = 5^m\ 5^s{\cdot}472$$
$$\Delta\delta = 17'\ 26''{\cdot}97$$

The corresponding distance and position-angle are :—

$$s = 4487''{\cdot}355$$
$$p = 76^\circ\ 30'\ 32''$$

The preliminary values adopted in the reduction of the observations were :—

$$s = 4486''{\cdot}930$$
$$p = 76^\circ\ 31'\ 9''$$

The necessary corrections to the unknowns to reduce to the finally adopted values are made in the next chapter.

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For convenience in making any corrections which may subsequently be found necessary, the following formulæ may be given :—

$$\text{Standards, 1901} \quad \begin{cases} ds = +0{\cdot}10\, d(\Delta\delta) + 13{\cdot}6\, d(\Delta\alpha) \\ dp = -31{\cdot}1\, d(\Delta\delta) + 42{\cdot}2\, d(\Delta\alpha) \end{cases}$$

$$\text{Standards, 1902} \quad \begin{cases} ds = +0{\cdot}23\, d(\Delta\delta) + 13{\cdot}9\, d(\Delta\alpha) \\ dp = -44{\cdot}7\, d(\Delta\delta) + 153{\cdot}2\, d(\Delta\alpha) \end{cases}$$

in which $d(\Delta\delta)$ is expressed in seconds of arc, and $d(\Delta\alpha)$ in seconds of time.

We now turn to the investigation of the systematic difference between long and short distances observed with the heliometer.

Since the scale-value for the distance observations of the satellites was determined by observation of standard distances which were roughly ten times the average distance of the satellites, it is essential to satisfy ourselves that the scale-value, *i.e.* the value in arc of one division of the heliometer scales, is the same for large distances as it is for small. Mr. Hough found in his discussion of the South Polar Triangulation that a self consistent geometrical figure could be built up out of a series of measures of distances ranging from 917″ to 7299″ only by assuming that direct measures of distance required a correction which could be analytically expressed by the formula

$$x + y\sigma + z\sigma^2.$$

The square term is the only term of importance, and the correction became zero for distances of a little over 1000″. Mr. Hough found that when σ was expressed in terms of 1000″ as unit, the coefficient z had the following values :—

Observer L	+ 0·0224	Mean 0·0210
Observer *g*	+ 0·0169	
Observer S	+ 0·0238	

If, then, the writer's observations were affected by a similar error, the measured distances of the satellites would require a correction of 0″·005, whilst those of the standards for 1901 would require a correction of as much as 0″·911. If this error existed unknown to the observer, the effect would be the same as that of adopting a distance of the standards which was in error by 0″·911, *i.e.* by nine times the limit aimed at. Though no such systematic correction had previously been detected in investigations with the Cape Heliometer, its discovery by Mr. Hough for three observers was unexpected and disconcerting, and the writer determined to take the first opportunity of returning to the Cape and making the necessary observations to see whether his measurements were similarly affected. The occasion of the visit of the British Association to South Africa offered a favourable opportunity ; the writer

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revisited South Africa with the Association and made the necessary observations in August and September, 1905.

Examination of the residuals of the distance equations had indicated no systematic error depending on the distance observed; in other words, the scale-value was the same for all distances up to 1000″. What was now necessary was therefore to compare the average distance of the satellites with that of the standards. In order to vary the conditions this was done in three ways.

(i.) Six or seven stars not fainter than 8·5 magnitude, lying as nearly as possible on a great circle and at distances of about 1000″, were chosen in a convenient part of the sky, thus:—

1 2 3 4 5 6 7

The distances $\overline{12}$, 23, $\overline{34}$, etc., were measured, also the position-angles of these pairs. Then if there were no systematic error, the sum of the projections of $\overline{12}$, $\overline{23}$, $\overline{34}$, etc., on the line $\overline{17}$ would be the same as the directly measured distance $\overline{17}$. (Groups I, II, and IV).

(ii.) Three stars were chosen with the same conditions as before, except that the distances between them were about 3500″ instead of 1000″. Distances and position-angles of the three pairs of stars were measured. (Groups C and E.)

(iii.) Three stars were chosen, the distances between them being about 700″ and 5000″. Since the systematic correction would be nearly the same for a distance of 5700″ as for one of 5000″, this grouping of the stars would afford a direct comparison of a distance of 700″ with a distance of 5350″. (Groups α, γ, δ, ϵ, η, θ, ι.)

If a systematic error existed, the first two methods of grouping would afford means of determining its analytical expression, whilst the third would serve as a check on the accuracy with which it represented the observations. The third method had, moreover, as already mentioned, the advantage of giving a direct comparison between the two distances in question. Seven hundred seconds was chosen as the length to aim at in selecting the stars, because it was the distance which entered most into the determination of the mass.

The individual measurements of the various groups of stars are given in the following table. The correction for refraction has been applied, but there was no necessity to apply the correction for aberration. The correction for "crossing through," or the κ correction, has been applied to the short distances in the third method of grouping, though it rarely amounted to 0″·02: κ was not measured in the other groups, since its effect would be practically inappreciable.

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Group I.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–4.	Pair 4–5.	Pair 5–6.	Pair 6–1.
August 10	1447″·595	1254″·254	1584″·323	1610″·298	766″·237	6586″·421
12	·700	4·026	·374	·655	·170	5·899
20	·719	4·029	·635	·993	·138	6·673
21	·573	4·281	·401	·504	·216	6·249
21	·689	3·967	·842	·434	·399	6·073
23	·674	3·953	·439	·706	·161	5·831
25	·374	3·972	·541	·670	·253	6·197
25	·670	4·015	·435	·441	·277	6·440
28	·682	4·116	·485	·660	·253	6·067
28	1447·500	1254·159	1584·503	1610·726	766·182	6586·125
Mean.....	1447·618	1254·077	1584·498	1610·609	766·229	6586·198

Group II.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–4.	Pair 4–5.	Pair 5–1.
August 17	1385″·349	1030″·641	2670″·374	901″·238	5935″·666
17	5·014	·602	·907	0·656	·001
19	4·711	·553	·496	0·996	·378
19	4·926	·744	·476	1·182	·541
20	4·782	·747	·479	1·013	·737
20	4·693	·404	·665	1·208	·881
21	4·760	·619	·512	1·146	·465
21	4·938	·420	·575	0·855	·622
23	4·682	·446	·371	1·266	·219
23	4·549	·652	·532	1·044	·433
25	4·648	·508	·450	1·072	·107
25	4·891	·559	·606	1·224	·633
24	4·754	·205	·343	1·135	·649
27	4·436	·543	·481	1·082	·536
27	1384·858	1030·561	2670·628	901·278	5935·424
Mean.....	1384·799	1030·547	2670·526	901·098	5935·553

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Group IV.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–4.	Pair 4–5.	Pair 5–6.	Pair 6–1.
August 30	360″·858	886″·904	1145″·478	1765″·080	500″·737	4614″·467
30	·838	·840	·291	·311	·697	·343
September 5	·925	·800	·384	·197	·585	·276
5	360·728	886·923	1145·284	1765·461	500·660	4614·242
Mean.....	360·837	886·867	1145·359	1765·262	500·670	4614·332

Group C.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–1.
Aug. 30	3383″·251	3485″·721	6834″·835
31	·172	·963	·937
31	·707	·815	·856
Sept. 5	·401	·693	·593
5	3383·671	3485·829	6834·826
Mean.....	3383·440	3485·804	6834·814

Group E.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–1.
Sept. 5	2911″·508	4006″·085	6916″·165
5	2911·733	4005·987	6916·363
Mean.....	2911·620	4006·036	6916·264

Group α.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–1.
Sept. 11	5162″·715	743″·313	5900″·529
11	5163·524	743·323	5900·999
Mean.....	5163·120	743·318	5900·764

Group δ.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–4.
Sept. 8	419″·636	6038″·057	6457″·717
8	9·702	7·769	·745
8	[420·068]	[6038·422]	[6457·400]
Mean.....	419·669	6037·913	6457·731

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Group γ.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–1.
Sept. 8	5858″·817	766″·069	6585″·937
8	9·342	·286	6·006
8	9·328	·206	6·153
8	9·160	·256	6·290
13	9·037	·204	6·231
13	5859·104	766·287	6586·114
Mean.....	5859·131	766·218	6586·122

Group ϵ.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–1.
Sept. 8	5047″·528	901″·006	5935″·771
8	·693	·016	6·079
8	·581	·133	5·712
8	·738	·218	5·916
11	·345	·166	5·834
11	·685	·245	5·911
13	·697	·150	5·929
13	5047·540	901·105	5935·691
Mean.....	5047·601	901·130	5935·855

Group ι.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–1.
Sept. 11	552″·551	5566″·340	6116″·317
13	·736	·394	·320
13	·670	·567	·751
Mean.....	552·652	5566·434	6116·463

Group η.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–1.
Sept. 10	789″·796	5893″·302	6674″·385
10			·458
11		·621	·559
11	·603	·284	·398
11	·644	·448	·578
11	·856	·584	·284
12	·717	·224	·663
12	·598	·202	·572
13	·598	·229	·198
13	·647	·322	·531
13	·648	·314	·577
13	789·676	5893·220	6674·362
Mean.....	789·678	5893·341	6674·464

Group θ.

Date.	Pair 1–2.	Pair 2–3.	Pair 3–4.
Sept. 11	585″·946	5941″·564	6524″·906
11	·790	·358	5·156
11	·835	·645	4·932
11	·894	·569	5·064
13	·847	·372	5·054
13	·996	·177	5·027
9	585·717	5941·600	6524·761
Mean.....	585·861	5941·469	6524·986

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The position-angles of all these pairs of stars were observed by Mr. Whittingdale (the distances, of course, by the writer): the individual observations need not be given here, but the mean observed position-angles corrected for refraction are tabulated below since they are required for the calculation of coefficients.

Pair.	No. of Obs.	Position-Angle.	Pair.	No. of Obs.	Position-Angle.	Pair.	No. of Obs.	Position-Angle.
	I.			II.			IV.	
1–2	8	51° 28′ 8″	1–2	8	176° 1′ 25″	1–2	8	17° 46′ 54″
2–3	8	37 21 40	2–3	8	162 23 0	2–3	8	24 3 39
3–4	8	50 54 38	3–4	8	180 51 19	3–4	6	18 37 8
4–5	8	38 52 25	4–5	8	186 20 23	4–5	8	31 51 57
5–6	8	25 15 41	5–1	8	177 24 35	5–6	8	7 15 57
6–1	8	42 38 34				6–1	8	23 23 52
	C.			E.			a.	
1–2	4	154° 44′ 6″	1–2	2	81° 47′ 55″	1–2	2	29° 51′ 2″
2–3	4	143 6 55	2–3	2	79 11 21	2–3	2	37 15 50
3–1	4	148 49 57	3–1	2	80 17 6	3–1	2	30 46 48
	γ.			δ.			ε.	
1–2	8	224° 52′ 44″	1–2	2	217° 30′ 12″	1–2	4	355° 48′ 52″
2–3	8	205 15 13	2–3	2	216 47 3	2–3	4	6 20 13
3–1	8	222 38 33	3–1	2	216 49 48	3–1	4	357 24 9
	η.			θ.			ι.	
1–2	4	104° 58′ 39″	1–2	2	211° 35′ 17″	1–2	2	337° 53′ 6″
2–3	4	95 51 10	2–3	2	216 38 59	2–3	2	343 48 23
3–1	4	96 55 46	3–1	2	216 11 26	3–1	2	343 16 30

It was desired first of all to arrive at some idea of the magnitude of the systematic error which we are investigating. For this purpose the means of the observed distances were taken and treated as follows. In the triangle formed by the stars 1, 2, 3, the sides $\overline{12}$ and $\overline{23}$ represent the first two of the observed distances. The observed position-angles are the position-angles of these sides at their middle points: by applying the necessary corrections (see *Chauvenet*, vol. ii. p. 396) we deduce the angles P12, P21, P23, P32, etc., where P is the pole, and hence the angle 123. Thus the triangle 123 is known; and having calculated by the formulæ of spherical trigonometry the side $\overline{13}$ and the angle 231, we proceed, if necessary, to the triangle 134 and find the side $\overline{14}$ and the angle 341. We finally arrive in this way at the side $\overline{16}$, which we

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now compare with the directly observed distance $\overline{16}$. It is evident that this process is equivalent to projecting in plane trigonometry the distances $\overline{12}$, $\overline{23}$, etc., on to the line $\overline{16}$ and taking their sum.

The distances were generally measured each night in the order 1, 2, 3, 3, 2, 1, so that the mean times of observation of the various distances were the same. Thus if any instrumental changes were taking place during the making of a set of observations their effect was eliminated. As each night's observation of a group forms a set complete within itself, the mean result from any number of such sets may be used regardless of changes of scale-value from night to night.

The following table contains the comparison of the distance calculated from the shorter distances with that directly observed.

	I.	II.	IV.	C.	E.	a.
Calculated Distance	6586″·082	5935″·560	4614″·276	6834″·617	6916″·376	5900″·861
Observed Distance	·198	·553	·332	·814	·264	·764
O−C	+0″·116	−0″·007	+0″·056	+0″·197	−0″·112	−0″·097
No. of Observations	10	15	4	5	2	2

	γ.	δ.	ε.	η.	θ.	ι.
Calculated Distance	6586″·136	6457″·546	5935″·854	6674″·700	6525″·084	6116″·328
Observed Distance	·122	·731	·855	·464	4·986	·463
O−C	−0″·014	+0″·185	+0″·001	−0″·236	−0″·098	+0″·135
No. of Observations	6	2	8	10	7	3

Inspection of the differences O−C points unmistakably to the conclusion that if there is any systematic error, it is much less than 0″·1: the weighted mean of the groups consisting of a long and a short distance, *i.e.* those named with Greek letters, is −0″·067, that is to say the short distance is measured too long relatively to the long distance. But the evidence from Groups I, II and IV points to exactly the opposite. After careful consideration it was concluded that the results so far derived were sufficient to prove that the systematic error under investigation was practically zero and that from a practical point of view it was not worth while proceeding further.

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CHAPTER VII.

RESULTS.

THIS chapter contains a discussion of the values of the unknowns and of some results that they lead to. Some of the quantities have not the same values as those assigned to them in the writer's paper in the *Monthly Notices, R.A.S*, lxiv. 728–47, the reason being that they have been subjected to a final revision chiefly on account of corrections to the preliminarily adopted places of the standard stars.

The first unknown to be determined is the correction to the constant of polar compression, ΔJ, since all the others have been expressed in terms of it. The best way to find it is from the secular motion of the node of the second satellite. This is about 12° per annum in a retrograde direction, and is principally due to the compression of Jupiter; by a comparison of the results of earlier observations with those of the present series the mean motion of the node can be found and hence ΔJ.

The quantity to be determined is the yearly motion of the node of II on its invariable plane, *i.e.* the quantity b_1 of Laplace or Souillart. The invariable plane is the plane defined by Souillart; referred to his adopted plane of reference, which is Jupiter's orbit of 1850·0, its node is $b_4 t + \gamma_4$ and its inclination N'_4. The observational data were first of all referred to this plane.

If we add the values found for Δp_2 and Δq_2 to the adopted values of p_2 and q_2 we have for the two epochs 1901·61 and 1902·60 :—

	1901·61.	1902·60.
p_2	−0°·2934	−0°·3502
q_2	−0°·4138	−0°·3339;

whence the corrected node and inclination referred to the fundamental planes of the epochs are :—

	1901·61.	1902·60.
Node	144°·66	133°·64
Inclination	0°·5073	0°·4838.

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We further take from Dr. de Sitter's paper, p. 71, the node and inclination referred to his adopted plane of reference, viz.:—

1891·75 . . .	250°·108	0°·5199.

The only other observations that could be made use of were Schur's for the epoch 1879·0. The inclination and node reckoned from the earth's mean equinox and referred to Jupiter's moving orbit are given on page 169 of his paper, *Bestimmung der Masse des Planeten Jupiter*, viz. :—

Node = 308°·8925
Inclination = 2°·7394.

These are the observational data from which to find ΔJ.

It is necessary first to consider what are the planes to which these inclinations and nodes are referred. The ultimate plane of reference is to be the plane of Jupiter's orbit as defined by Leverrier. Schur's quantities are already referred to this plane. The fundamental plane for the epoch 1891·75 is defined by Marth's figures referring to the ecliptic, and from them we find that for the fundamental plane referred to Jupiter's orbit we have the following values :—

Node.	Long. of O.	Inclination.
315°·321	135°·359	3°·068.

For the epoch 1901·61 we took

Node.	Long. of O.	Inclination.
315°·510	135°·510	3°·068 ;

and since the preliminarily adopted position-angle of the standards is practically in need of no correction, that is, since the measurements were correctly referred to the earth's equator, the plane of reference for the epoch 1901·61 is correctly defined.

But it has been shown that the position-angle of the standards adopted in the reductions of 1902 is in error by 37″. The effect of this on any absolute term is equivalent to an increase of the angle P by 37″. This angle P is the position-angle of Jupiter's axis of rotation, and since the planet's equator was adopted as plane of reference, it is necessary to ascertain what change in the position of the plane of reference corresponds to an increase of 37″ in the angle P. The only quantity that is changed is P ; the point from which longitudes are reckoned, the point O, is not changed nor are the jovicentric co-ordinates of the earth. In the spherical triangle

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KK′E, let K be the pole of the earth's equator, K′ the pole of Jupiter's equator, and E the centre of the earth. Then

$$\begin{aligned} K'E &= 90^\circ - \text{jovicentric latitude of earth.} \\ &= 90^\circ - B \text{ (see p. 61).} \\ KE &= 180^\circ - \text{geocentric N.P.D. of Jupiter.} \\ KEK' &= \text{the angle P.} \end{aligned}$$

The two sides KE, K′E are constant, and hence

$$\begin{aligned} \delta(KK') &= \sin K'E \sin KK'E\, \delta(KEK') \\ &= \cos B \sin KK'E\, \delta P. \end{aligned}$$

The value of the angle KK′E, namely, 316° 6′, was taken from the calculations for the ephemeris. Hence

$$\delta(KK') = -26''.$$

Let now γM be the plane of the earth's equator, MO the plane of Jupiter's equator: O is the point from which longitudes are reckoned, and is to be regarded as fixed. Hence the side γO and the angle $O\gamma M$ are constant, and we find that

$$\begin{aligned} \delta(MO) &= -\cot \gamma MO \tan MO\, \delta(\gamma MO) \\ \text{and } \delta(\gamma M) &= \sec \gamma MO\, \delta(MO). \end{aligned}$$

But $180^\circ - \gamma MO$ is the inclination of Jupiter's equator to the earth's equator, denoted just now by KK′: hence $\delta(MO)$ becomes known and also $\delta(\gamma M)$. The numerical values of γMO and MO can be found from p. 60. Thus we get

$$\delta(MO) = -50'' \quad : \quad \delta(\gamma M) = +56''.$$

Applying these corrections, we find that the plane of reference to which the observations of 1902 are referred, is thus defined relatively to the earth's equator:—

Node.	Long. of O.	Inclination.
$-1^\circ\ 59'\ 9''$	$135^\circ\ 19'\ 36''$	$25^\circ\ 25'\ 49''$

instead of

$-2^\circ\ 0'\ 5''$	$135^\circ\ 19'\ 36''$	$25^\circ\ 26'\ 15''$.

If the quantities P, B, L − O, etc., are calculated from these new values, it will be found that the only change is to increase P by 37″, the others remaining unchanged. If we refer the quantities to the ecliptic we have:—

Node.	Long. of O.	Inclination.
$336^\circ\ 33'\ 33''$	$135^\circ\ 30'\ 15''$	$2^\circ\ 11'\ 38''$,

and referred to Jupiter's orbit,

$315^\circ\ 51'\ 5''$	$135^\circ\ 31'\ 30''$	$3^\circ\ 6'\ 22''$
or $315^\circ{\cdot}852$	$135^\circ{\cdot}525$	$3^\circ{\cdot}106$.

This is the plane to which the observations of 1902 were referred.

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Collecting results, we find that the planes of reference at the different epochs are defined relatively to Jupiter's orbit by the following quantities :—

Epoch.	Node.	Long. of O.	Inclination.
1879·0	0°·000	0°·000	0°·000
1891·75	315 ·321	135 ·359	3 ·068
1901·61	315 ·510	135 ·510	3 ·068
1902·60	315 ·852	135 ·525	3 ·106.

The nodes and inclinations of the orbits of the satellites were given by the observations relatively to these planes, and it was next necessary to transfer to the plane of Jupiter's orbit and to refer to it the data for the last three epochs. This was done as follows :—

Let p denote generally inclination multiplied by sine node, q inclination by cosine node : let there be three planes, and let $\phi_{1,2}\ \theta_{1,2}$ denote the inclination and node of plane 1 referred to plane 2, $\phi_{2,3}\ \theta_{2,3}$ those of plane 2 referred to plane 3, and so on. Then

$$\sin \phi_{3,2} \sin (\theta_{3,2} - \theta_{2,1}) = \sin \phi_{3,1} \sin (\theta_{3,1} - \theta_{2,1}),$$
$$\sin \phi_{3,2} \cos (\theta_{3,2} - \theta_{2,1}) = -\sin \phi_{2,1} \cos \phi_{3,1} + \cos \phi_{2,1} \sin \phi_{3,1} \cos (\theta_{3,1} - \theta_{2,1}),$$

provided all the nodes are reckoned from the same origin : let this be the earth's mean equinox. If the inclinations are small, we have

$$\phi_{3,2} \sin (\theta_{3,2} - \theta_{2,1}) = \phi_{3,1} \sin (\theta_{3,1} - \theta_{2,1})$$
$$\phi_{3,2} \cos (\theta_{3,2} - \theta_{2,1}) = \phi_{3,1} \cos (\theta_{3,1} - \theta_{2,1}) - \phi_{2,1};$$

whence

$$\phi_{3,2} \sin \theta_{3,2} = \phi_{3,1} \sin \theta_{3,1} - \phi_{2,1} \sin \theta_{2,1}$$
$$\phi_{3,2} \cos \theta_{3,2} = \phi_{3,1} \cos \theta_{3,1} - \phi_{2,1} \cos \theta_{2,1};$$

or

$$p_{3,2} = p_{3,1} - p_{2,1}$$
$$q_{3,2} = q_{3,1} - q_{2,1}.$$

If the orbit of II is referred to the fundamental planes of the epochs and the longitude is reckoned from the earth's mean equinox instead of from the point O, we have—

	Node.	Inclination.
1891·75	25°·467	0°·5199
1901·61	280 ·170	0 ·5073
1902·60	269 ·165	0 ·4838;

and if in the above formula 1 denotes Jupiter's moving orbit, 2 the reference plane of the epoch, 3 the orbit of II, we get from these quantities and those at the top of this page—

	p_{32}.	q_{32}.	p_{21}.	q_{21}.
1891·75	+ 0·2236	+ 0·4694	− 2·1573	+ 2·1816
1901·61	− 0·4993	+ 0·0896	− 2·1504	+ 2·1890
1902·60	− 0·4837	− 0·0071	− 2·1634	+ 2·2287.

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Hence, by applying the preceding formula, we have the following determinations of the orbit of II referred to Jupiter's moving orbit:—

	p.	q.
1879·00	− 2°·1322	+ 1°·7198
1891·75	− 1 ·9337	+ 2 ·6510
1901·61	− 2 ·6497	+ 2 ·2786
1902·60	− 2 ·6471	+ 2 ·2216.

To refer these to Jupiter's orbit of 1850, *i.e.* Souillart's "plan fixe" instead of the orbit of the epoch, it is necessary to add the following small corrections (see Tisserand, *Méc. Cél.* IV. p. 59).

	δp.	δq.
1891	− 0°·0011	− 0°·0015
1901	− 0 ·0014	− 0 ·0017
1902	− 0 ·0014	− 0 ·0018.

In view of the large probable errors of the quantities for 1879 it was not considered worth while taking account of this correction for that year.

For the satellite's fixed plane referred to the "plan fixe," the following quantities, which are in Souillart's notation $N'_4 \sin(b_4 t + \gamma_4)$ and $N'_4 \cos(b_4 t + \gamma_4)$ respectively, were calculated from the figures given on p. 161 of his second memoir:—

	p.	q.
1879·00	− 2°·1506	+ 2°·1630
1891·75	− 2 ·1447	+ 2 ·1687
1901·61	− 2 ·1397	+ 2 ·1734
1902·60	− 2 ·1392	+ 2 ·1740

and hence, by another application of the foregoing formula, we have the following quantities referring the satelitte's orbit to its fixed plane:—

	p.	q.
1879·00	− 0°·0184	− 0°·4432
1891·75	+ 0 ·2099	+ 0 ·4808
1901·61	− 0 ·5114	+ 0 ·1035
1902·60	− 0 ·5093	+ 0 ·0458

Now the theoretical expressions of these quantities as given by Souillart are:—

$$p = N'_1 \sin(b_1 t + \gamma_1) + 102''\cdot3 \sin(b_2 t + \gamma_2) + 15''\cdot7 \sin(b_3 t + \gamma_3) + 2''\cdot8 \sin(2L - b_4 t - \gamma_4)$$

q = same, with cosines.

The value of N'_1 is about 1700″, or more than fifteen times the next largest

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coefficient, and if to the other coefficients and angles are given the numerical values of Souillart, we can derive values of N'_1 and $b_1t + \gamma_1$ by equating the observed value of p to its theoretical expression. In this way the following values of N'_1 and $b_1t + \gamma_1$ were found; with them are given the theoretical values of Souillart.

	Observed.		Souillart.	
	N'_1	$b_1t+\gamma_1$	N'_1	$b_1t+\gamma_1$
1879·00	0°·4733	179°·38	0°·4695	166°·17
1891·75	0 ·4956	25 ·45		12 ·61
1901·61	0 ·5010	278 ·65		253 ·87
1902·60	0 ·4926	267 ·91		242 ·04

N'_1 is a constant and the different determinations should agree. The probable errors of the last three values are about $\pm$ 0°·005, so that the differences between them are not larger than might be expected. The probable error of the first is, however, considerably over $\pm$ 0°·005.

To find the most probable value of b_1, it is necessary to ascertain what weights should be assigned to the different values of $b_1t + \gamma_1$. This was done by finding the probable errors of the node of the satellite's orbit on Jupiter's equator. In the case of the three last epochs the reference plane for the observations is practically the planet's equator, and hence the probable error of the node on the equator can be found at once by the formula on p. 162. Schur gives the probable error of the node on the orbit of Jupiter: this must be increased in the ratio of the inclination of the satellite's orbit relatively to the orbit of Jupiter to its inclination relatively to the equator. The following values of the probable errors of the node on the equator were determined in this way:—

1879	$\pm$ 2°·40
1891	$\pm$ 0 ·37
1901	$\pm$ 0 ·65
1902	$\pm$ 0 ·52.

The probable errors of $b_1t + \gamma_1$ are proportional to these numbers, and the weights are inversely proportional to their squares.

Comparing with Souillart's values, we get the following equations of condition from which to find corrections to his values of b_1 and γ_1 at the epoch 1891·75.

$-4657\,\Delta b_1 + \Delta\gamma_1$	$= +13°·21$	wt. 0·16	residual	$+5°·47$
$\Delta\gamma_1$	$= +12·84$	7·29		$-1·81$
$+3605\,\Delta b_1 + \Delta\gamma_1$	$= +24·78$	2·25		$+0·69$
$+3963\,\Delta b_1 + \Delta\gamma_1$	$= +25·87$	3·61		$+0·89$,

where b_1 is expressed in degrees per day.

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The solution of these gives :—

$$\Delta b_1 = + \ 0^{\circ}\cdot00300 \pm 0^{\circ}\cdot00037 \ ; \text{ whence } b_1 = - 0^{\circ}\cdot02997 \pm 0^{\circ}\cdot00037$$
$$\Delta \gamma_1 = + 13^{\circ}\cdot51 \quad \pm 0^{\circ}\cdot96.$$

The residuals given above are the result of the substitution of these values in the weighted equations of condition.

It would perhaps be better to omit the first equation. No details were given by Schur as to his method of observing position-angles; he found, moreover, that his observations were better represented by assuming the existence of a large empirical term depending on whether the satellite followed or preceded the planet. Also it will be remembered that the co-ordinates of the satellites were measured by Schur relatively to the limbs of the planet (see p. 3). If, then, we assign weight zero to the first equation we get

$$\Delta b_1 = + \ 0^{\circ}\cdot00330 \pm 0^{\circ}\cdot00012$$
$$\Delta \gamma_1 = + 12^{\circ}\cdot85 \quad \pm 0^{\circ}\cdot30.$$

These probable errors are calculated in the usual way from the sum of the squares of the residuals; but a truer estimate of the probable error of b_1 can be found from the consideration that the probable error of the difference between the mean $b_1 t + \gamma_1$ of 1901 and 1902 and that of 1891 is $\pm$ 1/203 of itself; whence the probable error of b_1 is $\pm 0^{\circ}\cdot00015$.

Now b_1 is very approximately equal to the quantity denoted by $\boxed{1}$: they differ by small quantities which for our purpose are negligible. The expression for $\boxed{1}$ is given by Souillart (B. p. 15), and from it we find that if corrections to the masses of the sun and of satellites I and IV are neglected, then

$$0\cdot8230 \frac{\Delta J}{J} + 0\cdot1297 \frac{\Delta m''}{m''} = \frac{\Delta \boxed{1}}{\boxed{1}} = - \frac{\Delta b_1}{b_1}.$$

This equation shows that 82 per cent. of the motion of the node of II is due to the polar compression of Jupiter, and 13 per cent. to the third satellite: the remaining 5 per cent. is due to the sun and the other two satellites. The adopted value of J is 0·02472, but as pointed out by Cohn (*Astr. Nach.*, 3403) this constant involves a knowledge of the planet's diameter; what is determined from observation is not J but Jb^2, b being the planet's equatorial radius. The value of b implied in this paper has been found by dividing the adopted values of the major-axes of the orbits by their values expressed in terms of b, as given by Souillart (II. p. 10): we get for the value of b at unit distance 96″·128, or at Jupiter's mean distance from the sun, viz., 5·2, 18″·486. The implied equatorial diameter of Jupiter is thus

$$36''\cdot97,$$

and it is to this value of the planet's diameter that our adopted value of J corresponds.

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It is rather more convenient, however, to refer J to a diameter of 39″·00, which is nearer the value given by actual measurements made with the filar micrometer.

The value of J corresponding to a diameter of 39″·00 is :—

$$0{\cdot}02472\left(\frac{36{\cdot}97}{39{\cdot}00}\right)^2 \text{ or } 0{\cdot}02221$$

and the value for any other diameter may be found from

$$J = 0{\cdot}02221\left(\frac{39''{\cdot}00}{d}\right)^2.$$

If, now, we substitute in the above equation the adopted values of J and b_1, namely, 0·02221 and − 0°·03297, we get

$$\Delta J = -\,0{\cdot}8212\ \Delta b_1 - 0{\cdot}00351\ \Delta m''/m'';$$

and neglecting $\Delta m''$ for the moment, we find by substituting the two values of Δb_1 obtained above

$\Delta J = -\,0{\cdot}00246$; whence $Jb^2 = 0{\cdot}01975 \pm 0{\cdot}00030$, including Schur's observations.
$\Delta J = -\,0{\cdot}00279$; whence $Jb^2 = 0{\cdot}01942 \pm 0{\cdot}00012$, excluding Schur's observations.

The weighted mean of these was taken as a first approximation ; thus

$$Jb^2 = 0{\cdot}01948 \pm 0{\cdot}00011.$$

An attempt was next made to find $\Delta m''$ from Bessel's, Schur's, Gill's, and the writer's observations, which altogether embrace a period of 66 years. Jb^2 being known, the mass of III can be obtained from the motion of the perijove of IV.

The values of h_4 and k_4 from the observations of 1891, 1901, and 1902 are:—

	h_4	k_4
1891·75	+ 0°·5255 ± 0°·0030	− 0°·6528 ± 0°·0044
1901·61	+ 0 ·4443 ± 0 ·0057	− 0 ·6998 ± 0 ·0064
1902·60	+ 0 ·4514 ± 0 ·0037	− 0 ·7145 ± 0 ·0062.

The values for 1891 are taken from de Sitter's paper, p. 70. From these we find Ω_4, viz. :—

	Referred to O.	Referred to Earth's Equinox.
1891·75	141°·17 ± 0°·25	276°·53 ± 0°·25
1901·61	147 ·58 ± 0 ·41	283 ·09 ± 0 ·41
1902·60	147 ·72 ± 0 ·41	283 ·24 ± 0 ·41.

Schur gives on pp. 169 and 170 of his paper the same angle at the epochs 1836·0 and 1876·0, viz., 239°·47 ± 0°·48 and 271°·10 ± 1°·25 respectively, the first being the value he derived from his re-reductions of Bessel's observations, and the second that from his own observations. These are reckoned from the earth's equinox in the plane of Jupiter's orbit ; the three values for the later epochs are reckoned in the fundamental

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planes of the epoch, but as the reduction from one plane to the other does not exceed 0°·01, the five values may all be taken as measured along the plane of Jupiter's orbit.

Souillart gives for the theoretical expressions of h_4 and k_4

$$2\,E\sin\varpi_4 = h_4 = -59''{\cdot}1\sin(g_2t+\beta_2) + 2995''{\cdot}7\sin(g_3t+\beta_3)$$
$$2\,E\cos\varpi_4 = k_4 = -59''{\cdot}1\cos(g_2t+\beta_2) + 2995''{\cdot}7\cos(g_3t+\beta_3);$$

whence

$$2\,E\sin(\varpi_4 - g_3t - \beta_3) = -59''{\cdot}1\sin(g_2t+\beta_2-g_3t-\beta_3)$$
$$2\,E\cos(\varpi_4 - g_3t - \beta_3) = -59''{\cdot}1\cos(g_2t+\beta_2-g_3t-\beta_3) + 2995''{\cdot}7$$

and $$\varpi_4 - g_3t - \beta_3 = -1°{\cdot}13\sin(g_2t+\beta_2-g_3t-\beta_3).$$

This was accepted as the best available expression for the motion of the perijove of IV; it shows that $g_3t + \beta_3$ is the mean value of the longitude of the perijove, and that g_3 is its mean motion. The values of the angles $g_2t + \beta_2$ and $g_3t + \beta_3$ were calculated for the several epochs from Souillart's figures (B. p. 40), and thence the values of $\varpi_4 - g_3t - \beta_3$. But ϖ_4 is the observed longitude of the perijove, and thus we get the following values of $g_3t + \beta_3$:—

1836·0	$g_3t+\beta_3 =$	238°·42 ± 0°·48
1879·0		271 ·37 ± 0 ·25
1891·7		277 ·22 ± 0 ·25
1901·6		284 ·03 ± 0 ·41
1901·6		284 ·20 ± 0 ·41.

Whence assuming $\beta_3 = 251°62$ at the epoch 1850 we have the following equations of condition for determining g_3 and $\Delta\beta_3$:—

		Weight.	$\sqrt{p}\,v.$
$-14{\cdot}0\,g_3 + \Delta\beta_3 =$	$-13{\cdot}20$	4·4	+ 0°·46
$+29{\cdot}0\,g_3 + \Delta\beta_3 =$	$19{\cdot}75$	0·6	− 2 ·42
$+41{\cdot}7\,g_3 + \Delta\beta_3 =$	$25{\cdot}60$	16·0	− 0 ·35
$+51{\cdot}6\,g_3 + \Delta\beta_3 =$	$32{\cdot}41$	5·8	− 0 ·15
$+52{\cdot}6\,g_3 + \Delta\beta_3 =$	$32{\cdot}58$	5·8	+ 1 ·10.

These equations were given weights which were inversely proportional to the squares of the probable errors and were solved by least squares. The solution is

$$g_3 = +0°{\cdot}6912 \pm 0°{\cdot}0087 \text{ per annum}: \Delta\beta_3 = -3°{\cdot}31 \pm 0°{\cdot}38,$$

and the residuals from the weighted equations of condition are given above.

Thus

$$g_3 = +0°{\cdot}001892 \pm 0°{\cdot}000024 \text{ per day};$$

whilst Souillart's value is

$$g_3 = +0°{\cdot}001999.$$

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The approximate theoretical expression for the correction Δg_3, which is the same as $\Delta[3]$, can be found from the expression for [3] given by Souillart (B. p. 15): it is:—

$$\frac{\Delta g_3}{[3]} = 0{\cdot}363\frac{\Delta J}{J} + 0{\cdot}480\frac{\Delta m''}{m''};$$

whence substituting the value already found for ΔJ and the above value of Δg_3, we get:—

$$\frac{\Delta m''}{m''} = -\,0{\cdot}022 \pm 0{\cdot}029.$$

This is not the same as the value given in the writer's paper in *Monthly Notices*, lxiv. p. 740: the change is due to a correction which was subsequently made in the absolute terms of the equations giving g_3 and $\Delta\beta_3$. Seeing that the perijove of IV has only a very slow motion—its period is about 490 years—it is not to be expected that any very reliable value of it can be found from observations covering only one eighth part of the period; all the above tends to show is, that if the adopted mass of III is in need of correction at all, it probably requires a negative correction not exceeding a twentieth of itself. It seems safer, however, from the evidence at hand to take $\Delta m''$ as zero, and

$$J = 0{\cdot}01948\left(\frac{39''{\cdot}00}{d}\right)^2 \pm 0{\cdot}00011.$$

This value of J is considerably less than the adopted value of Damoiseau, and it was interesting to see what confirmation of it could be derived from the motion of the node of III. For this the observations of Bessel in 1836 could be utilised. The following values of the node and inclination of III referred to Jupiter's orbit were taken from Schur's re-reduction of Bessel's observations (*Masse des Planeten Jupiter*, p. 169).

$$\text{Node } N = 315^\circ{\cdot}70 \pm 0{\cdot}19 \qquad \text{Inclination } J = 2{\cdot}8332 \pm 0{\cdot}0100;$$

whence

$$J \sin N = -\,1^\circ{\cdot}9785$$
$$J \cos N = +\,2^\circ{\cdot}0275.$$

From the observations of 1902 we find that the node and inclination of the orbit of III referred to the reference plane of the epoch are 200°·84 and 0°·1538; or reckoning the node from the first point of Aries instead of from O, 336° 36 and 0°·1538. The corresponding p and q are:—

$$p = -\,0^\circ{\cdot}0617$$
$$q = +\,0^\circ{\cdot}1409;$$

hence for the orbit of III referred to Jupiters orbit of 1850 we have:—

$$p = -\,2^\circ{\cdot}2265$$
$$q = +\,2^\circ{\cdot}3878.$$

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From Souillart we find that the invariable plane of III is defined relatively to Jupiter's orbit by the quantities

	1836.	1902.
p	$-2^\circ\cdot1266$	$-2^\circ\cdot0945$
q	$+2^\circ\cdot0963$	$+2^\circ\cdot1286$;

hence the orbit of III referred to its invariable plane is given by the quantities

	1836.	1902.
p	$+0^\circ\cdot1481$	$-0^\circ\cdot1320$
q	$-0^\circ\cdot0688$	$+0^\circ\cdot2592$.

These are the data of observation. The theoretical expressions for p and q are according to Souillart.

$$p = -57'' \sin(b_1t + \gamma_1) + 644'' \sin(b_2t + \gamma_2) + 97'' \sin(b_3t + \gamma_3)$$
$$q = -57'' \cos(b_1t + \gamma_1) + 644'' \cos(b_2t + \gamma_2) + 97'' \cos(b_3t + \gamma_3).$$

Terms with coefficients under $10''$ are neglected. Since the coefficient of $b_2t + \gamma_2$ is greater than the numerical sum of the other coefficients, it follows that $b_2t + \gamma_2$ is the mean value of the node of III on its invariable plane and that b_2 is its mean motion: this is the quantity sought. If then the theoretical values of the first and third terms in the foregoing expressions are accepted, the second term can be determined by equating it to the observed quantities at the two epochs. In this way the following values were found:—

	1836.	1902.
$N''_2 \sin(b_2t + \gamma_2)$	$+0^\circ\cdot1334$	$-0\cdot1448$
$N''_2 \cos(b_2t + \gamma_2)$	$-0^\circ\cdot0782$	$+0\cdot2220$;

whence

1836	$b_2t + \gamma_2 = 120^\circ\cdot38$
1902	$326^\circ\cdot89$.

Thus the motion is $153^\circ\cdot5$ in 66 years against Souillart's $168^\circ\cdot3$, *i.e.* the reduction is $1/11\cdot4$. The probable error of this is about 1/40 of itself. The theoretical expression for Δb_2 is (Souillart, B p. 15.):—

$$-\frac{\Delta b_2}{\boxed{2}} = 0\cdot752\,\frac{\Delta J}{J} + 0\cdot126\,\frac{\Delta m'}{m'} + 0\cdot088\,\frac{\Delta m''}{m''} + 0\cdot019\,\frac{\Delta m}{m},$$

if the correction to the sun's mass is neglected. It will be seen further on that the adopted mass of II m' is not much in error; $\Delta m'/m'$ is probably less than 1/15, and hence may be neglected in the foregoing equation. $\Delta m''/m''$, the percentage correction to the mass of III, does not exceed 1/20 and may also be neglected. $\Delta m/m$ may

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amount to as much as unity, but may be disregarded on account of its small coefficient. We have therefore with sufficient accuracy :—

$$0{\cdot}75\,\frac{\Delta J}{J} = -\frac{\Delta b_2}{[2]} = -\frac{\Delta b_2}{b_2}$$
$$= -\frac{1}{11{\cdot}4};$$

whence

$$Jb^2 = 0{\cdot}01961 \pm 0{\cdot}00051$$

This value agrees closely with that found from the motion of the node of II but has comparatively small weight. It indicates, however, that the adopted value of Jb^2 is in need of a considerable negative correction. A consideration of the evidence before us leads to the conclusion that

$$J = 0{\cdot}01950\left(\frac{39{\cdot}00}{d}\right)^2$$

is the value which best satisfies the heliometer observations of the satellites.

From the motion of the orbit of the fifth satellite Cohn (*Astr. Nach.*, 3403) found

$$J = 0{\cdot}02095\left(\frac{39''{\cdot}00}{d}\right)^2$$

corresponding to the observed annual motion of the perijove, viz., 912°. The latest observations of Barnard, however, indicate that this annual motion is too large, from which it follows that Cohn's J needs a negative correction (*Astronomical Journal*, No. 544).

It should be pointed out that this value of Jb^2 depends on the adopted position of Jupiter's equator, which in its turn can only be determined when we are given the values of the masses of the satellites. As will be seen further on, our knowledge of the masses is far from satisfactory, and hence this value of Jb^2, like other constants of the theory, cannot be considered final until it is brought into line with the final and correct values of the masses.

The annual motion of the node of V corresponding to the value of J found above is 851°, but it must not be forgotten that the motion of the node is not due to J alone. The complete expression for it is

$$\frac{\delta P}{\delta t} = -n\left(\frac{Jb^2}{a^2} + \frac{Kb^4}{a^4} + \ldots\right)$$

and in the case of the fifth satellite the second term amounts to about one fiftieth of the first, though for the other satellites it is quite insensible. We should expect then to

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find an annual motion of the node of V amounting to $868° \pm 5°$, and it will be of extreme interest to see how far this is borne out by a discussion of the observations of the fifth satellite. In this way it will be possible to determine the constant K, and thence to draw some conclusions as to the physical state of Jupiter.

Mass of Jupiter.

The values of the unknown μ lead to the following values of the reciprocal of the mass:

1901	$1047{\cdot}678 \pm 0{\cdot}093 + 5{\cdot}045\ \Delta J$
1902	$1047{\cdot}647 \pm 0{\cdot}059 + 3{\cdot}850\ \Delta J.$

This requires a small correction, since the observations were not reduced to the centre of the earth. The correction on this account to an observation in distance is:—

$$\delta s/s = -\pi\rho \cos\zeta,$$

and to one in position-angle

$$\delta p = \pi\rho \cos\phi^1 \tan\delta \sin t,$$

where

π = equatorial horizontal parallax,
ρ = radius vector, ϕ^1 = reduced latitude } of observer,
δ = declination, t = hour angle } of middle point
ζ = zenith distance } of objects observed.

It is quite sufficient to put

$$\pi = 2'', \ \phi' = -34°, \ \delta = -20°, \ \rho = 1.$$

Hence we get the following corrections for various hour angles:

	$\delta s/s$	δp
$\pm$ 1^h	$-9{\cdot}4 \times 10^{-6}$	$\mp 0''{\cdot}2$
2	$-8{\cdot}7$	$\mp 0{\cdot}3$
3	$-7{\cdot}4$	$\mp 0{\cdot}4$
4	$-5{\cdot}8$	$\mp 0{\cdot}5$

The correction in position-angle is quite inappreciable, but there will be a small systematic correction in distance. The observations were made between the hour angles 1^h and 4^h, and the mean was nearly $\pm 2^h\ 30^m$: the distances therefore require a correction of $-8 \times 10^{-6} \times$ s. The corresponding correction to the mass is only $+0{\cdot}025$.

Applying this correction and introducing the value of ΔJ, viz., $-0{\cdot}0025$, we get

1901	$1 : 1047{\cdot}691 \pm 0{\cdot}093 - 0{\cdot}4769\ \delta\sigma_1$
1902	$1 : 1047{\cdot}663 \pm 0{\cdot}059 - 0{\cdot}7005\ \delta\sigma_2,$

where $\delta\sigma_1$, $\delta\sigma_2$ are the corrections in seconds of arc to the adopted distances of the standards of 1901 and 1902 respectively.

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The quantities $\delta\sigma_1$, $\delta\sigma_2$ represent the corrections to the preliminary adopted values of the distances of the two pairs of standard stars. From pages 177, 181 we find

$$\delta\sigma_1 = + 0.''264 \qquad \delta\sigma_2 = + 0.''425.$$

Substituting these values we have for the reciprocal of the mass the two values

1901	1047·564
1902	1047·364.

But as mentioned in the footnote to p. 56, there is still a correction to be applied on account of errors in the major axes of the orbits of the satellites. The second and higher order terms have been applied twice, once by Marth and again by the writer, who applied perturbative terms of all orders under the impression that Marth had applied none at all. The second and higher order terms must therefore be subtracted from the values of the major axes which were adopted in computing the ephemeris. The values of these terms are as follows :—

I. . . . . $\frac{\delta a}{a} = + 96 \times 10^{-7}$,

II. . . . . $\frac{\delta a}{a} = + 421 \times 10^{-7}$,

III. . . . . $\frac{\delta a}{a} = + 9 \times 10^{-7}$,

IV. . . . . $\frac{\delta a}{a} = + 265 \times 10^{-7}$.

All the absolute terms require corrections on this account but it is not necessary to apply them, as the only unknown which will be appreciably affected is μ and the correction to it can be very simply made in the following manner.

If M is the reciprocal of the mass, we have

$$\frac{\delta M}{M} = - 3 \frac{\delta a}{a},$$

and considering any pair of satellites, we have for the corresponding correction of the mass as derived from observations of this pair

$$\begin{aligned} \delta M &= \frac{dM}{da_1} \delta a_1 + \frac{dM}{da_2} \delta a_2 \\ &= - 3M \left(\frac{\delta a_1}{a_1} + \frac{\delta a_2}{a_2} \right). \end{aligned}$$

The values of δM for the various pairs of satellites are as follows :—

I.– II.	– 0·16
I.–III.	– ·03
I.– IV.	– ·11
II.–III.	– ·13
II.– IV.	– ·21
III.– IV.	– 0·09.

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The question now arose: What were the relative weights with which these different pairs entered into the determination of the mass from the general solution? In other words: How were these six values of δM to be combined? The relative weights will clearly be nearly proportional to the coefficients of the unknown μ in the normals for μ which are given on pages 136 and 146. But a nearer approximation was made as follows. The weights of μ from the solution of the two sets of normals were 1163 and 2795, whilst the sums of the coefficients were 1660 and 3151. If, then, the coefficients are diminished in the ratio 1163 : 1660 and 2795 : 3151, we shall have a close approximation to the combining weights of the six values of δM. These coefficients and weights are given here.

	Coefficients.		Combining Weights.	
	1901.	1902.	1901.	1902.
I.–II. . . .	77	103	54	91
I.–III. . . .	173	184	121	163
I.–IV. . . .	369	705	259	625
II.–III. . . .	204	321	143	285
II.–IV. . . .	391	529	274	469
III.–IV. . . .	446	1309	313	1161
Sum . . .	1660	3151	1164	2794
Correction to Mass . . .	−0·12	−0·12	−0·125	−0·117

Thus it is indifferent which method of weighting is employed. The adopted value of M was therefore not 1047·70 but 1047·58, and hence the correct observed values of the mass are:—

1901 1 : 1047·439 ± 0·093
1902 1 : 1047·247 ± 0·059

The probable error given here is that derived from the interagreement of the observations and does not include that due to uncertainty in the adopted distances of the standards. If, however, the two values are assigned weights inversely proportional to the squares of their probable errors, the value of the mass is:—

1 : 1047·30 ± 0·05.

It is difficult to find any reliable values of the probable errors of the distances of the standards. There are not sufficient observations to make the values deduced from the interagreement of the several determinations in any way reliable. It is, however, very unlikely that the distances finally adopted are in error by as much as 0″·1. If we attempt to measure the expression "very unlikely" by estimating that the odds are 3 to 1 against the error exceeding ±0″·1, it follows from the theory of Least Squares that the probable error is ±0″·06. The probable error of the mass for the two years is then ±0·097 and ±0·072; whence the probable error of their weighted mean is ±0·06.

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We take as the mass of Jupiter from this investigation

$$1 : 1047{\cdot}30 \pm 0{\cdot}06.$$

Coefficients of the great Inequalities.

The substitution of the value of ΔJ gives for the corrections to these :—

	1901.	1902.	Mean.
Δx_1	$+0^\circ{\cdot}0509 \pm 0^\circ{\cdot}0503$	$-0^\circ{\cdot}0602 \pm 0{\cdot}0378$	$-0{\cdot}0201 \pm 0{\cdot}0302$
Δx_2	$+0{\cdot}0671 \pm 0{\cdot}0318$	$+0{\cdot}1444 \pm 0{\cdot}0205$	$+0{\cdot}1217 \pm 0{\cdot}0172$
Δx_3	$-0{\cdot}0196 \pm 0{\cdot}0214$	$-0{\cdot}0287 \pm 0{\cdot}0133$	$-0{\cdot}0259 \pm 0{\cdot}0113.$

Thus the writer's observations indicate no reliable correction to either x_1 or x_3 : and as these two coefficients are functions of the mass of II, it follows that its adopted mass is correct. The adopted values of x_1 and x_3 are $0^\circ{\cdot}4321$ and $0^\circ{\cdot}0633$; hence the probable error of this determination of the mass of II is about $\pm 1/14$ of itself.

The correction to x_2, however, seems to be real, but is very large, and is not confirmed by the value Dr. de Sitter found, which was :—

$$\Delta x_2 = -0^\circ{\cdot}0178 \pm 0^\circ{\cdot}0156.$$

In other words, he found that x_2 was practically in need of no correction. But the results of both the present series of observations support each other and point to a real correction of over 360″.

It is concluded that these results indicate that x_2 is in need of a positive correction of about $0^\circ{\cdot}050$ or 180″. This brings the adopted value of Souillart, viz., 3693″, into near agreement with Damoiseau's value, viz., 3867″.

This coefficient of Damoiseau's is a datum of observation, and was derived by him from eclipse observations ; whilst Souillart's coefficient was calculated from its algebraic expression by giving the quantities involved the values which he decided to adopt when he started on his numerical theory. This was Souillart's method of procedure throughout ; and seeing that he adopted Damoiseau's constants, it was to be expected that he would reproduce Damoiseau's coefficient. The reason, however, why this is not the case, has recently been suggested to the writer by a letter from Professor Sampson. This coefficient is important for the determination of the masses of I and III, and its first order terms are a linear function of these masses.

In his determination of the masses, Damoiseau disregarded some second order terms amounting to nearly 200″ and used Delambre's observed value of the coefficient, which of course included terms of all orders, and from it derived values

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of the masses. These were the values adopted by Souillart; but when they were substituted in the algebraic expression for x_2, they naturally failed to reproduce the observed value of x_2: the calculated value fell short of its observed value by 200″. On discovery of the second order terms, the correct procedure would have been to subtract them from Damoiseau's observed coefficient and to re-calculate the masses from the first order terms. The resulting values of the masses, when substituted in the algebraic expressions, would then have led to a value of the coefficient agreeing with that found from observation. This, however, appears not to have been done.

The conclusion is that Damoiseau's observed coefficient is correct, but that the theoretical value is incorrect owing to the adoption of erroneous values of the masses.

If it is assumed that the second order terms are correct, Δx_2 is the observed correction to the first order terms. Now the theoretical expression for the first order terms is a linear function of the masses of the first and third satellites, viz. :—

$$\Delta x_2 = 1{\cdot}09\ \Delta m_3 + 0{\cdot}62\ \Delta m_1,$$

where x_2 is expressed in degrees and the unit of m_1 and m_3 is ten thousand times the mass of Jupiter.

The numerical coefficients are functions of the mean distances and the mean motions and may accordingly be taken as correct (see Tisserand, IV. p. 79); whence by substituting the probable value of Δx_2, *i.e.* $+0{\cdot}050$, we get

$$1{\cdot}75\ \Delta m_3 + \Delta m_1 = +0{\cdot}081 \pm 0{\cdot}028.$$

It was seen above that the mass of III, m_3, is probably correct to within 1/20 of itself: if we neglect Δm_3, we find that the mass of I is about

$$0{\cdot}250$$

instead of the adopted value, viz. :—

$$0{\cdot}169.$$

The probable error of this determination is about $\pm 0{\cdot}030$. If, on the other hand, the value of Δm_3 found above is adopted, the mass of I is still further increased, and, curiously enough, exactly coincides with the value which Adams found: that is to say,

$$\text{Mass of I} = 0{\cdot}283 \pm 0{\cdot}053.$$

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Jupiter's Equator.

To determine this plane the orbits of satellites I, III and IV were first referred to Souillart's fixed plane, *i.e.* the planet's orbit of 1850. We have then the following data of observation referred to the fixed plane :—

TABLE.

	p_{I}.	q_{I}.	p_{III}.	q_{III}.	p_{IV}.	q_{IV}.
1891·75	− 2°·1439	+ 2°·2353	− 2°·1767	+ 2°·3768	− 1°·7890	+ 2°·1379
1901·61	− 2·1962	+ 2·2464	− 2·2129	+ 2·3239	− 1·8278	+ 2·1517
1902·60	− 2·2027	+ 2·2472	− 2·2129	+ 2·3286	− 1·7934	+ 2·1507

The values for 1891 were deduced from de Sitter's paper.

The theoretical expressions for these quantities are :—

$$\phi_{\mathrm{I}} \sin\theta_{\mathrm{I}} = p_{\mathrm{I}} = 4''\cdot8 \sin \mathrm{B} + 33''\cdot0 \sin \mathrm{B}_1 + 3''\cdot9 \sin \mathrm{B}_2 - 1''\cdot4 \sin \mathrm{B}_3 + 0''\cdot9 \sin(2\mathrm{L} - \mathrm{B}_4) + \mathrm{N}_4 \sin \mathrm{B}_4$$
$$\phi_{\mathrm{I}} \cos\theta_{\mathrm{I}} = q_{\mathrm{I}} = \text{same with cosines.}$$

$$\phi_{\mathrm{III}} \sin\theta_{\mathrm{III}} = p_{\mathrm{III}} = -57''\cdot4 \sin \mathrm{B}_1 + 644''\cdot2 \sin \mathrm{B}_2 + 97''\cdot3 \sin \mathrm{B}_3 + 6''\cdot4 \sin(2\mathrm{L} - \mathrm{B}_4) + \mathrm{N}^{\mathrm{II}}{}_4 \sin \mathrm{B}_4$$
$$\phi^{\mathrm{III}} \cos\theta_{\mathrm{III}} = q_{\mathrm{III}} = \text{same with cosines.}$$

$$\phi_{\mathrm{IV}} \sin\theta_{\mathrm{IV}} = p_{\mathrm{IV}} = -1''\cdot5 \sin \mathrm{B}_1 - 125''\cdot3 \sin \mathrm{B}_2 + 811''\cdot4 \sin \mathrm{B}_3 + 13''\cdot7 \sin(2\mathrm{L} - \mathrm{B}_4) + 1''\cdot1 \sin(2\mathrm{L} - \mathrm{B}_3) + \mathrm{N}^{\mathrm{III}}{}_4 \sin \mathrm{B}_4$$
$$\phi_{\mathrm{IV}} \cos\theta_{\mathrm{IV}} = q_{\mathrm{IV}} = \text{same with cosines.}$$

where B_n is written for $b_n t + \gamma_n$. If we substitute Souillart's values of the angles $\mathrm{B} \ldots \mathrm{B}_3$ and assume that the other numerical coefficients are correct, we can find the last term in each expression and thence N_4 $\mathrm{N}^{\mathrm{II}}{}_4$ and $\mathrm{N}^{\mathrm{III}}{}_4$ and the angle $b_4 t + \gamma_4$. The coefficients N_4 $\mathrm{N}^{\mathrm{II}}{}_4$ $\mathrm{N}^{\mathrm{III}}{}_4$ are equal to the inclination of Jupiter's equator multiplied by factors which are nearly unity and are functions of the masses. These factors are assumed to be known, and hence we get the inclination of the equator. $\mathrm{N}^{\mathrm{III}}{}_4$, which is the smallest coefficient of the three, has according to Souillart the value 9595″.

These three expressions require the following small corrections to allow for the effect of the displacement of Jupiter's equator by the action of the other planets (see Tisserand, *Méc. Cél.*, IV. p. 59).

$$\left.\begin{matrix}\delta p_{\mathrm{I}} \\ \delta q_{\mathrm{I}}\end{matrix}\right\} \text{ negligible : } \begin{matrix}\delta p_{\mathrm{III}} = -8''\cdot1 - 0\cdot002t : \delta p_{\mathrm{IV}} = -57''\cdot4 - 0''\cdot013t \\ \delta q_{\mathrm{III}} = +6''\cdot0 - 0\cdot003t : \delta q_{\mathrm{IV}} = +42''\cdot4 - 0''\cdot017t\end{matrix}$$

where t is reckoned in years from 1850.

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The easiest way to find N_4 is as follows. Let

$$\phi \sin\theta = \Sigma N \sin\beta + N_4 \sin\beta_4$$
$$\phi \cos\theta = \Sigma N \cos\beta + N_4 \cos\beta_4,$$

then

$$\phi \sin(\theta - \beta_4) = \Sigma N \sin(\beta - \beta_4) = x \text{ say}$$
$$\phi \cos(\theta - \beta_4) = -\Sigma N \cos(\beta - \beta_4) + N_4 = y + N_4,$$

and

$$N_4 = -y + \sqrt{\phi^2 - x^2}$$

or

$$N_4 = -y + \sqrt{p^2 + q^2 - x^2},$$

where p and q have been corrected by subtracting the above small quantities δp and δq. It was in this way that the following quantities were found :—

TABLE.

	I.				III.				IV.			
	x.	y.	ϕ.	N_4.	x.	y.	ϕ.	N_4^{II}.	x.	y.	ϕ.	N_4^{III}.
1891·75	+0°·0076	+0°·0046	3°·0974	3°·0928	+0°·1166	+0°·1515	3°·2201	3°·0665	+0°·1694	+0°·0998	2°·7684	2°·6634
1901·61	−0·0072	+0·0059	3·1416	3·1355	−0·0670	+0·1862	3·2058	3·0189	+0·1607	+0·1142	2·8040	2·6852
1902·60	−0·0087	+0·0050	3·1467	3·1417	+0·0618	+0·1921	3·2093	3·0166	+0·1637	+0·1186	2·7812	2·6578

The theoretical expressions for N_4^{II} and N_4^{III} are :—

$$N_4^{II} = \sigma_4^{II} N_4$$
$$N_4^{III} = \sigma_4^{III} N_4,$$

and the inclination of Jupiter's equator to the fixed plane is $\sigma_4^{IV} N_4$ (Souillart, B, p. 158). The multipliers σ_4 are all nearly unity, but they depend on the values of the masses and on Jb^2, and there is reason for believing that the values of the masses of I and IV adopted by Souillart are considerably in error. However, until the true masses have been found and Souillart's numerical theory has been reconstructed, his values must be accepted, namely :—

$$\sigma_4^{II} = 0{\cdot}97393$$
$$\sigma_4^{III} = 0{\cdot}86934$$
$$\sigma_4^{IV} = 1{\cdot}00055.$$

In any case, these factors will not be much changed by the adoption of new constants.

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We get in this way the following determinations of the inclination of Jupiter's equator to the fixed plane.

TABLE

	I.		III.		IV.	
	Inclination.	Weight.	Inclination.	Weight.	Inclination.	Weight.
1891·75.....	3°·0945	3·2	3°·1502	20·2	3°·0653	67·2
1901·61.....	3·1372	1·4	3·1013	9·0	3·0904	30·3
1902·60.....	3·1434	2·3	3·0990	16·0	3·0589	50·4
Weighted Mean.....	3·1195	6·9	3·1223	45·2	3·0683	147·9

The weights are inversely proportional to the squares of the probable errors: the weighted mean of all is 3°·0823, or

$$3^\circ\ 4'\ 56'' \pm 24'',$$

the value adopted by Souillart from Damoiseau being

$$3^\circ\ 4'\ 5''.$$

It will be noticed that the value from satellite IV is 0°·0540 or 3′ 14″ less than that from III, and it is a significant fact that Damoiseau found a difference of 2′ 47″ in the same direction. This is probably due to the adoption of erroneous constants and not to errors of observation: part of it may be explained as follows. The term in the latitude of a satellite which depends on Jupiter's equator is $\mu\ \omega \sin (v+\psi)$, when ω is the inclination of the equator and μ is a factor nearly equal to unity. If $\nu = 1-\mu$, the approximate values for the four satellites are according to Souillart:—

$$\begin{aligned} &\text{I.} && \nu &= 0{\cdot}00057 \\ &\text{II.} && \nu' &= 0{\cdot}00584 \\ &\text{III.} && \nu'' &= 0{\cdot}02702 \\ &\text{IV.} && \nu''' &= 0{\cdot}13227, \end{aligned}$$

and the equations for determining the last two are in the usual notation (Souillart, B, p. 140)

$$\begin{aligned} (2,0)\,\nu + (2,1)\,\nu' - \boxed{2}\,\nu'' + (2,3)\,\nu''' + [2] &= 0 \\ (3,0)\,\nu + (3,1)\,\nu' + (3,2)\,\nu'' - \boxed{3}\,\nu''' + [3] &= 0. \end{aligned}$$

If the numerical values of these coefficients are substituted, these equations become

$$\begin{aligned} 13\nu + 89\nu' - 710\nu'' + 62\nu''' + 10 &= 0 \\ 2\nu + 6\nu' + 96\nu'' - 1995\nu''' + 24 &= 0. \end{aligned}$$

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The unit in the coefficients is one in the fifth decimal place. These equations show that for the present purpose it is safe to neglect the first two terms: also [2] and [3] may be taken as accurately known, since they depend on the mean motions only. It follows that

$$\frac{\Delta\nu^{II}}{\nu^{II}} + \frac{\Delta\boxed{2}}{\boxed{2}} = 0 \quad : \quad \frac{\Delta\nu^{III}}{\nu^{III}} + \frac{\Delta\boxed{3}}{\boxed{3}} = 0.$$

But if M denotes the mass of the sun and m m^{I} m^{II} m^{III} the masses of the satellites, we have (see Souillart, B, p. 15):—

$$\frac{\Delta\boxed{2}}{\boxed{2}} = 0{\cdot}75\,\frac{\Delta J}{J} + 0{\cdot}02\,\frac{\Delta M}{M} + 0{\cdot}02\,\frac{\Delta m}{m} + 0{\cdot}13\,\frac{\Delta m^{I}}{m^{I}} + 0{\cdot}09\,\frac{\Delta m^{III}}{m^{III}},$$

$$\frac{\Delta\boxed{3}}{\boxed{3}} = 0{\cdot}36\,\frac{\Delta J}{J} + 0{\cdot}12\,\frac{\Delta M}{M} + 0{\cdot}01\,\frac{\Delta m}{m} + 0{\cdot}03\,\frac{\Delta m^{I}}{m^{I}} + 0{\cdot}48\,\frac{\Delta m^{II}}{m^{II}}.$$

In other words, 75 per cent. of the error in [2] is due to the error of J, 13 per cent. to that of the mass of satellite II, and so on. For present purposes it is quite sufficient to take

$$\frac{\Delta\nu^{II}}{\nu^{II}} = -\,0{\cdot}75\,\frac{\Delta J}{J}$$

$$\frac{\Delta\nu^{III}}{\nu^{III}} = -\,0{\cdot}36\,\frac{\Delta J}{J} - 0{\cdot}48\,\frac{\Delta m^{II}}{m^{II}}.$$

Further, we may here neglect the error of the mass of III and take $\Delta J/J$ as $-1/9$. This gives

$$\Delta\nu^{II}/\nu^{II} = 1/12 \text{ and } \Delta\nu^{III}/\nu^{III} = 1/25$$

or

$$\Delta\nu^{II} = +\,0{\cdot}00225 \text{ and } \Delta\nu^{III} = +\,0{\cdot}00529.$$

The observational data are $(1-\nu^{II})\,\omega$ and $(1-\nu^{III})\,\omega$, which are very approximately equal to N_4^{II} and N_4^{III} respectively: that is to say, to 3°·0392 and 2°·6660. Thus we get the following values of the inclination of Jupiter's equator:—

	III.	IV.	Difference.
With old value of J . . .	3°·122	3°·068	0°·054
With new value of J . . .	3°·131	3°·091	0°·040

Thus the new value of J reduces the difference by about 25 per cent.

The following values of $b_4 t + \gamma_4$ were found from the same equations as $N_4\,N_4^{II}\,N_4^{III}$:—

	I.	III.	IV.	Souillart.
1891·75	316°·07	315°·46	316°·68	315·32
1901·61	315 ·78	315 ·22	316 ·47	315·45
1902·60	315 ·73	315 ·37	316 ·91	315·46.

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$b_4t+\gamma_4$ is the mean value of the node of Jupiter's equator on his orbit. Assuming b_4 known from theory, we get the following corrections to Souillart's $b_4t+\gamma_4$ at the epoch 1891·75, namely :—

From I.	+ 0° 27′
From III.	− 0° 4′
From IV.	+ 1° 17′.

These determinations of the equator rest on the observations of position-angle, and we have seen that in 1901 and 1902 the observations of IV are affected by a systematic residual. The periodic part of this error, having a period of only eight days, can have no appreciable effect on the values of the unknowns, though it increases their probable errors; the constant part, however, must have some slight effect, and may in part account for the outstanding values of N_4 and $b_4t+\gamma_4$ from IV.

Longitudes of Epoch.

As is well known, the mean longitudes satisfy the relation

$$\epsilon_1 - 3\epsilon_2 + 2\epsilon_3 = 180°.$$

The mean longitudes adopted in this investigation satisfy it, and therefore the corrections to them ought to satisfy a relation of the same form, namely :—

$$\Delta\epsilon_1 - 3\,\Delta\epsilon_2 + 2\,\Delta\epsilon_3 = 0,$$

provided no neglected inequality or other correction is included in any of these quantities. It was proved by Laplace on theoretical grounds that there is an oscillation or libration about the state represented by the equation first written, *i.e.*, that

$$\epsilon_1 - 3\epsilon_2 + 2\epsilon_3 = \text{not } 180° \text{ but } 180° - \text{D} \sin (bt + \text{E}),$$

where D and E are two constants of integration to be determined by observation. This libration was searched for by Delambre without success, and it was concluded that the coefficient D was too small to be shown by observation.

The present observations, however, and also those of 1891, seem to show that this is not the case. If we put

$$\theta = \epsilon_1 - 3\epsilon_2 + 2\epsilon_3 - 180°,$$

we find from observation that

in 1901·61	. . .	$\theta = +0°{\cdot}0012 \pm 0°{\cdot}0230 - 0{\cdot}25\,\Delta J$
1902·60	. . .	$\theta = +0{\cdot}1644 \pm 0{\cdot}0170 + 0{\cdot}11\,\Delta J$;

whence

1901·61	. . .	$\theta = +0°{\cdot}002 \pm 0°{\cdot}023$
1902·60	. . .	$\theta = +0{\cdot}164 \pm 0{\cdot}017$.

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The value for 1891·75 given by Dr. de Sitter in *Handelingen van het Nederl. Nataur en geneesk. Congres* April 1905, is:—

$$1891{\cdot}75 \quad . \quad . \quad . \quad \theta = -0^{\circ}{\cdot}157 \pm 0{\cdot}014.$$

Before it is safe to conclude that this quantity θ is Laplace's libration, it will be necessary to reconstruct the numerical theory of the motions of the satellites; we must be assured that the adopted longitudes include all inequalities except the libration. In the writer's paper in *Monthly Notices*, lxiv. p. 746, it was pointed out that in the longitude of II there are some large inequalities with coefficients amounting to 0°·04 and with long periods (500 days), and that any possible correction to these would be included in the observed value of $\Delta\epsilon_2$. The terms referred to are those in the arguments $2l^1 - l - gt - \beta$, and Professor Sampson has recently pointed out in a letter to the writer that the coefficients of these terms are incorrectly given by Souillart. It is, moreover, especially important that all inequalities should be correctly applied to the mean longitude of II since it enters with a coefficient of 3 into the expression for the libration. This being so, it does not at present seem safe to draw any conclusions from the values found for the quantity θ.

At the same time, it may be pointed out that until the masses of the first three satellites are determined, it is not to be expected that the libration can be revealed except by a large series of observations extending over only a short period of time; eclipse observations, for instance, cannot be expected to disclose it. The duration of the observations must be short compared with the period of the libration so as to avoid the necessity of combining the observations by assuming a knowledge of the period. For the period depends on the masses of the first three satellites: the period in years is $2\pi/b$ where b is given by the expression

$$b^2 = \kappa \left[\frac{m_{\mathrm{I}} m_{\mathrm{II}}}{a^2} + 9 \frac{m_{\mathrm{II}} m}{a_{\mathrm{I}}^2} + 4 \frac{m m_{\mathrm{I}}}{a_{\mathrm{II}}^2} \right],$$

and κ is a function of the mean motions and of the mean distances $a, a_{\mathrm{I}}\, a_{\mathrm{II}}$, and where m, $m_{\mathrm{I}}\, m_{\mathrm{II}}$ are the masses of I, II, III respectively. When Jupiter's equatorial radius and his mass multiplied by 10^4 are taken as units of length and mass, the value of κ is 50. The determinations of the masses made hitherto are as follows:—

	Laplace.	Damoiseau.	Souillart.	Adams.
I.	0·1733	0·1688	0·3773	0·2831
II.	0·2324	0·2323	0·2453	0·2324
III.	0·8850	0·8844	0·8218	0·8125
IV.	0·4266	0·4248	0·2312	0·2149.

The values adopted by Souillart in his numerical theory of the satellites are the same as those of Damoiseau.

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In this table the values under Souillart's name as well as those under Adams's are the values which arise from the introduction of Damoiseau's revised value, viz., $65^s{\cdot}073$ of the second eccentricity of III instead of his earlier value, viz., $116^s{\cdot}73$. From this table it appears that the mass of II is the only one that is known with any degree of certainty: the mass of III is doubtful to the extent of not more than 10 per cent., but the masses of I and IV are at present so uncertain that they may almost be looked upon as unknown.

The period of the libration corresponding to Souillart's values of the masses is 4·5 years, whilst that corresponding to Laplace's values is 6·2 years. This shows how vague is our knowledge of the period, and that at present we can hope to detect the libration only by some method which is independent of a knowledge of its period.

The mean longitudes then must be determined by observations made within a period not exceeding three or four months. Unfortunately, owing to the comparative rareness of eclipses this can only be done by some micrometric method; and for the determination of longitudes micrometric methods are inferior to eclipse methods. But the loss in weight can be compensated by increase in the number of observations.

If the quantities $\Delta\epsilon$ could be shown to include no unknown inequalities or corrections to inequalities already known, the conclusion to be drawn from the heliometer observations would be that the libration has a coefficient of $0^\circ{\cdot}150$ or $9'$. This may at first sight appear surprisingly large, but although it can easily be discovered by heliometer measurements, yet the old eclipse observations which Delambre discussed could not be expected to disclose it. In the first place, the maximum effect would be an acceleration or retardation of the time of an eclipse amounting to 15^s, 32^s and 5^s for satellites I, II and III respectively, and these are quantities considerably smaller than the probable errors of the observations. Moreover, as already pointed out, eclipse observations must be combined by assuming the period to be known. Hence the fact that the libration has not yet been detected by eclipse observations does not demonstrate the insignificance of the coefficient.

It has already been pointed out by Dr. de Sitter in his paper published in April 1905 that a determination of the period of the libration would afford the means of finding the mass of the first satellite. We have seen that the period varies with the masses. The mass of II is given by the great inequalities in the motions of I and III. Hence in the expression for the period of the libration the value of the mass of II is known and we get the following linear relation between corrections to the masses of I and III,

$$12{\cdot}96\ \Delta m_1 + 3{\cdot}27\ \Delta m_3 = -\Delta T$$

where Δm_1, Δm_3 are corrections to Damoiseau's values of m_1, m_3 and T is the cor-

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responding period of the libration, viz., 6·1 years. Another relation between Δm_1 and Δm_3 is provided by the great inequality of II and is of the form

$$0{\cdot}63\ \Delta m_1 + 1{\cdot}09\ \Delta m_3 = n.$$

By solution of these two equations we could find Δm_1 and Δm_3, and we should then have the masses of the first three satellites. Until they have been determined, and also the mass of the fourth, it is useless to push this discussion any further. Indeed all that it is profitable to do is to present the results of observation in a convenient and serviceable form; the derivation of isolated constants which implicitly depend upon certain ill-determined quantities is of little advantage, and since the determination of a self-consistent set of constants would involve the reconstruction of the numerical theory of the motion of the satellites, it is not intended to pursue this side of the subject further.

It will be convenient to collect here the constants and elements of the orbits as corrected by the writer's observations.

Mass of Jupiter.

1901	. . .	1 : 1047·44 ± 0·093
1902	. . .	1 : 1047·25 ± 0·059
Mean	. . .	**1 : 1047·30 ± 0·06**

Constant of Compression of Jupiter.

$$J = 0{\cdot}01950 \left(\frac{39{\cdot}00}{d}\right)^2$$

where d is Jupiter's equatorial diameter.

Eccentricities and Perijoves.

The values of

$$h = e \sin \Omega$$
$$k = e \cos \Omega$$

for the four satellites are :—

		I.	II.	III.	IV.
1901·61	h	+ 0°·0766	− 0°·0294	− 0°·0263	+ 0°·4441
	k	− 0 ·0856	− 0 ·0152	− 0 ·2492	− 0 ·6997
1902·60	h	− 0 ·0421	− 0 ·0559	− 0 ·0626	+ 0 ·4513
	k	− 0 ·0063	− 0 ·0044	− 0 ·1850	− 0 ·7143

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If the longitudes are reckoned from the earth's mean equinox instead of from the point O, these quantities become :—

		I.	II.	III.	IV.
1901·61	h	− 0°·1146	+ 0°·0103	− 0°·1559	− 0°·8072
	k	+ 0 ·0073	+ 0 ·0314	+ 0 ·1960	+ 0 ·1876
1902·62	h	+ 0 ·0256	+ 0 ·0368	− 0 ·0851	− 0 ·8225
	k	+ 0 ·0340	+ 0 ·0423	+ 0 ·1758	+ 0 ·1931

Nodes and Inclinations.

When the nodes are reckoned from the point O and the inclinations are referred to the reference plane of the epoch, the values of

$$p = i \sin(-\Omega)$$
$$q = i \cos(-\Omega)$$

are as follows :—

		I.	II.	III.	IV.
1901·61	p	+ 0°·0098	− 0°·2934	+ 0°·0522	+ 0°·2062
	q	− 0 ·0734	− 0 ·4138	− 0 ·1404	+ 0 ·2524
1902·60	p	+ 0 ·0050	− 0 ·3502	+ 0 ·0547	+ 0 ·2294
	q	− 0 ·0785	− 0 ·3339	− 0 ·1437	+ 0 ·2771.

The planes to which these quantities are referred are defined relatively to the ecliptic and the earth's mean equinox by the following quantities :—

	Node.	Long. of O.	Inclination.
1901·61	336° 27′ 6″	135° 29′ 20″	2° 9′ 10″
1902·60	336° 33′ 33″	135° 30′ 15″	2° 11′ 38″

and relatively to Jupiter's orbit by the quantities :—

	Node.	Long. of O.	Inclination.
1901·61	315° 30′ 36″	135° 30′ 36″	3° 4′ 5″
1902·60	315° 51′ 5″	135° 31′ 30″	3° 6′ 22″

Referred to Jupiter's orbit at the epoch and the earth's mean equinox, the inclinations and nodes of the satellites orbits are as follows :—

		I.	II.	III.	IV.
1901·61	Node	315° 42′	310° 42′	316° 27′	319° 42′
	Inclination	3° 8′ 31″	3° 29′ 39″	3° 12′ 32″	2° 49′ 24″
1902·60	Node	315° 57′	310° 1′	316° 49′	320° 31′
	Inclination	3° 11′ 4″	3° 27′ 22″	3° 15′ 2″	2° 50′ 24″

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It will be of interest to see how far these values are confirmed by the photographs, which are now being reduced by the writer. The photographs were taken in 1902 at the same time as the heliometer measurements were being made, with the object of comparing the two methods and their results.

NOTE.

By an oversight it was not stated on p. 134 that the following equations of condition were omitted :—

1901.

Nos. 15 and 16, June 25. The wrong pair of satellites were observed on one side of the segments, and the absolute terms were $-3''{\cdot}3$ and $+10''$ respectively.

No. 39, July 2. Probably the wrong pair of wires was used in setting on the scales. $O - C = +2''{\cdot}4$.

No. 270, August 27. Discordant pointings.

1902.

No. 27, June 16. Discordant pointings.

No. 177, July 20. Discordant pointings.

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ANNALS OF THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE.

VOL. XII. PART III.

LICK OBSERVATORY
UNIVERSITY
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OF CALIFORNIA
MT. HAMILTON, CAL.

A DETERMINATION OF THE INCLINATIONS AND NODES OF THE ORBITS OF JUPITER'S SATELLITES.

BY

DR. W. DE SITTER,

FROM PHOTOGRAPHIC PLATES TAKEN AT THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE,

UNDER THE DIRECTION OF

SIR DAVID GILL, K.C.B., LL.D., D.Sc., F.R.S., ETC.,

HIS MAJESTY'S ASTRONOMER AT THE CAPE OF GOOD HOPE.

Published by Order of the Lords Commissioners of the Admiralty in obedience to His Majesty's Command.

EDINBURGH:
PRINTED FOR HIS MAJESTY'S STATIONERY OFFICE
BY NEILL & CO., LIMITED, BELLEVUE.

And to be purchased, either directly or through any Bookseller, from
WYMAN & SONS, LIMITED, FETTER LANE, E.C.; or
OLIVER & BOYD, EDINBURGH; or
E. PONSONBY, 116 GRAFTON STREET, DUBLIN.

1906.

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INTRODUCTION.

THE plates discussed in the present paper were taken at the Royal Observatory at the Cape of Good Hope in the years 1891, 1903, and 1904. The first series was taken simultaneously with the series of heliometer observations of the satellites which were made by Sir David Gill and Mr Finlay in 1891, and reduced by me and published in my inaugural dissertation.* When Sir David, in 1898, entrusted me with the reduction of these observations, he at the same time encouraged me to study the whole problem of the determination of new elements of the satellites, and especially requested me to investigate whether more observations were required, and if so at what epoch. In compliance with this request, I stated in the fourth chapter of my dissertation as my opinion that the determination of the positions and motions of the orbital planes must be based exclusively on heliometric or photographic observations; and I suggested the the mean epoch 1903·7 for a new series of observations, as being not too remote to allow the determination of new values of these elements before the expiration of the tables which are at present in use, and still promising a fair accuracy in the determination of the motions of the nodes. The plates of 1903 and 1904 were taken in accordance with this suggestion.

In the meantime Mr Bryan Cookson, M.A., had in 1901 and 1902, during his stay at the Cape Observatory, made a fine series of heliometer observations of the satellites, which are extremely valuable, and strengthen the determination of the different unknowns considerably. Mr Cookson's discussion of his observations has been provisionally published in *Monthly Notices*, June 1904, pages 728–747.† The results contained in this communication have been of much use to me, and will be included in the final discussion which is given at the end of the present paper.

The plates taken in the oppositions of 1903 and 1904 were sent to Groningen immediately after each opposition was completed, and were measured as soon as possible. All the measures were made by myself. The greater part of the numerical work connected with the discussions was also done by me, though I received much valuable

* Discussion of Heliometer Observations of Jupiter's Satellites, made by Sir David Gill, K.C.B., and W. H. Finlay, M.A.: by W. de Sitter. Groningen: J. B. Wolters, 1901. The results of this paper will be republished, after the correction of some mistakes, in Vol. XII. Part I. of these Annals.

† The final results will be published as Vol. XII. Part II. of these Annals.

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assistance from the staff of the Astronomical Laboratory at Groningen. My thanks are due to Prof. Kapteyn, who thus liberally placed the resources of the Laboratory at my disposal, whenever they could be spared from other work.

My thanks are also due to Sir David Gill, who was always ready to carry out all my wishes with regard to the plates to be taken. It has been a constant pleasure to me during the work, to think that in doing it I might contribute a little towards the work of the Royal Observatory at the Cape of Good Hope under his direction, and thus perhaps repay a small part of the great debt of gratitude which I owe to him and to the Cape Observatory.

GRONINGEN, *April* 1906.

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CHAPTER I.

THE PLATES.

TABLE I. contains the times of exposures of the plates. On each plate six (in some cases five) exposures of Jupiter and its satellites were taken near the centre of the plate, the telescope being moved a little in declination between every two exposures. For the purpose of the determination of the plate-constants a pair of standard stars was photographed on each plate, in such positions that the middle point between the two stars is in the centre of the plate. The standard stars for 1891 are the same as those used in the heliometer work, viz.—

B.D. -8° 5905 ($7^{m}\cdot0$) and B.D. -7° 5838 ($6^{m}\cdot2$).

Those used in 1903 are—

24 Piscium (6·0) and 27 Piscium (5·2);

and those used in 1904—

B.D. $+7^{\circ}$ 240 (6·9) and B.D. $+8^{\circ}$ 258 (6·7).

In 1903 and 1904 a trail of the planet with its satellites was also impressed on the plate. Three exposures of the satellites were first made; then the clock was stopped and the telescope pointed so that the planet should be at the following edge of the plate. The planet was then allowed to trail over the plate, the time when it reached the R.A. of the other exposures being recorded. After the trail was completed, three more exposures of the satellites were taken in the centre of the plate. The times given in the table are the local sidereal times of the middle of each exposure. The letters *a* to *f*, by which the different exposures of the satellites are distinguished, refer to the order in which the images were measured, and will be explained in Chapter III.

The plates were taken with the 13-inch astrographic telescope of the Cape Observatory.

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TABLE I.

TIMES OF EXPOSURES, 1891.

No. of Plate.	Date.	Barometer.	Thermometer.	Standard Stars.	Satellites.					
					a.	*b.*	*c.*	*d.*	*e.*	*f.*
	1891.	in.	°	h m s	m s	m s	m s	m s	m s	m s
295	Oct. 16	30·059	65·4	20 52 13·8	56 56·8	58 2·8	58 58·8	59 49·8	60 41·8	61 31·8
300	17	30·078	65·0	20 53 11·3		60 15·3	61 9·3	62 7·3	63 21·3	64 19·3
303	19	29·923	63·5	{22 59 43·4 / 21 1 15·4}	4 26·4	5 40·4	6 33·4	7 28·4	8 30·4	9 26·4
307	22	30·244	58·2	21 56 54·1	41 53·1	42 33·1	44 4·1	45 29·1	47 49·1	
308	23	30·207	53·5	21 28 7·3	33 13·3	34 55·3	36 14·3	37 37·3	38 50·3	40 11·3
311	24	30·323	55·1	21 42 5·7	31 23·7	32 49·7	33 59·7	35 9·7	36 15·7	37 17·7
312	26	30·132	59·5	21 64 29·2	52 39·2	54 15·2	55 30·2	56 41·2	57 41·2	58 41·2
313	27	30·287	54·9	21 45 33·4	36 59·4	38 13·4	39 5·4	40 16·4	41 8·4	42 5·4
314	28	30·227	58·0	23 17 10·5	20 47·5	22 6·5	23 0·5	23 46·5	24 53·5	25 38·5
315	29	30·263	58·5	21 78 38·0	59 38·0	63 3·0	64 21·0	65 38·0	67 8·0	68 34·0
316	29	30·263	58·5	22 25 9·0	28 35·0	29 29·0	30 11·0	30 54·0	31 34·0	32 22·0
317	30	30·197	60·3	22 14 31·7	6 27·7	7 27·7	8 14·7	9 14·7	10 3·7	10 54·7
318	31	29·995	66·5	22 21 52·4	12 22·4	13 35·4	14 22·4	15 22·4	16 58·4	18 6·4
319	Nov. 5	30·079	60·3	23 34 42·6	20 16·6	22 34·6	23 36·6	26 44·6	27 48·6	28 36·6
320	6	30·071	62·1	23 28 11·4	19 22·4	20 40·4	21 25·4	22 0·4	23 10·4	23 58·4
321	7	30·191	62·1	23 15 45·3	8 8·3	9 1·3	9 49·3	10 28·3	11 5 3	11 43·3
322	14	30·129	62·8	0 25 16·9	13 10·9	14 30·9	15 26·9	16 41·9	17 40·9	18 38·9
324	19	30·227	56·2	23 62 28·8	54 31·8	55 21·8	56 1·8	56 41·8	57 21·8	57 59·8
326	20	30·373	59·3	1 22 46·3	14 1·3	15 41·3	17 33·3	18 8·3	18 38·3	19 16·3
327	21	30·222	61·3	0 17 23·0	8 26·0	9 18·0	10 13·0	12 13·0	13 18·0	14 13·0
328	23	30·194	63·0	0 28 7·2	19 50·2	20 54·2	21 31·2	22 10·2	22 47·2	23 17·2
329	24	30·146	65·0	1 10 31·9	2 46·9	3 26·9	4 9·9	4 39·9	5 9 9	5 39·9
330	25	30·210	64·0	0 44 43·4	34 16·4	34 56·4	35 30·4	36 16·4	37 6·4	37 56·4
331	26	30·090	69·0	0 61 56·0	54 6·0	55 4·0	56 11·0	56 46·0	57 26·0	58 0·0
332	27	29·859	79·6	0 41 49·7	34 20·7	35 15·7	35 54·7	36 35·7	37 15·7	37 55·7
333	30	30·177	60·8	0 44 38·3	35 4·3	35 54·3	36 34·3	37 9·3	37 44·3	38 24·3
334	Dec. 2	30·030	68·6	2 58 41·2	51 9·2	52 14·2	52 48·2	53 26·2	54 0·2	54 31·2
335	5	30 029	57·5	1 27 26·9	18 1·9	19 1·9	19 43·9	20 21·9	21 1·9	21 41·9
337	7	30·135	60·3	2 40 0·9	31 50·9	32 40·9	33 15·9	33 50·9	34 30·9	35 20·9
338	8	30·065	61·3	1 33 50·4	24 10·4	24 50·4		28 20·4	29 0·4	29 30·4
339	9	30·232	63·2	1 29 20·0	21 10·0	22 0·0	22 45·0	23 55·0	24 30·0	25 10·0
340	10	30·144	65·0	1 43 37·6	36 9·6	36 49·6	37 29·6	37 59·6	38 29·6	39 14·6
341	11	29·926	71·5	1 37 41·3	40 42·3	41 50·3	42 39·3	43 24·3	44 19·3	44 59·3
342	14	30·031	61·5	2 9 27·6	1 2·6	1 57·6	3 22·6	3 57·6	4 42·6	5 17·6
343	15	30·116	61·5	2 5 56·9	10 1·9	10 36·9	11 21·9	12 1·9	12 31·9	13 6·9
344	16	30·051	62·4	1 54 56·7	46 56·7	47 46·7	48 26·7	49 6·7	49 46·7	50 26·7
345	18	30·136	63·0	1 54 55·6	59 36·6	60 25·6	61 5·6	61 40·6	63 15·6	64 5·6
346	19	30·105	61·0	2 11 6·2	3 45·2	4 40·2	5 15·2	5 55·2	6 35·2	7 10·2
348	22	30·035	66·0	2 23 47·0	26 29·0	27 34·0	28 14·0	28 44·0	29 14·0	29 44·0
349	23	29·953	63·8	2 31 53·4	25 8·4	26 3·4	26 33·4	27 13·4	27 43·4	28 13·4
350	24	30·087	65·9	2 33 32·8	37 7·8	37 42·8	38 22·8	38 57·8	39 27·8	39 57·8
352	29	30·155	63·0	2 46 50·2	39 10·2	40 5·2	40 50·2	41 30·2	42 20·2	42 55·2
353	30	30·025	63·0	3 0 49·7	4 59·7	5 59·7	6 44·7	7 29·7	8 14·7	8 59·7

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TABLE I.

Times of Exposures, 1903.

No. of Plate.	Date.	Barometer.	Att. Thermometer.	Ext. Thermometer.	Definition.	Standard Stars.			Satellites.			Trail.	Satellites.		
									f.	*e.*	*d.*		*c.*	*b.*	*a.*
	1903.	in.	°	°		h m s	m s	m s	m s	m s	m s	m s	m s	m s	m s
7510	Aug. 17	30·426	53	47·0	3–4	21 30 36·2	31 11·2	32 6·2	40 16·2	40 46·2	41 15·2	48 17·2	55 21·2	55 46·2	56 16·2
7513	18	30·471	54	49·8		21 51 22·7	52 17·7	53 12·7	64 32·7	65 27·7	66 17·7	80 18·0	90 37·7	91 52·7	92 42·7
7518	19	30·490	54	50		22 31 46·4	32 39·4	33 26·4	39 54·4	41 0·4	42 1·4	52 34·9	61 30·4	62 19·4	63 19·4
7522	20	30·414	54	53·5		21 48 24·7	49 29·7	50 24·7	55 24·7	56 44·7	57 49·7	64 44·7	71 54·7	72 54·7	73 39·7
7524	21	30·192	55·0	48·7	2–3	22 7 35·1	8 5·1	8 35·1	12 35·1	13 5·1	13 35·1	21 1·6	28 5·1	28 35·1	29 5·1
7533	26	30·314	53·0	51·4	3	22 10 29·8	11 9·8	11 34·8	16 39·8	17 29·8	17 59·8	24 21·3	30 59·8	31 29·8	32 29·8
7537	27	30·051	52·5	47·0		22 4 10·5	5 15·5	6 10·5	10 10·5	11 5·5	11 55·5	17 53·8	25 25·5	26 25·5	27 25·5
7538	28	30·064	55·2	55		22 12 55·1	14 5·1	14 40·1	20 50·1	21 50·1	22 25·1	27 43·6	34 0·1	34 50·1	35 40·1
7555	31	30·563	53·0	44·2		21 59 33·4	60 33·4	61 18·4	64 48·4	65 53·4	66 33·4	73 43·6	81 13·4	81 58·4	82 43·4
7565	Sept. 1	30·484	55·0	54·8	3	22 23 24·5	23 54·5	24 24·5	28 4·5	28 34·5	29 4·5	35 1·0	41 34·5	42 4·5	42 44·5
7569	2	30·339	57·9	57		21 23 30·9	24 20·9	25 0·9	29 5·9	30 0·9	30 43·9	40 11·7	47 25·9	48 5·9	48 48·9
7570	3	30·060	61·5	60·0	2–3	21 49 15·8	49 50·8	50 20·8	55 5·8	55 35·8	56 5·8	63 11·3	70 5·8	70 40·8	71 15·8
7580	10	30·150	56·0	54·1	2–3	22 14 27·3	15 32·3	16 22·3	21 42·3	22 47·3	23 37·3	30 14·3	35 47·3	36 37·3	37 22·3
7581	12	30·231	56·1	50·3	cloudy	20 18 8·6	18 53·6	19 43·6	25 48·6	26 23·6	26 53·6	48 46·1	55 28·6	55 58·6	56 33·6
7582	14	30·250	57·1	51·3		21 31 25·9	32 10·9	32 45·9	38 15·9	39 0·9	39 35·9	47 52·4	55 25·9	56 0·9	56 40·9
7593	15	30·398	55·0	51·0	3	21 52 41·3	53 46·3	54 41·3	58 11·3	59 1·3	59 41·3	66 5·8	72 31·3	73 26·3	74 6·3
7596	16	30·412	57·0	56·0	2–3	23 0 32·4	1 12·4	1 52·4	7 42·4	8 17·4	8 52 4	16 23·9	27 22·4	28 2·4	28 42·4
7601	17	30·292	59·0	59·0	3	21 15 8·3	16 13·3	16 58·3	19 33·3	20 28·3	21 18·3	28 16·8	35 13·3	35 58·3	36 48·3
7604	18	30·130	61·0	64·0	3 & 3–4	21 33 13·7	33 53·7	34 23·7	40 13·7	40 53·7	41 28·7	49 23·7	59 28·7	59 58·7	60 33·7
7606	25	30·237	57·1	54·0		21 14 40·3	15 22·3	15 58·3	20 5·3	20 50·3	21 37·3	30 41·8	37 25·3	38 10·3	38 56·3
7608	26	30·077	60·5	56·8	2–3 & 3	21 27 44·8	28 14·8	28 44·8	33 44·8	34 14·8	34 44·8	41 38·3	48 24·8	48 54·8	49 24·8
7634	Oct. 2	30·338	57·0	53·2		21 39 7·4	40 2·4	41 7·4	48 2·4	48 57·4	49 37·4	56 22·9	62 32·4	63 32·4	64 22·4
7645	7	30·271	53·5	48·6	thin clouds, defin. poor	21 34 4·9	34 54·9	35 29·9	40 29·9	41 24·9	42 4·9	52 37·4	59 29·9	60 24·9	61 19·9
7647	8	30·088	56·0	52·2		21 43 51·1	44 56·1	45 36·1	48 26·1	49 16·1	49 56·1	56 57·3	62 56·1	63 46·1	64 36·1
7653	9	30·170	59·0	57·4	2 cloudy	21 15 51·7	16 21·7	16 51·7	21 21·7	21 51·7	22 21·7	29 54·2	35 51·7	36 21·7	36 51·7
7655	10	30·186	60·0	57·8		21 32 52·5	33 52·5	34 32·5	38 32·5	39 17·5	40 7·5	47 29·8	54 2·5	55 2·5	55 52·5
7656	13	30·128	57	52	3 cloudy	22 11 4·6	11 34·6	11 59·6	15 4·6	15 29·6	16 9·6	22 32·1	49 34·6	50 4·6	50 34·6
7657	15	29·956	59·5	55·0	3–4	21 12 25·6	12 55·6	13 25·6	18 25·6	19 5·6	19 30·6	26 32·6	33 25·6	34 0·6	34 40·6
7660	19	30·152	57·0	54·8	2–3	21 26 5·1	27 5·1	27 50·1	31 5·1	31 50·1	32 35·1	40 21·1	47 5·1	47 45·1	48 35·1
7662	21	30·306	57·0	56·8	3	21 26 30·7	27 0·7	27 30·7	30 40·7	31 5·7	31 30·7	38 15·2	45 0·7	45 35·7	46 10·7
7664	22	30·062	62·4	62·0	bad	21 24 31·0	25 21·0	25 51·0	29 13·0	29 52·0	30 39·0	40 43·0	48 26·0	49 16·0	49 59·0

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TABLE I.

TIMES OF EXPOSURES, 1904.

No. of Plate.	Date.	Barometer.	Att. Thermometer.	Ext. Thermometer.	Definition.	Standard Stars.			Satellites. *f.*	Satellites. *e.*	Satellites. *d.*	Trail.	Satellites. *c.*	Satellites. *b.*	Satellites. *a.*
	1904.	in.	°	°		h m s	m s	m s	m s	m s	m s	m s	m s	m s	m s
8123	Oct. 27	30·127	62·5	58·5	2–3	1 17 58·4	19 58·4	20 58·4	35 45·9	36 45·9	37 45·9	45 35·4	53 0·9	54 25·9	55 55·9
8124	28	30·233	62·2	60·0	v. poor	0 50 59·0	51 59·0	52 59·0	57 6·5	57 36·5	58 6·5	64 49·8	71 46·5	72 16·5	72 46·5
8126	29	30·157	66·2	68·1	poor	0 28 54·9	29 54·9	30 54·9	39 47·4	40 47·4	41 47·4	49 52·4	58 47·4	59 47·4	60 47·4
8127	30	29·907	68·0	60·7	poor	0 39 4·8	40 4·8	40 44·8	44 47·3	45 17·3	45 47·3	52 42·8	58 47·3	59 17·3	59 47·3
8130	Nov. 2	30·299	62·5	58·8	2–3	1 23 0·7	23 55·7	24 55·7	28 48·2	29 48·2	30 48·2	38 54·7	46 48·2	47 48·2	48 48·2
8137	4	30·079	64	60·6	3 & 3–4	0 50 6·4	50 46·4	51 26·4	54 48·9	55 18·9	55 48·9	62 18·4	68 58·9	69 28·9	69 58·9
8141	5	30·087	64·1	61·1	bad thin cloud	23 16 22·2	17 37·2	18 30·2	21 17·8	22 1·8	22 39·8	32 6·8	39 34·8	40 28·8	41 9·8
8142	6	30·041	64	58·2	2–3*	1 18 17·9	19 2·9	19 42·9	22 10·4	22 40·4	23 10·4	29 55·9	36 30·4	37 10·4	37 40·4
8143	7	30·121	64·1	56·7	v. bad	1 11 33·0	12 48·0	13 53·0	16 45·5	17 52·5		25 20·0	33 20·5	34 45·5	35 42·5
8144	8	30·087	64	57·2	3 cloudy	0 18 58·5	19 58·5	20 58·5	23 51·0	24 21·0	24 51·0	31 46·0	38 36·0	39 6·0	39 31·0
8145	9	30·205	64	56·5	3	0 10 28·7	11 18·7	11 59·7	15 6·2	15 36·2	16 1·2	23 2·2	30 6·2	30 36·2	31 6·2
8146	10	30·053	63·5	60·5	3–4	0 4 59·4	5 59·4	6 59·4	9 51·9	10 21·9	10 51·9	18 8·6	25 1·9	25 36·9	26 11·9
8148	13	30·343	60	49·6	2–3 cloudy	1 11 0·5	11 45·5	12 35·5	15 58·0	16 23·0	16 58·0	23 46·2	28 58·0	29 23·0	29 53·0
8154	14	30·303	60	55·0	2–3	0 46 1·5	47 1·5	47 41·5	50 14·0	50 44·0	51 14·0	58 20·7	64 54·0	65 24·0	65 54·0
8158	15	30·231	61·7	59·7		0 20 46·9	21 53·9	22 56·9	27 1·4	27 47·4	28 24·4	36 0·4	43 2·4	43 43·4	45 54·4
8159	16	30·157	62·5	57·2	3&3–4*	0 18 2·4	19 2·4	20 2·4	24 24·9	24 54·9	26 49·9	36 25·4	43 4·9	43 34·9	44 4·9
8167	17	30·239	60·9	54·0		0 24 32·8	25 27·8	26 5·8	29 5·3	29 43·3	30 17·3	37 23·7	45 35·3	46 15·3	46 52·3
8181	22	30·287	62	56·1	2–3 & 3 cloudy	0 10 10·5	11 10·5	12 10·5	24 0·5	24 35·5	25 5·5	32 27·0	38 58·0	39 29·0	39 58·0
8182	23	30·285	61·5	56·1	2–3	0 5 36·4	6 16·4	6 56·4	10 58·9	11 28·9	11 58·9	18 54·9	25 58·9	26 28·9	26 59·9
8190	24	30·089	62·3	59·5		0 6 47·2	7 53·2	9 12·2	13 7·6	14 6·6	14 49·6	21 46·6	29 31·6	30 18·6	30 59·6
8192	25	30·055	65·9	60·1		0 6 2·2	7 27·2	8 22·2	11 34·7	12 27·7	13 11·7	21 49·2	28 24·7	29 17·7	30 8·7
8198	26	30·135	64	59·5	2–3 cloudy	0 56 37·6	57 27·6	58 12·6	61 35·1	62 0·1	62 25·1	69 20·6	75 42·6	76 17·6	76 47·6
8199	27	30·165	63	57·8	3–4	1 21 8·1	21 48·1	22 28·1	25 20·6	25 50·6	26 12·6	33 33·1	39 40·6	40 5·6	40 40·6
8200	28	30·105	63·4	58·5	v. bad at times	0 18 23·3	19 33·3	20 28·3	23 17·8	24 3·8	24 33·8	32 25·7	42 35·8	44 8·8	45 26·8
8209	29	30·021	65	57·5	2	0 59 8·9	59 58·9	60 53·9	63 51·4	64 26·4	64 56·4	71 42·9	78 11·4	78 46·4	79 26·4
8212	30	30·087	63	58·7	3–4	2 54 9·5	55 9·5	56 9·5	59 37·0	60 12·0	60 47·0	78 53·5		Cloudy.	
8213	Dec. 1	30·189	60	53·5	3–4 & 3 cloudy	0 50 20·0	51 15·0	52 15·0	55 2·5	55 42·5	56 17·5	63 51·0	70 22·5	70 57·5	71 27·5
8216	2	30·236	62	57·8		0 34 0·8	35 1·8	35 54·8	39 3·3	39 43·3	40 21·3	48 36·6	56 22·3	57 3·3	58 0·3
8222	3	30·219	64·2	61·5		1 5 11·4	6 11·4	7 11·4	12 3·9	12 48·9	13 23·9	23 24·9	32 3·9	32 38·9	33 13·9
8225	7	29·929	67	64·3	4–3	2 24 11·9	25 1·9	25 51·9	29 59·4	30 34·4	31 9·4	38 14·9	45 9·4	45 44·4	46 24·4
8226	9	30·023	66	60·4	2–3	1 41 32·2	42 12·2	43 2·2	45 34·7	46 4·7	46 34·7	53 54·2	60 44·7	61 24·7	62 4·7
8228	13	30·315	66	62·0	2–3	1 54 58·2	56 18·2	57 8·7	62 5·7	62 45·7	63 15·7	70 18·2	78 35·7	79 5·7	79 35·7

* Plate 8142. Thin cloud during last three exposures.

* Plate 8159. Cloudy at times.

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CHAPTER II.

METHOD OF MEASURING.

As was mentioned above, my intention in measuring and discussing the plates was only to determine the positions of the orbital planes. I have shown in Chapter IV. of my dissertation by arguments, which need not be repeated here, that the other elements are, or will most probably very soon be, determined with all desirable accuracy from other observations. Since that was written Mr Cookson's investigation was made, which also determines all elements, and a determination of the elements depending on the longitudes (*i.e.*, eccentricities, perijoves, longitudes at epoch and large inequalities) from the observations of 1903 and 1904 appeared entirely superfluous.

The fact that I had to do practically all the work myself, and the desirability of getting results as soon as possible, were further reasons for restricting my programme, and adopting methods of measurement and discussion which would shorten the work as much as possible without impairing the accuracy.

For these reasons I resolved not to measure the co-ordinates of the satellites in two directions (R.A. and decl.) but only those quantities, which alone, or nearly alone, determine the positions of the orbital planes, *viz.*, the co-ordinates perpendicular to the plane of Jupiter's equator. Since the principal interest of the determination of the orbital planes lies in the comparison with the observations of the large motions of the nodes which are demanded by the theory, and since these motions are almost entirely produced by the compression of the planet, the equator, and not the orbital plane of Jupiter, is the natural plane of reference. A system of co-ordinates is accordingly adopted, of which the axis of y is the projection on the plate of Jupiter's adopted axis of rotation, and the axis of x is perpendicular to this. The co-ordinates y of the satellites were alone measured. The *réseau* printed on the plate was therefore not used. By means of the position-circle with which the Repsold measuring machine of the Astronomical Laboratory at Groningen is provided, the plate was adjusted in a position-angle approximately at right angles to the computed position-angle of the axis of y, and the co-ordinates y were measured directly by the micrometer-screw. The whole of the measured quantity thus never exceeds a few revolutions of the screw.

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This screw has been repeatedly investigated and is practically free from error.* Since the measured quantities are so small, the scale-value is of little importance, and need not be known very accurately. All the errors, which would be introduced by referring the satellites to the scale of the instrument † and the *réseau*-lines (such as projection-error, division errors of the scale, errors of division and copying of the *réseau*, difference of scale-value in different directions, etc.) are thus entirely avoided. Of course, two elements enter into the measures, which would to a much smaller extent enter into measures of R.A. and decl. referred to the scale and the *réseau*, *viz.*, the division-errors of the position-circle, and the straightness of the cylinder which guides the plate-holder in its movement parallel to the measuring-wire of the microscope. The eccentricity of the position-circle is eliminated by the use of two microscopes. The division-errors of the circle are certainly very small ‡ and have been eliminated to a certain extent by using as many different divisions as was practicable. Their effect on the measures is undoubtedly absolutely negligible compared with the influence of the several sources of errors referred to above which would affect measures of R.A. and declination.

The straightness of the cylinder was investigated repeatedly by measuring with the micrometer-screw a long spider-line, stretched on a brass frame which could be put in the plate-holder in the same way as a plate. The result of these measures is given in Table II.

The first column of this table gives the divisions of the vertical scale of the instrument, and the body of the table contains the deviations of the cylinder from the spider-line (supposed to be a straight line) at that division, as measured on the dates mentioned in the heading. The unit in which they are given is one hundredth of a micron, *i.e.*, $0^{\text{rev}}\cdot0001$, or one hundredth of a division of the head of the micrometer-screw.

Some of the measures have been made by Mr T. W. de Vries of the Astronomical Laboratory, at my request. On each date two or more series of measures were made while the plate-holder was being moved in the direction of increasing readings of the scale (forward) and an equal number with decreasing readings (backward). It will be seen that there is a systematic difference between the forward and the backward series,

* See "Publications of the Astronomical Laboratory at Groningen," No. 1, pages 23 and 24.

† The field of our measuring microscope is too small to refer the satellites to the *réseau*-lines without using the scale as an intermediate stage.

‡ Messrs. Repsold have kindly informed me that the division of the circle is of the same standard of accuracy as, *e.g.*, that of the position-circle of the Cape Heliometer, whose errors have also never been investigated. The smallness of the division-errors is also proved by the residuals of Table VI. of the Appendix.

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TABLE II.

DEVIATIONS OF CYLINDER FROM STRETCHED SPIDERLINE, IN UNITS OF $\frac{1}{100}$ MICRON.

Scale.	1904 Oct. 14–15. de Sitter. 4 Series.		1904 Oct. 14–15. de Vries. 4 Series.		1904 Nov. 11. de Sitter. 2 Series.		1904 Nov. 12. de Vries. 2 Series.		1905 Jan. 28. de Sitter. 3 Series.		1905 Feb. 10. de Sitter. 2 Series.		1905 Feb. 12. de Vries. 2 Series.		1905 May 11. de Sitter. 2 Series.		1905 June 7. de Sitter. 3 Series.		Means.		General Mean.
m.m.	Forw.	Back	Forw.	Back	Forw.	Back	Forw.	Back	Forw.	Back	Forw.	Back	Forw.	Back	Forw.	Back	Forw.	Back	Forw.	Back	
38	−27	+ 13	+13	−27	− 20	−75	−66	−36	+ 3	−21	+47	−48	− 36	−11	− 19	−34	+43	− 10	− 4	−23	−14
40	−31	−108	−29	−46	− 48	−63	−47	−22	−50	−47	−42	−97	− 10	−30	− 71	−36	−40	− 36	−40	−57	−48
42	+13	+ 16	+11	−29	− 47	−37	−13	−13	+ 1	−43	+29	−21	− 10	−55	− 19	+31	+59	− 8	+ 6	−17	− 5
44	−15	0	+25	−33	+ 34	+ 4	−34	+11	+31	+ 5	+15	− 5	− 59	−34	+ 23	+ 3	+59	− 1	+11	− 7	+ 2
46	−13	− 93	+ 6	−87	+ 7	−98	−16	−21	−41	−38	−65	−50	− 74	−24	− 40	−50	−36	− 42	−26	−61	−44
48	+18	− 25	+ 7	− 7	+ 23	−42	−27	−12	−17	−24	+16	−29	− 28	− 3	− 33	+ 7	+ 2	− 18	− 2	−17	−10
50	+37	− 11	+57	−13	+ 64	+ 4	+17	+22	+27	+17	+27	+22	− 28	− 8	+ 44	−26	+36	+ 39	+34	+ 4	+19
54	+15	− 13	−11	−23	+ 28	−37	−10	−50	− 2	−35	−17	−42	+ 9	−31	+ 9	− 1	+ 8	− 12	+ 4	−26	−11
58	+23	− 25	+45	+15	+ 16	+ 1	+17	+27	− 1	−17	+35	−40	− 10	−15	+ 53	− 7	+64	+ 20	+28	− 4	+12
62	+ 9	− 19	+13	−27	+ 69	−21	+60	−35	+28	+ 8	+41	−14	− 9	+11	+ 32	+47	− 4	− 14	+23	− 9	+ 7
66	+14	− 13	−19	− 7	+ 32	−48	−32	+ 3	−44	−24	+ 3	−67	− 38	+77	− 29	+ 6	−23	− 13	−15	−12	−13
70	+47	+ 29	+49	+ 7	+ 10	+65	+30	0	+54	+38	− 6	−11	+ 53	+33	+ 86	+71	+60	+ 90	+45	+35	+40
74	+20	0	−13	+ 2	− 6	+14	− 2	−47	−18	+39	+16	− 4	+ 99	−16	− 10	+25	+29	+ 55	+11	+10	+10
78	+13	+ 51	+37	+25	+ 82	− 8	+66	−34	+77	+ 4	+62	−13	+115	+30	+ 34	+49	+44	− 12	+43	+14	+28
82	+23	+ 46	+51	+20	+ 20	+15	−22	+68	+32	+38	+44	+ 9	+ 76	+46	+ 34	+54	−23	+ 47	+26	+38	+32
86	+ 6	+ 9	+ 1	−19	+ 3	+13	+46	−29	−17	− 7	0	−30	+ 52	+17	− 12	−47	−25	− 72	+ 3	−18	− 7
90	+15	+ 47	+23	+12	+ 66	−19	+14	+54	+58	+21	+62	+62	+ 73	+73	+ 52	+37	+44	+ 37	+41	+34	+38
92	− 4	+ 31	+ 3	+45	+ 13	+13	+23	+38	+25	+32	+43	−12	+ 64	−41	− 31	−16	−19	− 26	+10	+12	+11
94	−17	− 3	−18	+27	− 6	+39	−34	+26	− 4	− 4	−12	−12	+ 29	+ 9	− 24	−29	−44	− 27	−22	+ 3	−10
96	+21	+ 31	+41	+33	+ 6	+31	−35	+35	+66	+26	+49	+24	+ 81	+31	+ 14	+14	+20	− 3	+31	+25	+28
98	+43	+ 41	+10	+32	− 13	+12	− 6	+19	+60	−23	+70	+25	+ 20	+50	+ 11	+16	+12	− 12	+25	+18	+21
100	−31	− 43	−21	+44	+ 29	−36	−22	+53	−36	−42	−19	−74	+ 11	+ 6	− 57	−47	−71	− 51	−27	−20	−23
102	− 1	+ 16	−17	+43	+ 66	−34	+ 2	+17	+12	+15	+ 6	− 9	− 8	− 8	− 15	+10	−24	− 64	0	+ 2	+ 1
104	+20	+ 45	−35	+25	+ 37	+27	+16	+16	+ 6	+ 2	+37	+52	+ 7	+42	− 33	− 8	−29	+ 1	0	+23	+11
106	−49	+ 7	−29	+31	− 16	−26	+70	+15	+13	+ 6	+23	− 2	+ 13	+38	+ 4	−66	−28	− 55	− 7	− 3	− 5
108	−22	− 15	−23	− 7	− 15	−30	+73	−12	− 3	−86	−36	−51	− 57	−12	− 44	−84	−50	− 40	−21	−35	−28
110	−11	+ 12	+ 1	− 6	+ 7	+42	+ 7	−28	+11	+ 4	+29	+49	− 36	+44	+ 14	−21	+14	+ 14	+ 3	+11	+ 7
114	−27	− 23	−21	−33	− 30	−30	+70	−25	− 4	−58	+26	+36	− 10	−15	− 47	−62	−27	− 47	−11	−30	−21
118	+27	− 7	+ 3	−17	+ 63	−22	−17	−12	+20	+ 7	+27	+72	+ 9	+21	− 8	+27	+18	− 2	+16	+ 4	+10
122	−31	− 11	+11	− 9	+ 21	+11	− 15	+50	−35	−35	−11	− 1	− 8	− 8	− 34	−24	−76	+ 10	−21	− 4	−13
126	−43	+ 29	+16	+43	+ 34	− 1	+58	+73	+13	− 4	+25	+35	+ 18	+53	+ 21	+41	+ 3	+ 36	+11	+33	+22
130	−31	− 11	+14	−11	+ 3	−12	−14	− 9	+38	+11	+ 2	−18	+ 14	+ 4	0	0	−35	+ 22	− 2	− 2	− 2
134	−25	+ 50	+32	−13	+101	+31	− 2	−22	+32	+29	− 2	−12	− 20	+35	+ 14	+74	−33	+ 87	+ 9	+26	+17
138	+25	− 9	−25	+13	+ 14	+34	−24	−34	−20	+10	−35	+15	+ 32	+31	+ 9	+ 9	−35	+ 5	− 7	+ 7	0
142	−69	+ 36	+ 1	− 7	+ 32	− 3	−41	−51	−24	−37	−74	−54	+ 2	−33	− 62	+58	−66	+ 34	−34	− 2	−18
146	− 6	+ 14	+ 6	+46	+ 10	+60	+36	+46	+44	+27	− 7	+33	+ 8	+23	− 28	+57	− 7	+ 57	+ 6	+39	+22
150	−31	+ 29	−28	−18	+ 13	+28	−21	+ 9	−15	+15	+ 9	+ 4	− 65	+10	− 28	+57	− 8	+112	−20	−27	+ 3
152	− 3	+ 93	+25	+31	+ 25	+40	+18	+18	− 4	+46	−10	+60	− 40	+75	+ 89	+39	+40	+110	+15	+60	+37
154	−10	+ 55	+ 5	+15	− 59	+11	−23	+ 2	−33	+10	−24	+ 6	− 9	− 4	− 24	+21	−10	+ 54	−18	+23	+ 3
156	−79	+ 29	−41	−27	−112	−52	−30	−25	−16	−26	−54	+11	− 19	− 9	− 62	+13	−58	+ 29	−52	− 4	−28
158	−17	+ 51	+ 3	−40	− 60	+ 5	−81	+54	−15	+31	−43	+ 7	− 13	−68	+ 29	+24	−17	+ 36	−20	+14	− 3
160	−68	+ 2	−33	+27	− 64	−44	−37	−22	+12	+22	− 7	−22	− 18	−48	− 23	+ 7	− 2	+ 44	−28	+ 2	−13
162	−77	− 21	−75	−37	− 52	−67	−68	−68	−51	−67	−71	−46	− 37	−42	−131	−46	−88	− 22	−76	−43	−60

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the difference Forw. – Back being positive for small and negative for large scale-readings. This means that the plate-holder carrying the spider-line suffers a slight rotation when the motion is reversed. The plate-holder rests on the cylinder with two V's and is pressed against it by its own weight. The rack and pinion by which it is moved are not placed in the vertical plane passing through the apices of the V's and the axis of the cylinder, and their action thus produces a moment, which effects the small rotation. The friction of the third support acts in the same direction.

The deviations from a straight line given in the last column are free from this rotation. It is almost certain that these deviations are mostly due, not to the cylinder, but to irregularities in the thickness of the spider-line, which was always placed on the instrument in the same position. This could be easily tested, but as this would involve a great labour, and as the matter is of no importance for our immediate purpose, I have not done so. The errors of the cylinder itself would be found by drawing a smooth curve through the values given in the last column of Table II. With the exception of the extreme ends, which are seldom or never used, the errors so found would be smaller than 0·2 micron, or 0″·012 in arc on the plate. But even if the deviations were wholly due to the cylinder, they would be much too small to have any effect on the measures.

The rotation of the plate-holder on the reversion of the motion also does not exceed 1″ of position-angle, and can be safely neglected. Moreover, its existence was suspected before its magnitude was known, and the observations were accordingly arranged in a systematical way so as to eliminate its effect entirely.

In conclusion, I believe that by the adopted method of measuring, not only an enormous gain is secured in the rapidity and ease of the measures and the subsequent reductions, as compared with measures of differences of R.A. and declination, but also the accuracy is considerably increased.

On a few plates of the 1891 series differences of R.A. and declination were measured in June 1902. These plates were afterwards remeasured by the method just sketched in June 1904, and the former measures have not been used, except for the determination of the scale-value (see Chapter V.).

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CHAPTER III.

THE MEASURES.

THE objects measured on the plates are the standard stars, the trails and the images of the satellites. I have not tried to measure the planet. It is certain that such measures would have been subject to very large systematic errors. All plates were measured in two positions, differing 180° in position-angle. The order in which the measures were made was the following:—

I.—1. The plate-holder being adjusted approximately in the position-angle of the standard stars, the distances of the images of these stars from a line parallel to the axis of the cylinder are measured by the micrometer-screw. [The position-angle of this line is determined under 2.] On the plates of 1891 there is only one image of each star, and four pointings were made on each image. On the plates of 1903 and 1904 there are three images and one pointing was made on each. The pointings were always symmetrically arranged with respect to the "forward" and "backward" motion of the plate-holder in passing from one star to the other.

2. Reading of the position-circle microscopes.—Two readings were taken, both of both microscopes, the plate-holder being moved between the two so that one reading was made after a "forward," and one after a "backward" movement of the plate-holder. In one reading the preceding division of the circle was used, and in the other the following division, so as to diminish the effect of both the run of the micrometer-screws (which was, moreover, frequently verified to be negligible) and the division-errors of the circle. [These rules were not always strictly adhered to in the latter part of the work, after it had appeared that these sources of error are insignificant.] The circle is divided to 10′ and one revolution of the screw is equal to one division of the circle. The heads of the screws are divided into sixty parts so that one division is 10″, and seconds are estimated.

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3. The plate is brought in the computed position-angle of the axis of x (corrected by an approximate index-error derived by inspection from the operation 2) and the position-circle is read as under 2.

4. One pointing on each image of each satellite. If the satellites are numbered A, B, C, D, in the order of increasing x, and the exposures are numbered a, b, c, d, e, f, in the order of increasing declination, then the order in which the pointings are made is :—aA, aB, aC, aD, bD, bC, bB, bA, cA, cB, etc.

Thus one half of the pointings (those of the exposures a, c, e,) are made with a "forward" motion of the plate-holder, and the other half with a "backward" motion.

In the series of 1903 and 1904 the last exposure is separated from the others by a larger interval in declination. The chronological order of the exposures is f, e, d, c, b, a. For the plates of 1891 there are no means of distinguishing the different exposures. On one or two plates it could be made certain that the chronological order is the same as the order a, b, c, d, e, f, and it is probable, though not certain, that this is the case with all plates.

The screw-head is divided into a hundred parts, and ten revolutions of the screw are approximately equal to one millimetre, *i.e.*, one minute of arc on the plate.

II.—5 to 8. The plate is rotated 180° and the same operations are repeated in the reversed order. The images of the satellites are measured in the same order as before, thus those of exposure a, which were in position I. measured with "forward" motion of the plate-holder, are now measured with "backward" motion, and similarly for the other exposures.

The measures of the trails were made in two positions differing by 180° before the first and after the last measure of the standard stars. They are not given in the following table, but will be treated separately in Chapter VII.

Table III. contains all the separate pointings actually made on the plan which has been described. The different operations are referred to in the table as follows :—1 and 8, *standard stars*; 2 and 7, *position-angle standard stars*; 3 and 6 *position-angle satellites*; 4 and 5, *satellites*. The measures on plates which were not included in the discussion are not given in the table.

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The sign : denotes that the image so marked is diffused or extraordinarily faint, or that for some other reason the measure was considered to be uncertain. The sign ‖ denotes that the measure was made difficult by a réseau-line overlapping the image. These signs are generally not repeated for position II. The upper line in the columns of position-angle give the readings of the right hand microscope, the lower line those of the left hand microscope. In the record of the pointings on the satellites and standard stars the whole number of revolutions is not repeated when it is the same for a whole line.

TABLE III.

MEASURES OF PLATES OF 1891.

Date.	Plate, Position.	Standard Stars.			Position Angle Standard Stars.			Satellites.							Position Angle Satellites.			Remarks.
									a.	*b.*	*c.*	*d.*	*e.*	*f.*				
1904.		rev.	rev.	rev.	°	′	″		rev.	rev.	rev.	rev.	rev.	rev.	°	′	″	
June 14	307	A. 12·712	·713	·725	68	4	30	II.	8·596	·513	·528	·452	·432		65	48	30	
	I.	B. 10·749	·728	·756		2	30	I.	7·769	·798	·726	·712	·650			46	34	
								IV.	6·120‖	6·050	5·988	6·068‖	5·955					
								III.	7·278	·258	·175	·240	·138					
	307	B. 7·268	·211	·216	248	4	59	II.	6·896	6·912	6·924	7·152	6·862		245	45	40	
	II.	A. 5·430	·420	·456		2	31	I.	7·760	·663	·741	·822	·722			49	48	
								IV.	9·441	·416:	·470:	·520‖	·427					
								III.	8·240	·180	·237	·295	·198					
	307	B. 11·022	·010		248	2	52	II.	8·205	7·376	7·492	7·671	7·549		245	48	21	Measured with a
	II.	A. 8·941	·920	·932		2	50	I.	8·990	·133	·261	·337	·330			49	2	higher power.
								IV.	10·668	9·860	10·010	9·992	10·028					
								III.	9·483	8·640	8·762	8·795	8·791					
	307	A. 9·222	·298		68	2	47	II.	9·072	9·008	9·070	8·951	8·981		65	48	50	Higher power.
	I.	B. 7·154	·190			4	1	I.	8·240	·251	·301	·233	·169			47	38	
								IV.	6·619‖	·550	·532	·568:	·479					
								III.	7·801	·732	·749	·721	·678					
June 15	314	A. 8·122	·078		68	2	30	IV.	9·401	·342	·318	·370	·302	·278	65	49	40	
	I.	B. 7·633	·582			5	11	I.	8·549	·496	·504	·466	·463	·370		49	52	
								II.	7·658	·615	·599	·582	·579	·539				
								III.	8·780	·772	·760	·845	·777	·788				
	314	B. 8·765	·761		248	5	13	IV.	6·938	7·118	6·939	6·972	7·050	7·072	245	49	31	
	II.	A. 8·250	·297			1	55	I.	7·809	·932	·752	·860	·910	·939:		49	59	
								II.	8·685	·798	·585	·763	·818	·786				
								III.	7·534	·601	·420	·481	·582	·622				
	315	A. 8·140	·082	·122	68	4	51	IV.	9·619	·662	·605	·602	·780	·752	65	49	28	
	I.	B. 7·650	·683			5	20	II.	8·210	·267	·168	·270	·389	·370		50	32	
								I.	7·771	·812	·739	·713	·972	·989				
								III.	7·390	·426	·329	·360	·559	·622				
	315	B. 8·168	·188		248	4	38	IV.	6·488	·481	·418	·522	·522	·508	245	49	31	
	II.	A. 7·740	·750			5	45	II.	7·877	·910	·879	·851	·948	·898		50	23	
								I.	8·319	·330	·260	·408	·366	·348				
								III.	8·698	·731	·712	·790	·738	·689				
June 16	316	A. 8·960	·918		68	8	21	IV.	9·767	·784	·728:	·660	·635‖	·618	65	48	27	
	I.	B. 7·300	·301			2	9	II.	8·351	·398	·237	·188	·200	·158		49	49	
								I.	7·750	·798	·710	·657	·592	·593				
								III.	7·350	·392	·249	·231	·172	·235				

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TABLE III.—Measures of Plates of 1891—*continued.*

Date.	Plate, Position.	Standard Stars.				Position Angle Standard Stars.	Satellites.	*a.*	*b.*	*c.*	*d.*	*e.*	*f.*	Position Angle Satellites.	Remarks.
1904.		rev.	rev.	rev.		° ′ ″		rev.	rev.	rev.	rev.	rev.	rev.	° ′ ″	
June 16	316	B. 8·231	·248			248 3 41	IV.	5·949	6·113	6·138	6·122	6·122	6·213	245 48 27	
	II.	A. 6·619	·627			6 57	II.	7·385	·478	·585:	·533	·578	·658	50 40	
							I.	7·942	8·027	8·106	8·081	8·060	8·190		
							III.	8·360	·463	·573	·522	·548	·579		
	317	A. 7·898	·972			68 4 38	IV.	9·975	9·969	9·973:	10·061	9·975:	9·916:	65 51 23	Bad plate.
	I.	B. 8·128	·130			4 52	I.	8·579	·498	·543:	·534	·420:	·422:	49 7	
							III.	6·703	·780	·711:	·710	·603	·557		
							II.	8·734	·745	·728:	·799	·610	·640		
	317	B. 8·042	·070			248 4 38	IV.	6·355	6·422	5·866	6·322	6·167	6·060	245 49 7	
	II.	A. 8·297	·338			5 10	I.	7·759	·950	·331	·701	·603	·563	49 39	
							III.	9·532	·725	·228	·532	·378	·393		
							II.	7·516	·702	·112	·504	·361	·373		
Oct. 18	295	A. 7·719	7·720	9·256	9·262	68 3 53 54	III.	6·643	·610	·529	·539	·503	·212	65 44 45 44	
	I.	B. 8·221	8·250	9·804	9·812	4 42 40	II.	8·743	·723	·630	·567	·567	·260	46 2 0	
							IV.	9·892	8·401	·792	·755	·652	·392:		
	295	B. 7·933	7·928	8·310	8·327	248 4 13 10	III.	9·539	·680	·670	·490	·500	·386	245 44 19 14	
	II.	A. 8·461	·450	·895	·880	4 39 40	II.	7·520	·622	·658	·460	·439	·314	46 0 2	
							IV.	6·369	·462	·319	·209	·331	·132		
	300	A. 7·248	7·251	6·760	6·776	68 4 27 18	III.		6·922	6·834	6·790	7·178	7·100	65 45 42 38	Satellite I very near to edge of planet. Satellite IV very faint.
	I.	B. 8·066	8·089	7·452	7·433	6 22 22	I.		7·331	·140	·160	·582	·490	47 40 42	
							II.		7·212	·072	·097	·450	·451		
							IV.		8·912	8·840	8·722	9·240	9·088		
	300	B. 8·090	8·070	7·122	7·128	248 5 51 64	III		8·792	·866	·805	·741	·852	245 46 26 38	
	II.	A. 8·910	8·909	7·980	7·973	4 59 45	I.		8·438	·580	·358	·271	·440	46 65 52	
							II.		8·530	·627	·479	·406	·499		
							IV.		6·733	·743	·830	·660	·792		
Oct. 19	303	A. 8·192	8·168	9·021	9·042	68 5 2 64	I.	7·857	·898	·711	·620	·580	·669	65 47 42 52	All images of Satellite I on réseau-line. Spotted plate.
	I.	B. 7·909	7·928	8·811	8·785	4 73 13	III.	9·458	·474	·300	·210	·205	·281	46 60 50	
							IV.	7·550	·587	·345	·335	·231	·380		
	303	B. 8·150	8·132	9·005	9·008	248 4 62 56	I.	8·450	·308	·492	·449	·571	·482	245 47 42 60	
	II.	A. 7·876	7·918	8·720	8·689	5 12 18	III.	6·840	·759	·948	·856	·904	·849	46 51 40	
							IV.	8·692	·670	·871	·816	·840	·782		
	308	A. 8·693	·735	·331	·329	68 5 32 29	I.	8·510	·515	·289	·150	·240	·363	65 48 83 18	
	I.	B. 8·058	8·062	7·690	7·680	4 13 12	II.	8·560	·522	·270	·260	·298	·352	47 58 129	
	308	B. 8·568	8·572	7·443	7·429	248 4 27 67	I.	7·670	·649	·842	·940	·820	·843	245 48 29 31	
	II.	A. 8·022	8·002	6·850	6·828	4 71 36	II.	7·595	·643	·798	·839	·802	·834	49 7 6	
Oct. 20	311	A. 8·022	·007	·642	·616	67 58 60 38	III.	7·200	7·005	7·317	7·152	7·143	6·984	65 42 50 79	Spotted plate.
	I.	B. 8·062	·070	·742	·720	58 12 40	IV.	6·401	·210	·588	·488	·392	·300	42 30 2	
							II.	7·661	·499	·792	·692	·655	·467		
	311	B. 8·742	8·748	7·543	7·513	247 58 61 40	III.	8·848	·838	·798	·658	·709	·799	245 42 32 68	
	II.	A. 8·907	8·892	7·652	7·648	58 2 33	IV.	9·664	·570	·510	·390	·485	·494	41 88 56	
							II.	8·340	·339	·267	·198	·272	·308		
	312	A. 8·009	·008	·308	·290	68 4 67 54	IV.	7·638:	·561:	·564:	·318	·481	·440:	65 50 2 0	Plate much spotted.
	I.	B. 7·740	7·712	8·011	8·045	4 49 57	III.	9·298	·194	·237	·000	·126	·061	49 9 19	
							I.	7·777	·642	·659	·412	·560:	·505		
	312	B. 8·207	·211	·630	·642	248 4 63 44	IV.	8·122	·189	·250	·143	·311	·329	245 49 72 54	
	II.	A. 7·930	7·890	8·323	8·322	4 37 53	III.	6·395	·536	·604	·455	·690	·670	49 10 32	
							I.	7·989	8·082	8·161	8·047	8·248	8·263		

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TABLE III.—Measures of Plates of 1891—*continued.*

Date.	Plate, Position.	Standard Stars.				Position Angle Standard Stars.	Satellites.						Position Angle Satellites.	Remarks.
							a.	*b.*	*c.*	*d.*	*e.*	*f.*		
1904.		rev.	rev.	rev.	rev.	° ′ ″ ‴	rev.	rev.	rev.	rev.	rev.	rev.	° ′ ″ ‴	
Oct. 20	313	A. 8·118	8·080	7·432	7·461	68 7 1 12	IV. 8·129	·409	·283	·248:	·188	·212	65 52 7 3	Plate very much spotted, showing colours of mother-of-pearl.
	I.	B. 8·270	8·268	7·580	7·594	5 61 45	I. 8·090	·359	·256	·200:	·152	·142	50 39 47	
							III. 9·245	·494	·333	·362	·322	·282		
							II. 8·030	·259	·110	·110	·088	·057		
	313	B. 7·359	7·321	8·342	8·318	248 6 63 31	IV. 7·539	·712	·721	·828	·919	·778	245 51 46 32	
	II.	A. 7·530	7·539	8·512	8·518	6 11 39	I. 7·539	·720	·639	·849	·928	·781	50 58 65	
							III. 6·405	·610	·555	·642	·729	·644		
							II. 7·622	7·909	7·840	7·908	8·025	7·942		
	318	A. 7·672	7·660	8·090	8·123	68 9 22 53	III. 7·136	7·002	6·982	6·848	6·959	6·971	65 55 28 71	Difficult to measure.
	I.	B. 7·816	7·795	8·221	8·198	8 80 42	IV. 10·152	·068	·060	·000	·058	·021	54 90 47	All images (réseau-lines included) have a ghost. Sat. I very near to planet.
							I. 7·719	·772	·610	·344:	·493	·482:		
							II. 7·790	·788	·758	·620	·696	·657		
	318	B. 8·062	8·050	7·541	7·537	248 9 11 32	III. 9·188	·269	·078	·039	·032	·040	245 55 17 31	
	II.	A. 8·097	8·041	7·570	7·590	8 70 48	IV. 6·142	6·168	6·067	5·892	5·900	5·943	55 31 18	
							I. 8·612	·673	·521	·462	·490	·484		
							II. 8·488	·532	·407	·269	·318	·347		
Oct. 21	319	A. 8·029	·038	·926	·910	68 8 27 49	II. 8·125	·028	·121	·161	·158	·204	65 54 29 30	Same remark as 318.
	I.	B. 8·004	7·967	8·923	8·874	8 41 28	I. 7·830	·842	·895	·929	·919	·955	54 31 29	
							III. 7·494	·478	·559	·641	·580	·647		
							IV. 7·271	·262	·269	·332	·317	·347		
	319	B. 7·710	7·710	8·188	8·200	248 8 40 57	II. 7·771	8·008	7·923	7·900	7·958	7·914	245 54 36 18	
	II.	A. 7·688	7·674	8·145	8·120	8 29 16	I. 7·998	8·181	8·160	8·117	8·231	8·152	54 26 52	
							III. 8·338	·558	·481	·433	·532	·450		
							IV. 8·600	·820	·769	·749	·840	·818		
	320	A. 8·198	8·220	7·879	7·870	68 5 37 69	I. 8·416	·360	·272	·209	·294:	·137	65 51 12 32	Same remark as 318.
	I.	B. 8·052	8·031	7·740	7·730	5 49 14	III. 6·818	·709	·650	·550	·662	·468	50 61 36	
							II. 8·729	·590	·540	·480	·529	·365		
							IV. 6·737	·569	·518	·489	·543:	·392		
	320	B. 7·992	7·978	8·482	8·449	248 5 58 25	I. 7·577	·553	·543	·560	·567	·649	245 50 81 39	
	II.	A. 7·820	7·789	8·291	8·330	5 22 49	III. 9·259	·238	·199	·232	·228	·254	50 33 68	
							II. 7·335	·300	·308	·272	·317	·329		
							IV. 9·361	·362	·300	·313	·300	·351		
	321	A. 8·028	·018	·423	·432	68 7 2 39	III. 6·879	·818	·909	·790	·769	·751	65 52 61 41	Same remark as 318. Images very diffused.
	I.	B. 7·958	7·907	8·311	8·330	6 69 38	I. 7·480	·430	·640	·531	·540	·518	52 40 50	
							II. 7·652	·601	·805	·591	·651	·580		
							IV. 6·172	·180	·310	·198	·265	·151		
	321	B. 8·112	8·108	7·658	7·650	248 7 38 1	III. 9·080	·183	·070	·200	·192	·168	245 52 32 37	
	II.	A. 8·005	8·008	7·518	7·543	6 30 64	I. 8·465	·513	·317	·476	·549	·452	53 8 4	
							II. 8·222	·300	·207	·411	·393	·351		
							IV. 9·681	·779	·611	·710	·810	·690		
Oct. 22	322	A. 7·367	7·368	8·107	8·116	68 3 42 9	IV. 9·080:	·248	·200	·170	·176	·070:	65 47 61 38	
	I.	B. 7·550	7·538	8·228	8·212	3 6 27	III. 6·854	·925	·920	·822	·923	·811:	47 0 31	
							I. 7·603:	·674	·729	·781	·677	·660:		
							II. 7·837	·891	·880	·794	·880	·872:		
	322	B. 7·900	7·892	8·685	8·690	248 3 41 30	IV. 6·982	·719	·770	·930	·953	·850	245 47 71 48	
	II.	A. 8·113	·112	·915	·890	2 48 29	III. 9·242	·072	·080	·198	·234	·144	47 19 38	
							I. 8·469	·331	·213	·382	·460	·290		
							II. 8·268	·090	·078	·220	·238	·112		

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TABLE III.—Measures of Plates of 1891—*continued.*

Date.	Plate, Position.	Standard Stars.				Position Angle Standard Stars.	Satellites.	*a.*	*b.*	*c.*	*d.*	*e.*	*f.*	Position Angle Satellites.	Remarks.
1904.			rev.	rev.	rev.	rev.	° ′ ″ ″		rev.	rev.	rev.	rev.	rev.	rev.	° ′ ″ ″	
Oct. 22	324	A.	8·098	·101	·593	·581	68 5 22 21	II.	7·913	·942	·950	·882	·921	·908	·889	65 47 48 74	Good plate.
	I.	B.	7·960	7·939	8·437	8·439	5 10 0	I.	8·278	·349	·323	·248	·317	·319	·211	47 32 10	In position I. one of the series of images was accidentally measured twice, probably the third or fourth.
								III.	8·020	·088	·016	·040	·000	·080	·000	
								IV.	9 113	·172	·149	·113	·111	·190ǁ	·043	
	324	B.	8·109	·130	·552	·548	248 5 24 35	II.	8·351	·277	·319	·271	·228	·245		245 47 45 54	
	II.	A.	8·005	7·972	8·378	8·390	4 52 50	I.	7·999	·862	·932	·902	·884	·929		47 23 10	
								III.	8·232	·123	·208	·162	·100	·220		
								IV.	7·179	·038	·118	·167	·110	·111		
Oct. 24	338	A.	7·858	·838	·499	·532	68 4 55 90	III.	9·018	·170		·215	·042	·162ǁ		65 36 1 11	Bad plate.
	I.	B.	8·377	·369	·101	·068	4 57 19	I.	8·448	·629		·669	·490	·595		35 33 36	
								II.	8·579	·778		·781	·688:	·712		
								IV.	7·692	·909		·824	·750	·750:		
	338	B.	7·648	·647	·322	·352	248 4 53 67	III.	6·868	7·032		6·857	6·869	6·930		245 35 27 46	
	II.	A.	8·218	8·196	7·848	7·872	4 57 54	I.	7·425	·602		·440	·392	·448		36 2 48	
								II.	7·248	·400		·267	·232	·339		
								IV.	8·230	·280		·219	·195	·208		
Oct. 25	326	A.	7·897	7·891	8·460	8·461	68 4 118 52	I.	8·380	·330	·350	·448	·452	·422		65 48 3 9	Spotted plate.
	I.	B.	8·013	·038	·532	·573	4 22 80	II.	8·837	·949	·838	·849	·951	·982		47 54 55	All images extremely faint and diffused.
								III.	7·151	·213	·128	·140	·333	·301		
								IV.	8·709	·621	·662	·665	·790	·812:		
	326	B.	7·258	·280	·842	·793	248 5 23 38	I.	7·472	·610	·723	·503	·523	·572		245 48 15 26	
	II.	A.	7·539	7·551	8·102	8·138	5 37 21	II.	7·091	·060	·303	·080	·080	·059		47 51 38	
								III.	8·780	8·781	9·018	8·820	8·779	8·769		
								IV.	7·284	·340	·477	·369	·258	·330		
	330	A.	8·032	·051	·352	·379	68 23 58 70	I.	7·476	·505	·549	·469	·458	·338		66 3 15 16	Images very diffused. Satellites II and IV very near together.
	I.	B.	7·788	7·767	8·078	8·103	23 32 8	II.	7·469	·500	·510	·412	·315	·379		3 21 28	
								IV.	6·220	·111	·102	·075	·110	·132		
								III.	8·928	·947	·884	·879	·803	·822		
	330	B.	8·550	8·559	7·713	7·717	248 23 26 52	I.	8·591	·502	·530	·541	·570	·562		246 3 21 28	
	II.	A.	8·360	8·370	7·568	7·567	23 77 39	II.	8·661	·592	·612	·515	·611	·544		3 12 6	
								IV.	9·911	·980	·882	·820	·904	·784		
								III.	7·180	·209	·153	·182	·161	·149		
Oct. 26	332	A.	7·551	7·562	8·038	8·080	68 3 27 37	IV.	6·972	·730	·769	·693	·849	·725		65 41 50 69	Good plate.
	I.	B.	7·587	7·596	8·090	8·118	2 44 28	I.	8·122	7·842	7·969	7·786	7·993	7·836		40 78 58	
								III.	7·212	7·068	7·070	6·992	7·199	7·061		
	332	B.	8·143	8·138	7·662	7·659	248 2 72 40	IV.	9·234	·358	·395	·272	·367	·269		245 41 31 52	
	II.	A.	8·120	8·134	7·680	7·681	2 48 75	I.	8·108	·179	·251	·210	·167	·181		32 6	
								III.	8·982	9·003	9·131	9·030	8·984	8·958		
Oct. 28	333	A.	7·648	7·660	8·091	8·093	68 2 52 40	IV.	8·581	·712:	·732	·477	·680	·599ǁ		65 39 42 17	Good plate.
	I.	B.	7·557	·528	·952	·963	2 44 56	III.	8·399	·360ǁ	·428	·177	·406	·370		39 6 26	
								II.	8·222	·173:	·268	·050	·261	·203		
								I.	7·641	·550:	·631	·508	·638	·628		
	333	B.	7·811	7·803	8·243	8·249	248 2 44 61	IV.	7·461	·210	·140	·450	·481	·379		245 39 41 48	
	II.	A.	7·601	7·633	8·027	8·028	2 40 9	III.	7·710	·489	·459	·692	·760	·638		38 52 42	
								II.	7·891	·648	·580	·811	·920	·790		
								I.	8·499	·316	·263	·464	·542	·400		
Oct. 29	337	A.	8·591	·652	·102	·043	68 4 70 57	III.	8·331	·368ǁ	·421	·161	·520	·428		65 37 41 39	Tolerably good plate.
	I.	B.	8·547	8·533	7·969	7·959	4 12 15	II.	8·219	·260	·381	·000	·412	·310		37 38 43	
								I.	7·712	·767	·820	·589	·888	·813		
								IV.	8·310	·370	·402	·290	·541	·343		

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TABLE III.—Measures of Plates of 1891—*concluded.*

Date.	Plate, Position.	Standard Stars.				Position Angle Standard Stars.	Satellites.						Position Angle Satellites.	Remarks.
							a.	*b.*	*c.*	*d.*	*e.*	*f.*		
1904.		rev.	rev.	rev.	rev.	° ′ ″ ″	rev.	rev.	rev.	rev.	rev.	rev.	° ′ ″ ″	
Oct. 29	337	B. 7·984	7·999	8·470	8·511	248 4 48 60	III. 7·694	·657	·689	·790	·548	·562	245 37 72 33	
	II.	A. 7·954	7·973	8·502	8·503	4 29 18	II. 7·820	·695	·691	·919	·642	·683	37 10 39	
							I. 8·298	·272	·272	·305	·150	·157		
							IV. 7·679	·612	·600	·632	·532	·535		
	340	A. 8·011	·000	·339	·342	68 8 28 20	II. 7·563	·549	·625	·320	·328	·583	65 38 3 13	Images of Satellites II and IV diffused.
	I.	B. 7·778	7·762	8·028	8·041	8 7 15	I. 8·490	·408	·460	·280	·171	·457	37 50 24	
							III. 8·500	·382	·470	·268	·199	·480		
							IV. 6·682	·620	·691	·461	·464	·719		
	340	B. 8·609	8·608	7·738	7·662	248 7 70 50	II. 8·341	·310	·307	·273	·248	·340	245 37 52 52	
	II.	A. 8·247	8·213	7·287	7·292	7 56 63	I. 7·471	·460	·488	·400	·460	·385	37 35 29	
							III. 7·522	·439	·490	·413	·497	·437		
							IV. 9·208	·211	·263	·159	·230	·191		
Nov. 1	342	A. 8·761	·760	·304	·272	67 52 22 21	IV. 7·017	·030	·169	·039	·032	·092	65 18 42 29	Very good plate.
	I.	B. 8·841	·822	·387	·393	51 52 50	III. 8·020	·059	·198	·100	·080	·117	18 26 37	
							II. 8·019	·077	·192	·050	·168	·091		
							I. 7·768	·852	·999	·855	·941	·916		
	342	B. 8·348	8·345	7·882	7·850	247 52 17 7	IV. 9·180	·122	·098	·157	·138	·168	245 18 20 19	
	II.	A. 8·388	8·380	7·892	7·882	51 58 60	III. 8·182	·087	·050	·092	·150	·173	18 39 34	
							II. 8·213	·124	·101	·122	·200	·129		
							I. 8·351	·300	·272	·300	·353	·311		
	343	A. 7·902	7·900	8·300	8·281	68 2 13 21	IV. 7·429	·328	·439	·260	·398	·320	65 28 31 53	
	I.	B. 7·867	7·800	8·228	8·208	1 52 48	III. 8·943	·763	·901	·723l	·812	·808	28 23 0	
							I. 8·332	·232	·364	·272	·238	·309		
							II. 8·601	·482	·635	·507	·502	·529		
	343	B. 8·540	8·537	7·622	7·611	248 2 40 62	IV. 8·688	·661	·698	·693	·754	·823	245 28 47 48	
	II.	A. 8·420	8·448	7·532	7·531	1 32 0	III. 7·232	·249	·217	·360	·359	·379	28 13 9	
							I. 7·798	·682	·732	·712	·863	·900		
							II. 7·530	·531	·510	·547	·604	·680		
Nov. 4	349	A. 8·307	8·317	7·804	7·783	68 11 4 17	I. 7·522	·381	·371	·389	·378	·429	65 29 50 52	Tolerably good plate.
	I.	B. 8·208	8·176	7·665	7·635	11 28 20	II. 7·849	·792	·672	·721	·708	·740	29 68 57	
							IV. 8·630	·569	·478	·501	·533	·550		
	349	B. 8·644	8·673	7·781	7·770	248 10 60 49	I. 8·382	·676	·661	·518	·618	·517	245 29 57 78	
	II.	A. 8·480	8·461	7·581	7·555	11 0 3	II. 8·053	·298	·361	·187	·298	·157	29 31 8	
							IV. 7·281	·472	·575	·426	·431	·369		
Nov. 5	352	A. 7·715	·708	·982	·997	68 18 5 13	III. 8·518	·530	·568	·607	·641:	·651:	65 31 37 17	Bad plate.
	I.	B. 7·769	7·763	8·069	8·057	17 55 41	I. 8·029l	·041:	·061:	·061l	·219:	·201:	30 37 45	
	352	B. 7·492	7·511	8·050	8·072	248 17 50 59	III. 7·242	·411	·485	·461	·363	·479	245 31 3 22	
	II.	A. 7·556	7·588	8·108	8·114	17 62 43	I. 7·681	·769	·981	·939	·740	·899	30 68 48	
	353	A. 8·061	8·050	7·594	7·600	68 5 39 38	IV. 6·719	·691	·949	·741	·804	·748	65 18 40 47	
	I.	B. 8·115	8·089	7·568	7·594	5 32 32	I. 7·695	·775	·938	·802	·870	·841	18 21 9	
							II. 8·091	·121	·354	·121	·232	·211		
	353	B. 8·432	8·438	7·708	7·708	248 5 32 38	IV. 9·155	·259	·299	·288	·300	·207	245 18 30 21	
	II.	A. 8·381	8·381	7·667	7·641	5 28 12	I. 8·107	·200	·268	·248	·253	·225	18 0 9	
							II. 7·752	·812	·875	·903	·873	·821		

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TABLE III.—Measures of Plates of 1903.

Date.	Plate, Position.	Standard Stars.			Position Angle Standard Stars.	Satellites.	a.	b.	c.	d.	e.	f.	Position Angle Satellites.	Remarks.
1905.			rev.	rev.	rev.			rev.	rev.	rev.	rev.	rev.	rev.		
Jan. 7	7513	A.	8·353	·321	·469	105° 26′ 52″·79	III.	7·918	·678	·772	·724	·848	·858	64° 32′ 70″·48	
	I.	B.	8·011	·018	·269	26 50 27	I.	7·400	·252	·325	·254	·310	·392	32 42 58	
							II.	6·398	·209	·332	·221	·352	·407		
							IV.	10·563	·324	·452	·372l	·461	·517		
	7513	B.	8·450	·371	·476	285 26 54 69	III.	8·231	·002	·120	·129	·072	·127	244 32 29 90	
	II.	A.	8·131	·051	·141	26 45 28	I.	8·718	·507	·580	·581	·599	·580	32 62 5	
							II.	9·700	·539	·590	·631	·568	·597		
							IV.	5·540	·420	·351	·517	·457	·499		
Jan. 9	7510	A.	8·391	·007	·123:	105 8 124 27	III.	6·140	·241	·187	·180l	·241	·214	64 15 2 1	
	I.	B.	8·499	·140	·471	8 8 90	I.	8·784	·917	·870	·888	·899	·819	14 59 60	
							IV.	11·009	·135	·078	·061	·168	·082		
							II.	7·810	7·950	7·900	7·898	8·009	7·948		
	7510	B.	7·691	·502	·442	285 8 69 58	III.	9·920	·720	·768	·799	·853	·849	244 14 52 64	
	II.	A.	7·970	·570	·570	9 5 8	I.	7·223	·055	·130	·090	·227	·184	14 52 47	
							IV.	4·977	·862	·821	·937	·972	·934		
							II.	8·209	·093	·059	·057	·124	·129		
Jan. 10	7518	A.	8·252	·270	·208	105 38 38 28	III.	9·747	·587	·536	·729	·768	·670	64 47 20 3	
	I.	B.	8·166	·178	·110	38 22 33	II.	8·918	·717	·748	·831	·919	·820	46 48 58	
							I.	8·304	·172	·147	·342	·452	·333		
							IV.	9·640	·554	·488	·632	·772	·673		
	7518	B.	7·972	·671	·829	285 38 36 42	III.	6·281	·330	·408	·301	·322	·327	244 46 56 87	
	II.	A.	7·929	·660	·773	38 30 29	II.	7·125	·179	·240	·164	·203	·140	46 80 51	
							I.	7·730	·750	·779	·626	·680	·637		
							IV.	6·392	·441	·468	·370	·341	·300		
Jan. 17	7522	B.	7·811	·908	·893	285 40 11 11	IV.	7·562	·522	·510	·460:	·399:	·533	244 47 40 34	
	II.	A.	8·039	·089	·021	40 21 20	II.	6·799	·734	·719	·712	·658	·798	47 3 8	
							III.	5·920	·841	·851	·792	·733	·842		
							I.	8·148	·122	·153	·127	·000	·129		
	7522	A.	8·692	·352	·150	105 40 35 21	IV.	8·485	·502	·577	·469	·420	·528	64 47 2 20	
	I.	B.	8·860	·531	·422	40 0 9	II.	9·270	·319	·433	·242	·209	·239	47 30 8	
							III.	10·148	·223	·288	·149	·112	·180		
							I.	7·912	·922	·957	·918	·839	·942		
	7524	A.	8·338	·450	·298	105 31 31 29	I.	7·690	·744	·776	·849	·909	·737	64 41 3 15	
	I.	B.	8·180	·212	·028	31 33 39	II.	6·748	·840	·850	·852	·943	·750	40 42 31	
							III.	9·280	·343	·381	·398	·443	·192		
							IV.	7·378	·413	·478	·468	·498	·300		
	7524	B.	8·422	·401	·363	285 31 29 64	I.	8·359	·468	·461	·213	·298	·325	244 40 43 72	
	II.	A.	8·160	·132	·150	30 89 50	II.	9·319	·341	·361	·208	·382	·322	40 52 27	
							III.	6·812	·803	·852	·677	·770	·808		
							IV.	8·748	·747	·798	·632	·714	·729		
Jan. 18	7533	A.	8·472	·657	·494	105 43 70 51	IV.	5·192	·319	·245	·189	·249	·320	64 55 21 9	
	I.	B.	8·165	·262	·069	43 50 68	III.	9·360	·500	·451	·368l	·453	·502	54 55 67	
							II.	8·419	·550	·401	·400	·462	·543		
							I.	8·542	·655	·632	·552	·672	·706		
	7533	B.	8·608	·547	·341	285 43 58 71	IV.	10·803	·662	·668	·651	·547	·687	244 55 30 41	
	II.	A.	8·165	·190	·022	43 60 51	III.	6·597	·471	·481	·471	·329	·474	54 54 39	
							II.	7·572	·429	·538	·462	·308	·482		
							I.	7·433	·298	·260	·269	·124	·308		
	7537	A.	7·956	·769	·799	105 32 1 30	IV.	5·862	·869	·880	·768	·828l	·782	64 40 46 68	
	I.	B.	8·191	·057	·077	31 50 21	I.	7·595	·655	·651	·510	·475	·483	40 53 31	
							II.	9·340	·419	·379	·342	·258	·330		

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TABLE III.—MEASURES OF PLATES OF 1903—*continued.*

Date.	Plate, Position.	Standard Stars.			Position Angle Standard Stars.	Satellites.						Position Angle Satellites.	Remarks.
						a.	*b.*	*c.*	*d.*	*e.*	*f.*		
1905.		rev.	rev.	rev.		rev.	rev.	rev.	rev.	rev.	rev.		
Jan. 18	7537	B. 7·861	·999	·920	285 32 21 7	IV. 9·871	·947	·870	·940	·906	·751	244 41 29 11	
	II.	A. 8·148	·292	·218	31 38 52	I. 8·090	·238	·171	·202	·143	·023	40 15 32	
						II. 6·338	·538	·392	·411	·351	·252		
Jan. 19	7538	A. 8·269	·134	·215	105 54 29 8	IV. 7·269	·215	·331	·189	·223	·423	65 4 68 50	
	I.	B. 8·100	·071	·060	54 8 27	I. 8·020	·027	·130	·061	·036	·292	4 52 68	
						II. 7·060	·042	·159	·072	·042	·263		
						III. 9·657	·568	·751	·679	·607	·798		
	7538	B. 8·061	·058	·129	285 54 0 18	IV. 8·650	·532	·591	·657	·706	·632	245 4 52 40	
	II.	A. 7·864	·857	·936	54 28 8	I. 7·850	·697	·768	·741	·869	·744	5 10 22	
						II. 8·880	·720	·761	·769	·900	·768		
						III. 6·302	·132	·221	·197	·368	·227		
	7555	A. 8·063	·137	·308	105 31 52 120	IV. 10·561	·689	·645	·636	·681	·674	64 42 11 48	
	I.	B. 8·264	·341	·452	31 110 40	III. 6·111	·155	·172	·173	·112	·221	41 83 49	
						I. 9·065	·154	·124	·128	·080	·140		
						II. 8·742	·830	·808	·810	·751	·860		
	7555	B. 7·612	·479	·745	285 32 38 73	IV. 5·372	·137	·235	·177	·306	·329	244 42 20 32	
	II.	A. 7·771	·761	·951	31 82 50	III. 9·863	·681	·721	·720	·889	·861	42 28 24	
						I. 6·889	·710	·719	·730	·869	·888		
						II. 7·209	·031	·072	·056	·214	·210		
Jan. 20	7565	A. 7·414	·551	·588	105 18 20 42	III. 7·258	·392	·350	·278	·347	·339	64 28 22 27	
	I.	B. 7·709	·771	·821	18 87 58	IV. 11·028	·139	·052	·079	·100	·114	28 58 51	
	7565	B. 8·152	·150	·045	285 18 27 30	III. 8·680	·546	·554	·469	·534	·536	244 28 38 38	
	II.	A. 8·388	·281	·305	19 1 0	IV. 4·888	·830	·822	·710	·801	·808	28 38 39	
	7569	A. 8·232	·123	·142	105 32 58 45	III. 9·100	·172	·098	·313	·226	·324	64 43 58 55	Images somewhat diffused.
	I.	B. 8·206	·130	·072	33 19 29	II. 8·001	·109	·072	·217	·107	·205	43 36 39	
						I. 8·695	·810	·842	·988	·904	·923		
	7569	B. 8·140	·190	·102	285 33 9 0	III. 6·837	·668	·758	·718	·820	·694	244 43 36 61	
	II.	A. 8·090	·170	·070	33 10 22	II. 7·952	·741	·833	·823	·948	·876	43 59 33	
						I. 7·191	·061	·024	·028	·197	·110		
Jan. 24	7580	A. 7·636	·832	·761	105 41 44 48	I. 7·261	·373	·372	·299	·222	·279	64 57 1 24	
	I.	B. 7·492	·628	·639	42 50 47	III. 10·062	·192	·199	·161	·064	·194	57 33 11	
						IV. 4·746	·860	·849	·848	·769	·841		
	7580	B. 8·494	·428	·331	285 42 36 2	I. 8·589	·572	·670	·738	·736	·654	244 57 23 12	
	II.	A. 8·425	·292	·269	42 7 41	III. 5·782	·731	·848	·885	·905	·823	57 27 39	
						IV. 11·101	·058	·199	·173	·243	·100		
	7581	A. 7·870	·932	·982	105 43 40 22	IV. 5·272	·540	·443	·468	·457	·428	64 59 51 33	
	I.	B. 7·615	·704	·682	43 46 54	I. 7·442	·758	·718	·652	·658	·551	59 23 39	
						II. 6·111	·500	·410	·438	·410	·282		
						III. 8·091	·403	·351	·401	·369	·318		
	7581	B. 8·486	·488	·475	285 43 48 54	IV. 10·431	·491	·521	·562	·612	·420	244 59 39 31	
	II.	A. 8·220	·300	·287	43 41 36	I. 8·201	·290	·310	·392	·368	·261	59 39 47	
						II. 9·539	·582	·543	·622	·642	·540		
						III. 7·599	·613	·676	·621	·662	·510		
Jan. 26	7582	A. 8·287	·312	·352	105 24 21 29	IV. 7·512	·419	·262	·241	·299	·278	64 41 61 42	All images of Sat. II on réseau-line, but not troublesome, réseau-line being very diffuse.
	I.	B. 8·067	·079	·088	24 38 29	I. 8·412	·327	·225	·212	·199	·182	42 3 18	
						III. 5·678	·623	·527	·519	·477	·518		
						II. 8·962	·939	·764	·841	·840	·832		

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TABLE III.—Measures of Plates of 1903—*continued.*

Date.	Plate, Position.	Standard Stars.			Position Angle Standard Stars.	Satellites.	*a.*	*b.*	*c.*	*d.*	*e.*	*f.*	Position Angle Satellites.	Remarks.
1905.			rev.	rev.	rev.		rev.	rev.	rev.	rev.	rev.	rev.		
Jan. 26	7582	B.	8·777	·721	·716	285 24 48 27	IV.	8·350	·459	·462	·486	·489	·442	244 41 46 50	
	II.	A.	8·469	·448	·483	24 19 30	I.	7·422	·480	·539	·515	·572	·531	41 54 50	
							III.	10·188	·219	·267	·228	·312	·220		
							II.	6·891	·910	·958	·863	·918	·881		
Jan. 27	7593	A.	8·153	·158	·258	105 44 28 60	IV.	8·550	·620	·628	·632	·619	·609	65 2 28 43	
	I.	B.	8·380	·330	·474	44 71 38	III.	6·580	·564	·580	·641	·611	·625	2 72 55	
							I.	7·098	·150	·200	·268	·288	·225		
							II.	6·350	·427	·486	·497	·560	·530		
	7593	B.	7·678	·732	·409	285 44 32 32	IV.	7·214	·328	·303	·272	·270	·284	245 2 63 55	
	II.	A.	7·860	·841	·620	45 8 2	III.	9·222	·324	·325	·300	·262	·282	2 31 42	
							I.	8·678	·710	·728	·627	·572	·688		
							II.	9·409	·432	·437	·378	·334	·379		
	7596	A.	8·311	·228	·192	105 17 17 0	IV.	9·767	·820	·803	·834	·849	·858	64 36 1 19	
	I.	B.	7·918	·900	·829	17 40 54	III.	8·550	·570	·529	·643	·578	·642	36 35 17	
							II.	7·411	·432	·470	·470	·495	·509		
	7596	B.	8·319	·445	·412	285 17 30 30	IV.	6·062	·147	·138	·130	·142	·073	244 36 12 19	
	II.	A.	8·008	·107	·025	17 28 28	III.	7·272	·393	·423	·345	·388	·352	36 49 38	
							II.	8·371	·509	·520	·482	·480	·431		
Jan. 28	7601	A.	7·801	·987	·941	105 20 17 12	IV.	10·428	·294	·290	·319	·331	·387	64 38 52 70	
	I.	B.	8·047	·071	·200	19 50 54	III.	9·800	·670	·668	·728	·680	·801	38 63 51	
							II.	9·418	·302	·362	·369	·390	·449		
	7601	B.	7·781	·948	·830	285 19 71 58	IV.	5·381	·527	·542	·642	·572	·575	244 38 42 54	
	II.	A.	8·004	·052	·069	19 48 59	III.	6·018	·158	·174	·288	·182	·258	39 20 18	
							II.	6·340	·489	·482	·577	·550	·548		
Jan. 30	7604	A.	7·958	·879	·957	105 16 48 102	IV.	10·910	·781	·829	·838	·930	·910	64 36 70 59	Images rather diffused.
	I.	B.	8·170	·084	·084	17 68 18	I.	8·611	·468	·525	·509	·602	·570	36 58 76	
							III.	9·904	·740	·812	·800	·958	·872		
							II.	7·810	·699	·751	·740	·895	·800		
	7604	B.	7·618	·700	·758	285 17 32 48	IV.	4·849	·830	·824	·892	·758	·790	244 37 1 11	
	II.	A.	7·802	·969	·974	17 57 40	I.	7·138	·112	·150	·247	·102	·168	37 11 1	
							III.	5·842	·860	·839	·948	·771	·868		
							II.	7·886	·900	·948	·951	·841	·870		
	7606	A.	7·815	·628	·671	105 36 58 73	I.	8·913	·882	·910	·762	·841	·889	64 59 44 67	
	I.	B.	7·979	·901	·800	36 62 49	III.	10·210	·182	·113	·030	·128	·160	60 21 1	
							II.	8·370	·372	·369	·362	·420	·370		
							IV.	5·747	·750	·679	·694	·760	·802		
	7606	B.	7·959	·951	·926	285 36 35 78	I.	7·142	·260	·158	·038	·111	·170	244 59 79 58	Images *d*, *e* and *f* of Sat. I near edge of planet.
	II.	A.	8·037	·057	·040	36 71 32	III.	5·930	·998	·902	·845	·868	·930	59 31 57	
							II.	7·689	·771	·670	·498	·572	·653		
							IV.	10·331	·352	·359	·238	·219	·229		
Jan. 31	7608	A.	8·280	·027	·152	105 34 68 33	I.	7·162	·030	·132	·072	·109	·114	64 58 41 60	
	I.	B.	8·503	·412	·262	34 46 81	IV.	4·921	·809	·798	·928	·869	·911	58 47 30	
							III.	8·667	·572	·562	·640	·619	·655		
	7608	B.	7·970	7·976	8·133	285 35 19 30	I.	8·782	·630	·659	·699	·712	·762	244 58 7 50	
	II.	A.	8·152	·370	·398	35 9 0	IV.	10·999	·850	·977	·900	·932	·952	58 49 10	
							III.	7·300	·153	·260	·162	·233	·182		
	7634	A.	8·091	·197	·250	105 28 11 33	IV.	8·792	·774	·765	·740	·803	·863	64 55 40 59	
	I.	B.	8·002	·107	·238	28 50 22	III.	9·882	·852	·800	·777	·850	·878	55 61 51	
							II.	8·437	·329	·351	·300	·332	·367		

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TABLE III.—Measures of Plates of 1903—*continued.*

Date.	Plate, Position.	Standard Stars.			Position Angle Standard Stars.	Satellites.	*a.*	*b.*	*c.*	*d.*	*e.*	*f.*	Position Angle Satellites.	Remarks.
1905.			rev.	rev.	rev.		rev.	rev.	rev.	rev.	rev.	rev.		
Jan. 31	7634	B.	8·041	·100	·350	285° 28′ 10″ 11″	IV.	6·963	·890	·826	·899	·930	·820	244 55 43 71	
	II.	A.	7·982	8·053	8·182	28 50 56	III.	5·910	·858	·732	·859	·929	·803	55 58 29	
							II.	7·422	·318	·207	·290	·410	·285		
Feb. 1	7645	A.	8·229	·256	·255	105 40 81 57	III.	7·790	·743	·747	·717	·611	·740	65 10 51 62	
	I.	B.	8·188	·202	·067	40 52 71	II.	6·789	·732	·780	·698	·758	·798	11 24 11	
							I.	8·437	·269	·411	·332	·328	·384		
							IV.	10·123	·118	·201	·220:	·137	·173		
	7645	B.	8·711	·749	·656	285 41 8 21	III.	8·361	·341	·272	·416	·343	·379	245 11 41 33	
	II.	A.	8·448	·541	·519	40 42 27	II.	9·300	·341	·247	·428	·289	·359	10 43 49	
							I.	7·659	·798	·580	·788	·688	·732		
							IV.	5·972	·902	·799	·961	·890	·889		
	7647	A.	8·118	·301	·178	105 31 100 58	III.	9·018	·120	·029	·083	·090	·048	65 3 18 34	
	I.	B.	7·880	·929	·870	32 2 48	II.	8·860:	·921	·851:	·890:	·835:	·909:	2 30 11	
							I.	7·218	·294	·163	·298	·228	·240		
							IV.	9·063	·151	·128	·201	·141	·192		
	7647	B.	8·017	·121	·208	285 32 38 59	III.	6·720	·690	·641	·700	·743	·712	245 2 30 27	
	II.	A.	7·740	·767	·980	31 70 49	II.	6·906	·878	·802	·870	·949	·870	3 2 14	
							I.	8·561	·521	·479	·527	·556	·503		
							IV.	6·711	·630	·576	·592	·690	·617		
Feb. 3	7653	A.	8·288	·123	·144	105 28 77 49	I.	8·430	·560	·594	·659	·696	·577	64 59 41 52	
	I.	B.	8·227	·170	·100	29 16 42	II.	8·587	·649	·655	·803	·829	·680	59 69 53	
							IV.	8·001	·114	·118	·167	·264	·130		
	7653	B.	8·046	·082	·140	285 29 30 9	I.	7·150	·298	·172	·255	·200	·311	244 59 46 28	
	II.	A.	8·033	·102	·142	29 2 23	II.	7·022	·212	·103	·133	·067	·153	60 7 19	
							IV.	7·597	·808	·652	·742	·630	·722		
	7655	A.	8·199	·131	·128	105 31 53 39	II.	6·368	·467	·496	·427	·432	·451	65 3 12 12	
	I.	B.	8·263	·231	·232	31 50 68	III.	9·082	·192	·190	·171	·149	·110	3 20 20	
							IV.	7·100	·173	·181	·179	·133	·122		
	7655	B.	8·178	·181	·159	285 31 23 63	II.	9·298	·292	·269	·404	·351	·078	245 3 1 0	
	II.	A.	8·248	·168	·158	31 82 43	III.	6·600	·597	·620	·706	·657	·418	3 13 17	
							IV.	8·612	·590	·604	·661	·639	·444		
Feb. 4	7657	A.	8·491	·332	·419	105 24 47 17	III.	8·953	·869	·973:	·718	·730	·670	64 58 27 11	Satellite IV rejected.
	I.	B.	8·672	·663	·710	24 19 48	II.	8·774	·653	·707	·535	·500	·494	57 58 75	
							I.	7·686	·538	·667	·468	·458	·410		
	7657	B.	7·842	·818	·850	285 24 17 17	III.	7·081	·140	·278	·341	·271	·279	244 58 17 11	
	II.	A.	8·132	·070	·038	24 57 57	II.	7·239	·351	·471	·550	·433	·419	58 2 8	
							I.	8·340	·499	·508	·521	·498	·498		
	7660	A.	8·129	·122	·139	105 41 51 49	IV.	9·009	·292	·240	·246	·150	·260	65 17 29 27	
	I.	B.	8·342	·378	·468	41 39 43	I.	7·133	·410	·327	·301	·192	·352:	17 2 9	
							II.	9·180	·452	·378	·363	·312	·380		
							III.	6·309	·520	·451	·538	·413	·479		
	7660	B.	7·677	·472	·500	285 41 54 39	IV.	6·622	·626	·649	·631	·651	·478	245 17 0 17	
	II.	A.	7·982	·712	·732	41 40 50	I.	8·469	·547	·625	·552	·599	·432	17 29 16	
							II.	6·440	·470	·509	·498	·481	·318		
							III.	9·353	·398	·427	·390	·396	·280		
Feb. 7	7662	A.	8·509	·631	·369	105 41 23 18	III.	7·186:	·221	·287	·191	·139	·197	65 17 47 30	
	I.	B.	8·434	·413	·262	41 37 40	IV.	10·494	·553	·581	·552	·487	·599	17 27 47	
							I.	7·844	·847	·878	·802	·788	·901		
							II.	6·551	·570	·621	·602	·564	·618		

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TABLE III.—MEASURES OF PLATES OF 1903—*concluded.*

Date.	Plate, Position.	Standard Stars.			Position Angle Standard Stars.	Satellites.	*a.*	*b.*	*c.*	*d.*	*e.*	*f.*	Position Angle Satellites.	Remarks.
1905.			rev.	rev.	rev.			rev.	rev.	rev.	rev.	rev.	rev.		
Feb. 7	7662	B.	8·089	·080	·090	285 41 9 11	III.	8·891	·853	·866	·832	·850	·834	245 17 21 32	
	II.	A.	7·951	·800	·932	41 43 47	IV.	5·567	·495	·491	·434	·500	·478	17 49 30	
							I.	8·181	·213	·182	·212	·178	·221		
							II.	9·463	·485	·427	·399	·472	·461		
Feb. 10	7664	A.	7·683	·702	·841	105 45 34 10	III.	8·418	·406	·503	·340	·513	·453	65 21 53 38	Images rather diffused.
	I.	B.	8·019	·061	·211	45 38 52	II.	8·242	·211	·260	·158	·241	·279	22 11 30	
							I.	7·635	·630	·660	·632	·759	·752		
	7664	B.	7·349	·592	·609	285 45 33 28	III.	7·298	·468	·488	·519	·480	·530	245 21 39 81	
	II.	A.	7·660	·929	·919	45 49 45	II.	7·548	·610	·742	·707	·690	·709	21 99 58	
							I.	8·143	·285	·325	·257	·237	·251		
Feb. 22	7570	A.	8·210	·199	·242	105 59 9 9	I.	7·608	·782	·701	·603	·680	·521	65 13 1 4	Originally measured on Jan. 23; re-measured to settle doubts in readings.
	I.	B.	8·054	·128	·139	59 40 40	II.	9·777	·948	·921	·769	·832	·742	13 19 15	
							IV.	11·072	·210	·240	·071	·130	·093		
	7570	B.	8·679	·551	·276	285 59 21 32	I.	8·361	·190	·356	·429	·390	·390	245 12 49 49	First measures rejected.
	II.	A.	8·620	·459	·158	59 28 10	II.	6·219	·058	·190	·153	·165	·159	13 17 21	
							IV.	4·907	·857	·899	·841	·936	·848		
						MEASURES OF PLATES OF 1904.									
May 9	8123	A.	8·049	·097	·069	45 28 28 40	III.	4·850	·881	·847	·817:	·814	·767	66 57 54 30	
	I.	B.	7·272	·307	·318	28 64 50	I.	9·174	·219	·188:	·228	·180	·158	58 2 30	
							II.	7·962	8·052	8·035	8·118	8·105	8·074		
							IV.	11·670	·679	·660	·740	·700	·698		
	8123	B.	7·970	8·041	8·191	225 28 58 53	III.	11·268	·372	·268	·423	·401	·389	246 58 2 2	
	II.	A.	7·218	·288	·348	28 19 19	I.	6·935	6·981	6·927	7·010	7·032	7·002	58 8 0	
							II.	8·091	8·138	8·062	8·083	8·112	7·928		
							IV.	4·439	·461	·394	·470	·471	·290		
May 10	8124	A.	7·938	·895	·837	45 39 41 39	III.	7·289	·312	·350	·420	·318	·294	67 8 20 18	
	I.	B.	8·099	8·070	7·965	40 52 51	IV.	10·028	·079	·161	·222	·181	·140	8 31 33	
	8124	B.	8·169	·198	·192	225 40 0 1	III.	8·541	·557	·609	·442	·463	·208	247 9 8 8	
	II.	A.	8·203	·244	·280	40 17 11	IV.	5·715	·738	·763	·653	·495	·390	7 58 59	
	8126	A.	8·488	·248	·167	45 38 32 38	III.	10·059	·165	·261	·261	·338	·383	67 8 58 54	Images diffused.
	I.	B.	8·321	·213	·029	39 8 4	II.	8·130	·350	·352	·323	·467	·430	9 7 11	Star A extremely faint.
							I.	9·038	·161	·142	·240	·349	·349		
							IV.	8·118	·323	·338	·368	·534	·520		
	8126	B.	7·883	·857	·910	225 38 37 37	III.	6·663	·837	·662	·773	·738	·602	247 9 1 1	
	II.	A.	7·680	·678	·705	38 51 51	II.	8·567	·670	·572	·752	·642	·544	9 8 7	
							I.	7·662	·840	·811	·790	·731	·667		
							IV.	8·568	·600	·639	·589	·561	·492		
	8127	A.	7·940	·848	·937	45 45 21 17	I.	6·923	·862	·990	·982	·983	·893	67 12 40 41	Images rather diffused.
	I.	B.	8·228	·100	·150	45 49 50	II.	10·013	·002	·148	·140	·104	·050	13 39 38	
							IV.	6·220	·352	·423	·366	·335	·378		
	8127	B.	8·000	·098	·053	225 45 38 38	I.	9·082	·132	·030	·078	·101	·092	247 13 20 22	
	II.	A.	8·203	·388	·276	45 30 30	II.	5·972	·991	·993	·972	·960	·959	13 21 18	
							IV.	9·728	·705	·682	·692	·692	·633		

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TABLE III.—Measures of Plates of 1904—*continued.*

Date.	Plate, Position.	Standard Stars.			Position Angle Standard Stars.	Satellites.						Position Angle Satellites.	Remarks.
						a.	*b.*	*c.*	*d.*	*e.*	*f.*		
1905.		rev.	rev.	rev.	° ′ ″ ″	rev.	rev.	rev.	rev.	rev.	rev.	° ′ ″ ″	
May 11	8130	A. 8·253	·222	·209	45 27 12 17	II. 9·702	·607	·551	·588	·449	·471	66 49 48 47	Good plate.
	I.	B. 8·146	·189	·197	27 34 33	IV. 2·432	·321	·271	·361	·182	·188	50 18 17	
						I. 6·940	·911	·833	·928	·844	·794		
						III. 4·470	·430	·423	·452	·277	·325		
	8130	B. 8·075	·071	·107	225 27 23 31	II. 5·899	·970	·883	·888	·962	·950	246 49 49 50	
	II.	A. 7·970	8·019	7·911	27 17 10	IV. 13·158	·232	·193	·159	·267	·251	50 2 2	
						I. 8·608	·672	·629	·551	·639	·570		
						III. 11·089	·112	·050	·052	·158	·110		
May 12	8137	A. 7·840	·850	·898	45 39 24 28	IV. 3·660	·545	·534	·540	·560	·529	66 58 19 22	Images rather diffused.
	I.	B. 8·162	·222	·163	39 38 38	III. 6·938	·751	·866	·827	·813	·790	58 45 40	
						I. 6·661	·520	·548	·596	·568	·633		
						II. 5·590	·500	·502	·537	·600	·567		
	8137	B. 8·059	·160	·208	225 39 17 17	IV. 12·212	·235	·382	·369	·369	·209	246 58 20 12	
	II.	A. 8·358	·547	·470	39 52 49	III. 8·927	9·033	9·191	9·138	9·077	8·964	58 38 39	
						I. 9·181	·273	·465	·351	·264	·178		
						II. 10·224	·293	·433	·413	·331	·231		
	8141	A. 7·506	·449	·290	45 43 16 12	IV. 5·233	·264	·283	·141	·056	·022	66 53 3 8	Last three exposures good; first three very diffused. Standard stars good.
	I.	B. 7·660	·608	·448	43 30 31	III. 9·810	·847	·889	·777	·690	·613	53 32 31	
						II. 7·774	·781	·808	·733	·601	·547		
	8141	B. 8·447	·509	·352	225 43 11 9	IV. 10·720	·878	·820	·780	·778	·651	246 52 61 59	
	II.	B. 8·471	·553	·344	43 9 7	III. 6·131	·268	·192	·282	·186	·165	53 31 32	
						II. 8·193	·351	·313	·282	·277	·212		
May 13	8142	A. 7·629	·680	·753	45 41 60 59	IV. 6·992	7·191:	7·201	6·870	6·912	6·747	66 57 51 58	First three exposures very good; last three extremely faint.
	I.	B. 7·803	7·822	8·052	42 22 28	I. 6·870	·785	·959	·705	·796	·496	58 39 38	
						III. 11·322	·368	·521	·348	·402	·162		
	8142	B. 8·325	·598	·550	225 42 7 5	IV. 8·771	·512	·469	·868	·731	·699	246 58 22 23	
	II.	A. 8·590	·698	·679	42 19 20	I. 8·925	·932	·721	·986	·869	·901	58 22 29	
						III. 4·458	·377	·282	·369	·320	·327		
May 15	8143	A. 7·843	·801	·718	45 52 11 17	IV. 8·910	·762	·832		·821	·879	67 7 14 10	
	I.	B. 8·127	8·118	7·948	52 48 48	I. 8·568	·410	·463		·572	·633	6 37 40	
						II. 6·730	·663	·723		·777	·860		
						III. 10·249	·122	·201		·220	·328		
	8143	B. 7·867	·873	·789	225 52 31 31	IV. 6·870	7·097	7·218		7·205	7·120	247 6 42 38	
	II.	A. 8·122	·153	·054	52 23 27	I. 7·242	·448	·540		·451	·370	7 20 14	
						II. 9·052	·223	·339		·272	·100		
						III. 5·633	·720	·862		·851	·705		
May 16	8144	A. 8·072	·203	·181	45 42 3 9	IV. 10·617:	·578	·630	·421	·639	·578	66 55 28 22	Images good, except last exposure.
	I.	B. 8·094	·219	·142	41 54 51	I. 7·309:	·314	·389	·153	·341	·349	55 8 10	
						II. 5·798:	·780	·832	·578	·823	·811		
						III. 7·490	·484	·542	·402	·628	·621		
	8144	B. 7·951	·999	·932	225 41 59 54	IV. 5·069	·213	·316	·410	·318	·327	246 56 13 18	
	II.	A. 7·945	·967	·879	42 2 5	I. 8·316	·469	·512	·641	·561	·572	56 39 35	
						II. 9·872	10·020	10·051	10·222	10·078	10·118		
						III. 8·200	·251	·357	·363	·278	·302		
	8145	A. 8·002	·069	·085	45 27 37 31	IV. 12·158	·141	·168	·182	·108	·002	66 40 45 45	
	I.	B. 7·617	·812	·731	27 32 32	II. 9·467	·482	·512	·539	·379	·334	40 52 54	
						I. 7·873	·870	·834	·958	·804	·792		
						III. 5·122	·150	·142	·229	·042	·078		

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TABLE III.—Measures of Plates of 1904—*continued.*

Date	Plate, Position.	Standard Stars.			Position Angle Standard Stars.	Satellites.	a.	b.	c.	d.	e.	f.	Position Angle Satellites.	Remarks.
1905.			rev.	rev.	rev.		rev.	rev.	rev.	rev.	rev.	rev.		
May 16	8145	B.	8·099	·125	·141	225 27 33 40	IV.	3·590	·560	·592	·519	·561	·630	246 40 36 34	
	II.	A.	7·761	·873	·836	27 42 39	II.	6·263	·182	·219	·203	·227	·268	41 2 3	
							I.	7·857	·889	·824	·811	·848	·788		
							III.	10·582	·602	·558	·482	·579	·537		
May 17	8146	A.	7·693	·627	·572	45 30 42 42	IV.	12·679	·884	·963	·710	·977	·781	66 42 37 34	
	I.	B.	7·698	·627	·632	30 30 24	I.	8·347	·409	·589	·307	·533	·323	42 31 34	
							III.	4·397	·528	·587	·363	·646	·385		
							II.	9·062	·128	·247	·004	·291	·096		
	8146	B.	7·972	8·081	8·140	225 29 59 61	IV.	2·671	·964	·837	·779	·873	·862	246 42 42 41	
	II.	A.	8·031	·048	·152	30 39 34	I.	7·098	·395	·241	·219	·241	·330	42 37 32	
							III.	11·007	·362	·181	·143	·178	·256		
							II.	6·409	·718	·572	·542	·562	·587		
	8148	A.	7·963	·880	·809	45 34 41 38	III.	11·142	·180	·056	·111	·038	·082	66 43 17 13	Very good plate Sat. I too near to planet.
	I.	B.	7·850	·782	·655	34 17 20	IV.	10·952	·931	·861	·893	·978	·929	43 8 1	
	8148	B.	7·957	·780	·758	225 34 28 28	III.	4·589	·631	·613	·477	·421	·403	246 43 7 4	
	II.	A.	7·757	·691	·660	34 39 37	IV.	4·731	·766	·814	·642	·540	·529	43 9 13	
May 18	8154	A.	8·109	·180	·111	45 35 31 32	I.	8·866	·811	·848	·862	·794	·798	66 43 17 13	Very good plate.
	I.	B.	8·028	·069	·012	35 28 27	III.	10·480	·447	·491	·520	·391	·408	43 11 12	
							II.	7·281	·230	·238	·302	·208	·218		
							IV.	9·227	·202	·231	·272	·180	·217		
	8154	B.	6·879	·920	·790	225 35 33 32	I.	6·887	·901	·820	·832	·801	·808	246 43 17 18	
	II.	A.	6·825	·873	·753	35 33 37	III.	5·241	·270	·176	·168	·160	·167	43 7 3	
							II.	8·481	·508	·439	·404	·345	·401		
							IV.	6·512	·512	·441	·423	·389	·395		
May 19	8158	A.	8·121	·157	·226	45 8 11 11	I.	7·171	·210	·222	·278	·182	·181	66 14 52 52	Images very diffused.
	I.	B.	8·022	·089	·174	8 20 19	II.	5·828	·883	·813	·921	·898	·880	15 2 7	
							III.	8·154	·218	·241	·379	·270	·331		
							IV.	7·568	·621	·628	·661	·691	·680		
	8158	B.	8·049	·061	·040	225 8 21 21	I.	8·698	·968	·884	·890	·796	·769	246 14 41 42	
	II.	A.	7·982	·952	·943	8 21 21	II.	10·082	·341	·240	·232	·080	·060	15 9 8	
							III.	7·747	·953	·838	·875	·704	·701		
							IV.	8·378	·472	·543	·522	·330	·318		
	8159	A.	7·925	·938	·878	45 30 45 43	II.	8·848	·651	·747	·658	·682	·673	66 36 30 32	Good plate.
	I.	B.	7·861	·732	·749	30 54 56	I.	7·961	·734	·894	·869	·889	·888	36 51 51	
							III.	5·219	·042	·179	·107	·135	·114		
							IV.	5·451	·279	·351	·321	·351	·376		
	8159	B.	7·997	8·113	8·108	225 30 60 57	II.	6·877	·909	·893	·978	·831	·763	246 36 19 18	
	II.	A.	7·861	·879	·938	30 42 57	I.	7·793	·809	·694	·730	·609	·511	36 57 56	
							III.	10·518	·528	·433	·552	·349	·312		
							IV.	10·241	·271	·258	·261	·141	·079		
May 20	8167	A.	7·852	·939	·789	45 25 0 0	I.	8·107	8·190	8·196	7·509	7·540	7·467	66 29 58 57	Pretty good plate.
	I.	B.	7·794	·820	·800	24 54 54	III.	4·525	4·563	4·559	4·028	4·048	3·991	30 1 2	
							II.	9·190	9·261	9·353	8·982	8·907	8·912		
							IV.	3·751	·794	·827	·773	·638	·669		
	8167	B.	8·406	·553	·563	225 24 61 58	I.	7·535	7·447	7·427	8·150	8·043	8·208	246 29 48 45	
	II.	A.	8·372	·422	·518	24 51 53	III.	11·130	·150	·108	·692	·620	·708	30 13 17	
							II.	6·427	·431	·363	·754	·680	·790		
							IV.	11·862	·825	·868	·918	·961	·968		

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TABLE III.—Measures of Plates of 1904—*continued.*

Date.	Plate, Position.	Standard Stars.			Position Angle Standard Stars.	Satellites.						Position Angle Satellites.	Remarks.
						a.	*b.*	*c.*	*d.*	*e.*	*f.*		
1905.		rev.	rev.	rev.	° ′ ″ ″	rev.	rev.	rev.	rev.	rev.	rev.	° ′ ″ ″	
May 27	8181	A. 7·571	·561	·621	45 25 50 50	IV. 5·850	·623	·661	·694	·736	·771	66 26 60 59	Tolerably good
	I.	B. 7·529	·499	·529	26 30 30	I. 6·869	·689	·719	·721	·731	·771	27 1 1	plate.
						III. 8·641	·432	·440	·510	·508	·553		
	8181	B. 8·050	·038	·085	225 26 20 18	IV. 10·182	·451	·320	·323	·292	·428	246 27 8 8	
	II.	A. 7·988	8·022	8·080	26 10 12	I. 9·169	·425	·262	·290	·252	·388	26 52 50	
						III. 7·398	·682	·539	·442	·490	·609		
	8182	A. 7·852	·634	·649	45 28 20 18	IV. 7·600	·419	·421	·350	·270	·330	66 28 11 4	Good plate.
	I.	B. 7·899	·829	·865	28 22 19	II. 8·553	·421	·406	·319	·207	·291	28 21 26	
						I. 8·569	·461	·402	·431	·271	·411		
						III. 5·857	·729	·730	·759	·558	·662		
	8182	B. 7·751	·844	·943	225 28 21 14	IV. 8·088	·201	·189	·328	·196	·237	246 28 5 3	
	II.	A. 8·000	·038	·067	28 24 22	II. 7·159	·210	·188	·299	·272	·237	28 22 21	
						I. 7·126	·162	·243	·210	·228	·157		
						III. 9·818	·869	·877	·897	·917	·842		
May 29	8190	A. 7·537	·649	·821	45 31 52 51	IV. 9·397	·402	·417	·294	·275	·151	66 31 10 8	Good plate, but
	I.	B. 7·637	·775	·845	32 8 4	I. 7·671	·652	·693	·520	·441	·364	31 13 14	some images elongated, especially star A.
						II. 9·682	·679	·691	·668	·588	·521		
	8190	B. 8·341	·199	·170	225 32 1 0	IV. 6·568	·649	·689	·640	·692	·703	246 30 59 59	
	II.	A. 8·331	·320	·249	32 1 2	I. 8·311	·411	·412	·430	·487	·461	31 18 13	
						II. 6·290	·399	·362	·290	·348	·332		
	8192	A. 7·966	·915	·968	45 36 29 28	IV. 11·120	·076	·039	·119	·089	·212	65 35 8 9	Good plate.
	I.	B. 8·060	·022	·009	36 28 29	III. 5·831	·808	·743	·859l	·827	·949	35 0 0	
						I. 7·789	·784	·694	·863	·895	·978		
						II. 6·547	·543	·468	·551	·634	·701		
	8192	B. 7·678	·706	·562	225 36 16 19	IV. 4·861	·841	·838	·868	·840	·889	246 35 7 2	
	II.	A. 7·717	·749	·613	36 34 29	III. 10·148	·101	·092	·182	·170	·150	35 10 11	
						I. 8·162	·145	·162	·108	·053	·092		
						II. 9·440	·410	·410	·383	·353	343		
May 30	8198	A. 7·423	·467	·696	45 33 31 32	III. 8·487	·408	·480	·520	·389	·458	66 31 24 22	Pretty good plate.
	I.	B. 7·338	·338	·478	33 59 57	IV. 12·131	·075	·171	·212	·127	·111	31 31 31	
						II. 6·748	·678	·712	·761	·711	·702		
						I. 8·738	·692	·729	·802	·712	·7 2		
	8198	B. 8·312	·222	·333	225 33 39 39	III. 7·395	·392	·368	·460	·547	·489	246 31 23 27	
	II.	A. 8·072	·130	·187	33 50 50	IV. 3·758	718	·688	·692	·781	·771	31 29 26	
						II. 9·179	·117	·111	·152	·283	·228		
						I. 7·178	·131	·092	·129	·287	·201		
	8200	A. 8·182	·268	·290	45 38 2 0	III. 10·842	·700	·741	·724	·797	·938	66 34 41 42	Bad plate; images
	I.	B. 8·351	·500	·500	37 55 57	IV. 12·388	·206	·318	·273	·373	·584	34 47 42	diffused.
						II. 8·213	·177	·200	·207	·331	·394		
	8200	B. 8·048	·389	·315	225 37 54 58	III. 4·947	·835	·812	·811	·798	·909	246 34 41 39	
	II.	A. 8·219	·462	·417	37 52 51	IV. 3·347	·222	·160	·253	·290	·224	34 48 49	
						II. 7·567	·420	·302	·292	·345	·411		
May 31	8209	A. 7·680	·617	·673	45 35 51 49	I. 6·772	·491	·523	·463	·421	·460	66 32 10 8	Good plate on the
	I.	B. 7·715	·649	·694	36 2 4	III. 8·921	·631	·708	·668	·607	·672	31 54 52	whole. Some images somewhat misformed.
						IV. 11·458l	·301	·401	·390	·236	·356		
	8209	B. 7·957	·934	·831	225 35 59 58	I. 9·211	·383	·155	·317	·329	·380	246 32 7 3	
	II.	A. 7·963	·969	·807	35 58 52	III. 7·071	·227	·017	·082	·121	·140	31 49 53	
						IV. 4·458	·634	·360	·439	·441	·489		

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TABLE III.—Measures of Plates of 1904—*concluded.*

Date.	Plate, Position.	Standard Stars.			Position Angle Standard Stars.	Satellites.	*a.*	*b.*	*c.*	*d.*	*e.*	*f.*	Position Angle Satellites.	Remarks.
1905.			rev.	rev.	rev.			rev.	rev.	rev.	rev.	rev.	rev.		
May 31	8213	A.	7·529	·452	·363	45 41′ 23″ 21″	I.	7·412	·147	·254	·199	·248	·052	66° 36′ 20″ 21″	First three exposures faint and diffused, especially of Sat. IV.
	I.	B.	7·386	·409	·274	41 30 34	III.	4·837	·587	·717	·751	·750	·509	36 32 31	
							II.	9·828	·600	·712	·668	·761	·482		
							IV.	8·691	·477	·600	·668:	·470:	·512:		
	8213	B.	7·949	·878	·818	225 41 38 38	I.	7·397	·400	·547	·664	·612	·784	246 36 21 20	
	II.	A.	7·819	·752	·663	41 35 34	III.	9·971	10·007	10·138	10·151	10·031	10·368	36 31 33	
							II.	4·950	4·981	5·132	5·168	5·050	5·347		
							IV.	6·110	·097	·217	·263	·245	·310		
June 2	8216	A.	8·003	·090	·175	45 44 20 24	III.	5·573	·638	·664	·566	·542	·623	66 39 17 11	Good plate.
	I.	B.	7·988	8·092	·139	44 55 52	I.	8·022	·102	·139	·117	·078	·172	39 10 12	
							II.	6·763	·881	·908	·850	·803	·932		
							IV.	7·011	·182	·170	·182	·058	·164		
	8216	B.	8·420	·320	·218	225 44 44 42	III.	10·281	·435	·452	·380	·458	·510	246 39 29 30	
	II.	A.	8·438	·370	·190	44 48 52	I.	7·777	·940	·970	·844	·947	·911	39 19 14	
							II.	9·080	·190	·232	·090	·194	·202		
							IV.	8·823	·880	·929	·798	·958	·879		
	8222	A.	7·837	·819	·837	45 29 12 12	III.	7·940	·689	·774	·691	·841	·871	66 23 37 32	Pretty good plate.
	I.	B.	7·746	·780	·826	29 48 48	II.	6·438	·158	·251	·148	·299	·312	23 39 41	
							I.	8·402	·083	·170	·014	·239	·269		
							IV.	5·329	·082	·190	·158	·290	·338		
	8222	B.	7·782	·833	·533	225 29 39 42	III.	7·862	·952	·879	·839	·768	·798	246 23 32 33	
	II.	A.	7·741	·767	·498	29 31 28	II.	9·367	·500	·401	·386	·287	·340	23 32 30	
							I.	7·382	·563	·488	·482	·411	·332		
							IV.	10·502	·506	·477	·358	·309	·327		
June 6	8226	A.	7·758	·749	·902	45 42 43 39	IV.	6·570:	·697	·836	·731	·622	·618	66 35 8 3	Good plate.
	I.	B.	7·701	·670	·854	42 51 53	III.	5·432	·502	·632	·473	·404	·411	34 57 56	
							I.	8·183	·229	·452	·349	·240	·243		
							II.	7·029	·071	·272	·189	·078	·101		
	8226	B.	8·064	·164	·310	225 42 48 47	IV.	9·140	·113	·073	·152	·092	·210	246 35 9 3	
	II.	A.	8·002	·099	·241	42 48 43	III.	10·293	·321	·260	·361	·257	·388	34 58 60	
							I.	7·529	·550	·441	·532	·395	·564		
							II.	8·665	·702	·641	·672	·580	·748		
June 7	8228	A.	7·571	·562	·471	45 37 31 30	IV.	12·056	·259	·143	·067	·150	·110	66 29 11 11	Pretty good plate.
	I.	B.	7·648	·622	·573	37 26 29	II.	6·109	·321	·187	·119	·190	·158	29 17 15	
							III.	9·614	·869	·740	·691	·762	·721		
	8228	B.	7·998	·997	·923	225 37 29 28	IV.	3·948	·798	·920	·850	·841	·802	246 29 12 11	
	II.	A.	8·141	·072	·018	37 27 22	II.	9·850	·743	·860	·778	·831	·710	29 13 18	
							III.	6·345	·183	·303	·227	·265	·182		

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CHAPTER IV.

Probable Errors of Measures of Satellites.

The comparison of the pointings on the different images and in the two positions of the plate allows a determination of the *p. e.* of measurement to be made. I will distinguish between that part of the error which has two different and independent values in the two pointings on the same image in the two positions of the plate, and that part which is the same in the two positions. The first kind of error I call the "error of pointing" and I denote its mean (or probable, as the case may be) value by ϵ. The other kind is the "error of the image," and is denoted by δ. I further put for brevity's sake :—

$$\zeta^2 = \epsilon^2 + 2\delta^2.$$

The splitting up of the error in this way is evidently always justifiable and does not involve any assumptions. Nor has it anything to do with the much-debated question of the elimination of errors by observing in two positions, which question I do not propose to discuss.

I distinguish the different images by combinations of letters and numerals, the letters a to f referring to the different exposures, as in the preceding articles, and the numerals 1 to 4 to the different satellites. The numerals therefore refer to the same images as the capital letters A to D in Chapter III., but not necessarily in the same order.* In deriving the probable errors, I neglect the effect of the motion of the satellite in latitude during the interval of time between the different exposures, or rather I include it in the error δ. Let there be p satellites and q images of each on a plate. The value of p is 2, 3 or 4; that of q, 5 or 6. Let the measured co-ordinate of the image $(a1)$ in position I of the plate be denoted by y'_{a1}, and the same co-ordinate measured in position II by y''_{a1}, both with their signs as measured, so that approximately $y'_{a1} = -y''_{a1}$. We then form the following means and differences :—

$$y'_1 = \frac{1}{q}(y'_{a1} + y'_{b1} + \ldots), \qquad \text{and similarly } y'_2 \ldots y''_1, y''_2 \ldots$$

$$y'_a = \frac{1}{p}(y'_{a1} + y'_{a2} + \ldots), \qquad \text{and similarly } y'_b \ldots y''_a, y''_b \ldots$$

$$y'_0 = \frac{1}{p}(y'_1 + y'_2 + \ldots) = \frac{1}{q}(y'_a + y'_b + \ldots), \qquad \text{and similarly } y''_0$$

$$n'_{a1} = y'_{a1} - \{y'_a + y'_1 - y'_0\}, \qquad \text{and similarly } n''_{a1}$$

$$S_{a1} = n'_{a1} + n''_{a1} \qquad D_{a1} = n'_{a1} - n''_{a1}$$

* Throughout the rest of this paper the quantities referring to the satellites I to IV will be distinguished by subscript figures 1 to 4. When any one satellite is referred to, the subscript is either dropped or replaced by the letter i.

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and similarly for the other combinations of letters and figures. Then assuming, as is usual, that the square of the error of the sum or difference of two quantities affected by *mutually independent* errors is the sum of the squares of these errors, it is easy to verify that we have :—

$$\Sigma D^2 = 2(p-1)(q-1)\zeta^2, \qquad \Sigma S^2 = 2(p-1)q\epsilon^2$$

where the sums extend over all combinations of letters and figures. If we suppose the errors ϵ and δ to be different for the different satellites, and if we still denote their mean values by ϵ and ζ, thus :—

$$\epsilon^2 = \frac{1}{p}(\epsilon_1^2+\epsilon_2^2+\ldots), \qquad \delta^2 = \frac{1}{p}(\delta_1^2+\delta_2^2+\ldots),$$

then we find for the sums of the squares of *all* the quantities S and D again the same equations :—

$$\Sigma D^2 = 2(p-1)(q-1)\zeta^2, \qquad \Sigma S^2 = 2(p-1)q\epsilon^2$$

and for the sums for *one satellite only* (*e.g.*, the first) :—

$$\Sigma_1 D^2 = 2(q-1)\left\{\frac{p-2}{p}\zeta_1^2+\frac{1}{p}\zeta^2\right\}, \qquad \Sigma_1 S^2 = 2q\left\{\frac{p-2}{p}\epsilon_1^2+\frac{1}{p}\epsilon^2\right\}$$

where the sums Σ_1 extend only over the combinations involving the figure 1. We have, of course, similar formulæ for the other satellites.

These formulæ have been applied to the measures made on the plates of 1891 and 1903. In the latter case, however, the images were divided into two groups of three exposures each, viz.: those taken before, and those taken after the trail, and the means y'_1, y'_2, y''_1, etc., were taken for the two groups separately. I shall not give these discussions in detail, but only mention the results. The formulæ, as they are written, give, of course, mean errors expressed in revolutions of the screw. I have, however, reduced these to *probable* errors expressed in seconds of arc.

From the plates of 1891 I found for the mean of all not rejected plates :—

$$\epsilon = \pm 0''\cdot 106 \qquad \delta = \pm 0''\cdot 086 \qquad \zeta = \pm 0''\cdot 161.$$

Weights were assigned to the plates in accordance with the values of ζ and corresponding to the following probable errors for weight unity :—

$$\epsilon_0 = \pm 0''\cdot 084 \qquad \delta_0 = \pm 0''\cdot 068 \qquad \zeta_0 = \pm 0''\cdot 127.$$

[This corresponds to the value 0·0010 of the square of the *mean* error ζ_0 expressed in revolutions of the screw.]

From the plates of 1903 I found for the mean of all plates :—

$$\epsilon = \pm 0''\cdot 083 \qquad \delta = \pm 0''\cdot 065 \qquad \zeta = \pm 0''\cdot 124$$

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ζ is the probable error of one distance between two objects measured on the plate, (*one* image of each, *two* positions of the plate). On the parallax and proper-motion plates taken at the observatory at Helsingfors with a similar instrument, and measured at the Astronomical Laboratory at Groningen, we found (see *Publ. of the Astr. Lab. at Groningen*, Nos. 10 and 14):—

$$\zeta = \pm 0''{\cdot}130.$$

The accuracy of the pointings on all these plates is thus practically the same.

The values of ζ_i for the different satellites were also determined. The inverse ratios of their squares, *i.e.* the relative weights of the satellites, were found to be :—

	Sat. I.	Sat. II.	Sat. III.	Sat. IV.
Plates of 1891	1	1·25	1·7	0·75
Plates of 1903	1	1·1	1·5	0·75

These weights were, however, not used, but the observations of the four satellites were supposed to be of the same accuracy, as was also done in the reduction of the heliometer observations. The introduction of different weights would have complicated the reductions very considerably, without in all probability sensibly affecting the resulting values of the unknowns. The relative weights vary largely from plate to plate, and I have thought it better not to prolong the labour of reduction and increase the possibilities of mistakes for the sake of a refinement of very doubtful value.

The quantities which are used in the subsequent discussions are the co-ordinates of the satellites referred to the mean of all the satellites occurring on the plate as origin, *i.e.*, in the notation of this chapter :—

$$y_i = \tfrac{1}{2}\{(y_i' - y_o') - (y_i'' - y_o'')\},$$

where i is any one of the subscripts 1 to 4. The square of the error of this quantity is—

$$\rho_y^2 = \frac{p-1}{2pq}\zeta^2.$$

If we wished to assign different weights to the different satellites, we would have for this error :—

$$\rho_{yi}^2 = \frac{1}{2q}\left\{\frac{p-2}{p}\zeta_i^2 + \frac{1}{p}\zeta^2\right\}.$$

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CHAPTER V.

Refraction and Scale-Value.

The corrections to the measured co-ordinates y_i for orientation, refraction and scale-value, are all of the linear form—

$$\Delta y = Ax + By.$$

It follows from the linear form that the co-ordinates x and y in the right-hand member may be referred to any origin, provided we subtract from the corrections so computed their mean for all the satellites in order to get the correction to the observed quantities y_i. If we refer the co-ordinates x and y to the mean of all the satellites as origin, the mean of the corrections becomes zero, and we get at once the required correction, with the check that their sum must be zero. This is what has been done, *i.e.* for x I have used $x_i - x_0$ (where x_0 is the mean of the values of x_i for the satellites occurring on the plate) and for y the observed value y_i, derived as explained in the preceding chapter.

The coefficients for the total correction Δy are :—

$$A = A' + \sin \Delta P \qquad B = B' + \beta.$$

The term $\sin \Delta P$ will be explained in the next chapter.

A′ and B′ are the coefficients for refraction. They are computed by the following formulæ, which are easily verified :—

$$A' = 0{\cdot}000280 \{\tan^2 \zeta \sin (p-q) \cos (p-q) + \tan \zeta \tan \delta \sin q\}$$
$$B' = 0{\cdot}000280 \{1 + \tan^2 \zeta \sin^2 (p-q)\}.$$

In these formulæ p is the position-angle of the axis of x, viz. $p = 90° + P$; q is the parallactic angle and ζ the zenith-distance. The total influence of refraction is so small that the corrections for barometer and temperature may be neglected.

The scale-value is determined from the measured distances of the standard stars. In measuring these distances, the images are projected on the scale of the instrument. The total correction for scale-value to be applied to the co-ordinates y, which were measured by the micrometer-screw, is therefore the sum of two corrections, viz.:—the

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correction for run, which is needed to convert revolutions of the screw into millimeters of the scale, and the correction for scale-value proper, which converts millimeters into minutes of arc. If the latter is denoted by B_1, so that $1^{mm} = (1 + B_1)$ minutes of arc; and the former by R, so that 10 revolutions = (1 + R) millimeters, then we have

$$\beta = B_1 + R,$$

and 10 revolutions = $(1 + \beta)$ minutes of arc. Since the correction β always occurs multiplied by the co-ordinates y, which are small quantities, it was not determined for every plate. For the plates of 1891 it was assumed constant, and equal to $-0{\cdot}0100$, which is the mean value derived from those plates on which differences of R.A. and declination were measured (see Chapter II.). The individual values were:—

No. of Plate.	β.
295	$-0{\cdot}0101$
307	·0100
313	·0100
314	·0099
315	·0102
316	·0100
317	$-0{\cdot}0100$

The coefficient B′ was computed for every plate. The sums $B = B' - 0{\cdot}0100$ are given in Table IV.

For the plates of 1904 the scale-value was determined for every plate for the purpose of another investigation, which will soon be published in the *Publications of the Astronomical Laboratory at Groningen*. The resulting values of β are given in Table IV, together with the values of B′ and $B = B' + \beta$.*

For the plates of 1903 β was assumed constant and equal to $-0{\cdot}0108$, which is the mean value for 1904. Since also B′ is constant throughout this series ($B' = +0{\cdot}0003$) the value of B for all the plates is constant, viz.: $B = -0{\cdot}0105$.

* Since this paper was written, a mistake has been found in the computation of the corrections for refraction of the standard stars for 1904, in consequence of which the adopted scale-value is also erroneous. All the values of β, and therefore of B, given in Table IV, must be diminished by 0·0003. The effect of this correction on the co-ordinates y is negligible, and can safely be ignored.

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CHAPTER VI.

Orientation from Standard Stars.

In order to derive the correction for orientation, let us imagine a system of axes of co-ordinates ξ, η, fixed to the microscope, of which the ξ-axis is parallel to the cylinder, and the η-axis to the direction of measurement.* The positive ξ-axis corresponds to large readings of the vertical scale of the instrument, and increasing η's correspond to increasing readings of the micrometer-screw. When the projection on the plate of the declination-circle through its centre coincides with the positive η-axis, then increasing ξ's correspond to increasing Right-ascensions. Let the reading of the position-circle for this position of the plate be—

$$90^\circ - \mathrm{I}.$$

The small angle I so defined is the index-correction of the position-circle, which, of course, varies from plate to plate. The system x, y of co-ordinate axes defined in Chapter II., which is fixed to the plate, coincides with the system ξ, η when the position-angle (corrected) is $90^\circ + \mathrm{P}$. The position-angle during the measures of the satellites is, however, $\mathrm{P}_0 + \mathrm{I}$, where P_0 is the observed instrumental position-angle (operations 3 and 6 of Chapter III.).

The measured co-ordinates are referred to the system ξ, η. The tabular places, with which they must be compared, are referred to the system x, y. We must thus apply to the measured co-ordinates a correction :—

$$\Delta y = x \sin \Delta \mathrm{P},$$

where

$$\Delta \mathrm{P} = \mathrm{P} + 90^\circ - (\mathrm{P}_0 + \mathrm{I}).$$

The index-correction I must be determined from the standard stars or from the trails. Let P_s be the apparent position-angle of the great circle joining the standard

* This supposes that the direction of measurement is truly perpendicular to the cylinder. Any defect in this adjustment is equivalent to a change of scale-value, and is, by the way in which the scale-value was determined, included in the constant β.

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stars, at the middle point and in the direction from A to B. In this chapter I will suppose that this middle point (as well as the mean of the six exposures of Jupiter) coincides with the centre of the plate. The effect of non-fulfilment of these conditions will be considered in the appendix. Let the instrumental position-angles of the plate during the measures of the standard stars be P'_{AB} and P''_{AB} in the two positions of the plate respectively (operations 2 and 7 of Chapter III.). In the operations 1 and 8, the co-ordinates η of the stars A and B are measured. Let the observed differences of these co-ordinates in the two positions be $(A-B)'$ and $(B-A)''$. The correction required to reduce the measured quantity to its tabular value is in position I. :—

$$(x_A - x_B) \sin [P_S - (P'_{AB} + I)].$$

The tabular value is zero : this correction must therefore be equal to $-(A-B)'$. Further, if s is the distance between the standard stars (or rather twice the tangent of half the distance) we have $x_A - x_B = -s$; therefore :

$$P_S = P'_{AB} + \frac{(A-B)'}{s \sin 1''} + I.$$

The angle

$$P'_{AB} + \frac{(A-B)'}{s \sin 1''}$$

is the instrumental position-angle for which the line joining the standard stars is parallel to the axis of the cylinder. In position II this angle is

$$P''_{AB} + \frac{(B-A)''}{s \sin 1''}.$$

These two angles must differ by 180° exactly apart from errors of observation. Therefore, if we put

$$P_{AB} = \tfrac{1}{2}\{P'_{AB} + P''_{AB} - 180°\}$$

$$\Delta = \tfrac{1}{2}\{(A-B)' + (B-A)''\},$$

we have finally

$$I = P_S - \left\{P_{AB} + \frac{\Delta}{s \sin 1''}\right\}.$$

The distance s must be expressed in the same unit as Δ, *i.e.* in revolutions of the screw. We then get I in seconds of arc.

The adopted position-angles and distances of the stars used in 1891 are those given in my dissertation, page 8, viz. (equinox 1891·0 ; epoch 1891·75) :—

$$s = 6705''{\cdot}36 \pm 0''{\cdot}065. \qquad p = 68°\,4'\,30'' \pm 2''{\cdot}1.$$

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Those of the stars used in 1903 and 1904 were derived from all available meridian observations.* The following catalogues have been used :—

1903, Star A. 24 Piscium. Mag. 6·0.

	No. Obs.	Epoch.	
Bradley (Auwers)	6, 4	1754·9	54·4
Mayer (Auwers)	4, 4	1757·2	57·0
Taylor (Downing)	7, 5	1833·6	33·9
Greenwich	3	1866·9	
Greenwich	4	1874·1	
Greenwich	15, 19	1892·27	92·38

The resulting proper motion is—

$$+ 0^s\cdot0045 \qquad - 0''\cdot025.$$

The position of Greenwich 1890 has been adopted.

The star occurs also in Piazzi (7 observations), which was not used for want of systematic correction. The R.A. agrees very well ; the declination leaves a residual of about $-6''$.

1903, Star B. 27 Piscium. Mag. 5·2.

Position taken from *Nautical Almanac.*

The resulting distance and position-angle are (equinox 1903·0 ; epoch 1903·7) :—

$$s = 537''\cdot0 \qquad p = 105^\circ\ 32'\ 52''\cdot5$$

1904, Star A. B.D. + 7°,240. Mag. 6·9.

Bessel (Weisse)	2	1825	
Glasgow	2, 3	1871·8	73·9
A. G. C. Leipzig	2	1884·8	
Greenwich	6	1894·0	

No proper motion could be derived. The adopted position is :—

$$1885\cdot0 \qquad 1^h\ 30^m\ 41^s\cdot947 \qquad +^\circ\ 14'\ 36''\cdot81.$$

1904, Star B. B.D. + 8°,258. Mag. 6·7.

Mayer (Auwers)	2	1756·8		
Piazzi	4	1800		
Bessel (Weisse)	1	1825		
Taylor (Downing)	5, 3	1832·8	32·0	
Königsberg	5	1833·8		
Greenwich	0, 3	1841		
Greenwich	6, 3	1845	43	
Glasgow	4, 2	1877·9	78·8	Two observations in δ rejected.
Gotha	5	1886·1	86·0	
A. G. C. Leipzig	2	1886·9		

* Special meridian observations of these stars were made at the Cape Observatory in the years 1901–1904. By some misunderstanding these observations were not forwarded to Groningen in time to be used for the purposes of this paper.

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The resulting position and proper motion are:—

1885·0	$1^h\ 34^m\ 31^s{\cdot}575$	$+8°\ 10'\ 38''{\cdot}40$
	$+0^s{\cdot}0056$	$+0''{\cdot}013$

The adopted distance and position-angle are (equinox 1904·0; epoch 1904·9):—

$$s = 4791''{\cdot}21 \qquad p = 45°\ 29'\ 50''{\cdot}4.$$

To the adopted position-angles corrections for precession, nutation, aberration, and refraction were applied by the usual formulæ. The resulting apparent position-angles P_s are given in Table IV. This table further contains all the quantities necessary for the computation of the corrections for refraction, scale-value and orientation. The notations in the headings are the same as those used in this and the preceding chapter. The degrees of P_{AB} and P_0 are the same as those of P_s and $P + 90°$ respectively.

For 1903 the multiplier $\frac{1}{s \sin 1''}$ is $230''{\cdot}46$, but instead of this the value $203''{\cdot}46$ was used by mistake. This was only discovered when it was too late to alter the values of I and $\sin \Delta P$. We must therefore add to $\sin \Delta P$, and consequently to A, the correction

$$\delta A = \sin 27'' \Delta.$$

These corrections are given in the last column of the table for 1903.

The square of the probable error of one value of I determined from the standard stars is:—

$$\psi^2 = \sigma^2 + \alpha^2 + \pi^2,$$

where

σ is the probable error of the adopted position-angle P_s,

α is the probable error of the quantity $\frac{\Delta}{s \sin 1''}$,

π is the probable error of the measured position-angle P_{AB}.

For 1891 we have $\sigma = \pm 2''$. For the other two oppositions the adopted position-angle is much more uncertain, chiefly owing to the proper motions of the stars, which are not known with sufficient accuracy.

From the agreement of the two readings of the position-circle in each of the two positions, I derived for the observations on the plates of 1903:—

$$\pi = \pm 0''{\cdot}52.$$

Since only seconds were estimated this is extremely satisfactory, the more so since it includes at least part of the division-errors of the circle and the run of the microscopes.

From the agreement of the three images of the standard stars I found for the plates of 1903:—

$$\alpha = \pm 4''{\cdot}93.$$

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TABLE IV.

ORIENTATION, REFRACTION AND SCALE-VALUE FOR PLATES OF 1891.

Plate.	P_S.	P_{AB}.	Δ.	I.	P + 90°.	P_O.	Sin ΔP.	A′.	A.	B.	Plate.
			rev.								
295	68° 4′ 36″	4′ 21″	− 0·540	+ 1′ 55″	65° 45′ 34″	45′ 16″	−·00047	+·00007	−·00040	−·0097	295
300	4 35	5 23⁵	− ·796	+ 1 39	46 1	46 43	− 68	+ 7	− 61	− 97	300
303	4 35	5 7⁵	+ ·260	− 1 21	46 50	47 20	+ 25	+ 7	+ 32	− 97	303
307	4 29	3 23	+ 2·054	− 5 13	48 0	48 3	+ 150	+ 5	+ 155	− 97	307
308	4 32	4 51	+ 0·614	− 2 12	48 19	48 45	+ 51	+ 5	+ 56	− 97	308
311	67 64 31	58 36	− ·123	+ 6 18	48 37	42 36	− 8	+ 5	− 3	− 97	311
312	68 4 28	4 53	+ ·291	− 1 19	49 8	49 40	+ 23	+ 4	+ 27	− 97	312
313	4 30	6 33	− ·173	− 1 31	49 21	51 22	− 15	+ 5	− 10	− 97	313
314	4 19	3 42	+ ·491	− 54	49 33	49 46	+ 20	0	+ 20	− 97	314
315	4 27	5 8	+ ·444	− 2 3	49 42	49 58	+ 52	+ 4	+ 56	− 97	315
316	4 26	5 17	+ 1·628	− 5 51	49 42	49 21	+ 180	+ 3	+ 183	− 97	316
317	4 27	4 50	− 0·228	+ 19	49 50	49 49	− 9	+ 4	− 5	− 97	317
318	4 26	9 15	− ·074	− 4 35	49 58	55 27	− 26	+ 3	− 23	− 97	318
319	4 17	8 36	+ ·040	− 4 26	50 3	54 31	− 1	− 1	− 2	− 96	319
320	4 18	5 40	+ ·161	− 1 52	49 58	51 3	+ 23	0	+ 23	− 97	320
321	4 20	7 5	+ ·107	− 3 5	49 53	52 49	+ 4	+ 1	+ 5	− 97	321
322	4 8	3 18	− ·178	+ 1 23	48 30	47 38	− 15	− 4	− 19	− 96	322
324	4 12	5 12	+ ·150	− 1 28	46 44	47 37	+ 17	− 3	+ 14	− 96	324
326	3 47	5 19	− ·200	− 55	46 17	48 1	− 24	− 13	− 37	− 95	326
330	4 2	23 45	+ ·219	−20 23	43 47	63 18	+ 25	− 6	+ 19	− 96	330
332	4 4	3 1	− ·022	+ 1 7	42 38	41 32	0	− 6	− 6	− 96	332
333	4 4	2 43	+ ·164	+ 51	40 44	39 19	+ 16	− 6	+ 10	− 96	333
337	2 36	4 39	+ ·052	− 2 13	35 36	37 39	+ 5	− 36	− 31	− 92	337
338	3 42	4 56	− ·544	+ 26	34 50	35 48	− 41	− 14	− 55	− 95	338
340	3 37	8 9	+ ·332	− 5 33	33 12	37 47	+ 28	− 16	+ 12	− 95	340
342	67 63 16	52 6	− ·058	+11 21	29 46	18 31	− 3	− 22	− 25	− 94	342
343	68 3 20	2 3⁵	+ ·082	+ 1 2	28 53	28 28	− 18	− 25	− 43	− 94	343
349	2 50	11 8	+ ·165	− 8 48	21 32	29 50	+ 15	− 29	− 14	− 93	349
352	2 25	17 56	− ·062	−15 20	15 43	31 5	− 1	− 34	− 35	− 92	352
353	1 48	5 31	+ ·020	− 3 47	14 45	18 22	+ 5	− 52	− 47	− 90	353

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TABLE IV.

ORIENTATION FROM STANDARD STARS, REFRACTION AND SCALE-VALUE FOR PLATES OF 1903.

Plate.	P_S.	P_{AB}.	Δ.	I.	P + 90°.	P_O.	Sin ΔP.	A′.	A.	δA.	Plate.
	° ′ ″	′ ″	rev.	′ ″	° ′ ″	′ ″					
7510	105 33 25	9 4	−·177	+24 57	64 39 52	14 57	−·000010	+·000082	+·000072	−·000023	7510
7513	33 18	26 50⁵	+·303	+ 5 26	40 8	32 50	+ 543	+ 68	+ 611	+ 40	7513
7518	33 7	38 32	+·064	− 5 38	40 24	47 5	− 305	+ 51	− 254	+ 8	7518
7522	33 18	40 16	−·192	− 6 19	40 40	47 18	− 92	+ 74	− 18	− 25	7522
7524	33 13	31 31	+·235	+ 54	40 57	40 51	− 233	+ 65	− 168	+ 31	7524
7533	33 12	44 0	+·374	−12 4	42 31	55 10	− 170	+ 62	− 108	+ 49	7533
7537	33 14	31 57⁵	−·280	+ 2 14	42 52	40 51	− 63	+ 65	+ 2	− 37	7537
7538	33 11	54 16	+·163	−21 38	43 13	65 0	− 44	+ 59	+ 15	+ 21	7538
7555	33 16	32 26	−·200	+ 1 31	44 21	42 22	+ 136	+ 65	+ 201	− 26	7555
7565	33 9	18 48	−·229	+15 8	44 45	28 39	+ 281	+ 54	+ 335	− 30	7565
7569	33 28	33 9	+·032	+ 12	45 8	43 47	+ 334	+ 81	+ 415	+ 4	7569
7570	33 18	59 24	+·100	−26 26	45 34	73 7	− 325	+ 69	− 256	+ 13	7570
7580	33 11	42 19	+·123	− 9 34	48 46	57 21	+ 286	+ 53	+ 339	+ 16	7580
7581	34 14	43 43	+·238	−10 17	49 43	59 38	+ 107	+ 108	+ 215	+ 31	7581
7582	33 25	24 30	+·255	+ 8 3	50 46	41 56	+ 228	+ 72	+ 300	+ 33	7582
7593	33 17	44 49	−·186	−10 54	51 17	62 49	− 184	+ 63	− 121	− 24	7593
7596	33 2	17 28	+·354	+14 22	51 50	36 24	+ 310	+ 29	+ 339	+ 46	7596
7601	33 30	20 1	−·192	+14 8	52 20	39 1	− 238	+ 80	− 158	− 25	7601
7604	33 23	17 37	−·202	+16 27	52 52	37 6	− 199	+ 69	− 130	− 26	7604
7606	33 31	36 57	−·144	− 2 57	56 44	60 0	− 92	+ 77	− 15	− 19	7606
7608	33 25	35 6	−·260	− 48	57 18	58 37	− 150	+ 70	− 80	− 34	7608
7634	33 21	28 30	+·078	+ 4 35	64 60 39	55 52	+ 58	+ 62	+ 120	+ 10	7634
7645	33 23	41 0	+·148	− 8 7	65 3 19	11 9	+ 82	+ 63	+ 145	+ 19	7645
7647	33 21	32 23	+·296	− 2	65 3 50	2 51	+ 296	+ 60	+ 356	+ 39	7647
7653	33 31	29 16	+·008	+ 4 13	64 64 20	59 54	+ 63	+ 72	+ 135	+ 1	7653
7655	33 24	31 53	−·054	+ 1 42	65 4 51	3 12	− 15	+ 64	+ 49	− 7	7655
7657	33 32	24 35	−·256	+ 9 49	64 67 12	58 11	− 233	+ 73	− 160	− 34	7657
7660	33 27	41 46	−·262	− 7 26	65 8 53	17 16	− 276	+ 65	− 211	− 34	7660
7662	33 27	41 28⁵	+·162	− 8 34	65 9 38	17 35	+ 179	+ 65	+ 244	+ 21	7662
7664	33 27	45 36	−·337	−11 0	65 9 59	22 3	− 310	+ 64	− 246	− 44	7664

The value of B is the same for all plates, viz. : B = −·0105.

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TABLE IV.

Orientation from Standard Stars, Refraction and Scale-Value for Plates of 1904.

Plate.	P_S.	P_{AB}.	Δ.	I.	P + 90°.	P_0.	Sin ΔP.	A′.	B′.	β.	A.	B.	Plate.
	° ′ ″	′ ″	rev.	′ ″	° ′ ″	′ ″							
8123	45 30 24	28 41	+ 0·778	− 1 38	66 60 30	58 1	+ ·001149	+ ·000071	+ ·0005	− ·0115	+ ·001220	− ·0110	8123
8124	30 22	40 11.5	− ·106	− 9 22	59 7	68 29	0	+ 96	+ 4		+ 96	− 104	8124
8126	30 20	38 47	+ ·154	− 9 7	57 43	69 3	− 645	+ 106	+ 4	− 107	− 539	− 103	8126
8127	30 21	45 34	− ·245	−14 10	56 19	73 15	− 805	+ 104	+ 4	− 102	− 701	− 98	8127
8130	30 24	27 22	+ ·084	+ 2 40	52 12	49 59	− 131	+ 73	+ 5	− 114	− 58	− 109	8130
8137	30 22	39 33	− ·318	− 7 49	49 38	58 29	− 301	+ 95	+ 4	− 114	− 206	− 110	8137
8141	30 11	43 16	− ·088	−12 42	48 27	53 17	+ 2288	+ 159	+ 4	− 104	+ 2447	− 100	8141
8142	30 24	42 12.5	− ·185	−11 0	47 6	58 20	− 68	+ 76	+ 5	− 110	+ 8	− 105	8142
8143	30 24	52 29.5	− ·272	−20 56	45 53	66 7	+ 204	+ 77	+ 5	− 105	+ 281	− 100	8143
8144	30 20	42 0	+ ·014	−11 44	44 44	55 52	+ 174	+ 112	+ 4	− 114	+ 286	− 110	8144
8145	30 19	27 36	+ ·316	+ 1 21	43 35	40 49	+ 412	+ 117	+ 4	− 105	+ 529	− 101	8145
8146	30 18	30 26	− ·018	− 3	42 27	42 36	− 29	+ 119	+ 4	− 98	+ 90	− 94	8146
8148	30 24	34 31	+ ·126	− 4 40	39 9	43 9	+ 194	+ 77	+ 5		+ 271	− 103	8148
8154	30 22	35 32	+ ·072	− 5 29	38 9	43 12	+ 126	+ 93	+ 4	− 111	+ 219	− 107	8154
8158	30 20	8 18	+ ·082	+21 41	37 10	14 57	+ 155	+ 106	+ 4	− 101	+ 261	− 97	8158
8159	30 20	30 52	+ ·156	− 1 12	36 12	36 39	+ 218	+ 106	+ 4	− 109	+ 324	− 105	8159
8167	30 20	24 56	+ ·063	+ 5 8	35 16	30 0	+ 39	+ 104	+ 4	− 108	+ 143	− 104	8167
8181	30 19	26 12.5	+ ·046	+ 3 55	31 3	27 0	+ 39	+ 105	+ 4	− 100	+ 144	− 96	8181
8182	30 18	28 20	− ·170	+ 2 42	30 18	28 14	− 184	+ 114	+ 4	− 106	− 70	− 102	8182
8190	30 18	32 0	− ·073	− 1 23	29 35	31 9	− 53	+ 111	+ 4	− 107	+ 58	− 103	8190
8192	30 18	36 26.5	− ·062	− 5 52	28 54	35 6	− 97	+ 112	+ 4	− 114	+ 15	− 110	8192
8198	30 23	33 45	+ ·152	− 4 1	28 13	31 27	+ 228	+ 81	+ 5	− 119	+ 309	− 114	8198
8200	30 19	37 56	− ·159	− 6 56	27 1	34 44	− 228	+ 102	+ 4	− 110	− 126	− 106	8200
8209	30 23	35 57	− ·018	− 5 29	26 27	31 59.5	− 15	+ 78	+ 4	− 112	+ 63	− 108	8209
8213	30 22	41 32	+ ·114	−11 40	25 26	36 26	+ 194	+ 84	+ 4	− 109	+ 278	− 105	8213
8216	30 21	44 42	+ ·002	−14 22	24 59	39 18	+ 15	+ 93	+ 4	− 114	+ 108	− 110	8216
8222	30 23	29 32.5	+ ·047	+ 39	24 33	23 34.5	+ 92	+ 71	+ 5	− 113	+ 163	− 108	8222
8226	30 24	42 46.5	+ ·063	−12 38	22 40	35 2	+ 78	+ 50	+ 5	− 115	+ 128	− 110	8226
8228	30 25	37 28	− ·092	− 6 39	22 6	29 13.5	− 136	+ 61	+ 5	− 108	− 75	− 103	8228

For the plates 8124 and 8148 the scale-value was not determined, and β = −0·0108 was adopted.

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If we put $\psi'^2_s = \alpha^2 + \pi^2$, then ψ'_s is the probable error of that part of the index-correction which varies from plate to plate. We find thus :—

$$\psi'_s = \pm 4''\cdot 96.$$

The probable error α is rather large. It corresponds to a probable error of the measured distance A − B (one image, two positions) $\zeta = \pm 0''\cdot 223$, while from the Helsingfors plates we should expect $\zeta = \pm 0''\cdot 130$ and from the measures of the satellites we found $\zeta = \pm 0''\cdot 124$. To see how this large probable error arises I have formed the differences—

$$D = \left\{ P'_{AB} + \frac{(A-B)'}{s \sin 1''} \right\} - \left\{ P''_{AB} + \frac{(B-A)''}{s \sin 1''} - 180^\circ \right\}$$

which must be equal to zero apart from the errors of observation.

From these differences we can determine the error of pointing ϵ, since the error of the image δ is eliminated from them. I thus find, by combination with the value of ζ just found :—

$$\epsilon = \pm 0''\cdot 121 \qquad\qquad \delta = \pm 0''\cdot 132.$$

Both ϵ and δ are larger than the values found for the satellites. The increase is most marked in the case of δ, which is exactly doubled. This may be due either to distortion of the photographic film, or to what Prof. Kapteyn calls guiding and hour-angle errors (*Publ. of the Astr. Lab. at Groningen No.* 1, *page* 67.), which depend on the magnitude. It should be remarked that the photographic difference of magnitude between the two stars is rather larger than the visual difference of 0·8 mag. Since the discussion of the trails, and of the measures which will be treated in the appendix, seems to prove the absence of photographic distortion, the second explanation must be accepted.

The increase in the error of pointing is probably also due to the great difference in brightness of the two stars. In this connection it is important to remark that the differences D are strongly systematic, the sums of the positive and negative residuals being

$$+ 54''\cdot 5 \text{ (11 plates)} \quad \text{and} \quad - 144''\cdot 0 \text{ (18 plates)}.$$

This points to a systematic error of pointing. A personal error of $0''\cdot 04$ in bisecting the bright image (star B) as compared with the fainter image (star A) would reduce the sum of the residuals to zero. The error is such that the bisection of the bright image is made too far to the right as seen under the microscope, or the bisection of the faint image too far to the left. A personal error of this kind is, of course, rigorously eliminated from the mean of the two positions, and I have not thought it necessary to investigate it more closely.

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CHAPTER VII.

Orientation from Trails.

The trails were measured in two positions of the plate. The plate was adjusted very nearly in the position-angle of the trail, (*i.e.* approximately so that the declination-circle through its centre coincides with the axis of η) and the position-angle was read off in the usual way. Pointings were made on eleven points on the trail, viz. that point which has the same R.A. as the image of Jupiter in the other exposures, and five points on each side of it at the distances $\pm 10'$, $\pm 20'$, . . . $\pm 50'$. By the effect of projection (normal distortion) the trail, which on the sphere is a small circle (parallel), is curved on the plate. The correction to be applied to the co-ordinates of its points to reduce it to a straight line is

$$\Delta\eta = + 0'{\cdot}000145 \tan\delta . \xi^2$$

where ξ and η are expressed in minutes of arc, and are counted from the middle point as origin. This formula is correct to the third order inclusive, and the maximum effect of the neglected terms of the fourth order is $0'{\cdot}00002 \tan\delta$.

After the application of these corrections the extremities of the co-ordinates η will lie in a straight line, which is the tangent to the trail at the middle point. The small angle which this tangent makes with the axis of ξ will be called ϕ. This angle was determined for each trail separately by a least square solution.* The coefficients of this solution are, of course, the same for all trails. If, further, we call the observed position-angle of the plate Π_0, then the instrumental position-angle of the tangent to the trail at the middle point is

$$\Pi_0 + \phi.$$

If the tabular position-angle of this tangent is denoted by $90^\circ + \Pi$, then the index-correction is

$$I' = 90^\circ + \Pi - (\Pi_0 + \phi).$$

* For the determination of ϕ the application of the corrections $\Delta\eta$ is not necessary, since these corrections are the same for points symmetrically situated on both sides of the middle point, and only the differences of the co-ordinates of such points enter into the determination. The corrections have only been applied to the measures on the plates of 1903 for the purpose of the derivation of probable errors.

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The difference II of the tabular position-angle from 90° is due to the following causes :—

1. *Refraction.*—The effect of this is* :—

$$R = 57''.7 \{ \tan^2 \zeta \sin q \cos q + \tan \zeta \sin q \tan \delta \},$$

2. *Motion of Jupiter* during the time of trailing over the plate. This gives—

$$J = \frac{\sec \delta}{360 \sin 1''} \cdot \frac{d\delta}{dt},$$

where $\frac{d\delta}{dt}$ is the daily motion of Jupiter in declination, expressed in degrees.

3. *Motion of the Satellite* relatively to Jupiter. This gives—

$$S = \frac{\sec \delta}{360 \sin 1''} \cdot \frac{d\delta'}{dt},$$

where δ' is the difference of declination of the satellite and Jupiter, expressed in degrees, and where the time in $\frac{d\delta'}{dt}$ is counted in days. The difference of declination is—

$$\delta' = -x' \sin P + y' \cos P.$$

In forming the differential coefficients I have neglected the term in y' which depends on the inclination of the satellite to the planet's equator. The effect of this term can never reach 1″ of position-angle for Satellite II and is entirely negligible for Satellites I and III. We then have the following expressions for x' and y' (see Chapter IX.) :—

$$x' = \frac{a'\rho}{\Delta'} \sin (v - L) \qquad y' = \frac{a'\rho}{\Delta'} \sin B \cos (v - L).$$

The longitude v is the only quantity in these expressions, which varies appreciably with the time, and its differential coefficient can be replaced by the mean motion. We then have—

$$\frac{d\delta'}{dt} = -n \sin 1' \left\{ \frac{y' \sin P}{\sin B} + x' \sin B \cos P \right\},$$

where the mean daily motion n is expressed in degrees and the co-ordinates x' and y' in minutes of arc.

Rigorously the co-ordinates x' and y' should be taken for the moment when the satellite reached the middle point of the trail. Actually the same co-ordinates which were used for the tabular places have been employed, *i.e.* those for the mean epoch

* I have verified, by actual computation, in an extreme case that the terms of higher order in the refraction are absolutely negligible. This applies also to the refraction of the standard stars and the satellites. The effects of barometer and temperature have been taken into account in extreme cases, but were always practically negligible.

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of the satellite-exposures. I have verified that the consequent error in the computed position-angle Π does never reach 1″, and is generally below 0″·1.

The measures of the trails are given in Table V. The trails of the satellites only were measured, and of these that of Sat. IV was always too faint, and that of I could only be measured on a few plates. The trails of II and III could nearly always be measured when they are not covered by the trail of the planet. On the plates 7664, 8126, and 8148 there were no measurable trails. Of the two lines given in the table for each trail the upper one contains the pointings made in position I. of the plate and the lower one those in position II. The point − 50 in position I. is the same as + 50 in position II., and the readings of position II. must be reversed in sign before the mean of the two positions is taken. The measures were always symmetrically arranged with regard to the "forward" and "backward" motions of the plate-holder.

Table VI. contains the quantities necessary for the derivation of the index-error. The notations used in the headings have been explained above. The values of Π_0, R and J are, of course, the same for all trails on one and the same plate.

On plate 7581 the trail is interrupted from $\xi = + 24'$ to $\xi = + 31'$ (position I). The pointing + 30 was made at $\xi = + 33'$, and a correction of $+ 0^{rev} \cdot 034$ has been applied to reduce it to + 30. The same remark applies to position II. with all signs reversed.

The probable error of one pointing on a trail was derived from the deviation of the measured co-ordinates, corrected for normal distortion, from a straight line. This gives the total error ζ. The comparison of the pointings on the same point in the two positions gives the error of pointing ϵ. (For the definition of these errors see Chapter IV.). I thus found from the plates of 1903 :—

$$\epsilon = \pm 0''\cdot 113 \qquad \delta = \pm 0''\cdot 151 \qquad \zeta = \pm 0''\cdot 242$$

The error of pointing is somewhat larger than in the case of the satellites, but this is what could be expected from the faint and diffused appearance of most of the trails. The error of the image δ is very much larger than for the satellites. This must be due either to distortion of the photographic film, or to the fact that the displacements produced by the unsteadiness of the atmosphere are not cancelled by guiding as they are in the case of the satellites. The first cause would affect points having the same ξ co-ordinates on the plate to the same extent, the latter cause would equally affect points corresponding to the positions of the different satellites at the same epoch of time, and therefore separated on the plate by a considerable distance in the direction of ξ.

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TABLE V.

Observations of Trails on Plates of 1903.

Plate.	Position Angle. Position I.	Position Angle. Position II.		Observations of Trails. −50	−40	−30	−20	−10	0	+10	+20	+30	+40	+50	Remarks.
	° ′ ″ ″	′ ″ ″		rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	
7510	89 34 23 60	34 39 40	II.	8·474	·463	·678	·712	·730	·792	·670	·698	·671	·463	·382	Trail II, Position I pointings −50 to −10 inclusive require correction of +0·064 rev., the microscope having been accidentally displaced. Trail II very faint.
	34 73 34	35 2 3		8·562	·421	·163	·200	·220	·131	·088	·102	·233	·365	·309	
			III.	8·097	·064	·349	·410	·455	·456	·441	·340	·290	·124	·041	
				8·488	·413	·304	·150	·118	·060	·119	·168	·260	·423	·537	
7513	89 52 30 19	51 58 88	I.	8·604	·614	·724	·842	·830	·798	·762	·580	·601	·446	·344	Position I of both trails mean of two measures.
	51 42 53	51 73 49		8·795	·650	·505	·472	·320	·292	·419	·340	·471	·569	·468	
			III.	8·653	·796	·850	·827	·815	·845	·824	·657	·568	·538	·288	
				8·913	·742	·636	·598	·403	·390	·459	·450	·400	·456	·581	
7518	90 5 48 82	5 72 59	I.	8·282	·378	·481	·567	·665	·680	·692	·717	·614	·605	·480	
	5 99 49	5 58 69		8·282	·152	·131	·022	·020	·069	·046	·140	·248	·304	·391	
			II.	9·252	·397	·460	·567	·618	·670	·645	·647	·602	·452	·420	
				7·463	·350	·280	·202	·181	·298	·295	·278	·399	·412	·560	
			III.	8·280	·347	·458	·532	·605	·571	·620	·588	·551	·440	·348	
				8·554	·442	·321	·298	·280	·250	·329	·312	·420	·511	·615	
7522	90 5 58 82	6 34 28	II.	8·372	·490	·581	·711	·789	·802	·788	·760	·712	·641	·582	
	5 66 47	5 41 51		8·237	·187	·152	·091	·059	·013	·068	·132	·259	·318	·469	
			III.	8·013	·162	·228	·317	·388	·358	·372	·381	·352	·259	·193	
				8·747	·698	·620	·591	·588	·578	·579	·650	·759	·817	·953	
7524	89 59 40 20	59 22 54	I. II.	8·200	·361	·424	·531	·534	·622	·550	·400	·354	·298	·158	Satellites I and II have nearly same declination; combined trail consists of two dark bands with a lighter band between them, on which pointings were made.
	59 9 38	59 31 3		8·719	·552	·538	·451	·326	·287	·299	·354	·404	·490	·610	
			III.	8·290	·462	·526	·538	·541	·595	·637	·478	·343	·350	·159	
				8·610	·531	·521	·379	·169	·220	·314	·219	·311	·338	·582	
7533	90 12 17 23	12 11 29	II.	8·271	·393	·403	·518	·533	·651	·648	·534	·300	·353	·248	Trail of Satellite III on réseau-line, not measurable.
	12 12 2	11 77 45		8·572	·479	·438	·352	·231	·199	·249	·323	·441	·512	·584	
7537	89 57 21 46	57 33 42	I.	8·028	·070	·312	·285	·380	·416	·368	·408	·262	·271	·171	
	57 48 14	57 41 30		7·620	·520	·519	·422	·451	·438	·534	·610	·648	·758	·908	
			II.	8·041	·100	·239	·256	·397	·450	·200	·333	·230	·190	·041	
				7·801	·602	·582	·518	·642	·418	·532	·664	·598	·770	·818	
7538	90 20 48 95	21 12 16	II.	8·190	·262	·520	·700	·530	·712	·691	·733	·758	·574	·451	Trail II very faint, especially middle part.
	20 94 44	21 18 14		8·670	·550	·444	·395	·489	·432	·591	·544	·671	·861	·980	
			III.	8·275	·291	·488	·629	·599	·609	·537	·671	·592	·490	·437	
				8·908	·781	·641	·603	·566	·598	·719	·618	·756	·918	·897	
7555	89 57 15 60	57 8 11	II.	8·249	·354	·522	·549	·500	·662	·584	·568	·523	·370	·242	
	57 50 8	57 52 50		7·808	·722	·522	·481	·474	·380	·503	·531	·570	·712	·781	
			III.	8·258	·327	·530	·601	·683	·682	·592	·660	·633	·450	·339	
				7·531	·428	·259	·269	·251	·158	·160	·273	·336	·558	·562	
7565	89 43 5 21	43 17 31	III.	7·563	·677	·777	·870	·838	·735	·750	·690	·614	·419	·252	
	43 38 20	43 27 10		7·901	·804	·624	·563	·560	·501	·390	·382	·450	·588	·659	
7569	89 57 48 54	57 38 48	III.	8·531	·582	·692	·620	·530	·510	·502	·372	·301	·112	·040	Trail III very faint. II almost invisible, not measurable.
	58 37 31	58 38 28		8·706	·581	·404	·373	·259	·212	·352	·082	·032	·036	·050	

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TABLE V.—Observations of Trails on Plates of 1903—*continued.*

Plate.	Position Angle. Position I.	Position Angle. Position II.		Observations of Trails. −50	−40	−30	−20	−10	0	+10	+20	+30	+40	+50	Remarks.
	° ′ ″ ‴	′ ″ ″		rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	
7570	90 27 6 18	27 40 5	I.	8·138	·297	·392	·498	·594	·698	·760	·632	·631	·568	·502	
	26 72 57	27 4 33		7·283	·308	·151	·213	·038	·168	·242	·359	·407	·537	·751	
			II.	8·619	·711	·870	·863	·917	·995	·974	·940	·872	·747	·621	
				7·727	·643	·560	·482	·450	·420	·438	·531	·558	·758	·823	
7580	90 8 0 18	7 48 58	I.	7·530	·617	·751	·782	·851	·909	·938	·772	·808	·700	·533	
	7 72 57	8 29 21		8·650	·481	·358	·392	·270	·224	·264	·317	·447	·557	·632	
7581	90 9 42 51	9 28 66	III.	8·273	·379	·339	·531	·543	·472	·371	·434	·331	·229	·069	Trail very faint in places.
	9 42 31	9 62 24		8·913	·792	·618	·617	·600	·548	·463	·548	·628	·630	·752	
7582	89 51 58 53	51 56 69	I.	8·494	·647	·727	·838	·902	·782	·816	·711	·612	·423	·334	
	51 51 59	51 76 57		7·731	·689	·549	·378	·254	·312	·211	·222	·342	·387	·585	
			II.	8·450	·609	·629	·787	·773	·755	·782	·566	·512	·377	·251	
				8·706	·598	·479	·399	·237	·222	·259	·198	·337	·419	·603	
			III.	8·154	·341	·399	·552	·620	·618	·638	·477	·447	·311	·202	
				8·738	·669	·532	·439	·310	·378	·388	·388	·543	·622	·751	
7593	90 9 22 42	9 22 42	II.	8·110	·291	·472	·591	·660	·628	·515	·528	·455	·360	·292	Image of Satellite IV is troublesome in pointing—10 of trail II.
	9 62 42	9 45 27		7·829	·769	·570	·542	·489	·400	·449	·408	·638	·753	·856	
			III.	8·322	·448	·512	·592	·650	·666	·605	·541	·538	·407	·322	
				7·990	·891	·751	·704	·623	·661	·589	·621	·682	758	·860	
7596	89 44 29 38	44 43 42	III.	8·550	·690	·773	·862	·902	·900	·829	·734	·679	·549	·312	
	44 67 54	44 49 46		7·765	·547	·409	·331	·251	·157	·192	·198	·278	·288	·472	
7601	89 43 29 88	43 53 74	III.	8·372	·380	·496	·530	·594	·540	·522	·451	·389	·264	·050	
	43 60 0	43 41 21		7·951	·730	·618	·563	·472	·457	·417	·452	·562	·630	·740	
7604	89 41 37 62	41 50 70	III.	8·532	·620	·730	·702	·719	·630	·588	·540	·313	·130	·000	Trail very diffused.
	42 27 7	42 18 2		7·940	·772	·522	·442	·370	·289	·250	·250	·218	·317	·362	
7606	90 1 8 47	1 3 20	III.	8·636	·785	·871	·797	·850	·887	·770	·693	·451	·279	·098	
	0 90 55	1 41 31		7·901	·711	·572	·317	·208	·132	·189	·147	·091	·172	·330	
7608	90 1 32 47	1 59 52	III.	8·380	·497	·701	·770	·861	·902	·922	·973	·894	·861	·728	Trail very irregular.
	1 43 38	1 23 28		7·511	·420	·359	·262	·321	·389	·398	·558	·481	·717	·818	
7634	89 53 12 32	52 50 73	II.	8·450	·648	·758	·781	·800	·849	·813	·814	·607	·500	·277	
	52 67 43	52 78 52		7·715	·500	·428	·282	·201	·182	·141	·192	·248	·328	·458	
			III.	8·287	·423	·529	·528	·591	·643	·563	·528	·454	·209	·050	
				7·961	·762	·584	·457	·398	·341	·341	·422	·388	·502	·608	
7645	90 8 21 73	8 51 71	III.	8·152	·357	·462	·601	·623	·673	·632	·657	·445	·390	·182	
	8 80 19	8 49 35		8·821	·563	·518	·340	·324	·290	·361	·352	·431	·669	·800	
7647	89 57 50 47	58 21 32	II.	8·128	·401	·469	·579	·651	·608	·624	·637	·612	·578	·404	
	58 8 17	57 57 52		8·601	·430	·444	·369	·399	·369	·357	·459	·520	·620	·911	
			III.	8·310	·588	·698	·821	·869	·886	·881	·830	·788	·713	·627	
				8·444	·308	·261	·195	·111	·152	·169	·232	·361	·490	·790	
7653	89 55 52 63	55 52 42	II.	8·352	·492	·637	·723	·775	·769	·775	·722	·548	·448	·233	
	56 14 8	56 12 14		7·941	·742	·601	·480	·397	·398	·440	·491	·612	·682	·861	
7655	89 56 64 51	56 50 48	III.	8·180	·316	·479	·681	·648	·583	·660	·698	·600	·450	·237	Trail very faint and irregular.
	56 39 50	56 40 38		7·748	·592	·472	·368	·362	·380	·319	269	·403	·682	·718	

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TABLE V.—Observations of Trails on Plates of 1903—*concluded.*

Plate.	Position Angle. Position I.	Position Angle. Position II.		Observations of Trails. −50	−40	−30	−20	−10	0	+10	+20	+30	+40	+50	Remarks.
	° ′ ″ ″	′ ″ ″		rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	
7657	89 51 38 22	51 29 28	III.	8·206	·400	·580	·838	·777	·846	·959	·874	·727	·659	·450	Trail very faint and diffused.
	51 22 42	51 38 39		7·505	·331	·200	·121	·026	·190	·238	·211	·431	·662	·881	
7660	90 6 3 10	6 17 27	III.	8·248	·341	·443	·520	·614	·700	·574	·545	·318	·291	·103	
	6 13 9	6 21 11		7·821	·667	·619	·458	·388	·262	·350	·451	·544	·607	·775	
7662	90 8 78 51	9 22 3	III.	8·246	·502	·558	·630	·724	·767	·752	·687	·657	·441	·381	
	9 10 29	8 52 69		7·691	·632	·411	·440	·341	·301	·296	·438	·524	·601	·812	
Observations of Trails on Plates of 1904.															
8123	89 57 18 4	57 10 11	II.	7·950	·772	·526	·378	·284	·265	·113	·258	·251	·343	·462	
	56 38 51	56 44 41		7·647	·800	·880	·878	·977	·860	·868	·754	·598	·354	·211	
			III.	7·962	·790	·619	·420	·354	·303	·279	·210	·340	·432	·437	
				7·678	·703	·820	·939	·917	·941	·823	·709	·569	·436	·198	
8124	90 9 43 44	9 30 29	III.	8·974	·761	·512	·440	·250	·299	·347	·302	·446	·577	·598	
	9 7 3	9 20 19		8·509	·531	·729	·741	·752	·768	·750	·588	·499	·323	·024	
8127	90 16 9 9	16 24 28	II.	8·582	·479	·451	·429	·348	·410	·497	·532	·660	·798	·995	Trail very faint and diffused.
	16 41 46	16 24 20		8·022	·195	·341	·660	·668	·529	·732	·550	·519	·508	·447	
8130	89 57 20 17	57 49 43	I.	8·720	·537	·445	·358	·373	·382	·409	·447	·597	·778	·921	Trails of Satellites I and III coincide.
	57 51 51	57 31 31	III.	8·066	·229	·401	·551	·607	·683	·653	·588	·588	·402	·218	
			II.	8·965	·739	·641	·404	·473	·343	·385	·443	·482	·597	·816	
				8·251	·332	·481	·523	·606	·662	·616	·588	·452	·312	·100	
8137	90 8 38 38	8 43 42	III.	7·790	·531	·453	·317	·291	·228	·236	·339	·433	·570	·774	
	8 59 57	8 59 61		8·202	·359	·463	·541	·702	·752	·639	·560	·470	·394	·188	
8141	90 14 49 51	15 0 0	II.	7·522	·401	·302	·278	·161	·242	·257	·299	·448	·687	·731	Trails rather irregular.
	15 8 10	15 1 1		8·269	·370	·502	·683	·742	·750	·775	·832	·685	·578	·383	
			III.	7·799	·651	·460	·348	·334	·332	·430	·480	·555	·711	·905	
				8·138	·280	·450	·520	·551	·610	·704	·648	·553	·398	·233	
8142	90 10 47 49	11 18 19	I.	7·788	·572	·500	·320	·331	·357	·297	·242	·510	·668	·722	Trails rather faint.
	11 10 10	11 1 0		8·049	·211	·370	·626	·607	·543	·524	·550	·420	·285	·111	
8143	90 20 51 48	21 23 25	III.	7·648	·624	·539	·460	·342	·413	·393	·427	·568	·723	·821	
	21 27 28	21 8 3		8·017	·115	·278	·410	·469	·421	·451	·320	·223	·191	·149	
8144	90 11 42 43	11 57 61	III.	7·571	·471	·323	·182	·324	·381	·349	·489:	·618:	·607	·910	Trail very bad, almost invisible from +15 to +35.
	12 28 30	12 9 7		8·152	·288	·493:	·502:	·571	·540	·691	·742	·658	·548	·383	
8145	89 59 23 29	59 40 38	I.	7·483	·389	·247	·122	·148	·059	·067	·179	·333	·589	·812	
	59 54 51	59 51 52		8·162	·342	·634	·732	·903	·949	·918	·847	·670	·592	·402	
			II.	7·532	·377	·298	·202	·201	·267	·341	·273	·374	·559	·678	
				8·007	·072	·336	·417	·422	·565	·528	·611	·382	·331	·102	
			III.	7·561	·500	·389	·185	·292	·173	·254	·443	·502	·731	·922	
				8·298	·449	·659	·713	·882	·950	·875	·953	·739	·669	·545	

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TABLE V.—Observations of Trails on Plates of 1904—*concluded.*

Plate.	Position Angle. Position I.	Position Angle. Position II.		Observations of Trails. −50	−40	−30	−20	−10	0	+10	+20	+30	+40	+50	Remarks.
	° ′ ″ ″	′ ″ ″		rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	rev.	
8146	90 0 41 42	0 44 41	III.	7·590	·358	·259	·158	·088	·275	·140	·239	·441	·583	·695	Trail extremely diffused.
	0 50 51	0 56 51		8·206	·324	·564	·629	·720	·628	·762	·619	·622	·552	·260	
8154	90 5 49 48	5 32 36	I.	7·810	·619	·524	·450	·418	·372	·287	·373	·421	·511	·706	
	5 29 30	5 41 39		8·172	·374	·441	·549	·620	·597	·513	·467	·417	·272	·069	
			II.	7·720	·573	·425	·340	·271	·228	·312	·339	·383	·533	·683	
				8·272	·481	·619	·667	·700	·818	·753	·664	·548	·421	·229	
			III.	7·717	·519	·379	·304	·157	·112	·209	·206	·210	·385	·589	
				8·186	·373	·468	·543	·561	·648	·588	·453	·371	·280	·013	
8158	89 39 38 32	39 34 38	III.	7·559	·460	·382	·422	·330	·310	·431	·519	·577	·813	·987	Trail very diffused. It cuts réseau-line under a sharp angle, which is troublesome in pointings +10 and +20.
	39 40 43	39 51 50		8·072	·319	·544	·452	·720	·871	·713	·852	·779	·672	·582	
8159	90 0 49 50	0 59 61	III.	7·762	·680	·491	·312	·378	·313	·382	·449	·512	·680	·812	
	0 59 59	0 58 57		8·228	·388	·521	·582	·734	·753	·669	·724	·628	·450	·323	
8167	89 55 14 8	55 2 8	III.	7·709	·494	·342	·301	·240	·203	·291	·350	·372	·548	·790	
	55 6 6	55 7 3		8·111	·314	·502	·625	·582	·682	·720	·611	·532	·371	·210	
8181	89 56 31 27	56 42 42	III.	7·680	·479	·365	·228	·316	·170	·249	·267	·395	·539	·658	
	56 37 39	56 31 30		8·198	·320	·478	·618	·669	·687	·536	·607	·529	·393	·129	
8182	89 58 22 20	58 22 24	III.	7·850	·610	·478	·427	·393	·392	·441	·478	·541	·693	·820	
	58 27 27	58 30 24		8·162	·293	·457	·499	·530	·610	·567	·544	·559	·327	·125	
8190	90 1 57 51	2 0 2	II.	7·702	·535	·405	·331	·291	·160	·227	·314	·368	·393	·509	
	2 11 17	2 0 0		8·393	·508	·550	·602	·689	·698	·613	·581	·445	·318	·149	
8192	90 6 43 43	6 33 29	III.	7·790	·669	·495	·337	·341	·382	·424	·551	·588	·691	·827	
	6 33 31	6 42 43		8·163	·342	·468	·460	·641	·618	·656	·641	·491	·370	·238	
8198	90 3 52 50	4 12 13	III.	7·819	·530	·278	·394	·219	·127	·075	·170	·269	·327	·532	
	4 3 7	3 53 53		8·180	·384	·480	·578	·682	·651	·554	·381	·450	·196	·022	
8200	90 9 21 21	9 19 12	III.	7·566	·444	·377	·247	·231	·308	·231	·483	·628	·742	·978	Trail very diffused and misformed.
	9 16 10	9 7 11		8·059	·484	·431	·626	·783	·665	·869	·821	·751	·622	·469	
8209	90 5 53 56	6 7 1	III.	7·834	·708	·630	·398	·295	·259	·385	·409	·523	·681	·709	
	6 1 0	5 54 57		8·376	·429	·632	·664	·722	·822	·749	·710	·487	·400	·294	
8213	90 12 2 1	12 35 38	I.	7·582	·420	·390	·334	·279	·223	·239	·433	·578	·740	·796	Trails extremely faint. I in some places hardly measurable.
	12 53 51	12 23 27		8·171	·259	·468	·678	·790	·797	·730	·869	·732	·650	·439	
			II.	7·880	·805	·757	·699	·559	·670	·590	·662	·769	·900	·999	
				8·179	·342	·448	·519	·663	·632	·678	·591	·602	·468	·367	
8216	90 14 28 27	14 5 8	II.	7·982	·805	·661	·482	·570	·445	·410	·460	·588	·681	.850	
	14 44 49	14 40 39		8·133	·341	·408	·518	·611	·623	·573	·473	·364	·188	·041	
			III.	7·800	·644	·560	·472	·390	·320	·291	·437	·476	·638	·763	
				8·287	·388	·591	·659	·780	·710	·576	·558	·451	·341	·148	
8222	89 59 32 31	59 38 33	III.	7·721	·531	·409	·303	·197	·303	·264	·372	·432	·548	·757	
	59 49 50	59 48 49		8·118	·302	·451	·521	·522	·573	·621	·560	·408	·332	·121	
8226	90 12 49 47	12 61 58	III.	7·711	·611	·495	·330	·287	·250	·345	·397	·468	·535	·710	
	13 16 12	13 7 10		8·078	·252	·349	·432	·479	·544	·492	·422	·308	·232	·119	
8228	90 7 52 54	8 18 15	III.	7·857	·771	·549	·521	·546	·497	·581	·608	·659	·860	·981	
	8 33 31	8 16 19		8·062	·190	·460	·448	·499	·609	·581	·647	·539	·344	·278	

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TABLE VI.

ORIENTATION FROM TRAILS.

1903.									1904.								
Plate.	Sat.	Π_0	R	J	S	Π	ϕ	I′	Plate.	Sat.	Π_0	R	J	S	Π	ϕ	I′
7510	II.	89° 34′ 49″	+18″·5	−25″·0	− 3″·5	−10″·0	+ 29″·4	+24′ 32″	8123	II.	89° 56′ 57″	− 3″·0	−26″·8	+ 2″·4	−27″·4	+99″·0	+ ′ 57″
	III.				−15·5	−22·0	+ 1·6	+24 47		III.				−15·6	−45·4	+95·3	+ 43
7513	I.	89 52 6	+11·9	−25·5	−21·4	−35·0	+ 48·4	+ 6 31	8124	III.	90 9 24	+ 5·6	−26·5	− 0·1	−21·0	+68·0	−10 53
	III.				+ 1·7	−11·9	+ 72·9	+ 6 29	8127	II.	90 16 25	+ 7·9	−25·8	+23·4	+ 5·5	−75·7	−14 53
7518	I.	90 6 7	+ 6·1	−26·0	+ 7·4	−12·5	− 41·2	− 5 38	8130	I.	87 57 36	− 2·2	−24·8	−20·3	−47·0	−44·7	+ 2 22
	II.				+14·2	− 5·7	− 28·4	− 5 45		III.				−19·7			
	III.				+17·6	− 2·3	− 19·3	− 5 50		II.				+22·4	− 4·6	+24·2	+ 1 55
7522	II.	90 6 6	+14·8	−26·4	+19·8	+ 8·2	− 39·7	− 5 18	8137	III.	90 8 50	+ 5·4	−24·2	− 3·0	−21·8	− 0·7	− 9 11
	III.				+20·5	+ 8·9	− 35·6	− 5 21	8141	II.	90 15 0	+40·4	−23·8	− 2·4	+14·2	−47·3	−13 59
7524	I.	89 59 27	+11·4	−26·8	−16·3	−34·5	+ 21·9	− 23		III.				+12·9	+29·5	−28·9	−14 1
	II.				−22·0				8142	I.	90 11 4	− 0·7	−23·5	−25·0	−49·2	− 3·9	−11 49
	III.				+13·3	− 2·1	+ 31·8	− 1	8143	III.	90 21 12	− 0·4	−22·8	+12·3	−10·9	−18·1	−21 5
7533	II.	90 12 12	+10·2	−28·5	+ 8·7	− 9·6	+ 26·7	−12 49	8144	III.	90 12 5	+11·9	−22·4	− 4·2	−14·7	−66·1	−11 14
7537	I.	89 57 34	+11·1	−28·7	− 6·9	−24·5	− 45·2	+ 2 47	8145	I.	89 59 42	+13·8	−21 9	− 4·8	−12·9	−49·6	+ 55
	II.				+22·8	+ 5·2	− 12·1	+ 2 43		II.				+17·8	+ 9·7	−38·6	+ 1 7
7538	II.	90 21 13	+ 9·2	−29·0	−18·7	−38·5	− 69·1	−20 42		III.				−18·1	−26·2	−59·8	+ 52
	III.				+11·3	− 8·5	− 27·8	−20 53	8146	III.	90 0 47	+15·0	−21·4	−19·1	−25·5	−33·4	− 40
7555	II.	89 57 32	+11·7	−29·7	+ 6·4	−11·6	− 2·4	+ 2 19	8154	I.	90 5 38	+ 5·3	−19·3	+25·3	+11·3	+26·3	− 5 53
	III.				−18·7	−36·7	− 19·3	+ 2 11		II.				− 6·0	−20·0	+ 9·8	− 6 8
7565	III.	89 43 21	+ 6·8	−29·8	− 3·9	−26·9	+ 62·4	+15 9		III.				+14·5	+ 0·5	+32·9	− 6 10
7569	III.	89 58 10	+18·8	−30·0	+13·1	+ 1·9	+123·0	− 11	8158	III.	89 39 41	+10·0	−18·7	− 1·4	−10·1	−92·8	+21 42
7570	I.	90 27 14	+13·4	−30·1	−18·7	−35·4	− 80·4	−26 29	8159	III.	90 0 57	+10·3	−18·2	−16·5	−24·4	−17·4	− 1 4
	II.				+25·5	+ 8·8	− 13·1	−26 52	8167	III.	89 55 7	+ 9·4	−17·6	−19·7	−27·9	−16·7	+ 4 42
7580	I.	90 8 8	+ 6·9	−30·6	−25·5	−49·2	− 8·5	− 8 49	8181	III.	89 56 35	+10·3	−14·3	+ 1·6	− 2·4	− 1·6	+ 3 25
7581	III.	90 9 43	+35·2	−30·4	+ 0·4	+ 5·2	+ 32·9	−10 11	8182	III.	89 58 26	+13·4	−13·7	−14·2	−14·5	− 8·5	+ 1 28
7582	I.	89 52 0	+15·4	−30·3	+21·3	+ 6·4	+ 49·8	+ 7 16	8190	II.	90 2 2	+12·2	−13·0	+21·1	−20·3	+39·0	− 3 1
	II.				+16·9	+ 2·0	+ 44·4	+ 7 18	8192	III.	90 6 37	+12·9	−12·3	−11·2	−10·6	−18·7	− 6 29
	III.				−21·1	−36·0	− 0·1	+ 7 24	8198	III.	90 4 0	+ 2·1	−11·7	+ 2·6	− 7·0	+47·1	− 4 54
7593	II.	90 9 38	+11·6	−30·2	−24·6	−43·2	− 7·8	−10 13	8200	III.	90 9 15	+ 9·6	−10·2	+17·6	−17·0	−79·8	− 8 12
	III.				− 9·9	−28·5	+ 16·9	−10 23	8209	III.	90 5 59	+ 1·5	− 9·5	+ 4·0	− 4·0	+16·7	− 6 20
7596	III.	89 44 46	− 0·2	−30·0	+ 9·2	−21·0	+ 52·3	+14 1	8213	I.	90 12 29	+ 3·1	− 8·0	−10·3	−15·2	−66·5	−11 38
7601	III.	89 43 46	+19·3	−29·8	+20·2	+ 9·7	+ 42·4	+15 42		II.				+22·6	+17·7	−28·4	−11 42
7604	III.	89 42 4	+14·3	−29·5	+18·0	+ 2·8	+116·8	+16 2	8216	II.	90 14 30	+ 6·1	− 7·3	−13·6	−14·8	+27·3	−15 12
7606	III.	90 1 22	+18·0	−27·5	+19·2	+ 9·7	+122·6	− 2 15		III.				−13·9	−15·1	+21·9	−15 7
7608	III.	90 1 40	+15·2	−27·1	+ 5·4	− 6·5	− 75·0	− 32	8222	III.	89 59 41	+ 1·0	− 6·4	+ 3·1	− 2·3	− 6·6	+ 24
7634	II.	89 53 6	+11·9	−24·4	+ 7·4	− 5·1	+ 42·9	+ 6 6	8226	III.	90 13 3	− 9·1	− 2·1	−14·3	−25·5	+ 1·9	−13 30
	III.				+20·0	+ 7·5	+ 54·5	+ 6 7	8228	III.	90 8 15	−11·6	+ 1·2	+ 8·6	− 1·8	−36·8	− 7 40
7645	III.	90 8 50	+12·5	−21·7	− 0·5	− 9·7	− 4·5	− 8 55									
7647	II.	89 58 5	+11·1	−21·2	+19·9	+ 9·8	− 49·5	+ 2 54									
	III.				+15·7	+ 5·6	− 49·4	+ 2 50									
7653	II.	89 56 2	+16·8	−20·6	+12·9	+ 9·1	+ 14·8	+ 3 52									
7655	III.	89 56 47	+13·0	−20·0	+10·6	+ 3·6	− 9·3	+ 3 26									
7657	III.	89 51 32	+16·8	−16·8	+13·0	+13·0	− 67·9	+ 9 49									
7660	III.	90 6 14	+13·7	−14·0	−17·9	−18·2	+ 19·1	− 6 51									
7662	III.	90 9 9	+13·9	−12·6	− 6·4	− 5·1	− 15·4	− 8 59									

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The probable error of the angle ϕ corresponding to the above value of ζ is :—

$$\rho_\phi = \pm 5''.60.$$

This same probable error can also be derived from the comparison of the different trails on the same plate. In this way I find

$$\rho_\phi = \pm 5''.62.$$

The agreement between these two values proves that the trails have no common errors, or, in other words, that there is no distortion of the photographic film—at least not to an extent comparable with the accidental errors of observation, and that consequently the increase of the error δ, compared with its value for the satellites, must be explained by the effect of atmospheric unsteadiness.

Adopting the mean of the two values of ρ_ϕ, and combining it with the value of π found in the preceding article, we find for the probable error of one index-correction I′ derived from one trail

$$\psi_t = \pm 5''.63.$$

If the index-correction I′, found from the trails, is added to observed position-angle P_0 of the plate during the measures of the satellites (operations 3 and 6 of Chapter III.), the angle

$$P_0 + I' - 90^\circ$$

will be the angle which the direction of measurement makes with the projection on the plate of the declination-circle through the middle point of the trail.

The tabular places are referred to the axis of y, which is defined by its position-angle relatively to the declination-circle through the centre of Jupiter. Since the middle point of the trail was so chosen as to have the same R.A. on the plate as the mean of the images of Jupiter, the correction for orientation, computed in the manner explained in Chapter VI. by means of the constant

$$\Delta P = P - (P_0 + I' - 90^\circ),$$

therefore correctly reduces the measured co-ordinates to the tabular axis of y.

On the first half of the plates of 1904 however (plates 8123 to 8167 inclusive), the middle point of the trails was not the point of same R.A. as Jupiter, but the point of which the ξ-co-ordinate was the mean of those of all the satellites of which the trail was measured. The position-angle

$$P_0 + I' + 90^\circ$$

is, therefore, for these plates the angle which the direction of measurement makes with

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the declination-circle through the *satellite* (or the mean of the satellites, if the trail of more than one is measured), and the orientation-constant derived in the usual way requires a further correction, which is *

$$\Delta I' = -\xi_0 \tan \delta_0,$$

where ξ_0 is the mean of the ξ-co-ordinates of the satellites of which the trails were measured relatively to Jupiter. These corrections, which must be applied to the values of I′ contained in Table VI., are :—

TABLE VII.

Corrections ΔI′.

Plate.	ΔI′	Plate.	ΔI′	Plate.	ΔI′
8123	− 2″	8141	+ 32″	8146	+ 10″
8124	+ 44	8142	+ 9	8154	− 20
8127	− 8	8143	− 31	8158	− 43
8130	− 5	8144	− 44	8159	− 30
8137	+ 42	8145	− 7	8167	+ 5

* See Appendix.

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CHAPTER VIII.

Comparison of Orientation from Standard Stars and Trails.

The following Table VIII. contains, for every plate of the oppositions 1903 and 1904, in the second column the index-correction, I, derived from the standard stars, and in the third column the same correction, I′, derived from the trails. The mean of the corrections found from the different trails on the same plate is given, and the corrections of Table VII. have already been applied. The difference I′ − I, which is given in the fourth column of Table VIII., is the correction to the adopted position-angle of the standard stars, as derived from the measures on the plate in the assumption that the orientation derived from the trails is correct. Since the positions of the standard stars for 1903 and 1904 were derived from meridian observations, which in no case had a later epoch than 1894, it cannot be a matter for surprise that these corrections are rather large. The mean values of the corrections I′ − I are :—

for 1903	$\Delta P_s = +19''$
for 1904 (rejecting plate 8123) . .	$\Delta P_s = -43''$.

If these means are subtracted from the individual values of I′ − I, we get the residuals ΔI, which are given in the next column of Table VIII. The square of the probable error of one of these residuals is

$$\rho_{\Delta I}{}^2 = \psi'_s{}^2 + \psi_t{}^2,$$

from which we find :—

$$\rho_{\Delta I} = \pm 7''{\cdot}5.$$

It will be seen at once that the actual residuals for the plates of 1903 are far too large for this probable error. In fact, out of 29 residuals only one is smaller than this probable error, and only three are smaller than twice the probable error. For the plates of 1904 the residuals are much better, and in the first half of this opposition they can (with the exception of plate 8123, the large residual of which I cannot explain) very well be ascribed to the ordinary accidental errors of observation. In the second half of this series, two or three residuals occur which are of the same order as those of 1903. These large residuals have been the object of a special investigation, which will be

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TABLE VIII.

Comparison of Index Corrections from Standard Stars and Trails.

1903.								1904.							
Plate.	I	I′	I′—I	ΔI	Δ tan δ	Δ t	ΔA	Plate.	I	I′	I′—I	ΔI	Δ tan δ	Δ t	ΔA
						m								m	
7510	+24′ 57″	+24′ 39″	− 18″	− 37″	−·015	+36	+·000087	8123	− 1′ 38″	+ ′ 48″	+146″	+189″	+·001	+30	−·000708
7513	+ 5 26	+ 6 30	+ 64	+ 45	− 16	+44	− 310	8124	− 9 22	−10 9	− 47	− 4	0	+24	+ 228
7518	− 5 38	− 5 44	− 6	− 25	− 16	+40	+ 29	8127	−14 10	−15 1	− 51	− 8	−·001	+17	+ 250
7522	− 6 19	− 5 20	+ 59	+ 40	− 17	+36	− 286	8130	+ 2 40	+ 2 3	− 37	+ 6	− 4	+18	+ 179
7524	+ 54	− 12	− 66	− 85	− 18	+34	+ 320	8137	− 7 49	− 8 29	− 40	+ 3	− 5	+18	+ 194
7533	−12 4	−12 49	− 45	− 64	− 22	+36	+ 218	8141	−12 42	−13 28	− 46	− 3	− 6	+21	+ 223
7537	+ 2 14	+ 2 45	+ 31	+ 12	− 23	+36	− 150	8142	−11 0	−11 40	− 40	+ 3	− 7	+18	+ 194
7538	−21 38	−20 48	+ 50	+ 31	− 24	+38	− 242	8143	−20 56	−21 36	− 40	+ 3	− 7	+20	+ 194
7555	+ 1 31	+ 2 15	+ 44	+ 25	− 27	+39	− 213	8144	−11 44	−11 58	− 14	+ 29	− 8	+20	+ 68
7565	+15 8	+15 9	+ 1	− 18	− 28	+37	− 5	8145	+ 1 21	+ 51	− 30	+ 13	− 9	+20	+ 145
7569	+ 12	− 11	− 23	− 42	− 28	+42	+ 111	8146	− 3	− 30	− 27	+ 16	− 9	+21	+ 128
7570	−26 26	−26 40	− 14	− 33	− 29	+40	+ 68	8154	− 5 29	− 6 24	− 55	− 12	− 12	+22	+ 267
7580	− 9 34	− 8 49	+ 45	+ 26	− 36	+45	− 218	8158	+21 41	+20 59	− 42	+ 1	− 13	+25	+ 204
7581	−10 17	−10 11	+ 6	− 13	− 38	+61	− 29	8159	− 1 12	− 1 34	− 22	+ 21	− 13	+29	+ 104
7582	+ 8 3	+ 7 20	− 43	− 62	− 40	+49	+ 208	8167	+ 5 8	+ 4 47	− 21	+ 22	− 14	+24	+ 102
7593	−10 54	−10 18	+ 36	+ 17	− 41	+45	− 174	8181	+ 3 55	+ 3 25	− 30	+ 13	− 16	+34	+ 145
7596	+14 22	+14 1	− 21	− 40	− 42	+47	+ 102	8182	+ 2 42	+ 1 28	− 74	− 31	− 17	+26	+ 356
7601	+14 8	+15 42	+ 94	+ 75	− 42	+46	− 456	8190	− 1 23	− 3 1	− 98	− 55	− 17	+28	+ 475
7604	+16 27	+16 2	− 25	− 44	− 43	+50	+ 121	8192	− 5 52	− 6 29	− 37	+ 6	− 17	+27	+ 179
7606	− 2 57	− 2 15	+ 42	+ 23	− 50	+53	− 203	8198	− 4 1	− 4 54	− 53	− 10	− 18	+26	+ 259
7608	− 48	− 31	+ 17	− 2	− 50	+51	− 82	8200	− 6 56	− 8 12	− 76	− 33	− 19	+28	+ 378
7634	+ 4 35	+ 6 6	+ 91	+ 72	− 55	+57	− 441	8209	− 5 29	− 6 20	− 51	− 8	− 19	+27	+ 247
7645	− 8 7	− 8 55	− 48	− 67	− 59	+67	+ 233	8213	−11 40	−11 40	0	+ 43	− 19	+28	0
7647	− 2	+ 2 52	+174	+155	− 59	+55	− 843	8216	−14 22	−15 10	− 48	− 5	− 19	+29	+ 231
7653	+ 4 13	+ 3 52	− 21	− 40	− 60	+57	+ 102	8222	+ 39	+ 24	− 15	+ 28	− 20	+32	+ 73
7655	+ 1 42	+ 3 26	+104	+ 85	− 60	+57	− 504	8226	−12 38	−13 30	− 52	− 9	− 20	+28	+ 252
7657	+ 9 49	+ 9 49	0	− 19	− 63	+59	0	8228	− 6 39	− 7 40	− 61	− 18	−·021	+31	+ 296
7660	− 7 26	− 6 51	+ 35	+ 16	− 64	+60	− 170								
7662	− 8 34	− 8 59	− 25	− 44	−·066	+58	+ 121								

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communicated in the appendix to this paper. I will therefore not enter into details about this question here, but only quote the result, which is that the orientation derived from the trails is correct, and that the results from the standard stars and the trails must agree within the errors of observation, so long as the hour-angle and declination of the middle point are the same for both. I have included in Table VIII. the differences of the tangent of the declination and of the hour-angle in the sense trail—standard stars. A comparison of these differences with the residuals ΔI, shows that this conclusion is confirmed by the facts.

While the plates on which this conclusion is based were being taken at the Cape, I proceeded to make two separate solutions of each opposition, based on the index-corrections derived from the standard stars and the trails respectively. The last column of Table VIII. gives the correction which must be applied to the orientation constant A derived from the standard stars, and given in Table IV., to reduce it to the value which would be derived from the trails. This correction is

$$\Delta A = -(I' - I) \sin 1''.$$

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CHAPTER IX.

TABULAR PLACES.

THE theory with which the observations have been compared is that of Souillart. The tables published by Mr Marth in *Monthly Notices*, June 1891, pages 505–538, were used.*

The co-ordinates x' and y' and the differential co-efficients $\frac{\partial y'}{\partial p}$ and $\frac{\partial y'}{\partial q}$ were computed by the formulas :—

$$\Delta' = \Delta + a\rho \cos(v - \mathrm{L}) \cos \mathrm{B}$$

$$x' = \frac{a'\rho}{\Delta'} \sin(v - \mathrm{L})$$

$$y' = \frac{a'\rho}{\Delta'} \left\{ \cos(v - \mathrm{L}) \sin \mathrm{B} + \sin(v - \Omega) \sin i \cos \mathrm{B} \right\}$$

$$\frac{\partial y'}{\partial p} = \frac{a'\rho}{\Delta'} \sin 1^\circ \cos \mathrm{B} \cos v$$

$$\frac{\partial y'}{\partial q} = \frac{a'\rho}{\Delta'} \sin 1^\circ \cos \mathrm{B} \sin v.$$

The meaning of the different notations is as follows :—

$180^\circ + \mathrm{L}$ = jovicentric longitude
B = „ latitude
Δ = „ distance
} of the earth.

a = mean distance
ρ = radius-vector
v = true longitude
i = inclination
Ω = ascending node
} of the satellite.

$p = i \sin(-\Omega)$
$q = i \cos(-\Omega)$
$a' = \frac{a}{\sin 1'}$

The co-ordinates x' and y' are referred to a system of axes of which the axis of y is the projection on the sphere of a line through the centre of Jupiter perpendicular to the fundamental plane, which is Marth's adopted equator of Jupiter, and which is defined by its inclination and node referred to the ecliptic and mean equinox of date as follows :—

(1)
$$\begin{aligned} \text{inclination } J &= 2^\circ\ 9'\ 3''\cdot 94 + 0''\cdot 000\ 950\ t \\ \text{node } N &= 336\ 21\ 28\cdot 39 + 0\cdot 126\ 783\ t. \end{aligned}$$

* The notations and formulæ of Marth are also reproduced by Mr Cookson in Vol. XII. Part II. of these Annals (Chapter III.).

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The longitudes in this plane are counted from the point of which the longitude is :—

(1) $$O = 135^\circ\ 27'\ 2''{\cdot}55 + 0''{\cdot}129\ 894\ t.$$

The time is counted in days from 1900 Jan. 0 Greenwich mean noon.

The position-angle P of the axis of y and the jovicentric co-ordinates L, B of the earth have been computed from the values (1) and the apparent geocentric R.A. and declination of Jupiter given in the *Nautical Almanac*. The adopted values for Greenwich mean noon are given in Table IX.

TABLE IX.

Values of P, L and B.

Date.	P	L	B	Date.	P	L	B
1903.	° ′ ″	°	°	1904.	° ′ ″	°	°
Aug. 19	334 40 16	215·692	+ 2·0294	Oct. 27	337 1 9	248·439	+ 3·1537
27	42 42	214·777	+ 2·0200	Nov. 4	336 50 8	247·425	+ 3·1086
Sept. 4	45 48	213·769	+ 2·0008	12	40 37	246·517	+ 3·0594
12	49 32	212·713	+ 1·9725	20	32 55	245·761	+ 3·0081
20	53 45	211·657	+ 1·9364	28	27 12	245·188	+ 2·9565
28	58 13	210·654	+ 1·8944	Dec. 6	23 34	244·820	+ 2·9073
Oct. 6	335 2 37	209·757	+ 1·8494	14	22 3	244·672	+ 2·8623
14	6 36	209·000	+ 1·8032				
22	9 53	208·418	+ 1·7584				

These are *apparent* values by the nature of their computation. We want, however, *true* values for the time t_c, or what is the same thing, apparent values for the time t_0, where

t_0 = time of observation

$t_c = t_0 - 498^s{\cdot}46.\ \Delta.$

The values of P, L and B must therefore be interpolated from Table IX. with the argument: time of observation, *not* corrected for aberration. The values of P which have been actually used have been given in Table IV.

The longitudes and radii-vectores must, however, be computed for the *corrected* time t_c. The fact that Δ', and therefore t_c, is not the same for all satellites, can be ignored. The effect on the co-ordinates y' is altogether negligible.

The inclinations and nodes of the orbital planes, referred to the adopted fundamental plane, were computed by the formulas given by Marth on page 513 of his paper. These formulæ give :—

$$p = i \sin(-\Omega) = \text{Marth's } \gamma \sin(O - \Gamma)$$
$$q = i \cos(-\Omega) = \text{,,} \quad \gamma \cos(O - \Gamma).$$

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The adopted values of the angles G entering into these formulas are:—

$G_1 = 215^\circ.590 + 0^\circ.141\ 0897\ t$ $\quad G_3 = 344^\circ.8170 + 0^\circ.007\ 01936\ t$

$G_2 = 42^\circ.0932 + 0^\circ.033\ 00968\ t$ $\quad G_4 = 309^\circ.6231 + 0^\circ.001\ 89845\ t$

the time being again counted in days from 1900, Jan. 0, Greenwich mean noon.

The resulting inclinations and nodes are given in Table X.

TABLE X.

Ephemeris for 1903 and 1904.

Date.	l_1	S_1'	α_1	β_1	γ_1	δ_1	i_1	$-\Omega_1$	l_2	S_2'	α_2	β_2	γ_2	δ_2	ϵ_2	i_2	$-\Omega_2$
1903.	°	•					°	°	°	•						°	°
Aug. 11	5·452	−·012	·9418				0·0100	266·0	15·937	+·010	·4709	·235	·394	·387	·613	0·4638	263·76
21	240·342	−·012	·6148				·0100	266·9	309·684	+·008	·3074	·654	·209	·203	·429	·4638	264·13
31	115·231	−·012	·2878				·0101	267·7	243·431	+·005	·1439	·072	·024	·019	·245	·4637	264·49
Sept. 10	350·121	−·012	·9608				·0101	268·3	177·178	+·003	·9804	·490	·839	·835	·061	·4636	264·85
20	225·010	−·012	·6338				·0101	268·8	110·926	·000	·8169	·909	·653	·651	·877	·4636	265·20
30	99·900	−·012	·3069				·0101	269·2	44·673	−·002	·6534	·327	·468	·466	·693	·4635	265·54
Oct. 10	334·789	−·012	·9799				·0102	269·6	338·420	−·005	·4899	·745	·283	·282	·509	·4635	265·88
20	209·679	−·012	·6529				·0102	269·9	272·167	−·007	·3264	·163	·098	·098	·325	·4634	266·21
1904.																	
Oct. 27	151·060	−·010	·2562	·064	·786	·753	0·0109	287·7	284·940	+·019	·1281	·064	·087	·125	·358	0·4624	278·77
Nov. 6	25·949	−·009	·9292	·482	·434	·405	·0109	288·1	218·687	+·018	·9646	·482	·902	·941	·174	·4624	279·10
16	260·839	−·008	·6022	·901	·083	·058	·0110	288·6	152·434	+·016	·8011	·901	·717	·757	·990	·4624	279·43
26	135·728	−·006	·2753	·319	·731	·710	·0110	289·0	86·182	+·014	·6376	·319	·531	·573	·806	·4624	279·77
Dec. 6	10·618	−·005	·9483	·737	·379	·362	·0110	289·5	19·929	+·011	·4741	·737	·346	·388	·622	·4624	280·10
16	245·507	−·004	·6213	·155	·027	·015	·0110	290·0	313·676	+·008	·3106	·155	·161	·204	·438	·4624	280·44

Date.	l_3	S_3'	α_3	β_3	γ_3	δ_3	i_3	$-\Omega_3$	l_4	S_4	α_4	β_4	γ_4	δ_4	i_4	$-\Omega_4$
1903.	°	•					°	°	°	•					°	°
Aug. 11	111·179	+·006	·6516	·8776	·2354	·560	0·1270	153·79	89·626	+·011	·8176	·560	·092	·505	0·3297	30·52
21	254·355	+·005	·0491	·2752	·6537	·358	·1269	153·88	305·337	+·010	·4168	·358	·691	·100	·3297	30·52
31	37·531	+·005	·4466	·6729	·0720	·157	·1269	153·98	161·048	+·010	·0159	·157	·290	·694	·3298	30·53
Sept. 10	180·707	+·005	·8441	·0705	·4902	·955	·1269	154·07	16·758	+·010	·6150	·955	·889	·289	·3298	30·53
20	323·883	+·005	·2416	·4682	·9085	·754	·1268	154·16	232·469	+·009	·2142	·754	·488	·884	·3298	30·54
30	107·059	+·005	·6391	·8658	·3267	·552	·1268	154·26	88·180	+·009	·8133	·552	·087	·478	·3299	30·54
Oct. 10	250·235	+·004	·0367	·2635	·7450	·351	·1268	154·35	303·891	+·008	·4125	·351	·686	·073	·3299	30·54
20	33·412	+·004	·4342	·6611	·1632	·149	·1268	154·44	159·601	+·008	·0116	·149	·285	·667	·3300	30·54
1904.																
Oct. 27	81·880	−·004	·5613	·7937	·0641	·934	0·1251	158·70	285·612	−·008	·3596	·934	·627	·848	0·3314	30·13
Nov. 6	225·056	−·003	·9588	·1914	·4823	·733	·1251	158·83	141·323	−·009	·9588	·733	·226	·443	·3314	30·11
16	8·232	−·002	·3563	·5890	·9006	·531	·1250	158·96	357·033	−·009	·5579	·531	·825	·037	·3313	30·09
26	151·408	−·001	·7538	·9867	·3188	·330	·1250	159·10	212·744	−·010	·1570	·330	·424	·632	·3313	30·08
Dec. 6	294·585	+·001	·1513	·3843	·7371	·128	·1250	159·23	68·455	−·010	·7562	·128	·023	·227	·3312	30·06
16	77·761	+·002	·5488	·7819	·1553	·927	·1249	159·37	284·166	−·011	·3553	·927	·622	·821	·3312	30·04

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The adopted values of the mean distances are :—

$\log a_1' = 0.98638$ $\log a_2' = 1.18806$ $\log a_3' = 1.39086$ $\log a_4' = 1.63609$

They correspond to the reciprocal of the mass of the system

$$M = 1047.70,$$

which is the same value that was used in computing the tabular places for the heliometer-observations of 1891, and was also used by Mr Cookson.

For 1891 Mr Marth's ephemerides were used. All mistakes previously found in these, and in the tables, were of course corrected. For 1903 and 1904 Marth's tables were also used. The adopted values of the arguments are given in Table X. The inequalities of the longitudes and radii-vectores may be divided into three groups, according to the periods, viz. :—

I. Short periods (not exceeding 17 days). The inequalities of this group, which have been applied, are those depending on Marth's arguments

$$\alpha_1, [\beta_1, \gamma_1, \delta_1]; \quad \alpha_2, \beta_2, \gamma_2, \delta_2, \epsilon_2; \quad \alpha_3, \beta_3, \gamma_3, \delta_3; \quad \alpha_4, \beta_4, \gamma_4, \delta_4.$$

II. Long periods (between 400 and 500 days), depending on the arguments :—

$$[\epsilon_1, \zeta_1, \eta_1, \theta_1]; \quad \zeta_2, \eta_2, \theta_2; \quad [\zeta_3, \eta_3, \theta_3].$$

III. Very long periods (exceeding 12 years). These are given by Marth's formulæ on page 512, and are denoted by S_i.

The inequalities depending on the arguments inclosed in brackets have been neglected for 1891 and 1903. Those depending on the arguments of Marth not mentioned above were neglected throughout. The inequalities of the groups II. and III. were combined, and their sum is denoted in Table X. by S_i'.

For 1891 corrections were applied to the values of v and ρ computed from Marth's tables, corresponding to the corrections to the elements found from the heliometer observations. The corrections were applied by the differential formulas (II.) given in my inaugural dissertation on page 23. The adopted corrections were :—

$\Delta\epsilon_1 = +0.^{\circ}1024$	$\Delta h_1 = +0.0225$	$\Delta k_1 = -0.0667$	$x_1 = +0.0747$	
$\Delta\epsilon_2 = -0.0002$	$\Delta h_2 = +0.0095$	$\Delta k_2 = -0.0534$	$x_2 = -0.0171$	$y_2 = -0.1177$
$\Delta\epsilon_3 = -0.0256$	$\Delta h_3 = +0.0589$	$\Delta k_3 = -0.0490$	$x_3 = -0.0060$	$y_3 = +0.0454$
$\Delta\epsilon_4 = -0.0208$	$\Delta h_4 = +0.0619$	$\Delta k_4 = +0.0282$		$y_4 = -0.0190$

The total effect of these corrections to v and ρ on the tabular co-ordinates y' is generally very small, and they might probably as well have been neglected. The corrections to p and q found from the heliometer work were not introduced into the tabular places.

For the oppositions 1903 and 1904 no corrections to the elements adopted by Souillart and Marth were introduced.

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CHAPTER X.

Equations of Condition and Normal Equations.

The equation of condition for any one satellite is:—

$$\frac{\partial y'}{\partial p}\Delta p + \frac{\partial y'}{\partial q}\Delta q = (y - h) - y' = m. \tag{1}$$

In this equation y' is the tabular co-ordinate computed as explained in the preceding Chapter, and referred to the centre of the planet as origin; y is the observed value of this co-ordinate (corrected for refraction, orientation, and scale-value) referred to an arbitrary origin, and h is the y co-ordinate of the centre of Jupiter referred to the same origin. Since the image of the planet was not measured, the value of h is not known, but it is a constant for each plate.

In order to make the coefficients of the unknowns belonging to the different satellites approximately equal, I introduced the following new unknowns and coefficients:—

$$a_i = c_i \frac{\partial y_i'}{\partial p_i} \qquad b_i = c_i \frac{\partial y_i'}{\partial q_i}$$

$$\pi_i = \frac{1}{c_i}\Delta p_i \qquad \kappa_i = \frac{1}{c_i}\Delta q_i.$$

The subscript i has the values 1 to 4, and refers to the different satellites. The four constants c are the same for all plates of one opposition. The adopted values are

1891:	$c_1 = 200$	$c_2 = 110$	$c_3 = 93{\cdot}75$	$c_4 = 42{\cdot}5$
1903 and 1904:	$c_1 = 200$	$c_2 = 100$	$c_3 = 75$	$c_4 = 50$

The equations of condition now become:—

$$a_i \pi_i + b_i \kappa_i = m_i. \tag{2}$$

In order now to eliminate the constant h, the mean of all the equations (2) for every plate was subtracted from each individual equation. If there are p satellites on the plate ($p = 2$, 3 or 4), the equations now become:—

$$\begin{aligned}
&\frac{p-1}{p} a_1 \pi_1 + \frac{p-1}{p} b_1 \kappa_1 - \frac{1}{p} a_2 \pi_2 - \frac{1}{p} b_2 \kappa_2 \quad - \quad . \quad . \quad . \quad . \quad . \quad = m_1 - m_0 = n_1 \\
&-\frac{1}{p} a_1 \pi_1 - \frac{1}{p} b_1 \kappa_1 + \frac{p-1}{p} a_2 \pi_2 + \frac{p-1}{p} b_2 \kappa_2 \quad - \quad . \quad . \quad . \quad . \quad . \quad = m_2 - m_0 = n_2 \\
&\qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .
\end{aligned} \tag{3}$$

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The right-hand members are now :—

$$n_i = y_i - (y_i' - y_0'), \tag{4}$$

where y_i is the quantity defined at the end of Chapter IV. (but with the corrections for refraction, orientation, and scale-value added as explained in Chapter V.), and y_0' is the mean of the tabular co-ordinates y' for all the satellites on the plate. The sum $n_1 + n_2 + \ldots$ is zero, and since all subsequent operations are conducted so that this sum remains zero, this furnishes a useful check on the computations. The value of n_i can either be computed by (4), or the value of m_i can first be computed by (1) in which the constant h, which is rigorously eliminated afterwards, is entirely arbitrary, and we may adopt either zero or any value which is convenient. The latter method has been used.

If the weight of all the equations (3) was equal to unity, then the part which any one plate contributes towards the normal equations would be, as can be easily verified :—

$$\begin{aligned}
&+\tfrac{p-1}{p}a_1^2\pi_1 + \tfrac{p-1}{p}a_1b_1\kappa_1 - \tfrac{1}{p}a_1a_2\pi_2 - \tfrac{1}{p}a_1b_2\kappa_2 - \ldots = a_1n_1\\
&\quad + \tfrac{p-1}{p}b_1^2 \quad - \tfrac{1}{p}b_1a_2 - \tfrac{1}{p}b_1b_2 - \ldots = b_1n_2\\
&\qquad\qquad + \tfrac{p-1}{p}a_2^2 + \tfrac{p-1}{p}a_2b_2 - \ldots = a_2n_2\\
&\qquad\qquad\quad + \tfrac{p-1}{p}b_2^2 - \ldots = b_2n_2\\
&\quad\ldots\ldots\ldots\ldots\ldots
\end{aligned} \tag{5}$$

The weight is, however, not unity. We have seen in Chapter IV. that the square of the probable error of the measured co-ordinate y_i is :—

$$\rho_y^2 = \frac{p-1}{2pq}\zeta^2$$

If therefore k is the relative weight of any one plate, so that for that plate $\zeta^2 = \frac{1}{k}\zeta_0^2$, where ζ_0^2 is the adopted value for weight unity, then the weight of the equations (3) is

$$2qk\frac{p}{p-1}$$

To simplify the computations a new unit of weight is introduced, the value of which expressed in the old unit may be denoted by k_0. The partial normals (5) must thus be multiplied by

$$\frac{2qkp}{k_0p-1}.$$

I adopt for k_0 the mean value of $2q$, viz. :

$$k_0 = 12,$$

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and I introduce the further notations:

$$\frac{2q}{k_0}\frac{k}{} = \frac{1}{6} q\, k = \lambda^2$$

$$a_i\lambda = a_i' \quad b_i\lambda = b_i' \quad n_i\lambda = n_i'$$

The part which the plate under consideration contributes to the normal equations is thus

$$\begin{array}{lllll} a_1'^2\pi_1 + a_1'b_1'\kappa_1 - \frac{1}{p-1}a_1'a_2'\pi_2 - \frac{1}{p-1}a_1'b_2'\kappa_2 & - & . & . & . = \frac{p}{p-1}a_1'n_1' \\ + b_1'^2 \quad - \frac{p}{p-1}b_1'a_2' \quad - \frac{1}{p-1}b_1'b_2' & - & . & . & . = \frac{p}{p-1}b_1'n_1' \\ + a_2'^2 \quad + a_2'b_2' & - & . & . & . = \frac{p}{p-1}a_2'n_2' \\ + b_2'^2 & - & . & . & . = \frac{p}{p-1}b_2'n_2' \\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . & & & & \end{array} \tag{6}$$

For the plates of 1891 values of λ were adopted in accordance with the probable errors ζ found from the comparison of the pointings, as explained in Chapter IV. For the plates of 1903 and 1904 I have assigned equal weights to all plates, thus:—

$$\lambda = 1 \quad a_i' = a_i \quad b_i' = b_i \quad n_i' = n_i.$$

The formation of the normal equations is thus very symmetrical and simple. After forming the co-efficients a_i, b_i and the right-hand terms n_i for every plate (or a_i', b_i', n_i' in the case of the plates of 1891), all the products are multiplied out in the ordinary way, and added separately for the plates containing two, three, and four satellites respectively. These partial normals are then added together, the quadratic terms and the terms containing the two unknowns of one and the same satellite unaltered, all the other terms after being divided by $p-1$, and the sign being reversed. The right-hand members are also added for the three groups of plates separately, then multiplied by $\frac{p}{p-1}$, and the results of the three groups added together.

When values for the unknowns have been determined from the normal equations, these must be substituted in the equations of condition (3), and residuals must be formed. This substitution also is very simple. We compute the quantities:—

$$\mu_i' = a_i'\pi_i + b_i'\kappa_i$$

and their mean for every plate

$$\mu_0' = \frac{1}{p}(\mu_1' + \mu_2' + \ldots),$$

and form the differences

$$\nu_i' = \mu_i' - \mu_0'.$$

Then the difference

$$\Delta_i' = n_i' - \nu_i'$$

is evidently the required residual already multiplied by the square root of the weight λ. For the oppositions 1903 and 1904, where $\lambda = 1$, the accents can be dropped.

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The weight of the residual Δ' is evidently $\frac{p}{p-1}$. In order to derive the probable error corresponding to the weight unity, we must therefore form the squares of the residuals Δ', add them for each group of plates separately, multiply the sums by $\frac{p}{p-1}$, add the results for the three groups together and divide the sum by the number of equations *minus* the number of unknowns, *i.e.* by $(2r_2 + 3r_3 + 4r_4 - 8)$, if r_p is the number of plates containing p satellites ($p = 2$, 3 or 4). The probable error derived in this way will be called ρ_1. It corresponds to the new unit of weight. The square of the probable error corresponding to the old unit of weight is therefore

$$\zeta_1^2 = k_0 \rho_1^2 = 12 \rho_1^2 .$$

The interest attaching to the comparison of this probable error with the value of ζ_0^2 derived from the inter-agreement of the pointings (see Chapter IV.) is evident.

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CHAPTER XI.

SOLUTION FOR 1891.

THE plates of 1891 were measured and discussed first. They were considered at the beginning of the work as to a certain extent an experimental series, which would be useful to test the methods of measurement and reduction. They were very well suited for this purpose, since the results from the simultaneous heliometer measures of 1891 were available for comparison.

The images on the plates are not very good as a rule. The following plates have not been measured, the images being too faint or too diffused.

Plate.	Observer's remarks.
327	Impossible to measure; film full of spots from moisture, which cover images of satellites.
328	Same remark; spots occur only on centre part of plate, where the satellites are.
331	Spots in film of the form of canals of Mars; images too diffused.
334	Images too diffused; IV absolutely immeasurable.
339	Images too bad; II and III smudged together.

For all the plates which were measured the mean error ζ was derived in the way explained in Chapter IV., and weights were assigned to the plates in accordance with the resulting values of ζ, and corresponding to the value $\zeta_0^2 = 0{\cdot}0010$ for the square of the *mean* error of unit weight expressed in revolutions of the screw. This corresponds to a probable error $\zeta_0 = \pm 0''{\cdot}127$. The adopted weights were not strictly inversely proportional to the values of ζ^2 actually found, but were more uniform than the rigorous weights would be, and the notes made during the measures were also taken into consideration. All plates which gave a value of ζ^2 (mean error in revolutions) exceeding 0·0040 were rejected. These are the following:—

Plate.	ζ^2.	Observer's remarks.
329	·0080	Images diffused; III very near Jupiter.
335	·0124	Images very diffused; IV hardly measurable.
341	·0054	Images very diffused.
344	·0050	Images rather diffused.
345	·0050	Images very diffused.
346	·0044	Images very faint; IV almost immeasurable.
348	·0076	Images diffused, especially IV.
350	·0049	Rather good plate, III on *réseau* line; standard stars bad.

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On some plates there are spots in the film, evidently caused by moisture, and the film showed the colours of Newton's rings when looked at under a small angle with the surface. Some of the plates showing those spots were not measured, the spots interfering with the measures, but in some cases the part of the plate where the images of the standard stars and the satellites are, were free from spots. These plates have been measured, but were kept separate in the reductions. These are the plates 303, 311, 312, 313, and 326. They are in Table XII. distinguished by a dagger (†).

The multipliers c_i which were adopted are :—

$$c_1 = 200 \qquad c_2 = 110 \qquad c_3 = \frac{1500}{16} = 93{\cdot}75 \qquad c_4 = \frac{170}{4} = 42{\cdot}5.$$

The corrections Δp, Δq to the adopted values of the elements are thus

$$\begin{array}{llll} \Delta p_1 = 200\,\pi_1 & \Delta p_2 = 110\,\pi_2 & \Delta p_3 = 93{\cdot}75\,\pi_3 & \Delta p_4 = 42{\cdot}5\,\pi_4 \\ \Delta q_1 = 200\,\kappa_1 & \Delta q_2 = 110\,\kappa_2 & \Delta q_3 = 93{\cdot}75\,\kappa_3 & \Delta q_4 = 42{\cdot}5\,\kappa_4. \end{array}$$

The values of Δp and Δq found from the heliometer observations were introduced as a first approximation. The corresponding values of π_i and κ_i are :—

$$\begin{array}{llll} \pi_1 = +0{\cdot}00021 & \pi_2 = +0{\cdot}00066 & \pi_3 = -0{\cdot}00003 & \pi_4 = +0{\cdot}00150 \\ \kappa_1 = -0{\cdot}00013 & \kappa_2 = +0{\cdot}00077 & \kappa_3 = -0{\cdot}00074 & \kappa_4 = -0{\cdot}00031. \end{array}$$

From these values approximate values N′ of the right-hand terms of the equations of conditions were computed thus :—

$$M_i' = a_i'\,\pi_i + b_i'\,\kappa_i \qquad N_i' = M_i' - M_0',$$

and these were subtracted from the original right-hand terms n' which were derived in the manner explained in the preceding chapter.

From the differences $n_i' - N_i'$ we can derive the orientation of the plate, if we assume the heliometer solution to be correct. This determination of the orientation has, of course, a very small weight compared with that from the standard stars, but it is free from the objections to the latter which arise from the imperfect coincidence of the middle point of the standard stars with the mean of the images of Jupiter, and from the difference of hour-angle and declination between the exposures of the standard stars and the satellites. The corrections to the orientation-constants A, derived in this way (and multiplied by the square root of the weight λ), are given in Table XI.

The probable error of these corrections ΔA is, of course, different for different plates, since it depends largely on the values of the co-ordinate x'. I do not think any of them can considerably exceed its probable error. But they seem to have a systematic character, which is perhaps due to the difference of hour-angle in the exposures of the standard stars and Jupiter.

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TABLE XI.

ORIENTATION FROM HELIOMETER SOLUTION.

Plate.	λ. ΔA.	Plate.	λ. ΔA.	Plate.	λ. ΔA.
295	+·00001⁵	316	+·00003	332	−·00022
300	− 15	317	− 24	333	− 39
303	− 34	318	− 10	337	− 9
307	− 14	319	− 20	338	+ 7
308	− 22	320	− 60	340	− 17
311	− 22	321	− 10	342	+ 15
312	− 41	322	− 14	343	+ 16
313	− 21	324	− 11	349	+ 27
314	− 3	326	+ 1	352	+ 26
315	+ 4	330	+ 19	353	− 54

Two solutions were finally made, viz. :—

Solution I., in which the orientation-constants derived from the standard stars, and given in Table IV., were used ; and

Solution II., in which the corrections Δ A given in Table XI. were applied, the orientation thus being derived from the heliometer solution.

Table XII. contains :

the number of the plate and the square root of the weight λ ;

the date and Greenwich mean time, corrected for aberration ;

the tabular quantities v, ρ, x' and y', computed as explained in Chapter IX. ;

the coefficients of the equations of condition (see Chapter X.) :

$$a_i' = \lambda c_i \frac{\partial y_i'}{\partial p_i} \qquad b_i = \lambda c_i \frac{\partial y_i'}{\partial q_i},$$

the observed co-ordinates $y_{\text{obs.}}$ (mean of the six images and the two positions, *i.e.* the quantities which are called y_i at the end of Chapter IV.) ;

the combined corrections for orientation, refraction, and scale-value :

$$\Delta y = A(x_i' - x_0') + B y_{\text{obs.}}$$

(the values of A and B are given in Table IV.) ;

the approximate values N′ derived from the heliometer solution ;

the right-hand terms of the equations of condition $n' - N'$ for the two solutions I. and II. :

$$n_i' = m_i' - m_0' \qquad m' = y_{\text{obs.}} + \Delta y - y',$$

the residuals Δ_{II}' remaining after the substitution of the solution IIB.

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TABLE XII.

Reduction of Plates of 1891.

Plate λ.	Date G.M.T.	Sat.	v.	ρ.	x'.	y'.	a'.	b'.	$y_{obs.}$	Correction.	Heliometer N′.	$n'-N'$ I.	$n'-N'$ II.	$\Delta n'$.
	1891.													
295	Oct. 16	II.	266°885	1·00283	+ 3′·29	+′·0587	− 0·31	−5·61	+′·0296	−′·0006	−′·0007	−′·0012	−′·0012	−′·0014
0·8	5h 30m·57	III.	31·786	1·00096	− 0·87	−·1470	+ 6·51	+4·04	−·1762	+ 31	+ 6	+ 4	+ 5	+ 6
		IV.	234·642	1·00027	+ 5·34	+·1701	− 3·52	−4·96	+·1464	− 26	0	+ 9	+ 9	+ 10
300	Oct. 17	I.	37·655	0·99815	− 0·56	−·0585	+ 4·44	+3·42	−·0292	+·0012	+·0008	+·0015	+·0015	+·0014
0·7	5 29·45	II.	6·632	1·00835	+ 1·05	−·0688	+ 4·91	+0·57	−·0380	+ 3	+ 39	− 12	− 12	− 14
		III.	82·021	0·99983	− 4·99	−·0851	+ 0·93	+6·61	−·0678	+ 43	− 46	+ 6	− 3	− 1
		IV.	256·133	1·00290	+ 8·18	+·1034	− 1·29	−5·23	+·1350	− 58	0	− 10	+ 1	+ 1
303†	Oct. 19	I.	84·198	1·00125	− 2·00	−·0291	+ 0·80	+7·94	−·0428	−·0009	−·0033	+·0028	+·0014	+·0013
1·0	5 26·15	III.	182·749	0·99866	− 2·01	+·1307	− 9·44	−0·45	+·1172	− 25	− 19	0	− 14	− 15
		IV.	298·782	1·00666	+10·21	−·0595	+ 3·66	−6·66	−·0746	+ 33	+ 51	− 28	0	+ 1
307	Oct. 22	I.	337·843	0·99619	+ 1·60	−·0410	+ 6·57	−2·68	+·0355	−·0005	−·0014	·0000	·0000	+·0002
0·9	5 51·30	II.	155·578	0·99057	− 2·62⁵	+·0324	− 5·66	+2·57	+·1130	− 78	− 48	+ 5	− 1	0
		III.	334·579	1·00132	+ 4·31	−·0858	+ 7·66	−3·63	−·0141	+ 42	− 6	− 9	− 5	− 7
		IV.	2·995	1·00551	+ 3·48	−·2159	+ 6·75	+0·35	−·1344	+ 41	+ 69	+ 3	+ 6	+ 6
308	Oct. 23	I.	179·461	1·00337	− 0·90	+·0535	− 7·09	+0·07	−·0019	−·0011	+·0021	+·0004	·0000	·0000
0·9	5 39·77	II.	257·176	1·00205	+ 2·92	+·0650	− 1·39	−6·12	+·0018	+ 11	− 20	− 4	0	0
311†	Oct. 24	II.	356·620	1·00873	+ 1·60	−·0567	+ 6·30	−0·37	+·0574	−·0007	+·0028	−·0008	·0000	−·0001
0·9	5 33·40	III.	74·393	1·00020	− 4·48	−·0956	+ 2·26	+8·11	+·0084	0	− 72	+ 7	+ 2	+ 6
		IV.	45·602	1·00078	− 3·89	−·1813	+ 4·68	+4·77	−·0658	+ 7	+ 44	+ 1	− 3	− 6
312†	Oct. 26	I.	71·536	1·00082	− 1·68	−·0380	+ 2·98	+8·90	−·0492	+·0013	+·0005	−·0006	+·0005	+·0007
1·2	5 46·68	III.	175·638	0·99873	− 2·59	+·1173	−11·09	+0·84	+·1076	− 6	+ 8	− 14	− 6	− 6
		IV.	89·102	0·99553	− 9·11	−·0498	+ 0·14	+8·85	−·0584	− 7	− 14	+ 20	+ 1	− 1
313†	Oct. 27	I.	271·368	0·99789	+ 2·07	+·0201	+ 0·18	−7·76	−·0262	·0000	+·0016	−·0002	+·0003	+·0005
1·0	5 26·41	II.	301·214	1·00825	+ 3·54	+·0143	+ 3·56	−5·88	−·0374	0	− 16	− 15	− 6	− 6
		III.	225·364	0·99911	+ 2·17	+·1304	− 6·49	−6·58	+·0880	− 12	+ 53	− 3	+ 3	+ 1
		IV.	110·602	0·99371	− 9·89	+·0288	− 2·58	+6·87	−·0244	+ 11	− 54	+ 19	− 1	− 1
314	Oct. 28	I.	129·428	1·00366	− 2·14	+·0160	− 4·45	+5·42	−·0075	·0000	−·0022	+·0010	+·0010	+·0011
0·9	5 6·16	II.	48·007	1·00185	− 1·51	−·0777	+ 4·14	+4·60	−·0950	+ 11	+ 56	− 2	− 2	+ 1
		III.	279·200	1·00017	+ 5·48	+·0425	+ 1·33	−8·20	+·0244	+ 13	+ 55	− 6	− 6	− 9
		IV.	133·983	0·99272	− 9·18	+·1081	− 4·53	+4·70	+·0781	− 23	− 89	− 2	− 2	− 2
315	Oct. 29	I.	321·064	0·99618	+ 1·95	−·0263	+ 5·40	−4·36	−·0475	+·0021	+·0031	−·0009	−·0010	−·0009
0·9	5 43·35	II.	144·523	0·99062	− 2·97	+·0168	− 4·97	+3·54	−·0030	− 12	+ 8	− 3	− 2	− 1
		III.	326·561	1·00108	+ 4·69	−·0665	+ 6·93	−4·57	−·0861	+ 40	+ 46	+ 10	+ 8	+ 5
		IV.	154·611	0·99285	− 7·31	+·1625	− 5·87	+2·79	+·1368	− 50	− 83	+ 1	+ 4	+ 3
316	Oct. 29	I.	324·725	0·99617	+ 1·87⁵	−·0295	+ 6·30	−4·46	−·0529	+·0056	+·0035	−·0001	−·0002	·0000
1·0	6 9·05	II.	146·367	0·99059	− 2·91	+·0192	− 5·58	+3·71	+·0025	− 37	+ 8	0	+ 1	+ 2
		III.	327·458	1·00110	+ 4·64	−·0683	+ 7·78	−4·96	−·0954	+ 112	+ 51	+ 2	0	− 3
		IV.	155·003	0·99285	− 7·26⁵	+·1634	− 6·60	+3·08	+·1458	− 131	− 93	− 2	0	− 1

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TABLE XII.—Reduction of Plates of 1891—*continued.*

Plate λ.	Date G.M.T.	Sat.	v.	ρ.	x'.	y'.	a'.	b'.	$y_{obs.}$	Correction.	Heliometer N'.	$n'-N'$ I.	$n'-N'$ II.	Δ_{II}'.
	1891.													
317	Oct. 30	I.	164°·446	1·00362	− 1′·38	+′·0438	− 6·71	+1·87	+′·0034	′·0000	+′·0033	+′·0001	−′·0001	−′·0002
0·9	5 43^{m}·30	II.	247·058	1·00119	+ 2·46	+·0692	− 2·39	−5·65	+·0250	− 4	− 11	+ 8	+ 16	+ 15
		III.	16·752	1·00122	+ 0·59	−·1331	+ 7·93	+2·38	−·1784	+ 17	+ 29	− 23	− 20	− 18
		IV.	176·469	0·99394	− 4·34	+·1969	− 6·48	+0·40	+·1500	− 13	− 51	+ 15	+ 6	+ 7
318	Oct. 31	I.	8·713	0·99724	+ 0·53	−·0537	+ 4·55	+0·70	−·0518	+·0003	+·0026	−·0011	−·0010	−·0010
0·6	5^{h} 45·64	II.	347·183	1·00899	+ 2·06	−·0444	+ 3·98	−0·90	−·0372	− 2	+ 36	+ 9	+ 12	+ 11
		III.	67·056	1·00050	− 3·91	−·1033	+ 2·14	+5·06	−·1096	+ 19	− 21	− 3	− 6	− 4
		IV.	198·298	0·99591	− 0·76	+·2029	− 4·13	−1·37	+·1987	− 18	− 41	+ 7	+ 7	+ 6
319	Nov. 5	I.	312·768	0·99619	+ 2·03	−·0182	+ 5·12	−5·54	+·0176	−·0002	−·0018	+·0012	+·0009	+·0010
1·0	6 35·01	II.	137·744	0·99066	− 3·10	+·0072	− 4·86	+4·42	+·0409	− 3	− 34	+ 6	− 7	− 6
		III.	320·625	1·00105	+ 4·88	−·0516	+ 6·98	−5·72	−·0151	+ 2	+ 4	+ 1	+ 4	+ 1
		IV.	306·412	1·00712	+ 9·48	−·0820	+ 4·30	−5·82	−·0434	+ 3	+ 46	− 19	− 7	− 5
320	Nov. 6	I.	155·178	1·00371	− 1·60	+·0362	− 8·94	+4·13	+·0784	−·0017	−·0025	+·0030	+·0006	+·0004
1·3	6 27·80	II.	239·812	1·00079	+ 2·07	+·0701	− 4·30	−7·42	+·1046	− 11	− 86	− 1	− 3	− 5
		III.	10·556	1·00150	+ 1·16	−·1245	+11·47	+2·14	−·0865	+ 6	− 20	+ 1	− 6	− 5
		IV.	327·568	1·00730	+ 8·00	−·1420	+ 7·89	−5·02	−·0964	+ 23	+ 133	− 31	+ 3	+ 5
321	Nov. 7	I.	356·835	0·99697	+ 0·94	−·0478	+ 5·24	−0·29	+·0460	−·0005	−·0005	−·0002	−·0002	−·0002
0·7	6 12·06	II.	338·724	1·00913	+ 2·41	−·0329	+ 4·33	−1·69	+0·597	− 5	0	− 1	0	− 2
		III.	60·193	1·00089	− 3·33	−·1086	+ 3·12	+5·44	−·0235	0	− 57	+ 7	+ 2	+ 4
		IV.	348·611	1·00647	+ 5·45	−·1819	+ 4·98	−1·00	−·0821	+ 10	+ 62	− 2	+ 2	+ 1
322	Nov. 14	I.	346·519	0·99678	+ 1·25	−·0412	+ 5·70	−1·38	−·0212	−·0004	+·0037	+·0002	+·0006	+·0006
0·8	6 49·48	II.	331·036	1·00923	+ 2·67	−·0224	+ 4·54	−2·52	−·0033	− 8	+ 34	− 6	0	− 2
		III.	53·733	1·00111	− 2·74	−·1120	+ 4·15	+5·66	−·1022	+ 12	− 20	− 10	− 11	− 9
		IV.	140·539	0·99272	− 8·26	+·1161	− 4·27	+3·52	+·1266	− 1	− 52	+ 16	+ 7	+ 5
324	Nov. 19	I.	277·847	0·99665	+ 1·99	+·0135	+ 0·98	−7·14	−·0047	−·0001	−·0007	+·0030	+·0029	+·0030
1·0	6 9·54	II.	114·959	0·99119	− 3·27	−·0215	− 2·64	+5·68	−·0417	− 4	+ 9	− 9	− 15	− 13
		III.	304·229	1·00098	+ 5·17	−·0133	+ 4·85	−7·12	−·0304	+ 7	+ 34	+ 8	+ 11	+ 7
		IV.	248·606	1·00212	+ 6·59	+·1029	− 2·50	−6·39	+·0770	− 2	− 35	− 30	− 26	− 24
326†	Nov. 20	I.	132·351	1·00388	− 1·96	+·0163	− 2·43	+2·67	+·0118	+·0016	−·0001	+·0009	+·0009	+·0008
0·5	7 26·14	II.	223·145	0·99966	+ 1·12	+·0688	− 2·30	−2·16	+·0598	− 1	− 25	+ 1	+ 1	0
		III.	357·066	1·00176	+ 2·32	−·1040	+ 4·26	−0·22	−·1103	+ 11	+ 8	− 12	− 12	− 12
		IV.	271·170	1·00459	+ 8·58	+·0364	+ 0·07	−3·39	+·0387	− 26	+ 19	+ 1	0	+ 1
330	Nov. 25	I.	61·552	1·00184	− 1·25	−·0389	+ 1·69	+3·13	−·0005	−·0004	−·0024	−·0010	−·0006	−·0005
0·5	6 24·74	II.	3·779	1·00554	+ 1·10	−·0522	+ 3·11	+0·20	−·0042	0	− 1	+ 17	+ 17	+ 18
		III.	246·695	0·99928	+ 3·52	+·0928	− 1·65	−3·84	+·1389	− 8	+ 4	− 1	− 6	− 7
		IV.	16·779	1·00404	+ 1·08	−·1803	+ 3·20	+0·97	−·1341	+ 12	+ 21	− 8	− 8	− 7
332	Nov. 27	I.	107·131	1·00380	− 2·01	−·0058	− 1·66	+5·39	+·0654	−·0007	−·0019	+·0006	+·0005	+·0006
0·8	6 16·83	III.	346·905	1·00174	+ 3·08	−·0882	+ 6·55	−1·52	−·0174	− 1	0	− 11	− 1	− 1
		IV.	59·667	0·99894	− 5·29	−·1225	+ 2·71	+4·62	−·0480	+ 7	+ 18	+ 7	− 2	− 3

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TABLE XII.—Reduction of Plates of 1891—*concluded.*

Plate λ.	Date G.M.T.	Sat.	v.	ρ.	x′.	y′.	a′.	b′.	$y_{obs.}$	Correction.	Heliometer N′.	n′—N′ I.	n′—N′ II.	Δn′
	1891.		°		′	′	′	′	′	′	′	′	′	′
333	Nov. 30	I.	356·221	0·99783	+ 0·93	−·0425	+ 5·57	−0·37	−·0610	+·0011	+·0034	−·0019	−·0001	−·0001
0·8	6 5m·23	II.	150·558	0·99134	− 2·53	+·0186	− 4·22	+2·38	+.0009	+ 1	+ 12	+ 1	+ 6	+ 6
		III.	137·284	0·99872	− 4·65	+·0368	− 4·87	+4·49	+·0164	− 3	− 10	− 2	− 6	− 4
		IV.	124·786	0·99315	− 8·70	+·0638	− 3·00	+4·31	+·0436	− 9	− 36	+ 21	+ 2	0
337	Dec. 7	I.	353·145	0·99868	+ 1·02	−·0401	+ 5·41	−0·66	−·0430	+·0002	+·0012	+·0001	+·0001	·0000
0·8	7 33·24	II.	146·387	0·99144	− 2·62	+·0129	− 3·94	+2·61	+·0069	+ 9	− 6	0	− 3	− 4
		III.	132·579	0·99886	− 4·71	+·0257	− 4·38	+4·77	+·0166	+ 14	− 34	+ 7	+ 2	+ 3
		IV.	277·918	1·00517	+ 8·39	+·0190	+ 0·72	−5·15	+·0197	− 26	+ 27	− 7	0	0
338	Dec. 8	I.	185·815	1·00115	− 0·60	+·0448	− 5·42	−0·56	+·0017	+·0014	−·0009	+·0001	+·0003	+·0002
0·8	6h 22·95	II.	243·286	1·00486	+ 1·94	+·0627	− 2·15	−4·27	+·0170	− 2	− 46	+ 5	+ 5	+ 4
		III.	180·583	0·99840	− 1·94	+·1002	− 6·45	−0·07	+·0570	+ 16	+ 4	− 11	− 8	− 8
		IV.	298·194	1·00662	+ 8·73	−·0449	+ 2·45	−4·58	−·0756	− 29	+ 50	+ 6	+ 1	+ 3
340	Dec. 10	I.	233·185	0·99786	+ 0·91	+·0413	− 4·04	−5·40	+·0662	−·0009	−·0052	+·0008	+·0006	+·0006
1·0	6 25·33	II.	84·689	0·99245	− 2·64	−·0503	+ 0·54	+5·84	−·0209	− 4	− 2	+ 8	0	+ 3
		III.	281·469	1·00035	+ 4·78	+·0378	+ 1·60	−7·88	+·0654	− 4	+ 5	− 17	− 13	− 17
		IV.	340·763	1·00688	+ 6·08	−·1427	+ 6·10	−2·12	−·1107	+ 15	+ 48	+ 3	+ 9	+ 11
342	Dec. 14	I.	329·157	0·99696	+ 1·59	−·0255	+ 8·01	−4·79	+·0109	−·0012	+·0024	+·0010	+·0003	+·0004
1·4	6 34·71	II.	131·716	0·99096	− 2·90	−·0045	− 5·39	+6·05	+·0298	− 3	+ 12	+ 6	+ 6	+ 7
		III.	122·725	0·99931	− 4·81	+·0046	− 6·00	+9·34	+·0309	+ 2	− 66	− 21	− 18	− 14
		IV.	66·657	0·99811	− 5·62	−·1055	+ 3·51	+8·14	−·0718	+ 14	+ 29	+ 4	+ 8	+ 4
343	Dec. 15	I.	172·507	1·00168	− 1·04	+·0388	− 5·28	+0·69	+·0048	−·0006	+·0007	+·0002	·0000	·0000
0·8	6 38·85	II.	234·063	1·00424	+ 1·45	+·0648	− 2·75	−3·78	+·0282	− 20	− 28	+ 5	− 1	− 1
		III.	173·308	0·99849	− 2·58	+·0887	− 6·27	+0·74	+·0556	− 5	+ 16	+ 1	+ 1	+ 2
		IV.	88·430	0·99561	− 7·54	−·0493	+ 0·14	+5·03	−·0886	+ 30	+ 5	− 9	− 1	− 2
349	Dec. 23	I.	358·853	0·99913	+ 0·87	−·0405	+ 6·50	−0·14	−·0490	+·0009	+·0006	−·0005	+·0002	+·0003
1·0	6 21·55	II.	322·354	1·00781	+ 2·70	−·0064	+ 4·54	−3·50	−·0154	+ 3	− 5	− 5	− 3	− 5
		IV.	262·127	1·00376	+ 6·86	+·0702	− 0·85	−6·12	+·0644	− 11	− 2	+ 10	+ 1	+ 2
352	Dec. 29	I.	138·084	1·00308	− 1·65	+·0152	− 3·21	+2·88	−·0228	−·0001	+·0002	+·0003	·0000	−·0001
0·7	6 11·51	III.	156·717	0·99843	− 3·60	+·0624	− 4·87	+2·10	+·0228	+ 1	− 2	− 2	+ 1	+ 2
353	Dec. 30	I.	344·926	0·99855	+ 1·19	−·0320	+ 7·08	−1·90	+·0222	−·0006	−·0003	−·0016	−·0011	−·0008
1·2	6 33·33	II.	312·879	1·00812	+ 2·83	+·0055	+ 4·59	−4·95	+·0595	− 17	− 28	− 6	+ 8	+ 7
		IV.	51·745	0·99986	− 3·31	−·1388	+ 4·51	+5·71	−·0817	+ 24	+ 30	+ 22	+ 3	+ 2

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Two sets of normal equations were formed for each of the two solutions I. and II. viz. :—

A, rejecting, and

B, including the spotted plates 303, 311, 312, 313, 326.

These normal equations are :—

A.

$$
\begin{array}{rrrrrrrrrrrrrrrr}
+740{\cdot}9\,\pi_1 & -219{\cdot}8\,\kappa_1 & -\ 20{\cdot}6\,\pi_2 & -\ 95{\cdot}2\,\kappa_2 & -\ 20{\cdot}0\,\pi_3 & -\ 43{\cdot}2\,\kappa_3 & +\ 8{\cdot}0\,\pi_4 & -\ 22{\cdot}2\,\kappa_4 & = \Pi_1 \\
 & +296{\cdot}1 & -\ 47{\cdot}2 & +\ 54{\cdot}7 & +\ 20{\cdot}5 & -\ 35{\cdot}8 & +\ 2{\cdot}4 & -\ 12{\cdot}9 & = K_1 \\
 & & +367{\cdot}4 & -102{\cdot}4 & +\ 30{\cdot}4 & -\ 20{\cdot}8 & +\ 6{\cdot}3 & +\ 3{\cdot}7 & = \Pi_2 \\
 & & & +424{\cdot}6 & +\ 21{\cdot}1 & +\ 68{\cdot}2 & +\ 6{\cdot}2 & -\ 24{\cdot}6 & = K_2 \\
 & & & & +742{\cdot}2 & -185{\cdot}7 & +\ 10{\cdot}4 & +\ 42{\cdot}5 & = \Pi_3 \\
 & & & & & +581{\cdot}0 & +\ 8{\cdot}3 & -\ 7{\cdot}1 & = K_3 \\
 & & & & & & +450{\cdot}9 & -\ 62{\cdot}0 & = \Pi_4 \\
 & & & & & & & +461{\cdot}2 & = K_4
\end{array}
$$

B.

$$
\begin{array}{rrrrrrrrrrrrrrrr}
+\ 756{\cdot}3\,\pi_1 & -194{\cdot}8\,\kappa_1 & -\ 22{\cdot}7\,\pi_2 & -\ 96{\cdot}6\,\kappa_2 & +\ \ 4{\cdot}2\,\pi_3 & -\ 44{\cdot}0\,\kappa_3 & +\ 6{\cdot}6\,\pi_4 & -\ 35{\cdot}9\,\kappa_4 & = \Pi_1 \\
 & +505{\cdot}7 & -\ 35{\cdot}9 & +\ 41{\cdot}4 & +\ 86{\cdot}7 & -\ 54{\cdot}5 & -\ 19{\cdot}5 & -\ \ 5{\cdot}1 & = K_1 \\
 & & +425{\cdot}0 & -120{\cdot}6 & +\ 34{\cdot}3 & -\ 38{\cdot}7 & -\ \ 3{\cdot}3 & -\ 22{\cdot}0 & = \Pi_2 \\
 & & & +464{\cdot}0 & +\ 11{\cdot}9 & +\ 56{\cdot}6 & +\ \ 2{\cdot}1 & -\ 12{\cdot}7 & = K_2 \\
 & & & & +1019{\cdot}6 & -130{\cdot}7 & +\ 17{\cdot}5 & +\ 74{\cdot}4 & = \Pi_3 \\
 & & & & & +691{\cdot}1 & -\ 15{\cdot}6 & -\ 16{\cdot}9 & = K_3 \\
 & & & & & & +492{\cdot}9 & -\ 80{\cdot}8 & = \Pi_4 \\
 & & & & & & & +666{\cdot}7 & = K_4
\end{array}
$$

The values of the right-hand members are :—

	Solution I.		Solution II.	
	A.	B.	A.	B.
Π_1	− ·0721	− ·0735	− ·0166	− ·0147
K_1	− 66	+ 187	− 147	+ 77
Π_2	− 167	− 315	+ 118	+ 87
K_2	+ 182	+ 302	− 208	− 164
Π_3	− 188	+ 26	− 72	+ 141
K_3	− 150	− 52	− 435	− 433
Π_4	− 514	− 722	− 9	− 26
K_4	+ 934	+ ·1638	+ 284	+ 249

The normals were solved by the method explained in the appendix to my dissertation. The solutions are :—

A.

$$
\begin{array}{rrrrrrrrrrrrrrrr}
+{\cdot}00183\,\Pi_1 & +{\cdot}00140\,K_1 & +{\cdot}00037\,\Pi_2 & +{\cdot}00030\,K_2 & +{\cdot}00003\,\Pi_3 & +{\cdot}00021\,K_3 & -{\cdot}00003\,\Pi_4 & +{\cdot}00013\,K_4 & = \pi_1 \\
 & +{\cdot}00461 & +\ 64 & -\ 19 & -\ \ 1 & +\ 43 & -\ \ 4 & +\ 18 & = \kappa_1 \\
 & & +{\cdot}00303 & +\ 73 & -\ 14 & +\ \ 4 & -\ \ 5 & +\ \ 6 & = \pi_2 \\
 & & & +{\cdot}00269 & -\ 19 & -\ 34 & -\ \ 2 & +\ 16 & = \kappa_2 \\
 & & & & +{\cdot}00149 & +\ 49 & -\ \ 6 & -\ 14 & = \pi_3 \\
 & & & & & +{\cdot}00196 & -\ \ 5 & -\ \ 2 & = \kappa_3 \\
 & & & & & & +{\cdot}00226 & +\ 31 & = \pi_4 \\
 & & & & & & & +{\cdot}00224 & = \kappa
\end{array}
$$

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B.

$+\cdot00155\,\Pi_1$	$+\cdot00061\,K_1$	$+\cdot00026\,\Pi_2$	$+\cdot00033\,K_2$	$-\cdot00007\,\Pi_3$	$+\cdot00012\,K_3$	$+\cdot00003\,\Pi_4$	$+\cdot00012\,K_4$	$=\pi_1$
	$+\cdot00229$	$+\ 26$	$-\ 2$	$-\ 19$	$+\ 20$	$+\ 12$	$+\ 10$	$=\kappa_1$
		$+\cdot00262$	$+\ 71$	$-\ 12$	$+\ 10$	$+\ 5$	$+\ 14$	$=\pi_2$
			$+\cdot00244$	$-\ 8$	$-\ 16$	$+\ 1$	$+\ 9$	$=\kappa_2$
				$+\cdot00103$	$+\ 18$	$-\ 6$	$-\ 13$	$=\pi_3$
					$+\cdot00152$	$+\ 5$	$+\ 3$	$=\kappa_3$
						$+\cdot00209$	$+\ 27$	$=\pi_4$
							$+\cdot00156$	$=\kappa_4$

Substituting the values of the right-hand members Π_1, K_1 . . . in the solutions we find the following values of the unknowns :—

	Solution I.		Solution II.	
	A.	B.	A.	B.
π_1	$-\cdot000132$	$-\cdot000084$	$-\cdot000058$	$-\cdot000024$
κ_1	$-\ 133$	$-\ 5$	$-\ 93$	$+\ 1$
π_2	$-\ 58$	$-\ 58$	$+\ 6$	$+\ 6$
κ_2	$+\ 41$	$+\ 43$	$-\ 29$	$-\ 31$
π_3	$-\ 49$	$-\ 12$	$-\ 34$	$+\ 4$
κ_3	$-\ 63$	$-\ 20$	$-\ 92$	$-\ 58$
π_4	$-\ 82$	$-\ 109$	$+\ 10$	0
κ_4	$+\ 188$	$+\ 228$	$+\ 58$	$+\ 34$

The solutions A and B are in satisfactory agreement. There is thus no reason to reject the spotted plates. It will be seen from Table XII. that these plates do not give larger residuals than the others. I will therefore adopt Solution B.

The choice between the solutions I. and II. is more difficult. Solution II. certainly agrees better with the heliometer solution, especially for Satellite IV. This agreement is, however, artificial, since the orientation for this solution was determined from the heliometer solution, and the orientation-constant in this determination naturally depends chiefly on Satellite IV, which has on the average the largest values of x'. What has been done is thus practically this: Satellite IV was made to agree with the heliometer solution by adopting a suitable orientation-constant, and the measures of the other satellites were then reduced with this same constant. On the other hand, the determination of the orientation from the standard stars, on which Solution I. was based, is *a priori* subject to suspicion of systematic errors, while the heliometer solution is certainly systematically sound. The large corrections to the heliometer solution found from Solution I. increase this suspicion. It is thus probable that by determining the orientation from the heliometer solution instead of from the standard stars, we eliminate real systematic errors, and therefore improve our result, notwithstanding the fact that the orientation is determined with a much smaller weight from the heliometer solution than from the standard stars. The fact that the corrections to the heliometer solution

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are greatly diminished, not only for Satellite IV, but also for the other satellites, especially so for Satellite I which has relatively little influence on the determination of the orientation-constant, strengthens this conclusion. I will, therefore, finally adopt Solution IIB.

This solution was substituted in the equations of condition, and residuals were formed in the manner which was explained in the preceding Chapter. These residuals, which are already multiplied by the square root of the weight λ, are given in the last column of Table XII. under the heading Δ'_{II}. The probable error of weight unity derived from them is :—

$$\rho_1 = \pm 0'{\cdot}00063 = \pm 0''{\cdot}038.$$

According to Chapter X. we have $\zeta^2 = 12\rho^2$. From the adopted value of ζ_0 for weight unity derived from the agreement *inter se* of the different images on the same plate, we find thus :—

$$\rho_0 = \pm 0''{\cdot}037.$$

The agreement is eminently satisfactory, and shows the absence of systematic errors.

Adding the values of the unknowns found from this Solution IIB. to the first approximation, we have finally :—

	Weight.		Weight.
$\pi_1 = +\ {\cdot}000186$	645	$\kappa_1 = -\ {\cdot}000129$	437
$\pi_2 = +\ {\cdot}000666$	382	$\kappa_2 = +\ {\cdot}000739$	410
$\pi_3 = -\ {\cdot}000026$	971	$\kappa_3 = -\ {\cdot}000798$	813
$\pi_4 = +\ {\cdot}001500$	690	$\kappa_4 = -\ {\cdot}000276$	800

From these we find :—

$\Delta p_1 = +$	$0^s{\cdot}0372 \pm 0^s{\cdot}0050$	$\Delta q_1 = -$	$0^s{\cdot}0258 \pm 0^s{\cdot}0061$
$\Delta p_2 = +$	${\cdot}0733 \pm$ 36	$\Delta q_2 = +$	${\cdot}0813 \pm$ 35
$\Delta p_3 = -$	${\cdot}0024 \pm$ 19	$\Delta q_3 = -$	${\cdot}0748 \pm$ 23
$\Delta p_4 = +$	${\cdot}0638 \pm$ 12	$\Delta q_4 = -$	${\cdot}0117 \pm$ 11

The values found from the heliometer solution were :—

$\Delta p_1 = +$	$0^s{\cdot}0409 \pm 0^s{\cdot}0052$	$\Delta q_1 = -$	$0^s{\cdot}0259 \pm 0^s{\cdot}0056$
$\Delta p_2 = +$	${\cdot}0740 \pm$ 36	$\Delta q_2 = +$	${\cdot}0853 \pm$ 33
$\Delta p_3 = -$	${\cdot}0033 \pm$ 23	$\Delta q_3 = -$	${\cdot}0683 \pm$ 20
$\Delta p_4 = +$	${\cdot}0642 \pm$ 12	$\Delta q_4 = -$	${\cdot}0137 \pm$ 12

The probable errors of the values derived from the plates and from the heliometer observations are nearly the same. The former are, however, systematically not entirely

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independent of the latter. I will therefore combine them with the relative weights 2 (heliometer) and 1 (plates). We have then* :—

$\Delta p_1 = +$	$0^\circ\cdot0397 \pm 0^\circ\cdot0038$	$\Delta q_1 = -$	$0^\circ\cdot0259 \pm 0^\circ\cdot0043$
$\Delta p_2 = +$	$\cdot0738 \pm 27$	$\Delta q_2 = +$	$\cdot0840 \pm 25$
$\Delta p_3 = -$	$\cdot0030 \pm 17$	$\Delta q_3 = -$	$\cdot0705 \pm 16$
$\Delta p_4 = +$	$\cdot0641 \pm 9$	$\Delta q_4 = -$	$\cdot0130 \pm 9$

These are thus the final corrections to the elements p and q for the epoch 1891·75. This is the epoch of the heliometer observations; the mean epoch of the plates is 1891·87. In making the solution it is necessarily assumed that the corrections Δp and Δq do not vary during the period of observation. It is therefore allowed to consider the corrections derived from the plates as valid for the epoch of the heliometer observations, on which the orientation of the plates was based.

Practically, these corrections to the elements are such that the relative positions of the four orbital planes depend on the heliometer and on the plates, with relative weights 2 and 1, while the orientation of the whole system is determined by the heliometer alone.

* The probable errors given for the heliometer are those contained in my dissertation, which correspond to a probable error of weight unity of $\pm 0''\cdot0869$. The true probable error resulting from a new discussion is $\pm 0''\cdot0795$. The probable errors of the final values are therefore in reality slightly smaller than those given above.

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CHAPTER XII.

SOLUTION FOR 1903.

ALL plates taken during the opposition of 1903 have been measured. Plate 7656 was afterwards rejected: the images on this plate are very diffused, all images of Satellite IV and two images of Satellite III are on a réseau line, and the trail is interrupted in several places so as to be entirely useless. The measures of Satellite IV on plate 7657 were also rejected. All images are very faint, and two of them are on a réseau line, and the comparison of the pointings shows that these are affected with very large errors.

All the other plates were included in the discussion, and equal weights were assigned to all.

Plate 7570 was remeasured, the comparison of the pointings having given rise to the suspicion that in one or more pointings a mistake might have been made in recording the whole number of revolutions. The first measure was rejected.

The plates are on the whole considerably better than those of 1891.

The multipliers c are for the four satellites.

$$c_1 = 200 \qquad c_2 = 100 \qquad c_3 = 75 \qquad c_4 = 50.$$

We have therefore :—

$$\Delta p_1 = 200\,\pi_1 \qquad \Delta p_2 = 100\,\pi_2 \qquad \Delta p_3 = 75\,\pi_3 \qquad \Delta p_4 = 50\,\pi_4$$
$$\Delta q_1 = 200\,\kappa_1 \qquad \Delta q_2 = 100\,\kappa_2 \qquad \Delta q_3 = 75\,\kappa_3 \qquad \Delta q_4 = 50\,\kappa_4.$$

As a first approximation, I introduced the values

$$\pi_2 = +0{\cdot}001 \qquad \kappa_2 = -0{\cdot}002.$$

The Table XIII. is similar to Table XII., and contains the tabular quantities v, ρ, x', y' and the coefficients of the equations of condition :

$$a = c\frac{\partial y'}{\partial p} \qquad b = c\frac{\partial y'}{\partial q};$$

further, the observed co-ordinates $y_{\text{obs.}}$, and the corrections

$$\Delta y = \mathrm{A}(x_i - x_0) + \mathrm{B}y_{\text{obs.}}$$

The values of A are given in Table IV. B is constant: $\mathrm{B} = -0{\cdot}0105$.

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The next column of the table contains the right-hand members of the equations of condition, which were computed by the formulæ:—

$$n_i = m_i - m_0$$
$$m = y_{obs.} + \Delta y - y' \quad \text{for Satellites I, III and IV.}$$
$$m_2 = y_{obs.} + \Delta y_2 - \{y'_2 + 0{\cdot}0001 a_2 - 0{\cdot}0002 b_2\} \quad \text{for Satellite II.}$$

The column headed Δn contains the correction which must be applied to the values of n thus computed, to reduce them to what they would be if the orientation had been derived from the trails instead of from the standard stars. This correction is:—

$$\Delta n_i = (x'_i - x'_0)\,\Delta A.$$

The values of ΔA have been given in Table VIII.

The column δn gives the correction to be applied to n on account of the error committed in computing the index-correction from the standard stars which was mentioned above in Chapter VI., viz.:—

$$\delta n_i = (x'_i - x'_0)\,\delta A.$$

The values of δA are contained in Table IV.

To compute the effect on the solution of a constant correction to the adopted position-angle of the standard stars the differential coefficients $\frac{dn}{\sin 1'' dP_s}$ were used as right-hand members of the equations of condition. These differential coefficients are:—

$$\frac{dn_i}{\sin 1'' dP_s} = -(x'_i - x'_0)$$

They are not given in Table XIII., which contains already in another column the values of x'_i.

TABLE XIII.

Reduction of Plates of 1903.

Plate	Date G.M.T.	Sat.	v	ρ	x'	y'	a	b	$y_{obs.}$	Correction.	n	δn	Δn	Δ_2	Δ_2
	1903.		°		′	′			′	′	′	′	′	′	″
7510	Aug. 17	I.	233·774	1·00319	+ 0·73	+·08100	−4·94	−6·74	+·03464	−·00037	+·00323	·00000	−·00001	−·00077	−·0000 79
	$10^h 20^m 12$	II.	308·877	1·00250	+ 3·80	−·02378	+4 18	−5·18	−·06056	+ 86	− 50	− 7	+ 26	− 173	− 63
		III.	74·714	1·00052	− 3·81	−·17783	+2·09	+7·66	−·23181	+ 211	− 191	+ 11	− 40	+ 156	+ 27
		IV.	229·149	0·99795	+ 2·43	+·30583	−6·06	−7·00	+·25766	− 259	− 80	− 4	+ 14	+ 94	+
7513	Aug. 18	I.	81·594	0·99816	− 1·71	−·05903	+1·22	+8·24	−·06502	+·00003	−·00188	−·00004	+·00033	−·00128	−·000 41
	10 46·47	II.	50·578	1·00320	− 0·98	−·15376	+4·27	+5·19	−·16564	+ 154	− 15	− 1	+ 10	+ 71	+ 48
		III.	125·923	1·00006	− 6·07	−·01269	−4·66	+6·44	− 01712	− 313	− 348	− 22	+ 168	− 238	− 18
		IV.	251·141	1·00071	+ 6·17	+·24794	−3·00	−8·80	+·24779	+ 157	+ 550	+ 27	− 211	+ 293	+
7518	Aug. 19	I.	288·390	1·00012	+ 2·29	+·02510	+2·64	−7·94	−·08082	+·00056	−·00093	+·00001	+·00004	−·00055	−·0000 41
	11 15·60	II.	153·730	0·99364	− 3·34	+·08903	−5·93	+2·92	−·02786	+ 136	+ 67	− 4	− 13	+ 148	+ 99
		III.	177·279	0·99955	− 3·77	+·16216	−7·94	+0·38	+·05700	+ 62	− 11	− 4	− 14	+ 133	+ 79
		IV.	273·058	1·00338	+ 9·03	+·15317	+0·50	−9·34	+·05168	− 257	+ 37	+ 7	+ 23	− 225	− 36

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TABLE XIII.—REDUCTION OF PLATES OF 1903—*continued.*

Plate	Date G.M.T.	Sat.	v	ρ	x'	y'	a	b	$y_{obs.}$	Correction.	n	δn	Δn	Δ_s	Δ_t
	1903.		°		′	′									
7522	Aug. 20	I.	125·423	1·00103	− 2·40	+·00002	−4·86	+6·84	−·10620	+·00121	−·00807	+·00013	+·00151	−·00217	−·00128
	10h23m·92	II.	253·086	0·99449	+ 2·31	+·11847	−1·93	−6·34	+·03115	− 31	− 144	+ 1	+ 16	+ 10	+ 20
		III.	225·848	0·99934	+ 1·08	+·21625	−5·54	−5·71	+·12140	− 125	+ 84	+ 4	+ 51	− 55	− 25
		IV.	293·670	1·00547	+10·54	+·04231	+3·78	−8·60	−·04632	+ 35	+ 866	− 19	− 219	+ 262	+ 132
7524	Aug. 21	I.	330·032	0·99752	+ 2·18	−·03567	+7·26	−4·18	−·00602	+·00056	−·00124	−·00010	−·00096	+·00102	−·00012
	10 37·08	II.	354·964	1·00851	+ 2·51	−·13463	+6·71	−0·59	−·10128	+ 152	− 447	− 9	− 85	+ 216	+ 116
		III.	276·687	0·99956	+ 5·34	+·11649	+0·93	−7·92	+·15010	− 161	+ 55	+ 1	+ 5	− 50	− 44
		IV.	315·168	1·00689	+10·66	−·07996	+6·69	−6·65	−·04280	− 47	+ 518	+ 19	+ 176	− 267	− 59
7533	Aug. 26	I.	265·168	1·00131	+ 1·86	+·05466	−0·72	−8·42	+·06856	−·00122	+·00155	+·00023	+·00102	+·00062	+·00184
	10 21·32	II.	139·424	0·99483	− 3·71	+·05538	−5·08	+4·35	+·05120	− 44	− 197	− 4	− 20	− 203	− 227
		III.	167·510	0·99965	− 4·51	+·13806	−7·83	+1·73	+·14976	− 139	− 82	− 8	− 37	0	− 45
		IV.	61·683	1·00027	− 4·89	−·27883	+4·48	+8·32	−·26952	+ 305	+ 123	− 10	− 45	+ 142	+ 88
7537	Aug. 27	I.	108·129	1·00026	− 2·32	−·02433	−2·64	+8·06	−·00136	+·00001	−·00211	−·00002	−·00009	−·00080	−·00082
	10 11·66	II.	241·767	0·99359	+ 1·74	+·13743	−3·16	−5·89	+·17601	− 184	+ 403	− 17	− 69	+ 18	+ 9
		IV.	83·153	0·99758	− 8·09	−·19602	+1·12	+9·36	−·17466	+ 182	− 191	+ 19	+ 78	+ 62	+ 73
7538	Aug. 28	I.	311·597	0·99822	+ 2·40	−·01060	+5·60	−6·32	+·00699	−·00004	+·00259	+·00005	−·00057	−·00022	−·00062
	10 17·26	II.	343·301	1·00800	+ 3·04	−·11478	+6·50	−1·95	−·09348	+ 103	− 303	+ 6	− 72	− 117	− 168
		III.	268·210	0·99929	+ 4·95	+·14048	−0·26	−8·04	+·16296	− 164	+ 688	+ 10	− 118	+ 98	+ 15
		IV.	104·961	0·99520	−10·16	−·08433	−2·28	+9·09	−·07646	+ 65	− 644	− 21	+ 247	+ 39	+ 213
7555	Aug. 31	I.	198·707	1·00374	− 0·65	+·08288	−8·06	−2·72	+·04398	−·00025	+·00526	−·00003	−·00023	+ 00042	+·00032
	9 51·20	II.	286·119	1·00044	+ 3·68	+·03652	+1·88	−6·50	+·01106	+ 97	+ 564	− 14	− 115	+ 96	+ 45
		III.	58·183	1·00087	− 2·51	−·20580	+4·27	+6·88	−·25333	+ 251	− 61	+ 2	+ 16	+ 58	+ 65
		IV.	170·133	0·99300	− 7·49	+·24914	−9·24	+1·61	+·19768	− 324	− 1029	+ 15	+ 122	− 196	− 142
7565	Sept. 1	III.	109·038	1·00049	− 5·97	−·07002	−2·64	+7·65	−·18754	+·00164	+·00324	+·00003	·00000	+·00028	+·00016
	10 8·72	IV.	192·254	0·99427	− 4·01	+·30826	−9·18	−2·00	+·18754	− 164	− 324	− 3	0	− 29	− 17
7569	Sept. 2	I.	239·398	1·00245	+ 1·05	+·07741	−4·34	−7·34	+·01340	+·00140	−·00102	+·00001	+·00041	−·00237	−·00163
	9 8·64	II.	123·836	0·99628	− 3·86	+·01412	−3·76	+5·60	−·06170	+ 15	+ 88	0	− 13	+ 100	+ 76
		III.	157·244	0·99979	− 5·17	+·10819	−7·46	+3·13	+·04830	− 155	+ 15	− 1	− 28	+ 137	+ 87
7570	Sept. 3	I.	86·482	0·99918	− 1·94	−·05176	+0·52	+8·50	−·18892	+·00273	−·00241	−·00004	−·00020	−·00062	−·00105
	9 28·78	II.	228·252	0·99268	+ 0·96	+·15139	−4·47	−5·01	+·02995	− 30	+ 473	0	0	+ 110	+ 110
		IV.	235·108	0·99887	+ 3·92	+·29088	−5·41	−7·76	+·15898	− 243	− 231	+ 4	+ 20	− 47	− 4
7580	Sept. 10	I.	70·596	0·99850	− 1·49	−·06692	+2·84	+8·06	−·01178	−·00025	−·00623	−·00002	+·00024	−·00056	−·00040
	9 26·77	III.	200·469	0·99912	− 1·34	+·20726	−7·58	−2·83	+·27198	− 318	+ 42	− 1	+ 20	+ 19	+ 31
		IV.	24·624	1·00452	+ 1·59	−·32369	+8·72	+4·00	−·26018	+ 341	+ 580	+ 3	− 44	+ 36	+ 7
7581	Sept. 12	I.	101·209	1·00056	− 2·28	−·03074	−1·66	+8·38	+·06951	−·00104	−·00232	−·00004	+·00004	−·00004	−·00009
	7 30·74	II.	50·868	1·00619	− 1·22	−·15074	+4·32	+5·31	−·05778	+ 53	− 174	− 1	+ 1	+ 73	+ 71
		III.	297·208	0·99966	+ 6·17	+·03412	+3·70	−7·22	+·13864	+ 5	+ 304	+ 22	− 20	− 101	− 79
		IV.	65·837	0·99972	− 5·98	−·25245	+3·90	+8·70	−·15038	+ 47	+ 101	− 16	+ 15	+ 32	+ 16
7582	Sept. 14	I.	156·172	1·00343	− 2·04	+·04702	−7·82	+3·46	+·07604	−·00062	+·00428	+·00002	+·00013	+·00232	+·00249
	8 29·00	II.	259·544	0·99779	+ 2·84	+·09868	−1·23	−6·65	+·13652	+ 22	+ 187	+ 18	+ 114	− 297	− 150
		III.	39·781	1·00098	− 0·80	−·21448	+6·25	+5·20	−·19546	+ 262	− 248	+ 6	+ 38	− 98	− 49
		IV.	110·104	0·99486	−10·61	−·03973	−3·26	+8·90	−·01709	− 220	− 368	− 26	− 166	+ 160	− 53
7593	Sept. 15	I.	1·566	0·99623	+ 1·24	−·07197	+8·50	+0·24	−·00140	−·00052	+·00658	−·00011	−·00077	+·00133	+·00107
	8 43·67	II.	0·950	1·00926	+ 2·04	−·14625	+6·85	+0·11	−·07428	+ 15	+ 202	− 12	− 90	− 3	− 33
		III.	90·517	1·00068	− 5·28	−·12413	−0·08	+8·12	−·06259	+ 92	− 101	+ 5	+ 37	− 60	− 48
		IV.	132·157	0·99342	−10·66	+·08186	−6·34	+7·00	+·13827	− 54	− 760	+ 18	+ 130	− 69	− 26

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TABLE XIII.—Reduction of Plates of 1903—*concluded.*

Plate	Date G.M.T.	Sat.	v	ρ	x′	y′	a	b	$y_{obs.}$	Correction.	n	δn	Δn	Δ_2	Δ_4
	1903.														
7596	Sept. 16	II.	106°056	0·99756	− 3′72	−′03059	−1·87	+6·50	−′11552	+′00205	+′00269	+′00011	+′00025	−′00184	−′00147
	9h51m·53	III.	143·178	0·99986	− 5·78	+·06355	−6·49	+4·85	−·00432	+ 18	+ 301	+ 2	+ 4	− 16	− 10
		IV.	155·061	0·99291	− 9·07	+·19400	−8·54	+3·98	+·11984	− 224	− 570	− 13	− 29	+ 200	+ 158
7601	Sept. 17	II.	200·821	0·99133	− 0·75	+·15816	−6·28	−2·39	−·04297	+·00007	+·00512	−·00006	−·00111	−·00022	−·00105
	7 57·94	III.	189·574	0·99924	− 2·36	+·19044	−7·97	−1·34	−·00994	− 2	+ 428	− 2	− 38	− 46	− 74
		IV.	175·211	0·99330	− 6·47	+·26095	−9·38	+0·78	+·05291	− 5	− 941	+ 8	+ 150	+ 67	+ 179
7604	Sept. 18	I.	247·869	1·00129	+ 1·44	+·06720	−3·22	−7·90	−·07167	+·00075	+·00238	·00000	·00000	−·00062	−·00062
	8 17·15	II.	304·034	1·00531	+ 3·90	−·02031	+3·81	−5·04	−·14724	+ 123	− 29	− 7	+ 30	− 32	+ 28
		III.	240·055	0·99910	+ 2·97	+·19150	−3·96	−7·04	+·05825	− 81	+ 644	− 4	+ 19	+ 29	+ 66
		IV.	197·339	0·99475	− 2·72	+·30857	−9·00	−2·81	+·16068	− 115	− 854	+ 11	− 50	+ 64	− 35
7606	Sept. 25	I.	225·490	1·00229	+ 0·61	+·07869	−5·96	−6·06	+·05936	−·00059	−·00378	+·00006	+·00059	−·00050	−·00024
	7 28·58	II.	290·314	1·00407	+ 3·80	+·01549	+2·34	−6·34	+·01012	− 10	− 435	0	− 5	+ 49	+ 46
		III.	231·164	0·99909	+ 2·12	+·19787	−5·04	−6·26	+·18462	− 192	+ 97	+ 3	+ 29	+ 73	+ 86
		IV.	346·879	1·00699	+ 7·61	−·24251	+9·28	−2·16	−·25410	+ 260	+ 715	− 8	− 83	− 72	− 110
7608	Sept. 26	I.	70·885	0·99927	− 1·56	−·06182	+2·88	+7·98	+·02416	+·00010	−·00637	+·00015	+·00036	+·00064	+·00045
	7 36·64	III.	281·839	0·99946	+ 5·81	+·07979	+1·64	−7·87	+·17458	− 207	+ 27	− 10	− 25	− 17	− 4
		IV.	8·286	1·00597	+ 4·19	−·29532	+9·42	+1·37	−·19874	+ 198	+ 611	− 5	− 11	− 47	− 40
7634	Oct. 2	II.	279·888	1·00325	+ 3·60	+·04045	+1·15	−6·61	−·06388	+·00131	+·00591	+·00005	−·00235	+·00316	+·00125
	7 27·35	III.	223·335	0·99937	+ 1·39	+·19755	−5·80	−5·47	+·08416	− 52	+ 939	+ 3	− 137	+ 243	+ 132
		IV.	137·793	0·99308	−10·15	+·11754	−0·88	+0·24	−·02028	− 79	− 1531	− 8	+ 372	− 500	− 258
7645	Oct. 7	I.	144·109	1·00354	− 2·18	+·03218	−6·78	+4·90	+·01118	−·00029	+·00009	−·00002	−·00028	−·00023	−·00060
	7 2·12	II.	62·993	1·00205	− 2·09	−·11814	+3·01	+5·92	−·14987	+ 141	− 11	− 2	− 26	+ 19	− 15
		III.	113·964	1·00022	− 6·03	−·03271	−3·22	+7·25	−·05283	− 18	+ 108	− 10	− 117	+ 297	+ 144
		IV.	246·338	1·00008	+ 6·34	+·21300	−3·72	−8·48	+·19153	− 95	− 104	+ 14	+ 171	− 291	− 67
7647	Oct. 8	I.	347·709	0·99624	+ 1·58	−·05704	+8·10	−1·76	−·13406	+·00164	−·00018	+·00002	−·00054	+·00108	+·00058
	7 3·72	II.	165·504	0·99080	− 2·61	+·11505	−6·34	+1·04	+·03008	− 157	− 172	− 14	+ 299	− 231	+ 49
		III.	164·342	0·99970	− 4·28	+·12732	−7·00	+2·13	+·04834	− 237	− 615	− 20	+ 440	− 457	− 45
		IV.	267·870	1·00277	+ 9·05	+·12511	−0·34	−9·27	+·05506	+ 229	+ 804	+ 32	− 684	+ 581	− 60
7653	Oct. 9	I.	186·567	1·00345	− 0·93	+·07051	−8·26	−0·96	+·01140	−·00082	−·00066	−·00001	−·00053	+·00259	+·00160
	6 32·42	II.	265·687	1·00184	+ 3·15	+·07107	−0·50	−6·59	+·02292	− 39	− 255	0	− 11	− 63	− 84
		IV.	288·803	1·00500	+10·46	+·02293	+3·00	−8·80	−·03432	+ 120	+ 322	+ 1	+ 64	− 195	− 75
7655	Oct. 10	II.	6·598	1·00840	+ 1·48	−·14254	+6·61	+0·76	−·11316	+·00099	−·00855	+·00003	+·00209	−·00096	+·00072
	6 46·42	III.	264·489	0·99908	+ 4·93	+·12067	−0·75	−7·82	+·15040	− 168	+ 22	0	+ 35	− 56	− 27
		IV.	310·331	1·00064	+10·46	−·08471	+6·02	−7·08	−·04324	+ 69	+ 833	− 3	− 244	+ 150	− 46
7657	Oct. 15	I.	323·809	0·99634	+ 2·12	−·03116	+6·60	−4·82	−·07778	+·00016	+·00068	−·00014	0	−·00018	+·00033
	6 5·82	II.	150·877	0·99094	− 3·14	+·08635	−5·64	+3·14	+·02920	− 11	+ 180	+ 4	0	+ 91	+ 76
		III.	154·478	0·99998	− 4·84	+·09812	−7·02	+3·35	+·04856	− 5	− 247	+ 10	0	− 71	− 108
7660	Oct. 19	I.	57·905	0·99952	− 1·14	−·00292	+4·30	+6·88	−·07892	+·00065	+·00130	−·00003	−·00015	+·00025	+·00021
	6 3·02	II.	197·002	0·99287	− 0·74	+·13098	−6·12	−1·87	+·12824	− 161	+ 568	− 4	− 22	− 43	− 49
		III.	355·909	1·00049	+ 3·19	−·14734	+7·71	−0·56	−·16240	+ 60	+ 213	− 18	− 88	+ 24	+ 2
		IV.	143·316	0·99284	− 9·34	+·13928	−7·20	+5·36	+·11316	+ 36	− 911	+ 25	+ 125	− 6	+ 27
7662	Oct. 21	I.	103·442	1·00242	− 2·24	−·01841	−1·88	+7·88	−·01999	+·00040	+·00306	+·00002	+·00009	+·00043	+·00057
	5 53·51	II.	37·511	1·00430	− 0·58	−·13719	+5·12	+3·94	−·14544	+ 212	+ 87	+ 5	+ 29	− 130	− 84
		III.	96·007	1·00101	− 5·43	−·08134	−0·80	+7·05	−·08481	+ 30	+ 107	− 5	− 29	+ 68	+ 22
		IV.	186·917	0·99380	− 3·70	+·25067	−8·80	−1·08	+·25024	− 282	− 501	− 2	− 9	+ 20	+ 6
7664	Oct. 22	I.	306·010	0·99663	+ 2·28	−·00954	+4·72	−6·48	−·04470	−·00061	+·00078	−·00019		−·00088	
	5 50·71	II.	139·320	0·99118	− 3·39	+·06074	−4·81	+4·53	+·01208	+ 21	+ 197	+ 6		+ 140	
		III.	146·168	1·00012	− 5·18	+·07232	−6·35	+4·26	+·03262	+ 41	− 274	+ 14		− 50	

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The normal equations are :—

$$
\begin{array}{rrrrrrrrl}
+698{\cdot}2\,\pi_1 & -\ 52{\cdot}3\,\kappa_1 & +\ 23{\cdot}8\,\pi_2 & -\ 96{\cdot}7\,\kappa_2 & +\ 47{\cdot}5\,\pi_3 & +\ 37{\cdot}3\,\kappa_3 & -\ 31{\cdot}8\,\pi_4 & -\ 39{\cdot}5\,\kappa_4 & = \Pi_1 \\
 & +1062{\cdot}6 & -\ 19{\cdot}1 & +\ 32{\cdot}7 & -102{\cdot}0 & +\ 25{\cdot}8 & -\ 22{\cdot}7 & -\ 40{\cdot}5 & = K_1 \\
 & & +578{\cdot}5 & -\ 37{\cdot}0 & -\ 96{\cdot}7 & +\ 56{\cdot}7 & -\ 28{\cdot}3 & +\ 9{\cdot}9 & = \Pi_2 \\
 & & & +626{\cdot}2 & +\ 72{\cdot}9 & -\ 89{\cdot}2 & -\ 29{\cdot}8 & +\ 1{\cdot}3 & = K_2 \\
 & & & & +774{\cdot}6 & -\ 29{\cdot}0 & -\ 34{\cdot}4 & -\ 70{\cdot}0 & = \Pi_3 \\
 & & & & & +944{\cdot}4 & +\ 258{\cdot}9 & +\ 32{\cdot}8 & = K_3 \\
 & & & & & & +1126{\cdot}2 & -\ 74{\cdot}4 & = \Pi_4 \\
 & & & & & & & +1243{\cdot}2 & = K_4.
\end{array}
$$

The values of the right-hand members of the normal equations, resulting from the different right-hand members of the equations of condition, are :—

	n.	Δn.	δn.	$\frac{dn}{\sin 1''\, dP_s}$.
Π_1	+0·0057	−0·0370	−0·0051	− 183
K_1	− ·3264	+ ·0159	+ ·0007	+ 315
Π_2	− ·2937	− ·0150	+ ·0001	− 98
K_2	− ·1130	+ ·0425	+ ·0027	+ 268
Π_3	− ·0862	− ·0440	+ ·0022	− 350
K_3	− ·3175	+ ·0327	− ·0022	+ 429
Π_4	+1·3431	− ·1192	− ·0069	− 589
K_4	− ·7891	+ ·1884	− ·0125	+1275

In computing the values given under Δn, plate 7664, on which there is no trail, must be omitted. In order not to alter the left-hand sides of the normal equations, this was effected by using, for the right-hand members Δn of the equations of condition for this plate, the values resulting from the substitution of an approximate solution, which differed very little from the final solution.

The solution of the normal equations is :—

$$
\begin{array}{rrrrrrrrl}
+{\cdot}001486\,\Pi_1 & +{\cdot}000060\,K_1 & -{\cdot}000055\,\Pi_2 & +{\cdot}000228\,K_2 & -{\cdot}000107\,\Pi_3 & -{\cdot}000057\,K_3 & +{\cdot}000060\,\Pi_4 & +{\cdot}000048\,K_4 & = \pi_1 \\
 & +{\cdot}000964 & +\ 54 & -\ 59 & +\ 140 & -\ 47 & +\ 40 & +\ 44 & = \kappa_1 \\
 & & +{\cdot}001790 & +\ 56 & +\ 230 & -\ 116 & +\ 80 & +\ 7 & = \pi_2 \\
 & & & +{\cdot}001680 & -\ 168 & +\ 139 & +\ 13 & -\ 10 & = \kappa_2 \\
 & & & & +{\cdot}001370 & -\ 4 & +\ 49 & +\ 80 & = \pi_3 \\
 & & & & & +{\cdot}001159 & -\ 271 & -\ 50 & = \kappa_3 \\
 & & & & & & +{\cdot}000961 & +\ 70 & = \pi_4 \\
 & & & & & & & +{\cdot}000817 & = \kappa_4.
\end{array}
$$

The substitution of the right-hand members of the normal equations given above into the solution gives the values of the unknowns contained in Table XIV.

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TABLE XIV.—SOLUTIONS FOR 1903.

	S.	T.	S′.	M.	Weight.
π_1	+·000042	+·000010	+·000028	+·000019	673
κ_1	− 304	− 298	− 282	− 290	1037
π_2	− 430	− 474	− 457	− 466	557
κ_2	− 187	− 120	− 142	− 131	595
π_3	− 207	− 264	− 245	− 254	730
κ_3	− 659	− 589	− 601	− 595	863
π_4	+ 1273	+ 1166	+ 1217	+ 1191	1041
κ_4	− 567	− 418	− 480	− 449	1224

The values of the unknowns given in the column S are those of the solution in which the orientation is derived from the standard stars. They result from the substitution of the right-hand members $n + \delta n$. Those given under T correspond to the determination of the orientation from the trails. They result from the substitution of $n + \Delta n$.

The differences between the solutions S and T are given in Table XV., which also contains the coefficients of a correction to the adopted position-angle of the standard stars. These coefficients, of course, are derived by substituting in the solution the right-hand members $\frac{dn}{\sin 1'' \, dP_s}$. It will be seen at once that the differences T − S are roughly proportional to these differential coefficients. If for the correction to P_s we adopt the value +19″, which is the mean of the values of I′ − I given in Table VIII., the differences T − S are reduced to those given in the fourth column of Table XV. (A). The best possible agreement between the two solutions is obtained by adopting the correction $\Delta P_s = +29''$, which gives the differences B in Table XV.

TABLE XV.

DIFFERENCES OF SOLUTIONS T AND S.

	$\frac{dS}{\sin 1'' \, dP_s}$	T − S.	A. ($\Delta P_s = +19''$).	B. ($\Delta P_s = +29''$).
π_1	−0·147	−·000032	−·000018	−·000010
κ_1	+ ·235	+ 6	− 16	− 27
π_2	− ·301	− 44	− 17	− 2
κ_2	+ ·482	+ 67	+ 22	− 1
π_3	− ·413	− 57	− 19	0
κ_3	+ ·637	+ 70	+ 12	− 20
π_4	− ·613	− 107	− 51	− 21
κ_4	+ ·953	+ 149	+ 62	+ 15

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The residuals A, though on the whole larger than B, are still satisfactory enough, only two of them exceeding the probable error of the corresponding unknowns in solution T. I have adopted the correction

$$\Delta P_s = +19'',$$

which is derived from the direct comparison of the standard stars with the trails. Applying this correction, the solution S is changed to S′, which is also given in Table XIV.

For the determination of the probable error of weight unity it is now necessary to substitute the solutions S′ and T in the equations of condition in order to derive the quantities v, which must be subtracted from the right-hand members n to give the residuals. The two solutions are, however, so nearly alike, that the two values of v will be practically the same. I have therefore not substituted the two solutions separately, but I have used their mean, which is given in Table XIV. under M. From this solution M, two sets of residuals were formed, thus :—

$$\Delta_s = n + \delta n - (x - x_0)\sin 19'' - v$$
$$\Delta_t = n + \Delta n - v,$$

where

$$v_t = \mu_t - \mu_0 \qquad \mu = a\pi_M + b\kappa_M.$$

The residuals Δ_s and Δ_t are given in the last columns of Table XIII. The resulting probable errors of unit weight are :—

from Δ_s . . . $\rho_s = \pm 0'{\cdot}00130 = \pm 0''{\cdot}078$;
from Δ_t . . . $\rho_t = \pm 0'{\cdot}00078 = \pm 0''{\cdot}047$.

The value of ρ derived from the comparison of the pointings on one and the same plate would be :—

$$\rho_0 = \pm 0''{\cdot}036.$$

The probable error derived from the solution in which the orientation was derived from the trails is much smaller than the value derived from the solution based on the standard stars. It will be shown in the appendix that this is what could have been expected, since the orientation from the trails is free from systematic errors, due to inaccurate adjustment of the telescope; while the same thing cannot with certainty be said of the orientation determined from the standard stars.

The probable error ρ_t is still not inconsiderably larger than ρ_0. This need not necessarily lead to the conclusion that the determination of the orientation from the trails is still subject to greater uncertainty than can be explained by the purely accidental errors of the trails. There may remain a few undetected small numerical errors in the work, though all computations were repeatedly examined and checked in

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many different and independent ways. There is also one source of error for this opposition, which did not exist for the plates of 1891, viz. : the neglect of corrections to the other elements entering into the computation of the tabular longitudes and radii-vectores. These corrections, which may be considerable, have no influence on the final result, since their coefficients are all periodic functions of the time and will cancel each other in the solution depending on all the plates, but their effect on the tabular places for each individual plate may possibly be comparable in magnitude to the errors of observation.

It must also be remembered that only *one* plate-constant (the orientation-constant A) was determined for each plate. Without entering into a discussion of the relative merits of a four-constant or a six-constant solution, so much can be said with certainty, that there are more than one constants which differ from plate to plate and affect the position of the images. Of these, however, only the orientation-constant is *systematically* of importance for our purpose, and therefore that only has been determined. The others may, however, quite well increase the residuals of some plates without affecting the result of the solution.

I will adopt as the result, which probably is the nearest approach to the truth, the solution T. Adding the values of this solution to the original approximations,

$$\pi_2 = +0^{\circ}\cdot001000 \qquad \kappa_2 = -0^{\circ}\cdot002000,$$

and multiplying by the constants c, we get for the final result for the opposition 1903 :—

$\Delta p_1 = +0^{\circ}\cdot0021 \pm$	$0^{\circ}\cdot0060$	$\Delta q_1 = -0^{\circ}\cdot0597 \pm$	$0^{\circ}\cdot0048$
$\Delta p_2 = +\ \cdot0526 \pm$	33	$\Delta q_2 = -\ \cdot2120 \pm$	32
$\Delta p_3 = -\ \cdot0199 \pm$	22	$\Delta q_3 = -\ \cdot0442 \pm$	20
$\Delta p_4 = +\ \cdot0583 \pm$	12	$\Delta q_4 = -\ \cdot0209 \pm$	11

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CHAPTER XIII.

SOLUTION FOR 1904.

THE following plates were not measured :—

Plate.	*Observer's remarks.*
8199	Images too diffused; trail too faint.
8212	Only three exposures. Trail interrupted in several places; images bad.
8215	Images too faint. Satellite IV invisible.

The images are as a rule not so good as those on the plates of 1903, and not much better than the average of the plates of 1891.

The multipliers c_i are the same as those for 1903, viz. :—

$$c_1 = 200 \qquad c_2 = 100 \qquad c_3 = 75 \qquad c_4 = 50.$$

The Table XVI. again gives the tabular quantities v, ρ, x', y', the coefficients $a = c\frac{\partial y'}{\partial p}$, $b = c\frac{\partial y'}{\partial q}$, the observed co-ordinates $y_{\text{obs.}}$ and the correction

$$\Delta y = \text{A}\,(x'_i - x'_0) + \text{B}y_{\text{obs.}}$$

The values of A and B are given in Table IV.

The right-hand terms n_i of the equations of condition are then :—

$$n_i = m_i - m_0 \qquad m = y_{\text{obs}} + \Delta y - y'.$$

The following values were introduced as first approximations :—

$\pi_1 = 0{\cdot}0000$	$\kappa_1 = -0{\cdot}0003$
$\pi_2 = +\ {\cdot}0006$	$\kappa_2 = -\ {\cdot}0022$
$\pi_3 = \ {\cdot}0000$	$\kappa_3 = -\ {\cdot}0005$
$\pi_4 = +\ {\cdot}0008$	$\kappa_4 = -\ {\cdot}0004.$

The substitution of these values in the equations of condition gives :—

$$\text{N}_i = \text{M}_i - \text{M}_0 \qquad \text{M} = a\pi + b\kappa,$$

and we must then form the differences $n - \text{N}$. We may, however, quite as well compute these differences thus :—

$$n_i - \text{N}_i = (m - \text{M})_i - (m - \text{M})_0.$$

This has been actually done. The table accordingly does not give N, but M.

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TABLE XVI.

Reduction of Plates of 1904.

Plate.	Date G.M.T.	Sat.	v	ρ	x'	y'	a	b	$y_{obs.}$	Correction.	Approximation M	$n-N$	Δn	Δ_s	Δ_t
	1904.		°												
8123	Oct. 27	I.	232·400	1·00370	− 0·67	+·12930	−5·20	−6·76	+·07368	−·00395	+·00203	+·00122	+·00182	−·00044	+·00191
	9h35m·34	II.	325·527	0·99956	+ 3·78	+·01922	+5·57	−3·83	−·03738	+ 270	+ 1177	− 285	− 133	− 215	− 387
		III.	101·876	0·99993	− 3·42	−·29130	−1·67	+7·93	−·36302	− 250	− 396	− 744	+ 377	− 365	+ 123
		IV.	294·756	1·00538	+ 7·90	+·37754	+3·99	−8·65	+·32070	+ 372	+ 665	+ 905	− 425	+ 622	+ 72
8124	Oct. 28	III.	150·678	0·99929	− 6·12	−·05809	−7·05	+3·96	−·14122	+·00069	−·00198	+·00214	−·00185	+·00049	+·00032
	8 50·82	IV.	315·403	1·00668	+10·07	+·21715	+6·79	−6·69	+·14122	− 69	+ 811	− 213	+ 184	− 48	− 32
8126	Oct. 29	I.	270·175	1·00313	+ 0·92	+·12368	+0·03	−8·51	+·01684	−·00020	+·00255	−·00581		−·00035	
	8 32·24	II.	162·756	0·99742	− 3·85	+·04786	−6·43	+2·00	−·07043	+ 327	− 826	− 298		− 236	
		III.	200·430	0·99916	− 4·56	+·21715	−7·56	−2·82	+·12030	+ 168	+ 141	+ 720		− 100	
		IV.	336·399	1·00712	+10·93	+·02531	+8·74	+3·82	−·06070	− 474	+ 546	+ 157		+ 372	
8127	Oct. 30	I.	113·988	0·99797	− 1·74	−·09191	−3·44	+7·74	−·08421	+·00432	−·00232	−·00089	−·00124	−·00087	−·00107
	8 30·27	II.	265·527	0·99189	+ 1·16	+·19784	−0·52	−6·68	+·22822	− 78	+ 1439	− 2	− 52	+ 110	+ 101
		IV.	357·643	1·00659	+10·30	−·17146	+9·53	−0·39	−·14401	− 354	+ 778	+ 90	+ 176	− 23	+ 6
8130	Nov. 2	I.	8·662	0·99737	+ 2·07	−·06819	+8·34	+1·27	+·10964	−·00126	−·00038	+·00202	+·00022	+·00031	+·00027
	9 4·76	II.	211·151	0·99136	− 2·27	+·19051	−5·70	−3·45	+·37826	− 394	+ 416	+ 472	− 56	− 56	− 47
		III.	42·772	1·00067	+ 2·59	−·29820	+5·92	+5·48	−·13848	+ 141	− 274	− 1106	+ 31	− 11	− 16
		IV.	62·376	1·00041	+ 0·99	−·52500	+4·39	+8·39	−·34941	+ 379	+ 15	+ 430	+ 3	+ 37	+ 37
8137	Nov. 4	I.	49·669	0·99629	+ 0·73	−·12451	+5·44	+6·41	+·09547	−·00175	−·00192	−·00009	+·00066	−·00047	−·00052
	8 20·50	II.	51·232	1·00931	+ 1·08	−·21771	+4·24	+5·28	−·00822	− 68	− 908	− 235	+ 73	+ 27	+ 22
		III.	141·838	0·99955	− 5·91	−·10185	−6·31	+4·96	+·11914	− 63	− 248	+ 260	− 63	+ 12	+ 16
		IV.	105·038	0·99543	− 6·57	−·41779	−2·44	+9·07	−·20638	+ 307	− 558	− 18	− 76	+ 6	+ 11
8141	Nov. 5	II.	144·828	0·99950	− 3·75	−·01683	−5·48	+3·86	+·01516	+·00540	−·01178	−·00216	+·00051	−·00077	−·00073
	6 45·58	III.	188·896	0·99932	− 5·21	+·16185	−7·90	−1·24	+·22171	− 24	+ 62	+ 767	+ 18	+ 94	+ 95
		IV.	125·402	0·99377	− 9·10	−·28049	−5·42	+7·62	−·23688	− 517	− 739	− 550	− 69	− 18	− 23
8142	Nov. 6	I.	99·740	0·99750	− 1·30	−·10914	−1·42	+8·27	−·15916	+·00169	−·00248	−·00241	+·00054	·00000	−·00004
	8 39·96	III.	243·275	0·99956	− 0·41	+·32726	−3·59	−7·13	+·29678	− 309	+ 356	+ 631	+ 71	− 174	− 180
		IV.	149·015	0·99283	−10·57	−·08058	−7·99	+4·80	−·13762	+ 140	− 831	− 389	− 126	+ 174	+ 183
8143	Nov. 7	I.	301·631	1·00122	+ 1·96	+·07487	+4·41	−7·15	−·00506	+·00063	+·00214	+·00009	+·00040	+·00052	+·00049
	8 33·78	II.	357·091	1·00563	+ 3·62	−·10203	+6·71	−0·34	−·18334	+ 287	+ 478	− 169	+ 72	+ 103	+ 98
		III.	293·400	1·00006	+ 4·41	+·23599	+3·17	−7·32	+·16248	− 36	+ 366	+ 400	+ 87	− 58	− 65
		IV.	170·803	0·99301	−10·35	+·11467	−9·18	+1·49	+·02592	− 314	− 794	− 242	− 199	− 97	− 83
8144	Nov. 8	I.	137·459	0·99985	− 2·26	−·04259	−6·17	+5·67	−·04820	+·00035	−·00170	+·00334	−·00004	+·00002	+·00011
	7 33·73	II.	92·654	1·00669	− 1·67	−·18015	−0·31	+6·71	−·20256	+ 222	− 1495	+ 166	0	− 49	− 48
		III.	341·592	1·00049	+ 6·08	−·01342	+7·57	−2·52	−·02626	+ 250	+ 126	− 470	+ 52	+ 12	− 97
		IV.	191·744	0·99414	− 8·73	+·28565	−9·09	−1·89	+·27702	− 508	− 651	− 30	− 48	+ 35	+ 134
8145	Nov. 9	I.	338·375	0·99874	+ 2·39	−·00397	+7·77	−3·08	−·07862	+·00245	+·00092	+·00224	+·00045	+·00006	−·00014
	7 21·38	II.	193·286	0·99236	− 3·04	+·14890	−6·43	−1·52	+·08168	− 257	− 52	+ 609	− 49	− 86	− 66
		III.	31·415	1·00063	+ 3·53	−·25866	+6·81	+4·16	−·35105	+ 581	− 208	− 914	+ 62	+ 44	+ 17
		IV.	213·339	0·99610	− 5·86	+·42099	−7·75	−5·10	+·34798	− 569	− 416	+ 82	− 60	+ 37	+ 63
8146	Nov. 10	I.	181·299	1·00254	− 2·18	+·05391	−8·37	−0·19	−·02974	+·00015	+·00006	+·00587	−·00018	+·00214	+·00226
	7 12·31	II.	295·617	0·99627	+ 2·86	+·11594	+2·86	−5·97	+·04071	− 5	+ 1485	− 70	+ 46	− 234	− 263
		III.	81·364	1·00031	− 1·54	−·31825	+1·20	+7·88	−·42312	+ 391	− 394	− 759	− 10	− 114	− 108
		IV.	234·885	0·99861	− 2·18	+·49635	−5·34	−7·59	+·41212	− 400	− 123	+ 243	− 18	+ 131	+ 143
8148	Nov. 13	III.	234·265	0·99945	− 1·26	+·31029	−4·61	−6·40	+·00814	−·00141	+·00320	+·00625		+·00041	
	8 5·08	IV.	300·057	1·00590	+ 8·59	+·30559	+4·66	−8·06	−·00814	+ 142	+ 695	− 625		− 42	

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TABLE XVI.—REDUCTION OF PLATES OF 1904—*continued.*

Plate.	Date G.M.T.	Sat.	v	ρ	x'	y'	a	b	$y_{obs.}$	Correction.	Approximation M	$n-N$	Δn	Δ_s	Δ_t
	1904.		°		′	′	′	′	′	′	′	′	′	′	′
8154	Nov. 14	I.	277·994	1·00227	+ 1·25	+·10731	+1·15	−8·22	−·01108	−·0006;	+·00247	−·00178	−·00094	+·00015	−·00006
	7h36m·29	II.	342·758	1·00434	+ 3·77	−·05309	+6·33	−1·96	−·16968	+ 160	+ 811	− 338	− 26	+ 68	+ 62
		III.	283·615	0·99982	+ 3·65	+·26166	+1·85	−7·66	+·15246	− 187	+ 383	+ 482	− 29	− 21	− 27
		IV.	320·922	1·00706	+10·29	+·14046	+7·23	−5·87	+·02831	+ 92	+ 813	+ 36	+ 149	− 61	− 27
8158	Nov. 15	I.	118·550	0·99897	− 1·88	−·07637	−3·95	+7·27	−·00320	−·00136	−·00218	+·00165	−·00109	−·00026	−·00024
	7 10·26	II.	80·720	1·00744	− 0·95	−·19511	+1·07	+6·55	−·13694	+ 18	− 1377	− 22	− 90	− 12	− 10
		III.	333·016	1·00034	+ 6·00	+·03068	+7·01	−3·57	+·10128	− 31	+ 178	− 383	+ 52	+ 71	+ 70
		IV.	341·802	1·00727	+10·61	−·04262	+8·84	−2·91	+·03886	+ 149	+ 823	+ 240	+ 146	− 34	− 37
8159	Nov. 16	I.	320·429	0·99953	+ 2·28	+·03334	+6·36	−5·26	+·11100	−·00144	+·00158	+·00152	−·00009	+·00097	+·00105
	7 4·77	II.	181·643	0·99302	− 3·37	+·11482	−6·52	−0·19	+·19365	− 413	− 349	+ 507	− 67	− 33	+ 35
		III.	23·087	1·00068	+ 4·10	−·22325	+7·24	+3·09	−·16388	+ 204	− 154	− 1017	+ 10	+ 21	+ 11
		IV.	2·996	1·00645	+ 9·49	−·22114	+9·28	+0·49	−·14076	+ 354	+ 722	+ 358	+ 66	− 86	− 152
8167	Nov. 17	I.	164·635	1·00194	− 2·34	+·01920	−7·96	+2·19	+·16058	−·00223	−·00066	+·01302	−·00040	+·01015	+·01056
	7 4·23	II.	284·620	0·99540	+ 2·33	+·14173	+1·65	−6·31	+·28526	− 286	+ 1487	− 99	+ 8	− 208	− 216
		III.	73·317	1·00057	− 0·76	−·31535	+2·26	+7·53	−·19686	+ 172	− 377	− 281	− 24	+ 580	+ 604
		IV.	24·319	1·00472	+ 7·06	−·36840	+8·44	+3·81	−·24899	+ 338	+ 523	− 923	+ 56	− 1387	− 1445
8181	Nov. 22	I.	98·407	0·99810	− 1·26	−·10204	−1·19	+8·05	−·02454	+·00030	−·00242	+·00252	+·00006	+·00020	+·00018
	6 38·25	III.	324·141	1·00016	+ 5·80	+·07379	+6·28	−4·54	+·15221	− 39	+ 227	− 194	+ 108	+ 22	− 25
		IV.	132·097	0·99328	− 9·50	−·19716	−6·06	+6·71	−·12766	+ 10	− 753	− 57	− 114	− 40	+ 9
8182	Nov. 23	I.	298·705	1·00062	+ 1·86	+·07231	+3·90	−7·13	+·09416	−·00122	+·00214	+·00074	+·00132	−·00065	−·00010
	6 21·23	II.	168·091	0·99400	− 3·59	+·07146	−6·29	+1·33	+·08928	− 79	− 670	+ 598	− 62	− 25	− 51
		III.	13·817	1·00067	+ 4·64	−·18049	+7·52	+1·85	−·17538	+ 134	− 92	− 1038	+ 231	− 191	− 95
		IV.	153·714	0·99274	−10·31	−·02073	−8·07	+3·99	−·00806	+ 67	− 806	+ 365	− 301	+ 280	+ 155
8190	Nov. 24	I.	142·908	1·00096	− 2·27	−·02578	−6·47	+4·89	−·12818	+·00139	−·00147	+·00190	+·00056	+·00026	+·00058
	6 20·37	II.	271·066	0·99426	+ 1·59	+·16047	+0·12	−6·40	+·07998	− 53	+ 1415	+ 27	+ 239	− 12	+ 122
		IV.	175·579	0·99321	− 9·65	+·15840	−8·95	+0·69	+·04820	− 86	− 744	− 218	− 295	− 16	− 182
8192	Nov. 25	I.	344·897	0·99771	+ 2·28	−·02030	+7·79	−2·10	+·00010	+·00006	+·00063	+·00275	+·00071	+·00105	+·00093
	6 15·05	II.	11·708	1·00832	+ 2·99	−·14223	+6·35	+1·32	−·12637	+ 146	+ 91	− 67	+ 84	+ 25	+ 12
		III.	114·108	1·00021	− 4·42	−·21043	−3·15	+7·03	−·20080	+ 217	− 352	− 176	− 49	− 103	− 95
		IV.	197·337	0·99466	− 7·63	+ 31237	−8·55	−2·67	+·32707	− 369	− 577	− 30	− 106	− 27	− 9
8198	Nov. 26	I.	195·107	1·00349	− 1·78	+·07715	−7·81	−2·11	−·02850	+·00091	+·00064	−·00074	+·00049	+·00072	+·00081
	6 59·18	II.	114·675	1·00198	− 2·80	−·10737	−2·69	+5·85	−·22946	+ 289	− 1448	− 8	+ 23	+ 11	+ 15
		III	165·983	0·99955	− 5·76	+·04354	−7·44	+1·86	−·05572	− 1	− 93	+ 630	− 54	+ 61	+ 50
		IV.	219·763	0·99696	− 4·42	+·42318	−6·87	−5·72	+·31368	− 381	− 321	− 546	− 19	− 141	− 145
8200	Nov. 28	II.	316·457	1·00181	+ 3·47	+·03597	+4·63	−4·40	−·22196	+·00227	+·01246	−·00476	+·00024	−·00057	−·00046
	6 16·11	III.	265·256	0·99942	+ 2·00	+·28520	−0·63	−7·59	+·03174	− 24	+ 380	+ 586	− 31	− 107	− 121
		IV.	262·298	1·00223	+ 3·02	+·45006	−1·20	−8·86	+·19022	− 203	+ 258	− 109	+ 7	+ 165	+ 168
8209	Nov. 29	I.	84·391	0·99763	− 0·75	−·11094	+0·78	+7·94	−·23378	+·00225	−·00238	+·00412	−·00111	+·00278	+·00260
	6 49·56	III.	316·770	1·00012	+ 5·52	+·10582	+5·54	−5·21	−·01550	+ 28	+ 260	− 131	+ 44	+ 7	+ 14
		IV.	284·220	1·00462	+ 6·46	+·36666	+2·20	−8·67	+·24927	− 252	+ 523	− 281	+ 67	− 285	− 275
8213	Dec. 1	I.	129·162	1·00042	− 2·05	−·05054	−5·03	+6·17	−·03200	−·00092	−·00185	+·00183	·00000	−·00153	−·00059
	6 33·23	II.	261·539	0·99371	+ 1·02	+·17593	−0·92	−6·21	+·21396	− 266	+ 1311	+ 462	0	+ 206	+ 236
		III.	56·698	1·00082	+ 0·84	−·29159	+4·16	+6·34	−·28552	+ 254	− 317	− 1220	0	− 257	− 223
		IV.	326·599	1·00709	+10·13	+·07327	+7·46	−4·92	+·10357	+ 104	+ 794	+ 576	0	+ 203	+ 44
8216	Dec. 2	I.	329·181	0·99828	+ 2·26	+·01125	+6·80	−4·06	+·11872	−·00138	+·00122	+·00062	−·00018	−·00007	−·00011
	6 14·36	II.	1·366	1·00789	+ 3·26	−·11166	+6·36	+0·15	−·00705	+ 12	+ 349	− 301	+ 9	− 130	− 128
		III.	106·296	1·00028	− 3·81	−·22924	−2·12	+7·25	−·13254	+ 73	− 362	− 320	− 188	− 17	− 65
		IV.	347·582	1·00697	+ 9·97	−·09616	+8·71	−1·92	+·02088	+ 53	+ 774	+ 558	+ 197	+ 152	+ 202

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TABLE XVI.—Reduction of Plates of 1904—*concluded.*

Plate.	Date G.M.T.	Sat.	v	ρ	x'	y'	a	b	$y_{obs.}$	Correction.	Approximation M	$n-N$	Δn	Δ_s	Δ_t
	1904.		°		′	′	″	″	″	″	″	″	″	″	″
8222	Dec. 3	I.	177·539	1·00313	− 2·10	+·04498	−7·92	+0·34	+·13254	−·00170	−·00010	−·00208	−·00012	−·00024	−·00001
	$6^h44^m·50$	II.	103·303	1·00298	− 2·25	−·13303	−1·45	+6·14	−·06075	+ 36	− 1438	− 102	− 13	+ 62	+ 87
		III.	157·679	0·99954	− 5·74	+·00157	−6·96	+2·86	+·09244	− 187	− 143	+ 239	− 39	− 90	− 18
		IV.	9·324	1·00587	+ 8·40	−·25618	+8·77	+1·44	−·16422	+ 321	+ 644	+ 69	+ 64	+ 53	− 66
8226	Dec. 9	I.	318·797	0·99859	+ 2·14	+·03032	+5·83	−5·11	+·13970	−·00103	+·00153	+·00119	+·00100	−·00055	−·00037
	6 51·22	II.	353·623	1·00763	+ 3·38	−·08691	+6·19	−0·69	+·02342	+ 41	+ 523	− 12	+ 131	+ 33	+ 56
		III.	99·817	1·00036	− 3·25	−·23943	−1·26	+7·29	−·14122	+ 137	− 364	− 241	− 36	+ 56	+ 50
		IV.	139·095	0·99317	− 9·57	−·12414	−6·55	+0·58	−·02190	− 75	− 547	+ 133	− 195	− 35	− 69
8228	Dec. 13	II.	38·305	1·00899	+ 1·57	−·17334	+4·83	+3·82	−·31637	+·00322	−·00550	−·00327	+·00015	−·00165	−·00161
	6 52·14	III.	301·302	0·99997	+ 4·65	+·16276	+3·79	−6·23	+·03846	− 67	+ 312	+ 295	+ 107	+ 21	+ 53
		IV.	226·392	0·99766	− 3·06	+·40833	−5·87	−6·16	+·27791	− 255	− 224	+ 31	− 122	+ 144	+ 107

The further columns contain the right-hand members $n-N$ and the correction Δn, which must be applied to them if it is desired to use the orientation derived from the trails instead of that from the standard stars. This correction is :—

$$\Delta n_i = (x'_i - x'_0)\,\Delta A.$$

The values of ΔA are given in Table VIII.

The last two columns contain the residuals.

The normal equations are :—

$$\begin{aligned}
+797{\cdot}8\,\pi_1 - 289{\cdot}8\,\kappa_1 + 1{\cdot}6\,\pi_2 - 27{\cdot}4\,\kappa_2 - 38{\cdot}1\,\pi_3 - 13{\cdot}8\,\kappa_3 + 31{\cdot}4\,\pi_4 - 63{\cdot}4\,\kappa_4 &= \Pi_1\\
+827{\cdot}0 + 10{\cdot}5 + 15{\cdot}6 - 49{\cdot}6 + 67{\cdot}1 + 20{\cdot}6 - 41{\cdot}8 &= K_1\\
+548{\cdot}0 - 60{\cdot}9 + 13{\cdot}7 + 14{\cdot}6 + 30{\cdot}0 + 99{\cdot}6 &= \Pi_2\\
+477{\cdot}1 + 43{\cdot}1 + 51{\cdot}7 + 55{\cdot}6 - 46{\cdot}5 &= K_2\\
+784{\cdot}9 - 68{\cdot}1 + 73{\cdot}8 - 20{\cdot}2 &= \Pi_3\\
+855{\cdot}8 - 74{\cdot}1 - 43{\cdot}6 &= K_3\\
+1506{\cdot}9 - 134{\cdot}4 &= \Pi_4\\
+903{\cdot}3 &= K_4
\end{aligned}$$

The right-hand members of the normal equations are :—

	n	Δn	$\frac{dn}{\sin 1'' dP_s}$
Π_1	− ·0198	+ ·0282	− 295
K_1	+ ·1467	− ·0564	+ 179
Π_2	− ·3105	+ ·0445	− 312
K_2	− ·0582	− ·0144	+ 57
Π_3	− ·8938	+ ·1194	− 661
K_3	− ·8534	− ·0163	+ 373
Π_4	+ ·3910	+ ·3248	− 1759
K_4	+ ·0170	− ·0482	+ 497

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In computing the first set of right-hand members, the plate 8167 has ultimately been rejected. The residuals of this plate are entirely outside all limits of possibilities. There must be a mistake either in the measures or in the computations; but although I have partially remeasured the plate, and repeatedly examined all the numerical work, I have not been able to find the error. In order not to change the left-hand sides of the normal equations and the solution, the rejection was effected by replacing the values of $n - N$ for this plate by those resulting from an approximate solution, which was very nearly identical to the final solution.

In computing the second set of right-hand members the correction ΔA was for the plates 8126 and 8148, on which no trails were measured, assumed to be $+0{\cdot}000208$, which corresponds to a correction of $-43''$ to the adopted position-angle of the standard stars. This is the mean of the corrections $I' - I$ from all plates, rejecting plate 8123 (see Chapter VIII. and Table VIII.).

The solution of the normal equations is :—

$$
\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrr}
+\cdot001463\Pi_1 & +\cdot000526K_1 & -\cdot000031\Pi_2 & +\cdot000071K_2 & +\cdot000107\Pi_3 & -\cdot000009K_3 & -\cdot000033\Pi_4 & +\cdot000131K_4 & = \pi_1 \\
+\cdot001415 & - 47 & - 9 & + 114 & - 90 & - 30 & + 101 & & = \kappa_1 \\
+\cdot001898 & + 240 & - 56 & - 64 & - 65 & - 215 & & & = \pi_2 \\
+\cdot002173 & - 124 & - 147 & - 82 & + 68 & & & & = \kappa_2 \\
+\cdot001309 & + 103 & - 54 & + 39 & & & & & = \pi_3 \\
+\cdot001202 & + 68 & + 65 & & & & & & = \kappa_3 \\
+\cdot000684 & + 103 & & & & & & & = \pi_4 \\
+\cdot001167 & & & & & & & & = \kappa_4 .
\end{array}
$$

Substituting the right-hand members of the normal equations in the solution, we get the following values of the unknowns, which are given in Table XVII. :—

TABLE XVII.

SOLUTIONS FOR 1904.

	S	$\frac{d\,S}{\sin 1''\, d\,P_s}$	S'	T	M	Weight.
π_1	$-\cdot000045$	$-0{\cdot}275$	$+\cdot000012$	$-\cdot000040$	$-\cdot000014$	684
κ_1	+ 176	+ ·106	+ 155	+ 110	+ 133	707
π_2	− 534	− ·557	− 418	− 467	− 442	527
κ_2	+ 2	+ ·231	− 46	− 59	− 52	460
π_3	− 1239	− ·715	− 1090	− 1108	− 1099	764
κ_3	− 1072	+ ·291	− 1133	− 1056	− 1095	832
π_4	+ 281	− 1·071	+ 504	+ 490	+ 497	1462
κ_4	+ 45	+ ·447	− 48	+ 13	− 17	857

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The solution S is derived from the substitution of the right-hand members $n - N$, and therefore corresponds to the orientation determined from the standard stars, without any correction. If we adopt the correction

$$\Delta P_s = -43''$$

and apply this correction by means of the differential coefficients given in the third column of Table XVII., we get the solution S′, which is contained in the fourth column of the same table. The next column contains the solution T, in which the orientation is derived from the trails, and which results from the substitution of the right-hand members $n - N + \Delta n$. The agreement between the solutions S′ and T is satisfactory. The mean of the two solutions is given in the column M. This solution M was substituted in the equations of condition, and two sets of residuals were formed, corresponding to the two methods of determining the orientations, thus :—

$$\mu = a\pi + b\kappa \qquad \nu_i = \mu_i - \mu_0;$$
$$\Delta_s = n - N + (x_i - x_0)\sin 43'' - \nu;$$
$$\Delta_t = n - N + \Delta n - \nu.$$

These residuals are included in Table XVI.

The probable error of weight unity derived from them is :—

$$\rho_s = \pm 0'.00105 = \pm 0''.063;$$
$$\rho_t = \pm 0\ .00097 = \pm 0\ .058.$$

It will be remembered that we found in Chapter VIII. that, after applying the correction $\Delta P_s = -43''$, the index-corrections derived from the standard stars showed no marked discordances with those from the trails (with the exception of plate 8123). The middle point of the standard stars was never very far removed from Jupiter in hour-angle and declination, and consequently, according to the conclusions reached in the appendix to this paper, we may expect a fair agreement between the orientations as determined by the two methods, which we actually find. There is thus here no reason to prefer one or the other of the two solutions S′ and T, and I will adopt their mean, *i.e.* the solution M, and the probable error of weight unity :—

$$\rho_1 = \pm 0'.00100 = \pm 0''.060.$$

Adding the values of the unknowns in solution M to the first approximations, and multiplying by the constants c, we find :—

$\Delta p_1 = +$	$0^s.0028 \pm$	$\Delta q_1 = -$	$0^s.0335 \pm 0^s.0077$
$\Delta p_2 = +$	$.0158 \pm 44$	$\Delta q_2 = -$	$.2252 \pm 48$
$\Delta p_3 = -$	$.0824 \pm 28$	$\Delta q_3 = -$	$.1196 \pm 26$
$\Delta p_4 = +$	$.0648 \pm 13$	$\Delta q_4 = -$	$.0209 \pm 17$

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These are the final corrections to the adopted elements p and q, so far as the Satellites I, II, and IV are concerned. In computing the tabular places of Satellite III, however, a mistake was made, which was only detected after the solution was completed. Instead of the values of Ω_3 given in Table X., $270° - \Omega_3$ was actually used; or, which comes to the same thing, the numerical values of p_3 and q_3 were interchanged, the correct signs being retained. The theoretical values and those actually used are :—

	Theoretical.		Used.		Difference.
	p_3	q_3	p_3	q_3	
1904.	°	°	°	°	°
Oct. 27	+ 0·0455	− 0·1166	+ 0·1166	− 0·0455	+ 0·0711
Nov. 21	+ ·0448	− ·1168	+ ·1168	− ·0448	+ ·0720
Dec. 16	+ ·0440	− ·1169	+ ·1169	− ·0440	+ ·0729

The difference is not constant, but varies very little. I have recomputed all tabular co-ordinates y', using the correct values of Ω_3, and compared the results with the values which can be derived by adding to the originally computed co-ordinates the correction

$$\Delta y' = + 0·0720 \left(\frac{\partial y'}{\partial p} + \frac{\partial y'}{\partial q} \right).$$

The difference between the two values never exceeds 0′·00010, and is generally below 0′·00005. It can therefore be neglected, and it is not necessary to repeat the solution, but it is sufficient to add the correction $+0°·0720$ to the values of Δp_3 and Δq_3 found from the solution. The final corrections to the elements of Satellite III thus become :—

$$\Delta p_3 = - 0°·0104 \qquad \Delta q_3 = - 0°·0476.$$

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CHAPTER XIV.

Discussion of Probable Errors.

It is interesting to compare the probable errors of the different series. Table XVIII. contains the probable errors ϵ, δ, and ζ, defined in Chapter IV. It will be remembered that ϵ, the "error of pointing," is that part of the error which differs in the two positions of the plate, while δ, the "error of the image," is that part which is the same in the two positions. Further, $\zeta = \sqrt{\epsilon^2 + 2\delta^2}$ is the probable error of a distance between two objects determined by one pointing on each image in each position of the plate. The probable error $\rho_0 = \zeta/\sqrt{12}$ is the probable error of the right-hand term of an equation of condition of weight unity. All these probable errors are derived from the inter-agreement of the different pointings on one and the same plate. The probable errors of weight unity ρ_s and ρ_t, are those derived from the solutions based on the standard stars and the trails respectively. For the plates of 1904, the agreement of the different images on the same plate was not investigated; it is probable that the error ζ is larger for 1904 than for 1903.

For comparison, I add some probable errors derived from other sources. The Helsingfors plates are those measured at the Astronomical Laboratory at Groningen, and discussed in the Nos. 1, 10, and 14 of its Publications.

TABLE XVIII.

Probable Errors.

	ϵ	δ	ζ	ρ_0	ρ_s	ρ_t
Plates, 1891. Weight = 1.....	± ″·084	± ″·068	± ″·127	± ″·037	″ ± ″·038	″
,, Average.....	± ·106	± ·086	± ·161	± ·047	± ·048	
Plates, 1903	± ·083	± ·065	± ·124	± ·036	± ·078	± ·047
Plates, 1904	...	...	...	...	± ·062	± ·058
Helsingfors Plates	...	...	...	± ·038	...	
Heliometer (Gill).....	...	...	...	...	± ·070	
,, (Cookson)	...	...	...	...	± ·104	

It is interesting to compare the plates and the heliometer. The probable error given for Gill is that resulting from a new discussion of the observations of 1891, after

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the correction of a number of numerical errors. The value for Cookson is quoted from his paper, *Monthly Notices*, June 1904, page 736. It is not improbable that Mr Cookson's final discussion will considerably reduce his probable error. On the other hand, it is hardly fair towards heliometer observers generally to take Sir David Gill's observations of his best time as a standard.* We will probably somewhat over-estimate the accuracy of the heliometer if we adopt for the probable error of an average observation the mean of the two values contained in Table XVIII., viz., $\pm 0''{\cdot}087$. This agrees exactly with the probable error originally found in my dissertation, and on which the probable errors of the unknowns p and q from the heliometer solution given in Chapter XI. (page 73) are based. The determination of these unknowns depends on 198 observations. The unknowns determined from the plates depend on 30, 29, and 27 plates for the three epochs respectively. The probable errors of these unknowns are about the same as those for the heliometer solution, in the case of 1891 and 1903, and somewhat larger in the case of 1904. In the average it will, perhaps, be a fair estimate if we take 30 plates as equal to 150 heliometer observations, or

1 plate = 5 heliometer observations,

from the point of view of purely accidental error.

The time and labour to be expended to get the same accuracy is thus very much larger for the heliometric method. The measuring of the plates, of course, demands a considerable labour, but the subsequent reductions are so much simpler for the plates that this disadvantage is more than cancelled. The total labour necessary to derive final results of equal accuracy by the photographic method is certainly less than half of that necessary if the heliometer is used.

From the point of view of systematic soundness, I believe that the two methods are equally good. It will be shown in the appendix that the orientation derived from the trails is almost certainly free from any systematic errors.

It must not be forgotten, however, that this conclusion is only justified so long as *short* distances only are measured on the plate. When *réseau*-lines and scale-divisions and the consideration of scale-value enter into the work, it seems probable that much of the accuracy will be lost. The plates are, therefore, from this point of view, only equal to the heliometer for the determination of the inclinations and nodes, which depend on the small co-ordinates y. For the other elements, which must be derived from the measures of much longer distances, the heliometer is doubtlessly far superior.

* The probable error quoted for Gill is that of one observation in distance. The probable error of the position-angles is $\pm 0''{\cdot}093$, and the probable error of one observation derived from distances and position-angles indiscriminately is $\pm 0''{\cdot}079$. This latter probable error is comparable to the value quoted by Cookson.

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CHAPTER XV.*

Summary of Results.

The corrections to the elements p and q of the four satellites found in the preceding Chapters are brought together in Table XIX. I have also included the values found by Mr Cookson, and published in *Monthly Notices*, June 1904, pages 728–747.

TABLE XIX.

Summary of Results.

	1891·75	1901·61	1902·62	1903·72	1904·89
Δp_1	+ 0°·0397 ± °·0038	+ 0°·0170 ± °·0077	+ 0°·0137 ± °·0072	+ 0°·0021 ± °·0060	− 0°·0028 ± °·0078
Δp_2	+ ·0738 ± 27	+ ·1113 ± 56	+ ·0922 ± 44	+ ·0526 ± 33	+ ·0158 ± 44
Δp_3	− ·0030 ± 17	− ·0148 ± 36	− ·0072 ± 28	− ·0199 ± 22	− ·0104 ± 28
Δp_4	+ ·0641 ± 9	+ ·0456 ± 18	+ ·0658 ± 15	+ ·0583 ± 12	+ ·0648 ± 13
Δq_1	− 0°·0259 ± ·0043	− 0°·0695 ± ·0084	− 0°·0755 ± ·0065	− 0°·0597 ± ·0048	− 0°·0335 ± ·0077
Δq_2	+ ·0840 ± 25	− ·1770 ± 49	− ·1860 ± 40	− ·2120 ± 32	− ·2252 ± 48
Δq_3	− ·0705 ± 16	− ·0360 ± 33	− ·0334 ± 23	− ·0442 ± 20	− ·0476 ± 26
Δq_4	− ·0130 ± 9	− ·0359 ± 18	− ·0070 ± 14	− ·0209 ± 11	− ·0209 ± 17

The epochs were supposed to be Greenwich mean noon of the following dates:—

1891·75	October	2	= Julian day 2412008
1901·61	August	11	5608
1902·62	August	16	5978
1903·72	September	20	6378
1904·89	November	23	6808.

The time is in the following discussions counted in days from

1899 December 29·0 = Julian day 2415018,

unless otherwise stated.

For Mr Cookson's observations of 1902, the epoch 1902·62 was inadvertently adopted instead of 1902·60. The corrections to the elements may be assumed to be the same for the two epochs.

There is one point which must be specially mentioned. Mr Cookson found from the reduction of his observations that the residuals of the position-angles of the pairs

* The results of the discussion contained in this and the following chapters were communicated to the Academy of Sciences of Amsterdam (see *Proceedings of the meeting of 31st March* 1906). The results given here differ slightly from those there published, owing to a final revision of the numerical work.

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in which Satellite IV enters can be much improved by adding to the tabular latitude of IV an empirical inequality, of which the coefficient is about 50″ and the period is equal to one-half of the periodic time of Satellite IV. Mr Cookson remarks that there are one or two days in 1891 when the same change in the latitude of IV would reduce large residuals of Sir David Gill's heliometer observations. This is quite true; but there are many more days on which the same change would increase small residuals. I have searched for this inequality in the heliometer observations of 1891, and in the three series of plates treated in this paper. All the residuals of Satellite IV were plotted with the mean longitude of this satellite as argument, and as the result I can with entire confidence declare that there is not the slightest trace of any inequality having an amplitude at all comparable to the errors of observation, and a period at all related to the periodic time of Satellite IV in any of the years 1891, 1903, and 1904. The apparent inequality found by Mr Cookson must, therefore, have been produced by a special cause, which acted only for the oppositions of 1901 and 1902, and has no relation whatever to the general theory of the satellites. It need hardly be mentioned that a similar inequality cannot possibly be explained by the purely gravitational interaction of the satellites and the planet. Since nothing can be said with certainty about the cause which produced this apparent inequality, we also cannot be sure that it has not influenced the determination of the unknowns, at least of p_4 and q_4. It seemed, therefore, to be the safest course to reject the results for Satellite IV for 1901 and 1902.

The discussions which are contained in the following Chapters cannot be considered as final, for reasons which will be stated below. In fact, when I started them I did not expect that it would be possible to push them so far as I have been able to do. I publish them here chiefly because the very fact that they could be pushed so far is a strong proof of the inner consistency and freedom from systematic errors of the observations, but also because the results, if not final, still doubtlessly represent a fair approximation to the truth and a real increase of our knowledge of the elements of the satellites; and further, because some interesting conclusions could be drawn from them which throw a strong light on the general problem of the determination of the elements of the satellites.

I have used Mr Cookson's results as they are published by him in his paper in the *M. N.*, which has already been quoted. It is not probable that his final results will differ much from those then published. The whole discussion will anyhow have to be repeated at some future time, when better values of the theoretical constants entering into the discussion will be available. In the meantime, it is certain that the determination of the different unknowns is materially strengthened by including Cookson's provisional results.

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CHAPTER XVI.

Theoretical Relations.

Before these results can be further discussed, it is necessary to consider the theoretical expressions of the quantities p and q. According to Souillart's theory, the inclinations and nodes* of the orbital planes of the satellites, referred to the fixed plane of Jupiter's orbit of 1850, are defined by the following formulas: †—

$$
(1)\qquad
\begin{aligned}
I_1 \sin N_1 &= b_{11} \sin \theta_1 + b_{12} \sin \theta_2 + b_{13} \sin \theta_3 + b_{14} \sin \theta_4 + \mu_1 \omega \sin \theta_0 \\
I_2 \sin N_2 &= b_{21} \sin \theta_1 + b_{22} \sin \theta_2 + b_{23} \sin \theta_3 + b_{24} \sin \theta_4 + \mu_2 \omega \sin \theta_0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
I_1 \cos N_1 &= \text{same with cosines} \\
&\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
$$

The quantities b_{ij} are constants and the angles θ_i vary proportionally with the time. All angles are counted from the first point of Aries as zero. ω is the inclination and θ_0 is the node of Jupiter's mean equator (*i.e.* neglecting the nutation) on the fixed orbit of 1850.‡ The angles θ_i are Laplace's "noeuds propres" of the satellites. The ratios of the constants b_{ij} are given by the theory. The diagonal terms b_{ii} must be determined from observations, and are Laplace's "inclinaisons propres." If we denote these by γ_i, and introduce the ratios explicitly, we have

$$b_{ii} = \gamma_i \qquad b_{ij} = \sigma_{ij}\, \gamma_j\,.$$

The multipliers σ_{ij} depend on the masses and the mean motions of the satellites, and on the compression of Jupiter. Approximately σ_{ij} is proportional to m_j. The values of these constants, according to Souillart's computations, are given in Table XX.

* By node without further qualification I mean ascending node.

† I have purposely altered Souillart's notation to avoid confusion. In Souillart's work, the quantities referring to Satellite I have no suffix; those referring to the Satellites II, III, IV, and to the equator, have the suffixes 1, 2, 3, and 4 respectively. We have the following identities:—

$$
\begin{array}{lllll}
b_{11} = N & b_{12} = N_1 & b_{13} = N_2 & b_{14} = N_3 & \mu_1 \omega = N_4 \\
b_{21} = N' & b_{22} = N_1' & b_{23} = N_2' & b_{24} = N_3' & \mu_2 \omega = N_4' \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{array}
$$

$$\theta_1 = bt+\gamma \qquad \theta_2 = b_1 t+\gamma_1 \qquad \theta_3 = b_2 t+\gamma_2 \qquad \theta_4 = b_3 t+\gamma_3 \qquad \theta_0 = b_4 t+\gamma_4.$$

‡ If the same quantities referred to the moving orbit are denoted by ω' and θ', we have by Souillart's theory:—

$$\theta' = \theta_0 + 2''\cdot 9706\ T$$
$$\omega' = \omega + 0\cdot 0243\ T,$$

the time being counted in years from 1850·0.

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TABLE XX.

Theoretical Coefficients.

i	σ_{i1}	σ_{i2}	σ_{i3}	σ_{i4}	μ_i
1	1	+ ·0196	+ ·0061	− ·0017	0·9995
2	− ·0119	1	+ ·1588	+ ·0194	0·9943
3	− ·0009	− ·0340	1	+ ·1199	0·9734
4	− ·0001	− ·0009	− ·1944	1	0·8689

The masses are only very imperfectly known; consequently, these values of σ_{ij} can only be considered as rather rough approximations to the true values. Their influence on the results of the following discussion is, however, considerable, and this is the reason why these results cannot be considered as final. It is not here the place to discuss the means to derive better values of the masses, or the probability that we will in a near future have these means at our disposal. In the meantime it is not uninteresting to see what can be reached by adopting Souillart's values of σ_{ij}. When better masses are available the discussion will have to be repeated, and the results here derived will then furnish a valuable approximation.

The observations do not give I and N, but

$$p = -i \sin \Omega, \qquad \text{and} \qquad q = i \cos \Omega,$$

where i and Ω are the instantaneous inclinations and nodes referred to the adopted fundamental plane, the node Ω being counted from the descending node of this plane on the orbit of 1850.

Let ω_0 and ψ_0 be the inclination and descending node of the fundamental plane on the orbit of 1850, then we have, neglecting quantities of the third order in i, I and ω_0:—

$$\begin{aligned} p &= -\mathrm{I} \sin (\mathrm{N} - \psi_0); \\ q &= \mathrm{I} \cos (\mathrm{N} - \psi_0) + \omega_0. \end{aligned}$$

Therefore, if we introduce the further notations

$$\Gamma_i = \psi_0 - \theta_i \qquad \psi = \psi_0 - \theta_0 + 180^\circ$$
$$x_0 = \omega \sin \psi$$
$$y_0 = \omega \cos \psi - \omega_0,$$

we have

$$(2) \qquad \begin{aligned} p_i &= \sum_{j=1}^{4} b_{ij} \sin \Gamma_j - \mu_i x_0 \\ q_i &= \sum_{j=1}^{4} b_{ij} \cos \Gamma_j + (1 - \mu_i)\, \omega_0 - \mu_i y_0. \end{aligned}$$

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Marth evidently intended to adopt

$$\omega_0 = \text{Souillart's value of } \omega$$
$$\psi_0 = 180^\circ + \text{Souillart's value of } \theta_0 \quad (b_4 t + \gamma_4).$$

His formulæ for p and q given on page 513 of his paper are identical to (2), if we take $x_0 = y_0 = 0$, and remember the difference of notation, viz.:—

$$\text{our } i = \text{Marth's } \gamma$$
$$\text{,, } \Omega = \text{,, } \Gamma - O$$
$$\text{,, } \Gamma = \text{,, } G + 180^\circ.$$

His statements regarding the definition of his fundamental plane are, however, rather vague. The fundamental plane adopted throughout my work is defined by its inclination and node referred to the ecliptic and mean equinox of date, which are given by Marth on page 510 of his paper, and from which the values stated above in Chapter IX. have been extracted.

What Marth actually did, seems to be the following:—

1. He states the inclination and node of his fundamental plane referred to Leverrier's *moving* plane of Jupiter's orbit. The node stated there is, however, Souillart's node θ_0 of the equator on the *fixed* orbit of 1850. The difference is $2''{\cdot}9706$ yearly, amounting to $2'\ 28''{\cdot}53$ from 1850 to 1900. For the inclination he has taken Souillart's value of ω', in accordance with his statement.

2. In referring the fundamental plane defined by this node and inclination on the moving orbit (which thus does *not* coincide with Souillart's adopted equator), to the ecliptic, he has altered the longitude of the node O (*i.e.* the node of the fundamental plane on the moving orbit). The consequence is that, as a matter of fact, Marth's zero of longitudes is *not* the node of his fundamental plane on the *moving* orbit of Jupiter, as would be inferred from his statements, but a point whose longitude referred to this node is $+2'\ 28''{\cdot}21$, and which, therefore, is practically the node on the *fixed* orbit of 1850.

Practically, therefore Jupiter's orbit of 1900 has been substituted for that of 1850, in the definition of the position of the fundamental plane, and a fundamental plane practically coinciding with Souillart's equator has been adopted; but the longitudes in this plane are counted from its descending node on the orbit of 1850.

The consequence of this is that the adopted values of ψ, x_0, y_0, as defined above, are zero. Corrections to these adopted values must be introduced as unknowns. ψ_0 and ω_0 are not adopted elements of the systems, but absolute constants, introduced to

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simplify the equations, and not liable to correction by the observations. Consequently, we have with sufficient accuracy :—

$$x_0 = \omega \sin 1^\circ . \psi \qquad \psi = -\delta\theta_0$$
$$y_0 = \omega - \omega_0 = \delta\omega .$$

If we denote the inclination and node of the equator on the fixed orbit of 1900·0 by θ and ω, we have $\delta\theta = \delta\theta_0$, since it is impossible to derive from the observations a correction to the motion of the equator. Consequently :—

$$x_0 = -\sin\omega\,\delta\theta \qquad y_0 = \delta\omega .$$

The adopted values of θ and ω are :—

$$\theta = 315^\circ\ 24'\ 34''{\cdot}34 + 47''{\cdot}3727\ \mathrm{T}$$
$$\omega = \quad 3\quad 4\quad 4{\cdot}75 ,$$

the time being counted in years from 1900 Jan. 0·0 and θ being reckoned from the movable terrestrial equinox. The coefficient of T includes the precession, for which Newcomb's value has been taken, though Marth uses Leverrier's and Souillart Delambre's.

The adopted values of γ and Γ are :—

$$\gamma_1 = 0^\circ{\cdot}00133 \qquad \Gamma_1 = 35^\circ{\cdot}308 + 0{\cdot}1410897\ t$$
$$\gamma_2 = \cdot 46941 \qquad \Gamma_2 = 222{\cdot}0272 + 0{\cdot}03300968\ t$$
$$\gamma_3 = \cdot 17894 \qquad \Gamma_3 = 164{\cdot}8030 + 0{\cdot}00701936\ t$$
$$\gamma_4 = \cdot 22539 \qquad \Gamma_4 = 129{\cdot}6193 + 0{\cdot}00189845\ t ,$$

the time being counted in days from 1899 Dec. 29·0.

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CHAPTER XVII.

Formation of Equations of Condition.

The expressions (2) for p and q must now be compared with the observed values, and the values of the unknowns γ and Γ determined from them. In order further to simplify these formulas, I introduce the quantities x_i and y_i, defined by

$$x_i = \gamma_i \sin \Gamma_i$$
$$y_i = \gamma_i \cos \Gamma_i .$$

The expressions for p and q then become :—

$$(3) \qquad \begin{aligned} p_i &= \Sigma \sigma_{ij} x_j - \mu_i x_0 \\ q_i &= \Sigma \sigma_{ij} y_j + (1 - \mu_i)\, \omega_0 - \mu_i y_0 . \end{aligned}$$

The sums, of course, extend over the values of j from 1 to 4. If we denote the corrections to the adopted values of x_i and y_i by δx_i and δy_i, we have thus :—

$$(4) \qquad \begin{aligned} \Sigma \sigma_{ij}\, \delta x_j - \mu_i x_0 &= \Delta p_i \\ \Sigma \sigma_{ij}\, \delta y_j - \mu_i y_0 &= \Delta q_i , \end{aligned}$$

where for Δp_i and Δq_i the values contained in Table XIX. must be substituted. These are thus the equations of condition, and each opposition furnishes eight of these equations. The solution of these equations has been conducted in the following way.

I define the quantities Δx_i and Δy_i by the equations :—

$$(5) \qquad \begin{aligned} \Sigma \sigma_{ij} \Delta x_j &= \Delta p_i \\ \Sigma \sigma_{ij} \Delta y_j &= \Delta q_i . \end{aligned}$$

These equations can be solved once and for all, and the solution is :—

$$(6) \qquad \begin{aligned} \Delta x_i &= \Sigma \sigma_{ij}' \Delta p_j \\ \Delta y_i &= \Sigma \sigma_{ij}' \Delta q_j . \end{aligned}$$

Further, the quantities μ_i' are defined by

$$\mu_i' = \Sigma \sigma_{ij}' \mu_j .$$

The values of the coefficients σ_{ij}' and μ_i are given in Table XXI.

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TABLE XXI.

THEORETICAL COEFFICIENTS OF SOLUTION.

i	σ_{i1}'	σ_{i2}'	σ_{i3}'	σ_{i4}'	μ_i'
1	+ 0·9998	− 0·0197	− 0·0025	+ 0·0024	+ 0·9793
2	+ ·0117	+ ·9945	− ·1582	− ·0004	+ ·8461
3	+ ·0013	+ ·0329	+ ·9724	− ·1173	+ ·8786
4	+ ·0002	+ ·0073	+ ·1889	+ ·9774	+ 1·0404

Introducing these auxiliary quantities the equations (4) evidently become :—

$$\begin{aligned} \delta x_i - \mu_i' x_0 &= \Delta x_i \\ \delta y_i - \mu_i' y_0 &= \Delta y_i. \end{aligned} \tag{7}$$

The right-hand terms of these equations, computed by the formulas (6) and the coefficients of Table XXI., are contained in Table XXII.

TABLE XXII.

TRANSFORMED RIGHT-HAND MEMBERS OF EQUATIONS OF CONDITION.

Epoch.	Satellite I.		Satellite II.		Satellite III.		Satellite IV.	
	Δx_1	Δy_1	Δx_2	Δy_2	Δx_3	Δy_3	Δx_4	Δy_4
1891·75	+°·0384	−°·0274	+°·0744	+°·0944	−°·0079	−°·0656	+°·0626	−°·0257
1901·61	+·0149	−·0660	+·1132	−·1711	−·0160	−·0367	+·0426	−·0432
1902·62	+·0121	−·0717	+·0930	−·1806	−·0117	−·0379	+·0636	−·0145
1903·72	+·0013	−·0555	+·0554	−·2045	−·0245	−·0475	+·0536	−·0302
1904·89	−·0029	−·0291	+·0173	−·2169	−·0172	−·0512	+·0614	−·0308

The probable errors of these quantities Δx and Δy must be somewhat larger than those of the corresponding Δp and Δq, even if we consider the coefficients σ_{ij}' as absolutely exact, since each participates to some extent in the uncertainty of all the satellites. Weights were, however, assigned to them strictly in inverse proportion to the squares of the probable errors of the corresponding Δp and Δq, and assuming a probable error of weight unity of $\pm 0^\circ\cdot 0050$. The same weight was, however, assigned to Δx and Δy. Even if an entirely satisfactory representation of the observations can be obtained (which cannot be expected on account of the provisional character of the investigation due to the uncertainty of the coefficients σ_{ij}), the

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probable error resulting from the solution may be expected to be a little larger than $\pm 0^{\circ}\cdot 0050$. The adopted values of the weights are given in Table XXIII.

TABLE XXIII.

Adopted Weights.

Epoch.	Satellite I.	Satellite II.	Satellite III.	Satellite IV.
1891·75	1·2	2·8	6·7	25·0
1901·61	0·3	0·7	1·7	0
1902·62	0·4	1·1	2·9	0
1903·72	0·7	2·0	5·0	15·4
1904·89	0·3	1·0	2·5	8·7

The unknowns δx and δy for the different oppositions are not independent of each other. We have :—

$$x_i = \gamma_i \sin \Gamma_i \qquad y_i = \gamma_i \cos \Gamma_i \qquad \Gamma_i = \Gamma_{i0} + \frac{d\Gamma_i}{dt}(t - t_0).$$

In the following discussions t_0 is 1899 Dec. 29·0.

The values of $\frac{d\Gamma_i}{dt}$ are again not independent of each other. They are the four largest roots of an equation of the fifth degree.* They are, however, approximately equal to the quantities $\square$, $\square$, etc., of Souillart, and these again depend on the adopted value of the constant Jb^2 of the mechanical compression of Jupiter, and on the masses of the satellites. Of the masses we need only consider m_3. The adopted mass of II is certainly not in need of any considerable correction, while the influence of m_1 and m_4 on the quantities $\square$ is too small to be of importance for our present purpose. I denote the corrections to Jb^2 and m_3 in fractions of the whole by κ and λ, thus :—

$$\kappa = \frac{\delta Jb^2}{Jb^2} \qquad \lambda = \frac{\delta m_3}{m_3}.$$

We then have :—

$$\frac{d\Gamma_i}{dt} = \left(\frac{d\Gamma_i}{dt}\right)_0 + a_i \kappa + b_i \lambda.$$

The values of the coefficients a and b are by Souillart's numerical theory :—

$a_1 = +0\ 1375$	$b_1 = +0{\cdot}0011$
$a_2 = +0{\cdot}0271$	$b_2 = +0{\cdot}0043$
$a_3 = +0{\cdot}0053$	$b_3 = 0$
$a_4 = +0{\cdot}00073$	$b_4 = +0{\cdot}00097.$

* In Souillart's *second* method of integration, which was adopted by Marth, and is also preferred by Souillart himself.

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The adopted values of γ_i, Γ_{i0} and $\left(\frac{d\Gamma_i}{dt}\right)_0$ were given at the end of the preceding chapter.

We have thus the 12 unknowns :—

$$\gamma_i, \quad \Gamma_{i0}, \quad x_0, \quad y_0, \quad \kappa, \quad \lambda,$$

and for the determination of these unknowns we have 36 equations of the form (7). These equations must be solved by successive approximations. The conditions for the application of the method of least squares are far from being fulfilled. These approximations have been conducted in the following manner.

Let x_{i0} and y_{i0} be the originally adopted values of x_i and y_i, corresponding to the adopted values of γ_i and Γ_i given above at the end of Chapter XVI. The true values are then $x_i = x_{i0} + \delta x_i$ and $y_i = y_{i0} + \delta y_i$. The equations (7) can thus be written :—

$$\begin{aligned} x_i - \mu_i' x_0 &= x_{i0} + \Delta x_i \\ y_i - \mu_i' y_0 &= y_{i0} + \Delta y_i . \end{aligned} \tag{8}$$

The right-hand members of these equations are :—

TABLE XXIV.

RIGHT-HAND MEMBERS OF EQUATIONS (8).

Epoch.	Satellite I.		Satellite II.		Satellite III.		Satellite IV.	
	$x_{10} + \Delta x_1$	$y_{10} + \Delta y_1$	$x_{20} + \Delta x_2$	$y_{20} + \Delta y_2$	$x_{30} + \Delta x_3$	$y_{30} + \Delta y_3$	$x_{40} + \Delta x_4$	$y_{40} + \Delta y_4$
1891·75	+·0377	−·0262	+·4696	−·1590	+·0981	−·2098	+·2497	−·1514
1901·61	+·0161	−·0666	−·2994	−·3950	+·0183	−·2123	+·2134	−·1903
1902·62	+·0123	−·0730	−·3576	−·3122	+·0146	−·2149	+·2326	−·1637
1903·72	+·0003	−·0564	−·4133	−·2297	−·0069	−·2256	+·2206	−·1816
1904·89	−·0042	−·0287	−·4433	−·1264	−·0090	−·2300	+·2262	−·1846

Next adopt some approximation for x_0 and y_0 and let this be x_{00} and y_{00}, so that the true values are $x_0 = x_{00} + \delta x_0$, $y_0 = y_{00} + \delta y_0$.

We have then :—

$$\begin{aligned} x_i - \mu_i' \delta x_0 &= x_{i0} + \Delta x_i + \mu_i' x_{00} \\ y_i - \mu_i' \delta y_0 &= y_{i0} + \Delta y_i + \mu_i' y_{00} . \end{aligned} \tag{9}$$

Now suppose that the approximations x_{00} and y_{00} are already so near the truth

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that the further corrections δx_0 and δy_0 may be neglected, then the equations from which the other unknowns must be determined evidently are :—

$$\begin{aligned} \gamma_i \sin \Gamma_i &= x_{i0} + \Delta x_i + \mu_i' x_{00} \\ \gamma_i \cos \Gamma_i &= y_{i0} + \Delta y_i + \mu_i' y_{00} . \end{aligned} \tag{10}$$

I now compute the auxiliary quantities g_i and G_i by the equations :—

$$\begin{aligned} g_i \sin G_i &= x_{i0} + \Delta x_i + \mu_i' x_{00} \\ g_i \cos G_i &= y_{i0} + \Delta y_i + \mu_i' y_{00} . \end{aligned} \tag{11}$$

The equations then become :—

$$\gamma_i = g_i$$

$$\Gamma_{i0} + \frac{d\Gamma_i}{dt}(t - t_0) = \Gamma_{i0} + \left\{ \left(\frac{d\Gamma_i}{dt}\right)_0 + a_i \kappa + b_i \lambda \right\} (t - t_0) = G_i . \tag{12}$$

On these equations the method of least squares is applicable and the unknowns can be determined by the ordinary process.

If, however, the approximations x_{00} and y_{00} were not very near the truth, it will not be possible to get a satisfactory solution, and a new approximation must be made.

If we suppose the corrections δx_0 and δy_0 to be not entirely negligible, but very small, the method of least squares will still be applicable. The equations then become :—

$$\begin{aligned} &\gamma_i - \mu_i' \sin G_i \, \delta x_0 - \mu_i' \cos G_i \, \delta y_0 = g_i \\ &\Gamma_{i0} - \frac{\mu_i'}{\sin g_i} \cos G_i \, \delta x_0 + \frac{\mu_i'}{\sin g_i} \sin G_i \, \delta y_0 + \left\{ \left(\frac{d\Gamma_i}{dt}\right)_0 + a_i \kappa + b_i \lambda \right\} (t - t_0) = G_i . \end{aligned} \tag{13}$$

If we introduce the approximations

$$\gamma_i = g_{i0} \qquad \Gamma_{i0} = G_{i0} ,$$

so that the true values are

$$\gamma_i = g_{i0} + \delta\gamma_i \qquad \Gamma_{i0} = G_{i0} + \delta\Gamma_{i0} ,$$

and if we put

$$\begin{aligned} \Delta g_i &= g_i - g_{i0} \\ \Delta G_i &= \left\{ G_i - G_{i0} - \left(\frac{d\Gamma_i}{dt}\right)_0 (t - t_0) \right\} \sin g_{i0} , \end{aligned}$$

then the equations are :—

$$\begin{aligned} &\delta\gamma_i - \mu_i' \sin G_i \, \delta x_0 - \mu_i' \cos G_i \, \delta y_0 = \Delta g_i \\ &\sin g_{i0} \, \delta\Gamma_{i0} - \mu_i' \cos G_i \, \delta x_0 + \mu_i' \sin G_i \, \delta y_0 + (a_i \kappa + b_i \lambda) \sin g_{i0} \, (t - t_0) = \Delta G_i . \end{aligned} \tag{14}$$

These equations can be solved by the method of least squares, but if the assumption which has been made, and which alone makes this method applicable, viz., that the corrections δx_0 and δy_0 are small, is not fulfilled, a new approximation must again be made, starting with better values of x_{00} and y_{00}.

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CHAPTER XVIII.

First Solution.

The first approximation which was tried was :—

$$x_0 = y_0 = 0.$$

The right-hand members of the equations (9) are then the same as those of the equations (8), viz., $x_{i0} + \Delta x_i$, $y_{i0} + \Delta y_i$. The values of these right-hand members have been given above in Table XXIV. From them we compute the auxiliary angles g_i and G_i by the formulas (11). The result is :—

TABLE XXV.

Auxiliary Quantities. Solution I.

Epoch.	Satellite I.		Satellite II.		Satellite III.		Satellite IV.	
	g_1	G_1	g_2	G_2	g_3	G_3	g_4	G_4
1891·75	0·0459	124·8	0·4958	108·71	0·2316	154·94	0·2920	121·23
1901·61	·0684	166·4	·4957	217·16	·2131	175·07	·2859	131·72
1902·62	·0740	170·4	·4747	228·88	·2154	176·11	·2844	125·14
1903·72	·0564	179·7	·4728	240·94	·2257	181·75	·2858	129·46
1904·89	·0290	188·3	·4610	254·08	·2301	182·24	·2919	129·22

We now use the equations (12). The second equation was used in its first form, *i.e.* the values of $\frac{d\Gamma}{dt}$ for the four satellites were separately determined, and the unknowns κ and λ not introduced *ab initio*. The equations (12) then consist of two sets of five for each satellite, and each of these eight sets is independent of all others.

Solving the equations with the weights given in Table XXIII., we find :—

$$\gamma_1 = 0.0527 \qquad \Gamma_{10} = 161.6 \qquad \frac{d\Gamma_1}{dt} = 0.01213 \pm .00057$$

$$\gamma_2 = .4820 \qquad \Gamma_{20} = 199.79 \qquad \frac{d\Gamma_2}{dt} = .030266 \pm .000030$$

$$\gamma_3 = .2257 \qquad \Gamma_{30} = 172.44 \qquad \frac{d\Gamma_3}{dt} = .00587 \pm .00021$$

$$\gamma_4 = .2900 \qquad \Gamma_{40} = 126.64 \qquad \frac{d\Gamma_4}{dt} = .00179 \pm .00011.$$

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The residuals from this solution are given in Table XXVI. The probable errors added for comparison are those of the corresponding Δp and Δq, somewhat increased for the reason stated in the preceding chapter.

TABLE XXVI.

Residuals of Solution I.

Epoch.	Satellite I.			Satellite II.			Satellite III.			Satellite IV.		
	$\Delta\gamma_1$	$\sin\gamma_1\Delta\Gamma_1$	Probable Error.	$\Delta\gamma_2$	$\sin\gamma_2\Delta\Gamma_2$	Probable Error.	$\Delta\gamma_3$	$\sin\gamma_3\Delta\Gamma_3$	Probable Error.	$\Delta\gamma_4$	$\sin\gamma_4\Delta\Gamma_4$	Probable Error.
1891·75	−·0068	−·0003	±·0045	+·0138	+·0002	±·0030	+·0059	+·0007	±·0020	+·0020	+·0001	±·0010
1901·61	+ 157	− 32	± 85	+ 137	− 40	± 60	− 116	− 33	± 40	[− 41]	[+ 207]	± 20
1902·62	+ 213	− 38	± 75	− 73	+ 2	± 50	− 103	− 77	± 30	[− 56]	[− 164]	± 20
1903·72	+ 37	+ 34	± 60	− 92	− 1	± 40	0	+ 52	± 25	− 42	+ 19	± 15
1904·89	− 237	+ 48	± 80	− 210	+ 8	± 50	+ 44	− 30	± 30	+ 19	− 35	± 20

The probable error of weight unity derived from these residuals is :—

$$\pm 0''\cdot0084.$$

It is evident at once that the resulting values of $\frac{d\Gamma_i}{dt}$ are entirely incompatible with each other and with the theory. If we subtract from them the adopted values, we get for the determination of κ and λ :—

$$\begin{aligned}
+\ 0\cdot1375\,\kappa + 0\cdot0011\,\lambda &= -\ 0\cdot12896 \pm 0\cdot00057\\
+\ \cdot0271\,\kappa + 0\cdot0043\,\lambda &= -\ \cdot00274 \pm \cdot00003\\
+\ \cdot0053\,\kappa \qquad\qquad &= -\ \cdot00114 \pm \cdot00021\\
+\ \cdot0007\,\kappa + 0\cdot0010\,\lambda &= -\ \cdot00010 \pm \cdot00011.
\end{aligned}$$

The value found for the first satellite is, of course, utterly irreconcilable with any possible values of κ and λ. But also the other three equations cannot be satisfied within their probable errors by reasonable values of κ and λ. Moreover, the inclinations are very badly represented, and the probable error of unit weight, notwithstanding the neglect of the theoretical relations between the motions of the nodes, is 1·7 times the adopted value. All this proves that the approximation $x_0 = y_0 = 0$ is very far from the truth.

Mr. Cookson, in his paper in the *Monthly Notices*, which has already repeatedly been quoted, derived a value of $\frac{d\theta_2}{dt}$ ($= b_1$ of Souillart), and in this derivation he also neglected the correction to the position of the equator. It is thus to be expected

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that his result will agree closely with the value found here. The value of $\frac{d\Gamma_2}{dt}$ corresponding to Cookson's $\frac{d\theta_2}{dt}$ is:—

$$0^{\circ}\cdot02979 \pm 0^{\circ}\cdot00012 .$$

This does indeed agree approximately with the value found from our Solution I, still the difference is too large to be explained by the probable errors alone. The residuals $\sin\gamma_2 \Delta\Gamma_2$ are all much smaller than their probable errors. The difference is therefore not due to an inconsistency of the observations in 1903 and 1904 with those of 1901 and 1902, but must be explained by the circumstance that Mr. Cookson derived his result from the observed values of p_2 and q_2 directly, assuming the terms due to the other satellites to be correct, *i.e.* without eliminating the corrections to the elements of the other satellites by the transformation from Δp and Δq to Δx and Δy, as was done here.

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CHAPTER XIX.

Final Solution from Modern Observations alone.

A new solution must now be made, in which x_0 and y_0 can no longer be assumed equal to zero. It has been shown in Chapter XVII. that the values of x_0 and y_0 must be found by trial and error. I have made several solutions with different approximations x_{00} and y_{00}, which it is not necessary to mention here in detail. It appeared finally that the true values must be somewhere near the values

$$x_{00} = -0^{\circ}\cdot0180$$
$$y_{00} = +0\cdot0400.$$

These were therefore introduced as approximate values in the formulas (9). The corrections which must be applied to the right-hand members of the equations (8) contained in Table XXII., are thus:—

$\mu_1' x_{00} = -0^{\circ}\cdot0176$	$\mu_2' x_{00} = -0^{\circ}\cdot0152$	$\mu_3' x_{00} = -0^{\circ}\cdot0158$	$\mu_4' x_{00} = -0^{\circ}\cdot0189$
$\mu_1' y_{00} = +\ \cdot0392$	$\mu_2' y_{00} = +\ \cdot0338$	$\mu_3' y_{00} = +\ \cdot0351$	$\mu_4' y_{00} = +\ \cdot0416.$

With the right-hand members thus corrected the following values of g_i and G_i were computed by the formulas (11):—

TABLE XXVII.

Auxiliary Quantities. Solution VI.

Epoch.	Satellite I.		Satellite II.		Satellite III.		Satellite IV.	
	g_1	G_1	g_2	G_2	g_3	G_3	g_4	G_4
1891·75	0°·0239	57°·1	0°·4713	105°·41	0°·1931	154°·78	0°·2556	115°·44
1901·61	·0275	183·1	·4791	221·05	·1772	179·19	·2448	127·40
1902·62	·0342	188·9	·4653	233·25	·1798	180·38	·2462	119·74
1903·72	·0244	225·1	·4712	245·44	·1919	186·80	·2455	124·76
1904·89	·0242	295·7	·4677	258·58	·1965	187·25	·2518	124·60

The following first approximations were introduced:—

$g_{10} = 0^{\circ}\cdot0258$	$G_{10} = 77^{\circ}\cdot9$	
$g_{20} = \cdot4706$	$G_{20} = 201\cdot92$	$\kappa = -0\cdot0368$
$g_{30} = \cdot1933$	$G_{30} = 176\cdot05$	$\lambda = 0$
$g_{40} = \cdot2518$	$G_{40} = 121\cdot43$	

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It does not appear necessary to write out the equations of condition. They can be easily formed by the formulas (14) and the angles G_i contained in Table XXVII. It appeared, as could have been expected, that the unknown λ cannot be determined with any weight from the available material. It has therefore been transferred to the right-hand members.

It had appeared from all the preceding solutions that the equations in γ_3 for 1901 and 1902 left very large residuals, and that their rejection had very little effect on the final result, but naturally improved the representation considerably. I have therefore rejected the observations of satellite III in 1901 and 1902.* It is not impossible that the unknown cause of the apparent inequality of Satellite IV, which Mr Cookson found for those years, has also to a certain extent influenced Satellite III, through the large coefficient σ_{34}.

Normal equations were formed and solved. The solution is effected very easily by successive approximations, since each normal, with the exception of the last three, contains only one of the unknowns $\delta\gamma_i$ or $\sin\gamma_i\delta\Gamma_{i0}$, together with the unknowns δx_0, δy_0 and $\delta\kappa$ (the last only in the equations in $\sin\gamma_i\delta\Gamma_{i0}$). The following are the final values :—

Solution VI.

$$x_0 = -0.0180 \pm .0023 - .0049\,\lambda$$
$$y_0 = +\ .0388 \pm\ 44 - .0186$$
$$\kappa = -0.0396 \pm .0096 - .1815\,\lambda$$

γ_1 =	0°.0258	±	°.0032	− °.0005 λ	Γ_{10} =	77°.5	± 8°.6	− 3°.4 λ
γ_2 =	.4711	±	27	+ .0067	Γ_{20} =	201.83	± 0.36	− 0.28
γ_3 =	.1943	±	41	+ .0149	Γ_{30} =	176.05	± 0.78	+ 1.79
γ_4 =	.2524	±	35	+ .0046	Γ_{40} =	121.68	± 0.93	+ 5.16

The coefficients of λ are those resulting from another solution (Solution V), which is practically identical with Solution VI, and only differs from it in that satellite III for 1901 and 1902 was not rejected.

No weights of the unknowns were determined for Solution VI. The probable errors given have been derived with the weights found in Solution V, but using the probable error of unit weight of Solution VI.

* The residuals $\sin\gamma_3\Delta\Gamma_3$ in 1901 and 1902 show nothing extraordinary. It appeared, however, better to reject the whole satellite if the inclinations are rejected. Also the equations would have lost their symmetry if only the inclinations were rejected and not the nodes. The effect of the rejections on the resulting value of the node is negligible.

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The residuals from this solution are :—

TABLE XXVIII.

Residuals of Solution VI.

Epoch.	Satellite I.			Satellite II.			Satellite III.			Satellite IV.		
	$\Delta\gamma_1$	$\sin\gamma_1\Delta\Gamma_1$	Probable Error.	$\Delta\gamma_2$	$\sin\gamma_2\Delta\Gamma_2$	Probable Error.	$\Delta\gamma_3$	$\sin\gamma_3\Delta\Gamma_3$	Probable Error.	$\Delta\gamma_4$	$\sin\gamma_4\Delta\Gamma_4$	Probable Error.
1891·75	−·0024	+·0134	±·0045	+·0005	−·0014	±·0030	−·0002	−·0022	±·0020	+·0037	−·0015	±·0010
1901·61	+ 29	+ 114	± 85	+ 88	+ 24	± 60	[− 160]	[− 31]	± 40	[− 69]	[+ 213]	± 20
1902·62	+ 96	− 88	± 75	− 52	+ 54	± 50	[− 134]	[− 74]	± 30	[− 56]	[− 152]	± 20
1903·72	− 9	− 177	± 60	+ 5	+ 4	± 40	− 13	+ 49	± 25	− 62	+ 34	± 15
1904·89	− 24	− 120	± 80	− 32	− 45	± 50	+ 33	− 35	± 30	+ 1	− 7	± 20

The probable errors added for comparison are the same as those in Table XXVI.

The probable error of weight unity derived from the residuals is :—

$$\pm 0^{\circ}\cdot 0066.$$

The residuals of Solution V are very nearly equal to those of Solution VI. The probable error of unit weight is $\pm 0^{\circ}\cdot 0074$. The difference is due exclusively to the rejection of the large residuals of Satellite III. The representation of the observations by the Solution VI (and also by V) is fairly satisfactory. Only the residuals $\sin\gamma_1\Delta\Gamma_1$ show that the observations demand a smaller motion of the node of Satellite I than is theoretically consistent with the motions of the other nodes. The value of $\frac{d\Gamma_1}{dt}$ derived from the value of κ given by the solution is $0^{\circ}\cdot 1357$, while the value which best represents the observations would be $0^{\circ}\cdot 1250$. Though the agreement is not perfect, it is enormously improved compared with Solution I. The remaining difference is probably for the greater part due to the uncertainty of the adopted masses.

The inclinations are all extremely satisfactorily represented, only six of the sixteen not rejected residuals exceeding their probable error, and only two exceeding twice their probable error. The residuals $\Delta\gamma_4$ are rather large, but not systematic. The representation of the nodes is also satisfactory.

The most striking result of this solution is that the large negative correction to the adopted constant of the compression of Jupiter, about $\kappa = -\frac{1}{10}$, which was found by Mr Cookson and confirmed, so far as Satellite II is concerned, by Solution I, is reduced

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to little more than one third of this value, as a consequence of the introduction of the unknowns x_0 and y_0.

With regard to these unknowns, it may be remembered that Mr Cookson also found corrections to the position of the equator, from the Satellites III and IV, which are in the same direction as the values found here. In deriving the corrections corresponding to our unknowns x_0 and y_0 Cookson neglected the corrections to all the other unknowns entering into the expressions for p_3 and q_3, p_4 and q_4. It is to this neglect that the difference between his values of ω found from Satellite III and from Satellite IV is chiefly due. No doubt a part of the difference may also be due to errors in the adopted masses, as Mr Cookson suggested.

From the values of Γ_i, the "noeuds propres" θ_i may be derived by the formula which served as the definition of Γ_i, viz. :—

$$\theta_i = \psi_0 - \Gamma_i$$

from which

$$\frac{d\theta_i}{dt} = \frac{d\psi_0}{dt} - \frac{d\Gamma_i}{dt}.$$

The value of $\frac{d\psi_0}{dt}$ is $0^\circ{\cdot}000036$.

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CHAPTER XX.

Final Solution from Modern Observations, combined with Delambre.

Carrying the values of θ_i found in the previous solutions back from 1900·0 to 1750·0 by means of the values of $\frac{d\theta_i}{dt}$ derived from the solution, we get the following results :—

	Cookson.	Solution I.	Solution VI.	Damoiseau.	Delambre.	Solution VII.
θ_2	122°·9	151°·8 ± 1°·7	235°·4 ± 14°·3	282°·0	283°·3	281°·1
θ_3		282·9 ± 11·0	329·2 ± 2·9	353·5	352·5	337·8
θ_4		104·7 ± 6·1	114·0 ± 1·0	98·3	105·0	117·1

The last column (Solution VII) will be explained below.

The probable errors for Solution I, where no theoretical relation between the motions of the different nodes was introduced, are of course independent of each other. In the case of Solution VI, practically only $\frac{d\theta_2}{dt}$ was derived directly from the observations, and the probable error of θ_2 in 1750 is therefore a true measure of the accuracy of the computed value.* The motions of the other nodes are derived theoretically from $\frac{d\theta_2}{dt}$, and their probable error must be increased by the effect of the uncertainties of the constants (α_i) of Souillart's theory.

It is very difficult to estimate the probable error of Delambre's and Damoiseau's results. It is also not known in how far Damoiseau is independent of Delambre, and therefore the agreement between the two is no criterion. Both sets of nodes have naturally been derived from observations of the duration of eclipses, which give a rather weak determination of the latitude, especially in the case of Satellites II and IV, where ingresses and egresses of different eclipses must be combined. The determination of the latitude of III, of which complete eclipses have often been observed, is much stronger. Still, the observations embrace a very long period of time, in the case of II many revolutions of the node, and the nodes, especially θ_2 and θ_3, must be determined

* Neglecting the effect of the uncertainty of the coefficients σ_{ij}'.

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from them with fair accuracy. The unknown factor $1+\kappa'$, depending on the geometrical compression of the planet, which enters into all determinations of latitude from eclipses,* does, of course, only affect the inclinations, not the nodes.† On the whole, I do not think the probable errors of Damoiseau's or Delambre's values of θ_2 and θ_3 can be much larger than $\pm 5°$. The determination of θ_4 is, of course, much weaker. Solution I, therefore, is utterly irreconcilable with the values for 1750·0 found by Delambre and Damoiseau (at least if Souillart's theory is correct). In the case of Solution VI, the differences are between two and three times the sum of the probable errors, and are thus still explicable by the ordinary errors of observation, on the assumption that the theory is correct.

They can, however, only be so explained on the assumption that a considerable part of this difference is due to the value of $\frac{d\theta_i}{dt}$ derived from the modern observations (Solution VI), *i.e.* to the correction κ. The question thus arises if we cannot improve our results by deriving the motions of the nodes from a comparison with Delambre.

Led by these considerations, I have made another solution, in which κ and λ were not introduced as unknowns, but the values

$$\kappa = -0°·0126$$
$$\lambda = 0$$

were adopted. This solution (Solution IV) gave the following results :—

$$x_0 = -0°·0164$$
$$y_0 = +\ ·0469$$

$\gamma_1 = 0°·0252$	$\Gamma_{10} = 77°·5$
$\gamma_2 = \ ·4681$	$\Gamma_{20} = 202\ ·17$
$\gamma_3 = \ ·1841$	$\Gamma_{30} = 175\ ·03$
$\gamma_4 = \ ·2499$	$\Gamma_{40} = 119\ ·89.$

I have then made still another solution (Solution VII), starting from these values as a first approximation and rejecting Satellite III in 1901 and 1902.

For Solution VII we have thus :—

$$x_{00} = -0°·0164$$
$$y_{00} = +0\ ·0469$$

* See my dissertation, page 77.

† At least if it can be assumed constant for different observers and different instruments, which seems not at all certain. If it depends on the instrument, it also depends on the time, since instruments improve with the time. This would mean an apparent secular change in the latitudes, which can hardly be without effect on the resulting motion of the nodes.

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and the corrections to be applied to the right-hand members contained in Table XXIV. are:—

$$\mu_1'x_{00} = -^\circ\cdot0163 \quad \mu_2'x_{00} = -^\circ\cdot0140 \quad \mu_3'x_{00} = -^\circ\cdot0146 \quad \mu_4'x_{00} = -^\circ\cdot0173$$
$$\mu_1'y_{00} = +\ \cdot0459 \quad \mu_2'y_{00} = +\ \cdot0397 \quad \mu_3'y_{00} = +\ \cdot0412 \quad \mu_4'y_{00} = +\ \cdot0488.$$

The following values of the quantities g_i and G_i were derived by the formulas (11) :—

TABLE XXIX.

Auxiliary Quantities. Solution VII.

Epoch.	Satellite I.		Satellite II.		Satellite III.		Satellite IV.	
	g_1	G_1	g_2	G_2	g_3	G_3	g_4	G_4
1891·75	°·0290	47·4	°·4710	104·67	°·1881	153·65	°·2540	113·82
1901·61	·0207	180·6	·4737	221·41	·1711	178·76	·2418	125·81
1902·62	·0274	188·4	·4608	233·75	·1737	180·00	·2440	118·09
1903·72	·0191	236·7	·4676	246·02	·1857	186·65	·2428	123·16
1904·89	·0268	310·0	·4655	259·27	·1903	187·12	·2492	123·03

The values of the unknowns resulting from Solution IV were introduced as approximations, equations of condition were formed by the formulas (14), and normal equations were formed and solved. The resulting values of the unknowns are :—

$$x_0 = -0^\circ\cdot0206 \pm {}^\circ\cdot0024$$
$$y_0 = +0\ \cdot0490 \pm\ \cdot0024$$

$$\gamma_1 = 0^\circ\cdot0271 \pm {}^\circ\cdot0042 \qquad \Gamma_{10} = 76^\circ\cdot8 \pm 9^\circ\cdot7$$
$$\gamma_2 = \cdot4682 \pm 26 \qquad \Gamma_{20} = 202\cdot37 \pm 0\cdot32$$
$$\gamma_3 = \cdot1864 \pm 29 \qquad \Gamma_{30} = 176\cdot53 \pm 0\cdot88$$
$$\gamma_4 = \cdot2450 \pm 28 \qquad \Gamma_{40} = 119\cdot90 \pm 0\cdot63.$$

The residuals are given in Table XXX., where again the estimated probable errors derived from the observations are inserted for comparison.

The probable error of weight unity is :—

$$\pm0^\circ\cdot0069.$$

The residuals of Solution IV were practically the same, and the probable error of unit weight was $\pm0^\circ\cdot0076$. The difference is due exclusively to the rejection of Satellite III in 1901 and 1902.

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TABLE XXX.

Residuals of Solution VII.

Epoch.	Satellite I.			Satellite II.			Satellite III.			Satellite IV.		
	$\Delta\gamma_1$	$\sin\gamma_1\Delta\Gamma_1$	Probable Error.	$\Delta\gamma_2$	$\sin\gamma_2\Delta\Gamma_2$	Probable Error.	$\Delta\gamma_3$	$\sin\gamma_3\Delta\Gamma_3$	Probable Error.	$\Delta\gamma_4$	$\sin\gamma_4\Delta\Gamma_4$	Probable Error.
1891·75	+·0034	+·0089	±·0045	−·0010	+·0043	±·0030	−·0018	−·0024	±·0020	+·0043	−·0019	±·0010
1901·61	− 97	+ 129	± 85	+ 66	+ 20	± 60	[− 171]	[− 23]	± 40	[− 81]	[+ 212]	± 20
1902·62	+ 8	− 54	± 75	− 55	+ 37	± 50	[− 144]	[− 65]	± 30	[− 59]	[− 157]	± 20
1903·72	− 41	− 110	± 60	+ 20	− 33	± 40	− 22	+ 60	± 25	− 68	+ 36	± 15
1904·89	+ 36	− 97	± 80	+ 5	− 105	± 50	+ 26	− 19	± 30	− 4	− 5	± 20

The only important difference between these residuals and those of Solution VI is that the residuals $\sin\gamma_2\Delta\Gamma_2$ in Solution VII are slightly systematic, which is naturally due to the fact that a value of κ was adopted different from the value which best satisfies the observations.

The probable error of the adopted value of κ is difficult to estimate. I do not think it will be much larger than $\pm 1/300$, which is about the probable error corresponding to a probable error of $\pm 5°$ in Delambre's θ_2. If we adopt this probable error, then the difference between the values of κ in the Solutions VI and VII is about twice the sum of the probable errors. The difference of the two values of y_0 is one and a half times the sum of the probable errors. For all the other unknowns the differences between the two solutions are within the sum of their probable errors, or exceed this sum very little. In several cases the two values agree within the probable error of each of them. It may be remarked that, with only one exception (γ_2), all the corrections to the originally adopted values of the elements are many times larger than their probable errors.

The probable error of unit weight which was assumed in assigning the weights was :—

$$\pm 0°·0050.$$

The probable errors resulting from the solutions are :—

Solution VI . . . . . $\pm 0°·0066$;
Solution VII . . . . . $\pm 0·0069$.

The excess of these probable errors over the assumed value is entirely due to the large weights assigned to Satellite IV. The residuals of this satellite are, on the whole,

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not much smaller than those of the other satellites, notwithstanding their very high weights. It will be remembered (see Chapter IV.) that from the discussion of the measures on the plates, we found the following relative weights of the four satellites :—

	I.	II.	III.	IV.
Plates of 1891 :	1·0	1·25	1·7	0·75
Plates of 1903 :	1·0	1·1	1·5	0·75

From the residuals of the solutions these weights would be :—

	I.	II.	III.	IV.
Solution VI	1·0	3·6	1·5	0·4
Solution VII	1·0	1·1	0·6	0·1

If the probable error of unit weight is derived from the residuals of the Satellites I, II, and III only, we find :—

Solution VI . . . . . $\pm 0^{s}\cdot 0051$,
Solution VII . . . . . $\pm 0\cdot 0048$,

or exactly the assumed value.

It would thus appear that a far too high weight has been assigned to Satellite IV in assuming the observations of this satellite to be of the same accuracy as those of the other satellites.

We must, however, not attach too much importance to this remark. For on the one hand, the part of the residuals which is due to the uncertainty of the coefficients σ_{ij} (*i.e.* of the masses), even if assumed to be of the same order of magnitude as it is for the other satellites, has a much larger influence on the resulting probable error, on account of the large weight. On the other hand, the coefficient σ_{43} is the largest of all the coefficients σ_{ij}, and consequently an error in the adopted value of m_3 (to which σ_{43} is approximately proportional) has a larger effect on the quantities Δx_4 and Δy_4, than the same or any other error in the adopted masses has on the corresponding quantities belonging to the other satellites. Further, the geometrical conditions of the problem are such that Satellite IV, notwithstanding its high weight, has practically very little influence on the determination of any of the unknowns, except its own inclination and node. This is a consequence of the slowness of the motion of the node (about 9° between 1891 and 1904). The position of the equator and the correction κ, on which the motion of the nodes depend, are practically determined from the three inner satellites, and IV has to follow suit as best it can. This is clearly shown by the fact, which was actually verified in Solution IV, that it makes only an inappreciable difference in the values of x_0 and y_0 whether Satellite IV is included in the normal equations or not, although more than two-thirds of the

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coefficients of x_0 and y_0 in these equations is derived from the equations of condition in $\delta\gamma_4$ and sin $\gamma_4\delta\Gamma_4$. On the whole it seems very probable that the large residuals of Satellite IV are not due to a smaller accuracy of the observations of this satellite, but to the influence of errors in the adopted values of the masses. The same reasoning applies in a lesser degree to Satellite III.

It has already been remarked that the problem is not one which is by its nature well adapted to the application of the method of least squares. Therefore the conclusions drawn from this application must not be accepted without some qualification.

Practically the solution has been effected by trial and error, and when a fairly satisfactory solution was found, the method of least squares was applied to the small residuals, with the sole object of satisfying myself that the approximation was really sufficient. When the corrections found by this method were rather considerable, a new solution was made, and the method of least squares was again applied to its residuals as a mechanical check on the correctness of the solution. In cases of such delicate nature, one never feels quite sure that everything has received due consideration and all bias has been excluded, and an automatically working control, such as is afforded by the method of least squares, is very valuable.

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CHAPTER XXI.

Conclusion.

I give below the values of the elements resulting from the two solutions discussed in the two last chapters. It may be remembered that Solution VI is based entirely on the Cape observations of 1891, 1901, 1902, 1903, and 1904, while in Solution VII the unknown κ, on which the motions of the nodes depend, was derived from the comparison of the modern observations with Delambre (1750·0), and all other unknowns were determined from the modern observations alone. The coefficients of $\lambda = \frac{\delta m_3}{m_3}$ give the effect on the values of the unknowns of a correction to the adopted mass of III, so far as it affects the solution through the motions of the nodes. They do, of course, *not* contain the effect of the corresponding correction to coefficients σ_{ij}.

The epoch for which the elements are given is :—

1900 *January* 0·0 *Greenwich mean noon.*

	Solution VI.	Coefficients of $\lambda = \frac{\delta m_3}{m_3}$	Solution VII.	Adopted values.
	Inclinations (" *Inclinaisons propres* ").			
γ_1	$0^\circ 0258 \pm 0^\circ 0032$	$-0^\circ 0005$	$0^\circ 0271 \pm 0^\circ 0042$	$0^\circ 0013$
γ_2	$\cdot 4711 \pm 27$	$+ \cdot 0067$	$\cdot 4682 \pm 26$	$\cdot 4694$
γ_3	$\cdot 1943 \pm 41$	$+ \cdot 0149$	$\cdot 1864 \pm 29$	$\cdot 1789$
γ_4	$\cdot 2524 \pm 35$	$+ \cdot 0046$	$\cdot 2450 \pm 28$	$\cdot 2254$
	Nodes (" *Noeuds propres* ") *with signs reversed and counted from descending node of fundamental plane on Jupiter's orbit of* 1850·0.			
Γ_1	$77^\circ 8 \pm 8^\circ 6$	$-3^\circ 4$	$77^\circ 1 \pm 9^\circ 7$	$35^\circ 6$
Γ_2	$201 \cdot 89 \pm 0 \cdot 36$	$-0 \cdot 28$	$202 \cdot 43 \pm 0 \cdot 32$	$222 \cdot 09$
Γ_3	$176 \cdot 06 \pm 0 \cdot 78$	$+1 \cdot 79$	$176 \cdot 54 \pm 0 \cdot 88$	$164 \cdot 82$
Γ_4	$121 \cdot 68 \pm 0 \cdot 93$	$+5 \cdot 16$	$119 \cdot 90 \pm 0 \cdot 63$	$129 \cdot 62$

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	Solution VI.	Coefficients of $\lambda = \frac{\delta m_3}{m_3}$	Solution VII.	Adopted values.
Nodes ("Noeuds propres") counted from terrestrial equinox.				
θ_1	$57^\circ 6 \pm 8^\circ 6$	$+3^\circ 4$	$57^\circ 4 \pm 9^\circ 7$	$99^\circ 8$
θ_2	$293^\circ 56 \pm 0^\circ 36$	$+0^\circ 28$	$293^\circ 02 \pm 0^\circ 32$	$273^\circ 32$
θ_3	$319^\circ 39 \pm 0^\circ 78$	$-1^\circ 79$	$318^\circ 96 \pm 0^\circ 88$	$330^\circ 59$
θ_4	$13^\circ 77 \pm 0^\circ 93$	$-5^\circ 16$	$15^\circ 55 \pm 0^\circ 63$	$5^\circ 79$
Motions of nodes (daily).				
$\frac{d\theta_1}{dt}$	$-0^\circ 13561 \pm 0^\circ 00131$	$+0^\circ 0238$	$-0^\circ 13932$	$-0^\circ 14105$
$\frac{d\theta_2}{dt}$	$-0^\circ 031893 \pm 0^\circ 000259$	$+0^\circ 00064$	$-0^\circ 032633$	$-0^\circ 032974$
$\frac{d\theta_3}{dt}$	$-0^\circ 006757 \pm 0^\circ 000051$	$+0^\circ 00096$	$-0^\circ 006916$	$-0^\circ 006983$
$\frac{d\theta_4}{dt}$	$-0^\circ 001833 \pm 0^\circ 000007$	$-0^\circ 00084$	$-0^\circ 001854$	$-0^\circ 001863$
Plane of Jupiter's equator, referred to Leverrier's orbit of 1900·0.				
ω	$3^\circ 1068 \pm 0^\circ 0044$	$-0^\circ 0186$	$3^\circ 1170 \pm 0^\circ 0024$	$3^\circ 0680$
θ	$315^\circ 735 \pm 0^\circ 043$	$+0^\circ 090$	$315^\circ 789 \pm 0^\circ 045$	$315^\circ 410$
Correction to compression of Jupiter.				
κ	$-0^\circ 0396 \pm 0^\circ 0096$	$-0^\circ 1815$	$-0^\circ 0126$	

Finally, I have computed the residuals of the original data, viz.: Δp and Δq. The residuals of x_i and y_i were derived by actual computation of x_i and y_i from the elements given by the solution, and comparison with the observed values, thus :—

$$\text{Res. } (x_i) = x_{i0} + \Delta x_i - (x_i - \mu_i' x_0)_2$$

and similarly for y_i [compare formula (8) of Chapter XVII.]. The residuals of p_i and q_i were then derived by means of the coefficients σ_{ij}, thus :—

$$\text{Res. } (p_i) = \Sigma \sigma_{ij} \text{ Res. } (x_j).$$

These residuals are given in Table XXXI. The probable errors given for comparison are those derived directly from the observations.

The probable errors of unit weight derived from these residuals are :—

Solution VI . . . . . $\pm 0^\circ 0065$;
Solution VII . . . . . $\pm 0^\circ 0068$.

These values are practically the same as those found from the residuals $\Delta\gamma$ and $\sin\gamma\,\Delta\Gamma$.

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TABLE XXXI.

RESIDUALS OF Δp AND Δq.

Epoch.	Satellite I.			Satellite II.			Satellite III.			Satellite IV.		
	Solution VI.	Solution VII.	Probable Error.	Solution VI.	Solution VII.	Probable Error.	Solution VI.	Solution VII.	Probable Error.	Solution VI.	Solution VII.	Probable Error.
	Δp_1			Δp_2			Δp_3			Δp_4		
1891·75	−·0011	+·0094	±·0038	+·0014	−·0015	±·0027	+·0025	+·0036	±·0017	+·0036	+·0042	±·0009
1901·61	− 115	− 139	± 77	− 74	− 35	± 56	+ 9	+ 21	± 36	− 182	− 184	± 18
1902·62	+ 67	+ 59	± 72	+ 21	+ 36	± 44	+ 79	+ 69	± 28	+ 17	+ 15	± 15
1903·72	+ 83	+ 71	± 60	− 16	− 14	± 33	− 58	− 64	± 22	− 60	− 64	± 12
1904·89	− 52	− 57	± 78	+ 44	+ 19	± 44	+ 29	+ 20	± 28	+ 1	− 4	± 13
	Δq_1			Δq_2			Δq_3			Δq_4		
1891·75	−·0029	−·0042	±·0043	+·0016	−·0033	±·0025	+·0013	+·0031	±·0016	−·0004	−·0005	±·0009
1901·61	− 47	+ 67	± 84	− 25	− 3	± 49	+ 157	+ 157	± 33	− 164	− 163	± 18
1902·62	− 119	− 15	± 65	+ 100	+ 87	± 40	+ 149	+ 152	± 23	+ 133	+ 135	± 14
1903·72	− 148	− 67	± 48	+ 8	− 32	± 32	+ 21	+ 31	± 20	+ 3	+ 16	± 11
1904·89	− 107	− 5	± 77	− 40	− 108	± 48	− 34	− 23	± 26	+ 14	+ 14	± 17

The close agreement of the probable error of unit weight derived from the solution with the assumed value is a strong proof of the inner consistency of the observations, and the absence of systematic errors, especially since the observations of the different oppositions were made by different instruments and different observers. I doubt not that, when better masses and coefficients σ_{ij} will eventually be available, it will be possible to represent the observations still more closely than is now accomplished by the Solutions VI and VII.*

With regard to the often referred to uncertainty of the theoretical constants σ_{ij} and μ_i, it is a fact which seems worthy of the notice of mathematicians that these coefficients as determined by Souillart, from the *same* masses and elements by his two methods of integration, in several instances differ by amounts which in their effect on the resulting tabular values of p and q are quite comparable with the errors of observation. It seems improbable that this defect of the theory will be remedied so

* It must also be remembered, that for 1901 and 1902 Cookson's provisional results, as published in the *Monthly Notices* of June 1904, have been used. The assumption was necessarily made, that Cookson's fundamental plane coincides with the one used by me. It now appears from Mr. Cookson's final discussion of the places of his standard stars, that the values of Δp and Δq, as used above, are not in agreement with this assumption. If the corrections necessary to reduce them to the same plane are applied, and a new solution is made, the representation of the observations will doubtlessly be improved. I am much indebted to the courtesy of Mr. Cookson, who kindly communicated his final results to me in advance of publication. [Note added Sept. 1906.—W. de S.]

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long as the orbital plane of Jupiter is used as the fundamental plane of the theory, instead of its equator. The coefficients adopted by Marth and by me are those resulting from the second method of integration, which is also preferred by Souillart himself.

One of the most important results of the present work is certainly the compression of Jupiter, on which the motions of the nodes depend. The unknown κ, which was determined from the solutions, is the correction, in fractions of the whole, to the adopted value of Jb^2 :—

$$\kappa = \frac{\delta\, Jb^2}{Jb^2}.$$

J is the constant of the mechanical compression, which depends on the inner distribution of the mass and the ellipticity of the successive shells of which the planet is conceived to consist; b is the equatorial radius of the free surface. The motions of the nodes depend on $\frac{Jb^2}{a_i^2}$, where a_i is the semiaxis major of the satellite's orbit.

The value of J adopted in Souillart's theory is

$$J = 0{\cdot}0247187.$$

The value of b is not explicitly stated. The motions of the nodes were computed by means of the values of $\frac{a_i}{b}$ stated on page 10 of Souillart's second memoir. The value of b which was actually adopted therefore depends on the adopted values of a_i, and these again depend on the adopted mass of the system, since the mean motions can be considered as absolutely correct. The reciprocal of the mass adopted in computing the tabular places for all the observations treated in the present discussion is $M = 1047{\cdot}7$. By comparing the corresponding values of a_i with Souillart's values of $\frac{a}{b}$, we find :—

$$\log b = 6{\cdot}668410 - 10.$$

This corresponds to the diameter of Jupiter at mean distance ($\Delta = 5{\cdot}202800$) :—

$$d = 36''{\cdot}9512.$$

If it is desired to adopt another value d' of the diameter, retaining the same motions of the nodes, and therefore the same Jb^2, we must adopt another value J' of the constant J, viz.:—

$$J' = J\left(\frac{36''{\cdot}9512}{d'}\right)^2.$$

I give below the values of Jb^2 from different sources, expressed in two different units, viz.: (1) the semiaxis major of the earth's orbit, and (2) the value of b corresponding to the diameter $d = 39''{\cdot}00$. This value expressed in the first unit is $\log b = 6{\cdot}691847 - 10$. The numerical value of J does not depend on the units of mass,

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time, and length, but b of course does. The values of J and d for Damoiseau are quoted from Souillart, who quotes them from the "*Mémoire Manuscrit.*" *

Authority.	J	d	Jb^2 (Astronomical units).	Jb^2 ($b=1$ for $d=39''\cdot00$).
Laplace	0·0219013	39·00	0·000000005298 66	0·0219013
Damoiseau	0·0247187	36·74	530722	219368
Tabular places of this paper	0·0247187	36·9512	536841	221897
Cohn (A. N. 3403–4)	0·02095	39·00	5068	2095
Solution VI			5156 ± 51	2131 ± 21
Solution VII			5301	2191

The result of the present investigation is thus that the true value of Jb^2 is very nearly that adopted by Laplace (Delambre) and Damoiseau. The value of J itself can, of course, not be stated, since the radius b is very imperfectly known. (Compare also my dissertation, page 84.)

It has been repeatedly pointed out that the motion of the perijove of Satellite V affords a strong determination of the constant Jb^2. Now the latest determinations of the motion of this perijove seem to point to a very large negative correction to the adopted value. We have the following values of this motion (yearly.)

From observations, by Cohn (*Astr. Nachr.*, 3403–4)	911°·7
„ „ „ Miss Dobbin (*Astr. Journ.*, 562)	887·2
From value of Jb^2 in Solution VI	926
„ „ Jb^2 „ VII	952

I have shown in my dissertation (page 83) that the actual motion may be expected to be about $\frac{1}{50}$ *larger* than the theoretical value derived from the consideration of Jb^2 alone. Both Solution VI and Solution VII are, therefore, irreconcilable with the motions derived from the observations. Cohn's value depends practically on an interval of only one year, 1892–1893. Miss Dobbin used the observations of 1898, 1899, 1902, and 1903. I wish not to detract from the value of her investigation; too few details are published to enable the reader to form a judgment of the reliability of the resulting motion, without practically repeating a large portion of the work. The determination of a motion of the perijove, where the eccentricity is so small and

* Damoiseau, in the introduction (page i) to his tables, gives among the quantities derived by him from the observations only the motion of the node of Satellite II, not the value of Jb^2. On page ii he gives, among the quantities derived from those resulting directly from the observations by theoretical relations ("qui supposent les constantes de Delambre"), the ellipticity ρ of the planet. This was evidently derived from J by the well-known (approximate) formula $J=\rho-\frac{1}{2}\phi$, using Laplace's ϕ. Damoiseau here overlooked that, since he had adopted a diameter differing from the value adopted by Laplace, he should also have altered Laplace's ϕ accordingly.

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the observations so difficult, evidently is a very delicate problem, which for its solution demands mature judgment and careful criticism. Comparing the values of the longitude of the perijove given for the different oppositions, it seems very difficult to satisfy them all by one and the same yearly motion. And so much is at least certain, that a determination of Jb^2 from this motion can have no appreciable weight compared with that from the node of Satellite II.

It appears from the solution that the determination of Jb^2 from the observed values of p and q is intricately connected with that of the position of the equator. In the last chapter of my dissertation, while suggesting the epoch 1903·7 for a new series of observations, I estimated the accuracy with which Jb^2 could be determined from the comparison of that series with the observations of 1891 as $\frac{1}{300}$. In making this estimate, I overlooked the necessity of determining from the *same* observations the position of the equator. The actual probable error of κ is $\pm 0{\cdot}0096$, or about $\frac{1}{100}$. That this is really due to the influence of the corrections to the equator, is best shown by a comparison with Solution I, where these unknowns were not introduced, and where the probable error of $\frac{d\Gamma_2}{dt}$ is about $\frac{1}{1000}$.

In order clearly to see how this large depression of the weight of κ by the introduction of the unknowns x_0 and y_0 arises, consider the case that we had observations of one satellite only. As far as Jb^2 is concerned this is practically the case of nature, for the inclination of I is so small, and the motions of the nodes of III and IV are so slow, that these satellites contribute practically nothing to the determination of Jb^2. Now it is evident at once that, with one satellite observed at two epochs, it would be *impossible* to determine both the equator and the motion of the node, since we would have only four data (the values of p and q at each epoch) for the five unknowns γ, Γ_0, $\frac{d\Gamma}{dt}$, x_0, and y_0. If the interval between the two epochs is in the neighbourhood of half a revolution of the node (as it is in the present case, the motion of θ_2 between 1891 and 1904 being about 160°), it will be seen on a little consideration that the addition of a third epoch *between* these two will practically not alter the conditions of the problem. What is required for a complete separation of the motion of the node and the position of the equator, is three epochs with intervals of half a revolution of the node between each. Thus, to supplement the data discussed in the preceding chapters, a new series of observations taken about the year 1920 would be necessary.

That it has, notwithstanding this, been possible to derive corrections to the adopted position of the equator with very satisfactory probable errors, is due to the fact that

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we have not one, but four satellites. The equator is practically determined from the condition that it shall be the same for the four satellites, *i.e.* that its position shall be such that for all satellites the inclinations shall be constant, and the ratios of the motions of the four nodes shall be those demanded by the theory. If two or more of the satellites had quick-moving nodes and large inclinations, it is quite possible that even Jb^2 might have been determined with a satisfactory weight; as it is, however, its determination rests on Satellite II alone, and its weight is considerably reduced. That also the weight of the parameters determining the position of the equator is reduced by the influence of κ, is at once evident from a comparison of the probable error of γ_0 in the Solutions VI and VII.

It is at present very difficult to say which of the Solutions VI or VII is the nearest approach to the truth. It has already been pointed out that, with the exception of κ and γ_0, the values of all the unknowns agree within their probable errors, and that all the corrections to the adopted values (with the only exception of γ_2) exceed their probable errors many times. In the present state of our knowledge of the masses of the satellites, either of these solutions may be adopted, and either will represent a real addition to our knowledge of the elements. A change in the adopted values of the masses and in the coefficients of the theory may, however, alter the results considerably, and this alteration may be confidently expected to be in the direction of reconciling the two solutions.

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APPENDIX.

DISCUSSION OF PLATES TAKEN FOR THE INVESTIGATION OF THE METHODS OF DETERMINING THE ORIENTATION.

THE determination of the orientation of the plate is of the most vital importance for the investigation of the positions of the orbital planes. The bad agreement between the index-corrections derived from the standard stars and the trails, which has been shown in Table VIII., was therefore very disappointing. The following discussion was undertaken with a view to investigate these discrepancies, and, if possible, to find their cause, and to decide which of the two methods should be adopted. In the meantime, while the plates on which it is based were being taken at the Cape, I proceeded to make two solutions of each opposition—one based on the standard stars, and one on the trails, as has been explained in the preceding paper.

Consider a system X, Y of co-ordinate axes fixed to the plate, of which the origin is the centre of the plate, and the axis of Y is the projection on the plate of the declination circle through this centre. The centre of the plate is defined as that point in which a line joining it to the optical centre of the object-glass is perpendicular to the plate. Let, further, α_1, δ_1, α_2, δ_2, be the right-ascensions and declinations of any two points, of which the co-ordinates on the plate are X_1, Y_1, X_2, Y_2, and let X_0, Y_0 be the co-ordinates on the plate of the point of which the R.A. and decl. are $\alpha_0 = \frac{1}{2}(\alpha_1 + \alpha_2)$ and $\delta_0 = \frac{1}{2}(\delta_1 + \delta_2)$. Let, further, $\Delta\alpha_i = \alpha_i - \alpha_0$, $\Delta\delta_i = \delta_i - \delta_0$, $\Delta X_i = X_i - X_0$, $\Delta Y_i = Y_i - Y_0$, where the suffix i has one of the two values 1 or 2.

We have then, correct to quantities of the second order inclusive in X, Y, ΔX, and ΔY:—

$$\begin{aligned} \cos\delta_0\,\Delta\alpha_i &= \Delta X_i + \tan\delta_0\, X_0\,\Delta Y_i + \tan\delta_0\,\Delta X_i\,\Delta Y_i; \\ \Delta\delta_i &= \Delta Y_i - \tan\delta_0\, X_0\,\Delta X_i - \tfrac{1}{2}\tan\delta_0\,\Delta X_i^2. \end{aligned} \tag{1}$$

With the same order of accuracy we have:—

$$\tan p = \frac{(\alpha_2 - \alpha_1)\cos\delta_0}{\delta_2 - \delta_1}$$

$$\tan P = \frac{X_2 - X_1}{Y_2 - Y_1},$$

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where p is the position-angle on the sphere, at the middle point, of the great circle joining the points $\{\alpha_1, \delta_1\}$ and $\{\alpha_2, \delta_2\}$, and P is the position-angle on the plate of the line joining their images.

From the formulas (1) we find easily (neglecting quantities of the third order in ΔX and ΔY)

$$\tan p = \tan P + X_0 \tan \delta_0 \sec^2 P,$$

from which

$$P = p - X_0 \tan \delta_0. \tag{2}$$

The correction $-X_0 \tan \delta_0$ was neglected in Chapter VI., where the assumption was made that the middle point of the standard stars, as well as the mean position of the exposures of Jupiter, coincide with the centre of the plate. Instead of the angle P_s used for the determination of I, we should thus have used $P_s - X_0 \tan \delta_0$. The index-correction $I = P_s - \left(P_{AB} + \frac{\Delta}{s \sin 1''}\right)$ thus requires the correction:—

$$- X_0 \tan \delta_0.$$

The same reasoning applies to the position-angle P of the axis of y. This angle is the position-angle *at Jupiter* of the projection on the sphere of the adopted rotation-axis. The position-angle of the axis of x on the plate is therefore not $P + 90^0$, as supposed in Chapter VI., but $P + 90^0 - X_y \tan \delta_y$. The orientation-correction $\Delta P = 90^0 + P - (P_0 + I)$, as determined in Chapter VI., therefore requires the correction :—

$$X_0 \tan \delta_0 - X_y \tan \delta_y = (X_0 - X_y) \tan \delta_0 + X_y (\tan \delta_0 - \tan \delta_y). \tag{3}$$

The differences $X_0 - X_y$ were measured on all plates, and were found to be small and practically *constant*; $\tan \delta_0$ is also constant; the first part of the correction (3) is therefore constant, and is absorbed in the correction ΔP_s to the adopted position-angle of the standard stars. The actual differences $X_0 - X_y$ were :—

	Smallest. $X_0 - X_y$	Largest. $X_0 - X_y$	Mean. $X_0 - X_y$	Mean Correction. $(X_0 - X_y) \tan \delta_0$
1891	$-0'.1$	$-2'.3$	$-1'.6$	$+8''.7$
1903	$-2'.0$	$-3'.2$	$-2'.3$	$+9''.5$
1904	$-0'.5$	$-0'.9$	$-0'.8$	$-6''.5$

The second part of the correction, viz., $X_y (\tan \delta_0 - \tan \delta_y)$, cannot be estimated, since X_y is unknown, and must remain unknown, as there are no means of determining the true centre of the plate. The differences $\Delta \tan \delta = \tan \delta_y - \tan \delta_0$ are included in Table VIII.

In the case of the trails the correction is :—

$$X_{tr} \tan \delta_y - X_y \tan \delta_y,$$

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where X_{tr} is the X co-ordinate of the middle point of the trail. Since the point having the same R.A. as the exposures of Jupiter was chosen for this middle point, we have $X_{tr} = X_{\mathcal{U}}$, and the correction vanishes identically. In the case of the first half of the 1904 series, we have $X_{tr} = X_{\mathcal{U}} + \xi_0$, and the correction is * :—

$$\xi_0 \tan \delta_{\mathcal{U}}.$$

These corrections have been applied (see Chapter VII, Table VII).

So far as this argument goes, the orientation derived from the trails is thus rigorously correct in all cases. That derived from the standard stars contains the unknown second part of the correction (3).

Some assumptions have, however, been tacitly made in the above reasoning, which must be examined more closely.

The first assumption is that the origin of the co-ordinates X is the same in all exposures, *i.e.* that the position of the plate relatively to the object-glass is the same. Now this relative position depends on the flexure of the tube, which changes the inclination of the plate to the optical axis. This flexure in its turn is a function of the zenith-distance, *i.e.* of the hour-angle and declination. Further, this assumption supposes that the plate does not move in its frame, nor the frame relatively to the tube, between two exposures.

The second assumption is that the direction of the co-ordinate axes is the same in all exposures, *i.e.* that the co-ordinates of the image of the pole on the plate, supposed to be an infinite plane surface, are the same in all exposures. These co-ordinates are functions of the equatorial adjustment of the telescope and of the (instrumental) hour-angle and declination of the centre of the plate. If the equatorial adjustment is not perfect, the *instrumental* pole will retain its co-ordinates when the declination and hour-angle are changed, but the true pole will not. A similar reasoning applies to other instrumental constants, of which, perhaps, what might be called the lateral flexure of the tube, which was found by Bessel in the Königsberg Heliometer (*Astronomische Untersuchungen*, I. p. 45), is the most important for our purpose.

It will be seen that both assumptions amount to the neglect of corrections to the position-angles of the form

(4) $$K f(t, \delta),$$

where K is a constant depending on the mechanical state of the instrument and its adjustment, and $f(t, \delta)$ is a function of the declination and hour-angle of the centre of the plate. The second (unknown) part of the correction (3) also comes under this

* This is the correction to ΔP; the correction to I′ is, of course, $\Delta I' = -\xi_0 \tan \delta_{\mathcal{U}}$.

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general formula. It would not be difficult to write out the expression (4) explicitly for each separate instrumental constant, but it is not necessary to do so.

We have thus the following possible explanations for the discrepancies between the two methods of determining the orientation.

1. The constants K are different for different exposures.

2. The constants K are truly constant, at least for one and the same night, but they are so large that the change in $f(t, \delta)$ on passing from the standard stars to Jupiter is sufficient to explain the observed differences.

In the case 1, it is impossible to say without further investigation which of the two methods is the better one. In the case 2, the orientation from the trails must be adopted.

There are two further possible explanations, viz.:—

3. There may be errors in the computed angles P_s and Π.

4. There may be errors in the observed position-angles.

The explanation 3 is extremely improbable. It must be remembered that we are here only concerned with the variable part of the differences $I' - I$ of Table VIII. The constant part is due partly to the neglect of the correction $(X_0 - X_{\frac{1}{2}}) \tan \delta_0$, and chiefly to errors in the adopted position-angles of the standard stars. But the variable part ΔI cannot be explained by such errors. It is many times larger than any of the corrections for refraction, etc., which make up the tabular position-angles, and shows not the slightest parallelism with any of these corrections. If it was Π that was in error, the different satellites could, moreover, not agree so well as they do. All the computations of P_s and Π have been repeatedly and searchingly examined. The explanation 3 may be safely dismissed.

The explanation 4 is equally inadmissible. The discussion of the probable errors in Chapters VI. and VII. shows that the errors of observation are insignificant compared with the quantities which must be explained. This is confirmed with even greater stringency by the observations to be related below.

There thus remain only the explanations 1 and 2.

Of these, the explanation 1 appeared the most probable. Especially it appeared not impossible that the instrumental pole might be different when the instrument was at rest, as in taking the trail, and when it was being driven by the clock, as in the exposures of Jupiter and of the standard stars. This explanation is also the one which

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most needed further investigation to decide which method of determining the orientation must be adopted, while in the case of explanation 2 there is no doubt on this point, as has been already explained above. So soon as the discrepancies in question had been found, I therefore immediately proposed to Sir David Gill to test this explanation by taking some experimental plates. Sir David at once arranged that the plates which I proposed should be taken without delay.

On each plate four pairs of standard stars and three trails were to be photographed, under conditions which should be as much as possible equal to those under which the Jupiter plates were taken. The following stars were selected :—

			Mag.	R.A. 1905·0	δ 1905·0
				h m s	° ′ ″
a	B.D. − 5° 5640	C.G.A. 29813	7·0	21 42 26·53	− 5 2 52·8
b	− 6 5837	29848	6·8	44 3·48	− 5 50 37·8
c	− 4 5564	29930	7·2	47 47·77	− 4 26 19·0
d	− 6 5859		7·3	49 12·2	− 5 48·4
e	− 4 5568	29957	5·7	49 12·79	− 4 43 15·8

The positions have been carried forward with precession alone to 1905, from the epochs of the Argentine *General Catalogue* (1875) and the Bonn *Durchmusterung* (1855). From these stars the following pairs were formed :—

α	stars *c* and *d*	*s* = 5086″·0	*p* = 165° 38′ 24″
β	„ *a* „ *d*	6646·5	114 15 36
γ	„ *a* „ *c*	5279·5	65 26 49
δ	„ *b* „ *c*	6067·2	33 31 12

On each plate the following exposures were made :—

I. (*Exp.* α). The telescope was pointed to the middle point of the pair α, and the plate was exposed for one minute. The instrument was then unclamped, and

II. (*Exp.* β) pointed to the middle point of pair β, and an exposure of one minute taken. Instrument again unclamped, and

III. (*Exp.* γ) pointed to the middle point of pair γ, and plate exposed for one minute. The star *e* is then at the following edge of the plate. After the exposure γ the instrument is *not* unclamped, but

IV. (1*st trail*) the clock is stopped, and the star *e* is allowed to trail over the plate, the time of passing the centre being noted.

V. (2*nd trail*). The instrument is then unclamped, moved 2 mm. in declination, and in R.A. so much as to bring the star *e* back to the following edge of the plate. The instrument is then clamped again and a second trail of this star is taken, time at centre being noted.

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VI. (*3rd trail*). Instrument again unclamped, moved 2 mm. in decl. and a third trail of star *e* taken, time at centre being noted, and the instrument being moved slightly in declination at the end of the trail, so as to distinguish this trail from the others and leave no possible doubt about the chronological order of the trails.*

VII. (*Exp.* δ). The instrument is then unclamped and set on the middle point of pair δ, and an exposure of one minute is taken. Since this middle point of pair δ coincides nearly with the centre of the triangle formed by the three other pairs, those three pairs are also photographed on the plate in the same exposure δ.

We have thus on each plate the following means of determining the orientation, viz.: four pairs of standard stars α, β, γ, δ in different position-angles, and all with their middle points nearly coinciding with the centre of the plate; three pairs of standard stars unsymmetrically situated with regard to this centre; and three trails. The first four pairs will, in the following discussion, be denoted by α, β, γ, δ; the latter three pairs, *i.e.* the pairs α, β, γ photographed in the exposure δ, will be denoted by $(\alpha)_\delta$, $(\beta)_\delta$, and $(\gamma)_\delta$ respectively.

If the clamping and unclamping of the instrument is the cause of the change in the adjustments which was thought to be the explanation of the observed discrepancies, then the different pairs α, β, γ, δ must give different orientations; also, the three trails must differ from each other, but the pair γ and the first trail must agree. If the decisive circumstance is, whether the telescope is being driven by the clock or not, then the different pairs of standard stars will agree with each other, as will also the trails, but the trails will differ from the standard stars. In both cases the orientation determined from the four pairs $(\alpha)_\delta$, $(\beta)_\delta$, $(\gamma)_\delta$, and δ must agree with each other, at least if there is no optical distortion, and no distortion of the photographic film.

Table I. contains the details of the plates taken.

The plates were measured in two positions. The position-angles of the different pairs of standard stars were determined in exactly the same way as those of the standard stars on the Jupiter plates. On each image two pointings were made in each position, and only one reading was taken of the position-circle microscopes in each position. The position-angle of the plate was the same for the measures of the pairs α and $(\alpha)_\delta$, and similarly for β and $(\beta)_\delta$, γ and $(\gamma)_\delta$.

* The first trail is distinguished by the image of star *e* from Exp. γ at its beginning.

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TABLE I.

Exposures of Experimental Plates.

No. of Plate.	Date.	Barometer.	Att. Thermometer.	Ext. Thermometer.	Definition.	Exp. I. α	II. β	III. γ	IV. 1st Trail.	V. 2nd Trail.	VI. 3rd Trail.	VII. δ
	1905.	in.	°	°		h m s	m s	m s	m s	m s	m s	m s
8537	July 27	30·179	60·0	58·0	3	20 44 37·6	52 52·6	57 32·6	62 29·6	74 55·6	84 11·6	94 2·6
8540	Aug. 1	30·283	48·5	46·0	2–3	20 40 56·1	45 56·1	53 1·1	57 10·1	74 18·1	86 19·1	98 3·6
8543	7	30·297	47·2	45·2	Very poor.	22 4 48·0	7 48·0	13 3·0	17 4·0	29 10·0	40 44·0	49 3·0
8544	10	30·281	57·0	56·0	3	20 53 26·0	55 46·0	59 6·0	64 11·0	74 16·0	84 20·0	93 46·0
8546	24	30·329	54·0	50·0	3 {3rd trail incomplete}	20 57 22·1	60 12·1	62 42·1	68 2·1	78 4·6	90 3·1	97 37·1
8549	27	30·263	53·0	44·2	2–3	20 52 26·1	55 46·1	58 6·1	62 54·6	73 3·6	83 10·1	91 36·1
8550	28	30·137	52·0	47·4	2–3	20 54 7·4	56 32·4	58 52·4	63 43·9	74 4·9	84 3·4	93 57·4
8553	30	30·531	53·0	48·1	2–3	20 55 29·6	58 9·6	60 29·6	65 14·6	75 19·1	85 18·1	93 19·6
8556	31	30·377	53·9	52·4	Poor.	20 45 31·0	48 31·0	52 36·0	57 6·5	69 38·0	80 56·0	89 12·0
8557	Sept. 1	30·123	56·0	55·8	3–4	20 45 12·0	47 12·0	49 22·0	54 4·0	64 1·0	73 30·0	82 2·0
8560	5	30·039	58·0	54·1	2–3	20 43 35·5	46 15·5	48 35·5	53 24·0	63 19·5	73 15·5	81 25·5
8565	18	29·963	59·0	56·5		20 46 49·2	49 29·2	52 9·2	56 38·7	67 15·2	79 19·2	87 19·2
8570	19	29·937	60·0	56·2	2–3	20 49 40·0	51 50·0	53 50·0	58 29·5	68 10·0	77 59·0	85 20·0

The measures of the trails were also made exactly as those on the Jupiter plates.

On plate 8543 all images of the exposures α and γ are double. The first trail is covered by a *réseau*-line, and the other trails are extremely faint.

On plate 8556 all images of the exposures β and δ are double.

These two plates have not been measured.

On plate 8546 nearly one-half of the third trail is missing. This trail was not measured on this plate.

The plates 8537, 8540, and 8543 have a *réseau* impressed on them.

The measures were made by myself between 1905 December 13 and December 19.

To the adopted position-angles p corrections for refraction, precession, nutation, and aberration were applied by the usual formulas. The resulting apparent position-angles require the further correction

$$-X_0 \tan \delta_0$$

to reduce them to position-angles on the plate. Since the declinations of all the stars are very nearly equal, the co-ordinates of the centre (which are unknown) disappear

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from the differences of these corrections for any two pairs. We can therefore reduce all pairs to one of them, the middle point of which is then supposed to be the centre of the plate. For this the pair γ was chosen, and for X_0 the co-ordinates of the different middle points were used, referred to the middle point of γ as origin. These relative co-ordinates are given in Table II.

TABLE II.

Co-ordinates of Middle Points.

Plate.	α	$(\alpha)_\delta$	β	$(\beta)_\delta$	$(\gamma)_\delta$	δ
	′	′	′	′	′	′
8537	− 1·1	+ 37·4	+ 1·0	− 2·6	− 13·2	− 1·1
8540	+ 0·7	+ 43·2	+ 0·8	+ 3·2	− 7·5	+ 5·1
8544	+ 1·3	+ 42·7	+ 1·4	+ 2·7	− 7·9	+ 4·1
8546	− 0·4	+ 46·7	+ 3·1	+ 6·5	− 4·3	+ 7·9
8549	− 3·6	+ 35·4	− 4·4	− 4·8	− 15·2	− 3·2
8550	− 3·6	+ 38·0	− 2·1	− 2·1	− 12·7	− 0·6
8553	− 5·4	+ 39·0	− 4·4	− 1·2	− 11·8	+ 0·4
8557	− 1·2	+ 37·5	− 1·3	− 2·8	− 13·4	− 1·3
8560	− 1·2	+ 38·8	− 4·8	− 1·5	− 12·0	0·0
8565	− 2·4	+ 38·4	− 2·1	− 1·6	− 12·7	− 0·6
8570	− 3·1	+ 39·6	− 3·4	− 0·6	− 11·1	+ 1·0

Let now

P_0 = observed position-angle of the plate for the measures of any one pair (mean of two positions).

A − B = observed difference of the η co-ordinates of the two stars of this pair—see Chapter VI.—(mean of two positions).

R = the correction for refraction, precession, nutation, and aberration to be applied to the mean position-angle p to reduce it to apparent position-angle.

Δp = the correction to the adopted position-angle p.

Then the index-correction is evidently

(5) $$I = P - P' + \Delta v,$$

where

$$P = p + R - X_0 \tan \delta_0 ;$$

$$P' = P_0 + \frac{A-B}{s \sin 1''}.$$

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TABLE III.

MEASURES OF STANDARD STARS.

Plate.	P_0	A—B	P′	R $-X_0 \tan \delta_0$	P—P′	I	A—B	P′	R $-X_0 \tan \delta_0$	P—P′	I
$p = 165° 38' 24''$		Pair α					Pair $(\alpha)_\delta$				
		rev.					rev.				
8537	165° 38′ 11″	−0·022	165° 38′ 6″	− 8″ − 6″	+ ′ 4″	− 8′ 11″	+0·768	165° 41′ 18″	− 3″ +201″	+ ′ 24″	− 7′ 51″
8540	25 56	− ·080	25 37	− 8 + 4	+12 43	+ 4 28	+ ·944	29 46	− 3 +232	+12 27	+ 4 12
8544	38 40	+ ·260	39 44	− 8 + 7	− 1 21	− 9 36	+1·174	43 26	− 4 +230	− 1 16	− 9 31
8546	31 47	+ ·088	32 8	− 8 − 2	+ 6 6	− 2 9	+1·251	36 51	− 4 +251	+ 5 40	− 2 35
8549	38 1	− ·141	37 27	− 8 −19	+ 30	− 7 45	+ ·754	41 5	− 5 +190	+ 24	− 7 51
8550	34 44	− ·242	33 45	− 8 −19	+ 4 12	− 4 3	+ ·684	37 30	− 4 +204	+ 4 14	− 4 1
8553	37 27	− ·412	35 47	− 8 −29	+ 2 0	− 6 15	+ ·606	39 54	− 5 +210	+ 1 55	− 6 20
8557	39 17	− ·850	35 50	− 9 − 6	+ 2 19	− 5 56	+ ·034	39 25	− 6 +202	+ 2 15	− 6 0
8560	24 38	+ ·096	25 1	−10 − 6	+13 7	+ 4 52	+1·004	28 42	− 7 +209	+13 4	+ 4 49
8565	37 15	−1·028	33 5	−10 −13	+ 4 56	− 3 19	− ·089	36 53	− 6 +206	+ 4 51	− 3 24
8570	29 15	− ·845	25 49	− 9 −17	+12 9	+ 3 54	+ ·098	29 39	− 6 +213	+12 12	+ 3 57
$p = 114° 15' 36''$		Pair β					Pair $(\beta)_\delta$				
		rev.					rev.				
8537	114° 16′ 21″	−0·074	114° 16′ 8″	+ 2″ + 6″	− ′ 24″	− 7′ 53″	−0·190	114° 15′ 46″	− 3″ − 14″	− ′ 27″	− 7′ 56″
8540	5 56	− ·641	3 57	+ 3 + 5	+11 47	+ 4 18	− ·521	4 19	− 4 + 18	+11 31	+ 4 2
8544	17 41	+ ·020	17 45	+ 1 + 8	− 2 0	− 9 29	+ ·057	17 52	− 4 + 15	− 2 5	− 9 34
8546	10 12	+ ·120	10 34	0 +18	+ 5 20	− 2 9	+ ·302	11 8	− 4 + 37	+ 5 1	− 2 28
8549	15 50	− ·124	15 27	+ 1 −25	− 15	− 7 44	− ·139	15 24	− 4 − 27	− 19	− 7 48
8550	11 42	+ ·092	11 59	+ 1 −12	+ 3 26	− 4 3	+ ·066	11 54	− 4 − 12	+ 3 26	− 4 3
8553	13 58	+ ·014	14 1	0 −25	+ 1 10	− 6 19	+ ·192	14 34	− 4 − 7	+ 51	− 6 38
8557	14 33	− ·152	14 5	+ 2 − 7	+ 1 26	− 6 3	− ·165	14 2	− 3 − 16	+ 1 15	− 6 14
8560	2 30	+ ·062	2 42	+ 1 −27	+12 28	+ 4 59	+ ·198	3 7	− 4 − 9	+12 16	+ 4 47
8565	11 6	+ ·074	11 20	0 −12	+ 4 4	− 3 25	+ ·064	11 18	− 4 − 9	+ 4 5	− 3 24
8570	3 37	− ·010	3 35	0 −19	+11 42	+ 4 13	+ ·071	3 50	− 4 − 3	+11 39	+ 4 10
$p = 65° 26' 49''$		Pair γ					Pair $(\gamma)_\delta$				
		rev.					rev.				
8537	65° 34′ 44″	+0·025	65° 34′ 50″	+13″	− 7′ 48″	− 7′ 47″	−0·221	65° 33′ 52″	+10″ − 73″	− 8′ 6″	− 8′ 5″
8540	24 23	− ·383	22 53	+14	+ 4 10	+ 4 11	− ·501	22 26	+10 − 41	+ 3 52	+ 3 53
8544	35 39	+ ·220	36 31	+12	− 9 30	− 9 29	+ ·033	35 47	+ 9 − 44	− 9 33	− 9 32
8546	29 3	+ ·098	29 26	+12	− 2 25	− 2 24	+ ·002	29 3	+ 9 − 23	− 2 28	− 2 27
8549	35 39	− ·223	34 47	+12	− 7 46	− 7 45	− ·588	33 21	+ 9 − 84	− 7 47	− 7 46
8550	30 40	+ ·108	31 5	+12	− 4 4	− 4 3	− ·208	29 51	+ 9 − 70	− 4 3	− 4 2
8553	33 51	− ·068	33 35	+12	− 6 34	− 6 33	− ·392	32 19	+ 9 − 65	− 6 26	− 6 25
8557	33 8	− ·064	32 53	+13	− 5 51	− 5 50	− ·368	31 42	+10 − 74	− 5 57	− 5 56
8560	24 6	− ·549	21 58	+12	+ 5 3	+ 5 4	− ·860	20 45	+ 9 − 67	+ 5 6	+ 5 7
8565	30 40	− ·056	30 27	+12	− 3 26	− 3 25	− ·370	29 14	+ 9 − 70	− 3 26	− 3 25
8570	23 37	− ·133	23 6	+12	+ 3 55	+ 3 56	− ·483	21 44	+ 9 − 62	+ 4 12	+ 4 13

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TABLE III.

MEASURES OF STANDARD STARS—*continued.*

$p = 33^\circ\ 31'\ 12''$		Pair δ					
Plate.	P_0	A—B	P′	R $-X_0 \tan \delta_0$		P—P′	I
	° ′ ″	rev.	° ′ ″	″	″	′ ″	′ ″
8537	33 39 54	− 0·087	33 39 36	+ 10	− 6	− 8 20	− 7 43
8540	28 33	− ·050	28 23	+ 10	+ 27	+ 3 26	+ 4 3
8544	41 10	+ ·204	41 52	+ 9	+ 22	− 10 9	− 9 32
8546	35 8	+ ·019	35 12	+ 9	+ 43	− 3 8	− 2 31
8549	39 32	− ·015	39 29	+ 9	− 17	− 8 25	− 7 48
8550	35 49	+ ·015	35 45	+ 9	− 3	− 4 27	− 3 50
8553	39 23	− ·305	38 21	+ 9	+ 2	− 6 58	− 6 21
8557	38 28	− ·167	37 54	+ 8	− 7	− 6 41	− 6 4
8560	26 45	+ ·044	26 54	+ 7	0	+ 4 25	+ 5 2
8565	35 45	− ·100	35 25	+ 7	− 3	− 4 9	− 3 32
8570	27 44	+ ·093	28 2	+ 7	+ 5	+ 3 22	+ 3 59

The corrections Δp were derived by adding to the mean P′ − P for every pair (*i.e.* the mean P − P′ with reversed sign), the mean of the index-corrections found from the trails, viz. : − 3′ 7″. I found in this way the following values :—

Pair	α	$(\alpha)_\delta$	β	$(\beta)_\delta$	γ	$(\gamma)_\delta$	δ
Δp	− 8′ 17″	− 8′ 13″	− 7′ 33″	− 7′ 25″	0″	+ 2″	+ 37″

The large corrections for the pairs α and β need be no matter for surprise, since these pairs involve the star *d*, whose position was derived from the Bonn *Durchmusterung*. The agreement between the corrections found for α and $(\alpha)_\delta$, β and $(\beta)_\delta$, γ and $(\gamma)_\delta$, is satisfactory. The adopted values of Δp are :—

Pair	α and $(\alpha)_\delta$	β and $(\beta)_\delta$	γ and $(\gamma)_\delta$	δ
Δp	− 8′ 15″	− 7′ 29″	+ 1″	+ 37″

Table III. contains all the quantities entering into the formula (5). It has not been thought necessary to publish every separate pointing, as was done for the Jupiter plates, and the results of the measures only are given.

For the trails, the small angle φ was derived in the manner explained in Chapter VII. If the correction for refraction with reversed sign (*i.e.* the correction to be

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applied to the apparent angle to reduce it to the true angle) is denoted by R′, we have for the index-correction :—

(6) $$I' = 90^\circ - P',$$

where

$$P' = P_0 + R' + \phi.$$

The values of P_0, R′, ϕ, and I′ are given in Table IV.

TABLE IV.

Measures of Trails.

Plate.	P_0	1st Trail.			2nd Trail.			3rd Trail.			ΔI	I_0
		R′	φ	I′	R′	φ	I	R′	φ	I′		
8537	90° 8′ 42″	− 8″	−32″	− 8′ 2″	−5″	−36″	− 8″ 1′	−4″	−17″	− 8′ 22″	+ 5″	−8′ 3″
8540	89 55 23	− 9	+31	+ 4 15	−5	+36	+ 4 6	−4	+43	+ 3 58	0	+4 6
8544	90 10 36	− 7	− 6	−10 23	−5	−25	−10 6	−4	−22	−10 10	+ 37	−9 36
8546	90 2 54	− 7	− 2	− 2 45	−5	− 1	− 2 48	Not measured.			+ 22	−2 24
8549	90 7 26	− 8	+ 1	− 7 19	−6	− 6	− 7 14	−4	+ 8	− 7 30	− 17	−7 38
8550	90 3 50	− 7	+ 6	− 3 49	−5	−10	− 3 35	−4	+18	− 4 4	− 11	−4 0
8553	90 5 58	− 7	+22	− 6 13	−6	+26	− 6 18	−4	+39	− 6 33	+ 5	−6 16
8557	90 5 25	−10	+33	− 5 48	−7	+45	− 6 3	−5	+44	− 6 4	− 15	−6 13
8560	89 54 40	−11	+18	+ 5 13	−7	+20	+ 5 7	−5	+18	+ 5 7	− 9	+5 0
8565	90 3 56	− 9	−47	− 3 0	−6	−33	− 3 17	−4	−52	− 3 0	− 5	−3 11
8570	89 56 12	− 9	−15	+ 4 12	−6	− 1	+ 3 55	−5	− 8	+ 4 1	− 4	+3 59

In measuring the trails, the fact that they also require a correction $-X \tan \delta$ was not at first realised. The middle point was taken arbitrarily at the reading 100 of the vertical scale. Since this point cannot be identified on the plate after it has been taken out of the measuring apparatus, it was necessary, in order to compare the index-corrections derived from the trails with those from the standard stars, to remeasure one of the pairs and one of the trails on each plate. Accordingly the pair γ and the first trail were remeasured between 1906 January 2nd and 4th, the middle point of the trail being now taken at the same R.A. reading as the middle point of the pair γ.

For the pair γ the angle P is, of course, the same as in the first measure. The difference

$$\Delta_\gamma = P'_I - P'_{II}$$

of the angles P′ found in the first and in the second measures is the difference of orientation of the plate in the plate-holder on the two occasions, and must be applied to all observed angles in the second measure to make them comparable to those of

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the first measure. If the index-correction derived from the second measure of the first trail by the formula (6) be denoted by I'_{II}, then the correction which must be applied to all index-corrections derived from the trails in the first measure evidently is :—

$$\Delta I = I'_{II} - \Delta_\gamma - I'_I,$$

where I'_I is the value of I′ from the first trail in the first measure, which has been given in Table IV.

The measures for the determination of ΔI are given in Table V.

TABLE V.

Second Measure of Pair γ and First Trail.

Plate.	Pair γ				1st Trail.				ΔI
	P_0	A—B	P′	Δ_γ	P_0	R′ ϕ		I′	
		rev.							
8537	65° 34′ 44″	+ 0·016	65° 34′ 48″	+ 2″	90° 8′ 41″	− 8″	−38″	−7′ 55″	+ 5″
8540	24 21	− ·431	22 40	+ 13	89 55 22	− 9	+19	+4 28	0
8544	35 40	+ ·107	36 5	+ 26	90 10 33	− 7	−66	−9 20	+37
8546	29 0	+ ·090	29 21	+ 5	90 2 53	− 7	−28	−2 18	+22
8549	35 41	− ·365	34 16	+ 31	90 7 30	− 8	−17	−7 5	−17
8550	30 44	+ ·044	30 54	+ 11	90 3 52	− 7	+ 4	−3 49	−11
8553	33 50	− ·271	32 47	+ 48	90 6 0	− 7	−33	−5 20	+ 5
8557	33 5	− ·029	32 58	− 5	90 5 30	−10	+48	−6 8	−15
8560	24 8	− ·622	21 42	+ 16	89 54 39	−11	+12	+5 20	− 9
8565	30 41	+ ·044	30 51	− 24	90 4 6	− 9	−28	−3 29	− 5
8570	23 35	+ ·028	23 42	− 36	89 56 9	− 9	+28	+3 32	− 4

The corrections ΔI are also included in Table IV., together with the corrected mean of the index-corrections from the three trails, viz. :—

$$I_0 = \tfrac{1}{3}(I'_1 + I'_2 + I'_3) + \Delta I.$$

The mean of all these index-corrections I_0 has been used for the derivation of the corrections Δp, as was explained above.

To derive the final index-correction for every plate, I have first formed the mean of the values given by the pairs δ, $(\alpha)_\delta$, $(\beta)_\delta$, and $(\gamma)_\delta$. This gives the index-correction for the exposure δ. The agreement between the four separate values is excellent, which shows that the index-correction does not depend on the position-angle of the pair which is used for its determination, nor on the distance of the middle point from the centre of the plate, provided the proper correction is applied. For the final index-correction I

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then adopted the mean of the seven separate values derived from the four exposures α, β, γ, δ, and the three trails (the latter after correction by ΔI). Subtracting these means from the individual values, we find the residuals, which are given in Table VI.

TABLE VI.

Residuals.

Plate.	Mean I	α	β	γ	δ	$(\alpha)_\delta$	$(\beta)_\delta$	$(\gamma)_\delta$	1st Trail.	2nd Trail.	3rd Trail.
8537	−7° 59′	− 12″	+ 6″	+ 12″	+ 16″	+ 8″	+ 3″	− 6″	+ 2″	+ 3″	− 18″
8540	+4 11	+ 17	+ 7	0	− 8	+ 1	− 9	− 18	+ 4	− 5	− 13
8544	−9 33	− 3	+ 4	+ 4	+ 1	+ 2	− 1	+ 1	− 13	+ 4	0
8546	−2 20	+ 11	+ 11	− 4	− 11	− 15	− 8	− 7	− 3	− 6	
8549	−7 42	− 3	− 2	− 3	− 6	− 9	− 6	− 4	+ 6	+ 11	− 5
8550	−4 1	− 2	− 2	− 2	+ 11	0	− 2	− 1	+ 1	+ 15	− 14
8553	−6 20	+ 5	+ 1	− 13	− 1	0	− 18	− 5	+ 12	+ 7	− 8
8557	−6 5	+ 9	+ 2	+ 15	+ 1	+ 5	− 9	+ 9	+ 2	− 13	− 14
8560	+4 59	− 7	0	+ 5	+ 3	− 10	− 12	+ 8	+ 5	− 1	− 1
8565	−3 18	− 1	− 7	− 7	− 14	− 6	− 6	− 7	+ 13	− 4	+ 13
8570	+4 1	− 7	+ 12	− 5	− 2	− 4	+ 9	+ 12	+ 7	− 10	− 4

With regard to these residuals, it must be remarked that there is only one image of each standard star, and only two pointings were made on each image. Further, only one reading of the position-circle microscopes was taken. The residuals, therefore, contain the full effect of the division-errors of the circle and the run of the microscopes. In the case of the trails, the error of the correction ΔI is superimposed on the error of I′. The probable error of one index-correction must, therefore, be larger than in the case of the Jupiter plates, and it will certainly not be overestimated if we assume it to be about $\pm 6''$. Now, of the 109 residuals there are 55—*i.e.* exactly one-half—smaller than $\pm 6''$, if those exactly equal to $6''$ are counted as halves; and not one residual exceeds three times this amount.

This proves that the residuals of Table VI. can be naturally explained by the ordinary errors of observation. There are, however, some signs of a systematic tendency in them. Those of the first trail have a strong tendency to be positive, and those of the third trail to be negative. This must most probably be explained by the difference of hour-angle, which is more than 20^m on the average. If the polar distance and hour-angle of the instrumental pole are denoted by π and τ, then all position-angles measured on the plate require the correction

(7) $$\pi \sin (\tau - t) \sec \delta,$$

which is of the general form (4), and changes with the hour-angle t. To explain the

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observed difference of about 9″ for a difference of 20^m in t, π must be assumed to be about 1″·5 (if τ differs little from t), which is not at all an improbable value.

In the case of the Jupiter plates the hour-angle of the trail is the same as the mean hour-angle of the exposures of the satellites, and since the correction (7) does not depend on the position-angle, it does not affect the determination of the orientation from the trails.

From the index-corrections determined from the pairs α, β, γ, δ, the effect of the correction (7) has practically been eliminated, so far as it is the same for all plates, by the use of the constant corrections Δp. The difference between the values of Δp found for the pairs α and $(\alpha)_\delta$, β and $(\beta)_\delta$, is in the same direction as that between the first and the third trail, and the difference of hour-angle is also in the same direction.

We are, however, only concerned with differences which are *not* the same for all plates, and such differences, between the different pairs of standard stars or the different trails, or between the standard stars and the trails, do certainly not exist in the residuals of Table VI.—at least not to an amount which is at all comparable to the differences found on the Jupiter plates.

So far as it is allowed to apply conclusions drawn from plates taken in 1905 to plates taken in 1903 under entirely identical circumstances, the explanation 1 for the observed discrepancies can therefore not be accepted, and there remains only the explanation 2, viz.: that the effects of the adjustments or other quantities connected with the mechanical state of the instrument and with the position of the plate relatively to the object-glass are so large that their variation with the hour-angle and declination can produce the observed differences ΔI, which were given in Table VIII. The difficulty in the way of this explanation is not so much that these quantities must be large, but that they must be assumed to vary irregularly and considerably from night to night, while they are constant for one night. This makes the explanation 2 very improbable, but it is the only remaining possible explanation.

The only conclusions which can be drawn with fair certainty from these experimental plates are:—that the orientation is independent of the position-angle of the standard stars by which it is determined, and of the distance of the middle point from the centre of the plate. This conclusion is equivalent to the absence of optical, as well as photographic, distortion. Further that, so long as the hour-angle and declination of the centre remain the same, the orientation also remains the same, whether determined from standard stars in different exposures or from trails.

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Now, in the case of the trails on the Jupiter plates, both the hour-angle and the declination *are* the same for the trail and for the exposures of the satellites. The orientation derived from the trails is, therefore, entirely reliable. The same thing cannot be said of the standard stars. It is a significant fact that the discrepancies in question only appear in 1903, and to a certain extent in the latter half of the 1904 series—*i.e.* in those cases where the declination and hour-angle of the standard stars differed considerably from those of Jupiter. This can be verified by inspection of the columns Δt and $\Delta \tan \delta$ in Table VIII.

A satisfactory explanation for the observed differences has, however, not been found.

It must be remembered that the plates treated in this Appendix were only taken to elucidate the origin of these differences, and to decide which of the two methods should be adopted. The latter aim has been reached. For an exhaustive investigation of the whole problem of determining the orientation of photographic plates they are, however, entirely insufficient. For such an investigation stars of higher declination should be selected, many more plates should be taken, and the circumstances of the exposures (hour-angle, order of exposures, etc.) should be varied as much as possible, instead of being, as in the present instance, chosen as much as possible equal to those of the Jupiter plates. It is hardly necessary to point out the interest attaching to such an investigation.

POSTSCRIPT.

(*September* 1906.)

THE recent observations of the standard stars, which were referred to in the footnote to page 38, are contained in the *Cape Catalogue for* 1900, of which advance sheets were kindly presented by Sir David Gill to the Astronomical Laboratory at Groningen.

For the standard stars of 1903, this Catalogue contains fourteen and twenty-four observations, at the mean epochs 1902·94 and 1902·61, of the stars A and B respectively.* Reducing these to the epoch 1903·7 with the aid of the proper motions

* The seconds of declination of star A (24 Piscium) are 38″·60, not 37″·48 as printed, the corrections for flexure and colatitude having been applied with the wrong sign. I am indebted to the kindness of Sir David Gill for communication of this erratum.

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derived in Chapter VI., we find for the distance and position-angle :—

$$s = 5372''{\cdot}4 \qquad p = 105^\circ\,33'\,10''{\cdot}4.$$

Adding the correction $-X_0 \tan \delta_0 = +9''{\cdot}5$ (see page 124), we find the position-angle on the plate :—

$$P = 105^\circ\,33'\,19''{\cdot}9.$$

The adopted value was :—

$$P = 105^\circ\,32'\,52''{\cdot}5.$$

The correction is thus :—

$$\Delta P = +27''{\cdot}4.$$

In Chapter XII. we found that the correction, which produces the best possible agreement between the solutions based on the orientation-constants derived from the standard stars and from the trails, was :—

$$\Delta P = +29''.$$

The agreement is eminently satisfactory. It is worthy of remark, that the correction derived through the intermediary of the solutions ($\Delta P = +29''$) is apparently nearer the truth than the direct mean of the individual corrections $I' - I$ of Table VIII. ($\Delta P = +19''$).

Of the stars A and B used in 1904, the Cape 1900 Catalogue contains three and five observations respectively. There are also two observations of star B in *Publication* 15 *of the Cincinnati Observatory*. These observations were combined with those quoted on page 38, and the following distance and position-angle were derived :—

$$s = 4791''{\cdot}12 \qquad p = 45^\circ\,29'\,3''{\cdot}5.$$

Applying the correction $-X_0 \tan \delta_0 = -6''{\cdot}5$ from page 124, we find for the position-angle on the plate :—

$$P = 45^\circ\,28'\,57''{\cdot}0.$$

The adopted value was :—

$$P = 45^\circ\,29'\,50''{\cdot}4.$$

The correction therefore is :—

$$\Delta P = -53''{\cdot}4 \pm 10''.$$

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The estimated probable error of $\pm 10''$ corresponds to a probable error of $\pm 0''{\cdot}25$ in the differences of R.A. and declination. Considering the very small number of observations on which the adopted co-ordinates, especially of star A, are based, this seems not too high an estimate.

The correction found from the plates was :—

$$\Delta P = -43''.$$

The agreement is satisfactory.

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ANNALS OF THE CAPE OBSERVATORY.

VOL. XII. PART IV.

LICK OBSERVATORY
UNIVERSITY
OF CALIFORNIA
OCT 4 1907
MT. HAMILTON, CAL.

DETERMINATION OF THE ELEMENTS OF THE ORBITS OF JUPITER'S SATELLITES,

FROM PHOTOGRAPHS TAKEN AT THE CAPE IN 1902.

By BRYAN COOKSON, M.A.

UNDER THE DIRECTION OF

Sir DAVID GILL, K.C.B., LL.D., D.Sc., F.R.S., Etc.,

HIS MAJESTY'S ASTRONOMER AT THE CAPE OF GOOD HOPE.

Published by Order of the Lords Commissioners of the Admiralty in obedience to His Majesty's Command.

EDINBURGH:

PRINTED FOR HIS MAJESTY'S STATIONERY OFFICE

By NEILL & CO., Limited, Bellevue.

And to be purchased, either directly or through any Bookseller, from
WYMAN & SONS, Limited, Fetter Lane, E.C.; or
OLIVER & BOYD, Edinburgh; or
E. PONSONBY, 116 Grafton Street, Dublin.

1907.

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ERRATA.

Page 100, line F_8, col. 2, *for* " $-\cdot 0383$ " *read* " $-\cdot 0283$ ".

" " col. 4, *for* " $-\cdot 0395$ " *read* " $-\cdot 0295$ ".

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CHAPTER I.

General Introduction.

This paper contains a discussion of the measurements of thirty-five photographs of Jupiter and his satellites, which were taken during the opposition of 1902, when the writer was also making a series of visual observations of the relative positions of the satellites with the Cape Heliometer. It is to be regarded as supplementary to the paper in which the heliometer observations are discussed (Part II. of the present volume), and for this reason reference is here frequently made to the latter for more complete explanation of several points. It is sometimes referred to in this paper simply as "Heliometer."

Excepting the measurement of the plates, nearly the whole of the present work was carried out after the writer had left the Cape and had finished the heliometer reductions. In July 1904 the writer applied as a candidate for one of the Mackinnon Studentships of the Royal Society, and by the award which was made in that year and renewed in the following year, he has been able to carry out the reduction and discussion of the measurements of the Cape photographs, which was one of the two investigations undertaken in connection with the studentship.

The photographs were all taken by Mr Woodgate with the astrographic telescope, generally whilst the writer was at work with the heliometer. The object in view in taking the photographs was a comparison of the two methods and a confirmation and strengthening of the heliometer results. The same ephemeris that was used in the discussion of the visual observations served also for the photographs, so that the discussion of the photographs did not involve a disproportionate addition of labour. The measurements were made by Mr Löwinger and the writer. Mr Löwinger also carried out at the Cape, after the writer had left, some of the reductions of the measurements, and in many ways rendered much valuable assistance. I wish to record my appreciation of the enthusiastic aid he was ever ready to give, both in this and the former work with the heliometer, and to thank him most sincerely for it.

For the determination of the scale-value and orientation of the plate, the images of the same pair of standard stars as were used in the reduction of the heliometer

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observations were impressed on every plate, the exposure on them being made immediately before the exposure on Jupiter. The orientation was also determined from measurements of the trails of one of the satellites, and the values of the unknowns were found for both values of the orientation. The only other correction that had to be applied to the measured co-ordinates of the satellites was that for refraction, and this was calculated by means of Kapteyn's formulæ.

I wish to express again my sincere thanks to Sir David Gill for all the innumerable acts of kindness and true friendship he has shown me; also to thank Mr Hough and all the members of the staff of the Royal Observatory for their courtesy and readiness to help and assist in every way.

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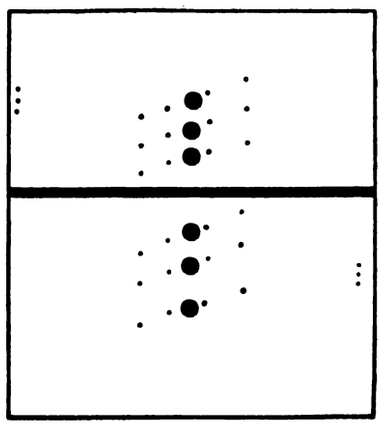
CHAPTER II.

Observed Co-ordinates.

The method of taking the plates which are discussed in this paper was as follows :—The aperture of the astrographic equatorial was cut down to 6 inches. This was found by experiment to be the aperture which gave a measurable trail of the satellites and at the same time good images of the satellites. First of all the telescope was set to the mean position of the standard stars, and three separate exposures of 10 seconds each were made, the instrument being moved slightly in declination between each exposure. The telescope was then directed so that the centre of Jupiter came as nearly as possible on the y axis of the plate at a distance of 10′ from the centre of the plate, and three exposures of 5 seconds each were made, the telescope being moved in declination between each exposure by the same amount, namely, about 3′. After these three exposures the telescope was unclamped and moved just sufficiently to allow Jupiter to trail centrally from one side of the plate to the other, and was left at rest until this had been effected. The telescope was next re-connected with the driving clock, and three more exposures were made on Jupiter similar to the first three, except that the images were not quite equally spaced: this was done in order to distinguish them from the first set. The exposures thus took place in the following order :—

3 of 10 seconds each on standards ;
3 of 5 seconds each on Jupiter ;
Trail of Jupiter ;
3 of 5 seconds each on Jupiter ;

and the appearance of the plate may be roughly represented thus—



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It was not possible to increase the length of the separate exposures without making the images of the satellites too large for accurate measurement, nor was it possible to cut down still further the aperture without making the trails of the satellites too faint. So a compromise had to be adopted, and in order to avoid possible errors arising from refraction-displacement of the images during the 5 seconds exposure, six exposures were made as stated and an observation of weight unity depended on the mean of the six, which might reasonably be supposed to give a reliable position of the satellite.

The plates were measured at the Cape Observatory on one of the Repsold measuring machines, which were made for measuring the plates of the Astrographic Catalogue. The estimated place on the micrometer heads of these machines represents 0·0001 of a *réseau* interval, which corresponds to 0″·006, and every image was set upon once in two positions of the plate by two observers. Since there were in general six images of every satellite, it follows that the absolute term in an equation of condition depends upon twenty-four pointings. A few cases in which there was a difference of over about 0″·4 between the measurements of the two observers, were re-examined by Mr Löwinger and the mean of the three readings was adopted. In the tables that follow on pp. 25–42 the mean co-ordinates for the first three and for the last three exposures are tabulated. The corresponding mean epochs differ by 17 or 18 minutes and since it could not be assumed that during this interval the apparent motion of a satellite was a linear function of the time, it was necessary to calculate absolute terms for both epochs. The mean of these two is the value which appears in the final equations of condition.

The image of the *réseau* marked G*x* was imprinted with parallel light on all the plates, and it was examined to see whether its division errors were appreciable. First the straightness of the lines was tested by stretching alongside of them a spider-web, and measuring the distance between the *réseau* line and the spider-web at a series of points. No appreciable deviation from straightness could be detected and the lines were accordingly taken to be straight. Next the division errors of the four outside lines were determined, and were again found to be practically zero: a very large series of observations would have been necessary to determine them with any precision. No corrections have therefore been applied for *réseau* errors of any kind.

The corrections that have to be applied to the co-ordinates on the plates are those for Refraction, Scale-value, and Orientation.

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Refraction.

The formulæ used for calculating the refraction were Kapteyn's, viz. :—

$$\Delta x = ax + by + cx^2 + dxy + ey^2 + \ldots$$
$$\Delta y = Ax + By + Cx^2 + Dxy + Ey^2 + \ldots$$

where

$$a = k' \sec^2 q + \Delta \sin 1' \tan^2 q + \epsilon$$
$$b = k' \tan q (\tan p - \tan \delta) + \Delta \sin 1' \tan p \tan q + \theta$$
$$A = k' \tan q (\tan p + \tan \delta) + \ldots$$
$$B = k' \sec^2 p + \ldots$$

and

$$\tan N = \cot \phi \cot t$$
$$p = 90^\circ - (\delta + N)$$
$$\tan q = \tan t \sin N \sec p$$
$$\cos \xi = \sin \phi \sec N \cos p$$
$$k' = 1{\cdot}0156\, k \sin 1''$$
$$\Delta = \frac{1{\cdot}0156}{3600 \sin 1'} \frac{dk}{d\xi} \cot \xi$$

k being Bessel's quantity $\alpha' \beta^{\lambda'} \gamma^{\lambda'}$.

In the present case the terms of the second and higher orders could be neglected owing to the comparatively small zenith distances and to the small values of x and y. The formulæ then become—

$$\Delta x = ax + by$$
$$\Delta y = Ax + By$$

where

$$a = k' \sec^2 q$$
$$b = k' \tan q (\tan p - \tan \delta)$$
$$A = k' \tan q (\tan p + \tan \delta)$$
$$B = k' \sec^2 p$$

and k', p, and q are derived as above. These quantities, a, b, A, B, were computed for eastern hour-angles between $0^h\ 30^m$ and $4^h\ 10^m$ at intervals of ten minutes. The declination was taken as $-17^\circ\ 46'$, which is that of the standards, and the value of k is that corresponding to a barometer of 30·100, attached and outside thermometers both 50·0 F. The following table contains these quantities. The unit is one in the sixth place of decimals.

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TABLE I.

Coefficients of Refraction.

Computed for B 30·100, t 50°·0 F, T 50°·0 F, and $\delta - 17° 46'$.

Hour Angle. East.	a.	b.	A.	B.
h m				
0 30	292	− 1	20	313
	3		7	
40	295	− 1	27	313
	3		7	1
0 50	298	− 1	34	314
	5		8	1
1 0	303	− 1	42	315
	6		7	1
10	309	− 1	49	316
	7		8	2
20	316	0	57	318
	8		9	2
30	324	+ 1	66	320
	9	1	9	2
40	333	+ 2	75	322
	11	2	10	2
1 50	344	+ 4	85	324
	12	2	11	2
2 0	356	+ 6	96	326
	14	3	11	3
10	370	+ 9	107	329
	15	4	13	4
20	385	+ 13	120	333
	18	4	14	4
30	403	+ 17	134	337
	20	6	16	5
40	423	+ 23	150	342
	24	7	17	6
2 50	447	+ 30	167	348
	27	8	19	7
3 0	474	+ 38	186	355
	31	10	22	8
10	505	+ 48	208	363
	35	12	25	10
20	540	+ 60	233	373
	41	15	29	11
30	581	+ 75	262	384
	48	19	33	14
40	629	+ 94	295	398
	56	23	38	17
3 50	685	+117	333	415
	66	28	45	20
4 0	751	+145	378	435
	78	33	53	23
10	829	+178	431	458

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The quantities from this table give the mean refraction, and to allow for a different state of atmosphere they must be multiplied by the factor $(1 + \kappa)$ where

$$\log \kappa = A + B + C$$

and A, B, and C are taken from the following table :—

TABLE II.

Table of Meteorological Factors.

Logarithms in units of the 5th place of decimals.

A. Barometer.	Thermometer.	B. For Attached Thermometer.	C. For Ext. Thermometer.
	°		
in.	44	+ 24	+ 511
30·0 — 144	45	+ 20	+ 426
·1 0	46	+ 16	+ 340
·2 + 144	47	+ 12	+ 255
·3 + 288	48	+ 8	+ 170
·4 + 431	49	+ 4	+ 85
·5 + 574	50	0	0
	51	— 4	— 85
	52	— 8	— 169
	53	— 12	— 255
	54	— 16	— 338
	55	— 20	— 422
	56	— 24	— 506
	57	— 28	— 590
	58	— 32	— 674
	59	— 36	— 757
	60	— 39	— 840
	61	— 42	— 923

The declination of Jupiter was nearly the same as that of the standards, so that with small changes the foregoing table served also for computing the refraction of the

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satellites. The next small table contains the change in the factors a, b, A, B due to an increase of 1° of southern declination.

Hour Angle.	a.	b.	A.	B.
h				
1	0	− 2	0	− 3
	1	3	1	1
2	− 1	− 5	− 1	− 4
	2	4	1	1
3	− 3	− 9	− 2	− 5
	3	4	2	2
3½	− 6	−13	− 4	− 7
	5	5	3	3
4	−11	−18	− 7	−10

From these tables the four refraction-factors were calculated for (i.) the mean epoch of exposure on the standards, (ii.) the mean epoch of the first three exposures on Jupiter, (iii.) the epoch when Jupiter in trailing over the plate crossed its centre, and (iv.) the mean epoch of the last three exposures on Jupiter. All these quantities need not be given here: the quantities ax, by, Ax, By only are printed in the table of the measured co-ordinates. But it is convenient to tabulate here the Cape Sidereal Times of the epochs referred to just now. The times of the exposures on the satellites were observed to tenths of a second and are so given in this table. It was from the Greenwich Mean Times corresponding to these sidereal times that the tabular co-ordinates of the satellites were calculated. The other epochs are only required for the calculation of refraction.

TABLE III.

Sidereal Times of Exposures of Plates.

Date.		Rotation No. of Plate.	Standards.			Mean Epoch of First Set of Three Exposures on Jupiter.			Epoch of Trail Passing Centre of Plate.			Mean Epoch of Second Set of Three Exposures on Jupiter.		
			h	m	s	h	m	s	h	m	s	h	m	s
July	8	6942	19	16	58	19	22	9·7	19	31	50	19	40	9·4
	14	6955	18	14	58	18	22	2·4	18	31	31	18	40	48·0
	15	6970	17	39	33	17	43	46·3	17	52	35	18	1	59·0
	16	6984	17	42	36	17	45	57·7	17	54	24	18	2	52·3
	19	6990	18	20	58	18	23	49·3	18	33	0	18	40	52·0
	20	6991	18	22	27	18	25	50·0	18	34	46	18	44	32·7
	24	6997	19	50	44	19	56	26·5	20	5	48	20	17	41·5
	26	7000	18	32	5	18	35	51·4	18	44	13	18	52	21·1
	27	7001	18	6	39	18	11	3·3	18	19	13	18	27	15·6
	28	7002	18	42	9	18	47	5·1	18	55	45	19	3	54·6

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TABLE III.—*continued.*

Date.		Rotation No. of Plate.	Standards.			Mean Epoch of First Set of Three Exposures on Jupiter.			Epoch of Trail Passing Centre of Plate.			Mean Epoch of Second Set of Three Exposures on Jupiter.		
			h	m	s	h	m	s	h	m	s	h	m	s
Aug.	3	7005	18	41	9	18	43	52·1	18	52	16	19	0	4·1
	4	7007	22	34	17	22	42	49·5	22	53	8	23	1	27·5
	6	7010	18	58	8	19	1	23·2	19	10	14	19	17	33·2
	7	7013	19	38	32	19	41	24·5	19	49	5	19	56	2·9
	17	7017	18	29	3	18	32	26·5	18	41	23	18	49	36·5
	18	7018	20	14	13	20	17	35·9	20	26	23	20	33	59·9
	19	7019	19	34	51	19	37	35·8	19	45	39	19	53	22·5
	21	7020	18	14	47	18	19	17·2	18	27	57	18	36	15·5
	24	7023	18	9	55	18	14	48·3	18	23	11	18	31	16·7
	25	7024	16	53	51	16	57	31·2	17	6	20	17	14	21·2
	27	7025	17	1	24	17	4	50·7	17	13	40	17	22	5·4
	28	7028	17	14	28	17	17	50·1	17	26	59	17	35	30·1
	31	7029	18	42	33	18	45	56·7	18	54	47	19	2	18·4
Sept.	1	7030	19	14	55	19	18	26·5	19	28	5	19	36	39·8
	2	7031	17	49	56	17	53	16·1	18	2	25	18	11	36·1
	3	7033	17	28	47	17	32	0·6	17	41	7	17	49	24·0
	4	7034	17	35	24	17	38	43·2	17	47	22	17	55	58·2
	5	7036	18	42	44	18	45	57·3	18	54	38	19	2	3·0
	6	7039	17	47	35	17	51	1·4	17	59	44	18	8	1·5
	7	7043	18	4	0	18	7	43·8	18	16	18	18	27	15·5
	8	7045	18	29	27	18	32	18·7	18	40	40	18	47	41·7
	10	7046	18	11	10	18	15	42·2	18	24	4	18	32	5·2
	12	7047	19	53	52	19	57	9·2	20	5	35	20	13	14·2
	13	7048	19	11	7	19	14	34·5	19	23	14	19	29	59·5
	14	7055	19	0	6	19	7	16·5	19	16	18	19	24	40·8

Scale-Value and Orientation.

The instantaneous scale-value was determined from standard stars whose positions had been observed with meridian circles. A discussion of the observations of these stars is contained in Chapter V. of Vol. XII., Part II., of these Annals, and the positions there derived were adopted in the present reductions. In the next table are contained the measured co-ordinates of the two standards. Every co-ordinate is the mean of twelve separate pointings, two in reversed positions of the plate by each of two observers on three images of the standards. Then follows the refraction, which has already been explained; and next the aberration, for which Kapteyn's formulæ were used, viz. :—

$$\Delta x = Hx - Ly$$
$$\Delta y = Hy + Lx$$

where

$$H \operatorname{cosec} 1'' = A(\sin\alpha\cos\delta + \tan\epsilon\sin\delta) - B\cos\alpha\cos\delta$$
$$L \operatorname{cosec} 1'' = (A\cos\alpha + B\sin\alpha)\tan\delta$$

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and α, δ are the right ascension and declination of the plate, ϵ is the obliquity of the ecliptic, and A, B are the aberration factors from the *Nautical Almanac*. The values of α and δ were taken as 21^h and $-17^\circ\ 46'$ respectively, and the corresponding values of H and L were

$$H = -(3{\cdot}90\,A + 3{\cdot}26\,B)\,10^{-6}$$
$$L = -1{\cdot}10\,(A - B)\,10^{-6}.$$

The only other correction applied was that for proper motion. It was not, however, applied to the measured co-ordinates, but to their differences, $x_1 - x_2$, $y_1 - y_2$, so that it does not appear in the following table. The values adopted were those given on p. 181 of *Cape Annals*, Vol. XII., Part II., and expressed in decimal parts of one minute the corrections were :—to the observed value of $x_1 - x_2$ $(= +72{\cdot}7\pm)$

from July 8 to July 20	− 0·0010
„ „ 24 „ Aug. 7	− 0·0011
„ Aug 17 „ „ 28	− 0·0012
„ „ 31 „ Sept. 14	− 0·0013

to the observed value of $y_1 - y_2$ $(= +17{\cdot}4\pm)$

from July 8 to Aug. 18	+ 0·0002
„ Aug. 19 „ Sept. 14	+ 0·0003.

TABLE IV

Measured Co-ordinates of Standards.

Date 1902.	Co-ordinate on Plate.	Refraction. *ax.*	Refraction. *by.*	Aberration.	Corrected measured Co-ordinate.	Co-ordinate on Plate.	Refraction. Ax.	Refraction. By.	Aberration.	Corrected measured Co-ordinate.
July 8	+31·9035	+0·0108	0·0000	+0·0016	+31·9159	+ 5·6752	+0·0025	+0·0019	−0·0006	+ 5·6790
	−40·8048	− 138	0	− 21	−40·8207	−11·6994	− 32	− 38	+ 7	−11·7057
14	+31·5356	+ 136	+ 2	+ 13	+31·5507	+ 6·6790	+ 50	+ 23	− 7	+ 6·6856
	−41·1350	− 178	− 3	− 17	−41·1548	−10·7508	− 65	− 37	+ 8	−10·7602
15	+31·4372	+ 168	+ 4	+ 13	+31·4557	+ 6·6550	+ 73	+ 25	− 7	+ 6·6641
	−41·2360	− 221	− 6	− 17	−41·2604	−10·7679	− 96	− 41	+ 8	−10·7808
16	+31·4418	+ 162	+ 4	+ 12	+31·4598	+ 6·6049	+ 69	+ 24	− 7	+ 6·6135
	−41·2334	− 213	− 6	− 16	−41·2569	−10·8130	− 94	− 40	+ 9	−10·8252
19	+31·5950	+ 134	+ 2	+ 11	+31·6097	+ 6·6225	+ 47	+ 23	− 7	+ 6·6288
	−41·0892	− 173	− 3	− 14	−41·1082	−10·7855	− 61	− 37	+ 9	−10·7944
20	+31·5743	+ 133	+ 1	+ 10	+31·5887	+ 6·6770	+ 46	+ 23	− 7	+ 6·6832
	−41·1207	− 173	− 2	− 14	−41·1396	−10·7547	− 60	− 37	+ 9	−10·7635

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TABLE IV.—*continued.*

Date 1902.	Co-ordinate on Plate.	Refraction. $ax.$	Refraction. $by.$	Aberration.	Corrected measured Co-ordinate.	Co-ordinate on Plate.	Refraction. Ax.	Refraction. By.	Aberration.	Corrected measured Co-ordinate.
July 24	+31·5686	+ 97	0	+ 8	+31·5791	+ 6·6793	+ 15	+ 21	− 8	+ 6·6821
	−41·1304	− 126	0	− 11	−41·1441	− 10·7227	− 20	− 34	+ 10	− 10·7271
26	+31·4402	+ 125	+ 1	+ 7	+31·4535	+ 6·6188	+ 41	+ 22	− 9	+ 6·6242
	−41·2476	− 164	− 2	− 10	−41·2652	− 10·8002	− 53	− 36	+ 10	− 10·8081
27	+31·6567	+ 143	+ 2	+ 7	+31·6719	+ 6·6612	+ 54	+ 23	− 9	+ 6·6680
	−41·0222	− 185	− 3	− 9	−41·0419	− 10·7406	− 69	− 37	+ 10	− 10·7502
28	+31·6185	+ 118	+ 1	+ 6	+31·6310	+ 6·5998	+ 36	+ 22	− 9	+ 6·6047
	−41·0927	− 154	− 1	− 8	−41·1090	− 10·7836	− 47	− 35	+ 11	− 10·7907
Aug. 3	+31·5848	+ 119	+ 1	+ 3	+31·5971	+ 6·6222	+ 36	+ 22	− 10	+ 6·6270
	−41·1055	− 155	− 1	− 4	−41·1215	− 10·8249	− 47	− 36	+ 13	− 10·8319
4	+31·4951	+ 104	0	+ 2	+31·5057	+ 6·6360	− 23	+ 21	− 10	+ 6·6348
	−41·2016	− 136	0	− 4	−41·2156	− 10·7597	+ 30	− 35	+ 13	− 10·7589
6	+31·6285	+ 111	0	+ 1	+31·6397	+ 6·6629	+ 30	+ 21	− 10	+ 6·6670
	−41·0632	− 145	− 1	− 2	−41·0780	− 10·7588	− 39	− 35	+ 13	− 10·7649
7	+31·5216	+ 100	0	+ 1	+31·5317	+ 6·6595	+ 18	+ 21	− 10	+ 6·6624
	−41·1755	− 131	0	− 2	−41·1888	− 10·7491	− 23	− 34	+ 13	− 10·7535
17	+31·5360	+ 127	+ 1	− 4	+31·5484	+ 6·6169	+ 42	+ 22	− 10	+ 6·6223
	−41·1502	− 166	− 1	+ 5	−41·1664	− 10·8234	− 54	− 36	+ 14	− 10 8310
18	+31·4786	+ 93	0	− 5	+31·4874	+ 6·6277	+ 9	+ 21	− 10	+ 6·6297
	−41·2106	− 122	0	+ 6	−41·2222	− 10·7925	− 12	− 34	+ 14	− 10·7957
19	+31·6284	+ 102	0	− 5	+31·6381	+ 6·6359	+ 19	+ 21	− 11	+ 6·6388
	−41·0729	− 132	0	+ 7	−41·0854	− 10·7690	− 25	− 35	+ 15	− 10·7735
21	+31·5407	+ 134	+ 1	− 6	+31·5536	+ 6·6030	+ 48	+ 22	− 11	+ 6·6089
	−41·1440	− 175	− 3	+ 8	−41·1610	− 10·8138	− 63	− 36	+ 15	− 10·8222
24	+31·5243	+ 138	+ 2	− 8	+31·5375	+ 6·7076	+ 52	+ 23	− 11	+ 6·7140
	−41·1587	− 181	− 3	+ 10	−41·1761	− 10·7250	− 68	− 37	+ 15	− 10·7340
25	+31·6838	+ 248	+ 11	− 8	+31·7089	+ 6·8080	+ 126	+ 30	− 11	+ 6·8225
	−40·9702	− 320	− 17	+ 11	−41·0028	− 10·5997	− 164	− 47	+ 16	− 10·6192
27	+31·6805	+ 230	+ 9	− 9	+31·7035	+ 6·7876	+ 115	+ 29	− 11	+ 6·8009
	−40·9824	− 298	− 14	+ 12	−41·0124	− 10·6280	− 148	− 46	+ 16	− 10·6458
28	+31·6804	+ 205	+ 7	− 10	+31·7006	+ 6·6959	+ 97	+ 27	− 12	+ 6·7071
	−40·9931	− 266	− 11	+ 12	−41·0196	− 10·7121	− 126	− 43	+ 16	− 10·7274
31	+31·6128	+ 120	+ 1	− 11	+31·6238	+ 6·6627	+ 36	+ 22	− 12	+ 6·6673
	−41·0610	− 156	− 1	+ 14	−41·0753	− 10·7538	− 47	− 36	+ 16	− 10·7605

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TABLE IV.—*continued.*

Date 1902.	Co-ordinate on Plate.	Refraction. *ax.*	Refraction. *by.*	Aberration.	Corrected measured Co-ordinate.	Co-ordinate on Plate.	Refraction. Ax.	Refraction. By.	Aberration.	Corrected measured Co-ordinate.
Sept. 1	+31·5418	+ 107	0	− 12	+31·5513	+ 6·6437	+ 25	+ 22	− 12	+ 6·6472
	−41·1540	− 139	0	+ 15	−41·1664	−10·7837	− 33	− 35	+ 16	−10·7889
2	+31·6483	+ 157	+ 3	− 12	+31·6631	+ 6·6387	+ 64	+ 24	− 12	+ 6·6463
	−41·0323	− 204	− 5	+ 16	−41·0516	−10·7877	− 83	− 39	+ 16	−10·7983
3	+31·5773	+ 181	+ 5	− 13	+31·5946	+ 6·6000	+ 81	+ 25	− 12	+ 6·6094
	−41·1137	− 235	− 8	+ 16	−41·1364	−10·7752	− 105	− 41	+ 16	−10·7882
4	+31·6101	+ 173	+ 4	− 13	+31·6265	+ 6·6422	+ 75	+ 25	− 11	+ 6·6511
	−41·0787	− 224	− 7	+ 17	−41·1001	−10·7867	− 97	− 40	+ 16	−10·7988
5	+31·6039	+ 119	+ 1	− 14	+31·6145	+ 6·6517	+ 36	+ 22	− 11	+ 6·6564
	−41·0994	− 155	− 1	+ 17	−41·1133	−10·7412	− 47	− 35	+ 16	−10·7478
6	+31·6766	+ 158	+ 3	− 14	+31·6913	+ 6·6719	+ 65	+ 24	− 11	+ 6·6797
	−41·0038	− 204	− 5	+ 18	−41·0229	−10·7457	− 84	− 38	+ 17	−10·7562
7	+31·6370	+ 144	+ 2	− 15	+31·6501	+ 6·6487	+ 55	+ 23	− 12	+ 6·6553
	−41·0517	− 187	− 4	+ 18	−41·0690	−10·7777	− 72	− 38	+ 17	−10·7870
8	+31·5656	+ 126	+ 1	− 15	+31·5768	+ 6·6597	+ 42	+ 22	− 12	+ 6·6649
	−41·1198	− 164	− 2	+ 19	−41·1345	−10·7611	− 55	− 36	+ 17	−10·7685
10	+31·6456	+ 140	+ 2	− 16	+31·6582	+ 6·6887	+ 52	+ 23	− 12	+ 6·6950
	−41·0343	− 182	− 3	+ 20	−41·0508	−10·7677	− 67	− 38	+ 16	−10·7766
12	+31·6227	+ 97	0	− 17	+31·6307	+ 6·6942	+ 15	+ 21	− 12	+ 6·6966
	−41·0827	− 126	0	+ 21	−41·0932	−10·7303	− 18	− 33	+ 16	−10·7338
13	+31·5632	+ 108	0	− 17	+31·5723	+ 6·6896	+ 26	+ 22	− 12	+ 6·6932
	−41·1288	− 140	0	+ 22	−41·1406	−10·7414	− 34	− 35	+ 17	−10·7466
14	+31·6001	+ 111	0	− 18	+31·6094	+ 6·6513	+ 30	+ 21	− 12	+ 6·6552
	−41·0966	− 145	0	+ 22	−41·1089	−10·7706	− 39	− 35	+ 17	−10·7763

From the corrected co-ordinates of the standards the scale-value S and the orientation P were derived as follows. Let x, x' and y, y' denote the measured co-ordinates, x_0, x_0' and y_0, y_0' the co-ordinates calculated from the adopted right ascension and declination for 1902·0, the plate-centre being at the mean right ascension and declination. Then each star provides a pair of equations of the form

$$x + a + Sx + Py = x_0$$
$$y + b - Px + Sy = y_0$$

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where a, b are corrections to the adopted plate-centre. The four equations are sufficient to determine a, b, S, P. We get

$$S = \frac{\Delta x\,(x-x') + \Delta y\,(y-y')}{(x-x')^2 + (y-y')^2.}$$

$$P = \frac{\Delta x\,(y-y') - \Delta y\,(x-x')}{(x-x')^2 + (y-y')^2}$$

where

$$\Delta x = (x_0 - x_0') - (x - x')$$
$$\Delta y = (y_0 - y_0') - (y - y').$$

Since the co-ordinates were calculated from the stars' places for the beginning of 1902, the orientation P is the angle between the x-axis of the plate and a parallel to the earth's mean equator of 1902·0.

The adopted places of the standards were—

21 Capricorni	$20^h\ 55^m\ 20^s{\cdot}801$	$-17^\circ\ 54'\ 47''{\cdot}65$
23 Capricorni	$21\ \ 0\ \ 26{\cdot}273$	$-17\ 37\ 20{\cdot}68.$

It is important that their calculated abscissæ should not differ more than 0′·1 from the corrected observed abscissæ: for if the adopted plate-centre were in error by an amount Δa^s, the effect would be the same as if we had let the whole field trail across the plate for Δa seconds and measured star-images at the ends of the trails instead of at their beginnings. Owing to the curvature of the trails this would have a considerable effect on the ordinates of stars; and it is chiefly from the ordinates of the standards that the orientation was determined.

For various plate-centres we have the following values of the co-ordinates of the standards.

TABLE V.

Calculated Co-ordinates of Standards.

x_0	x_0'	$x_0 - x_0'$	y_0	y_0'	$y_0 - y_0'$
+ 31·7080	− 41·0124	+ 72·7204	+ 6·6422	− 10·8399	+ 17·4821
·6127	·1075	·7202	·6423	·8404	·4827
·5174	·2027	·7201	·7093	·7740	·4833
+ 31·4221	− 41·2979	+ 72·7200	+ 6·6428	− 10·8409	+ 17·4837

The variation in $x_0 - x_0'$ is inappreciable, but the value of $y_0 - y_0'$ was taken according to the value of x_0 or x_0'.

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The values of S and P found in this way are given in Table VII.

The orientation P was also derived in another way, namely, from measurements of the ordinate of a trail of one of the satellites at points at equal distances and on opposite sides of the centre of the plate. Six pointings were made between abscissæ whose values were $+44'$ and $+46'$, and six between $-44'$ and $-46'$, and the mean of the two sets gave the ordinates for abscissæ with the values $\pm 45'$: but more frequently the pointings were made so that their mean abscissæ were $\pm 47'{\cdot}5$. The orientation of the plate can be found from the differences of the two ordinates $y_W - y_E$, but it is first necessary to apply three corrections to $y_W - y_E$.

(1) *Correction for refraction.*—The point at which the trail was measured may be regarded as an instantaneous photograph of the satellite, the right ascension of the plate-centre being equal to the right ascension of the satellite plus or minus $45'$ sec δ, and its declination being the declination of the satellite plus the ordinate of the trail. The refraction correction to $y_W - y_E$ is—

$$(A_W x_W + B_W y_W) - (A_E x_E + B_E y_E).$$

The part depending on $y_W - y_E$ vanishes and we get simply

$$x\,(A_W + A_E).$$

Practically, it was quite sufficient to take the value of A corresponding to the time when Jupiter crossed the centre of the plate and to apply the correction $2A_0 x$.

(2) *Correction for change in declination of Jupiter and his system.*—The declination of Jupiter was changing at the rate of from $1'$ to $2'$ a day, and the effect of this was far from negligible. The correction to be applied to $y_W - y_E$ is:—Hourly change of N.P.D. in minutes of arc $\times \frac{90}{15 \times 60}$ sec δ.

(3) *Correction for motion of satellite round its primary.*—If Δx, Δy denote the change in the calculated abscissæ and ordinate of the satellite referred to Jupiter's equator, the correction is $+ (\Delta x \sin P + \Delta y \cos P)$. Δy was always negligible. Δx was easily found from the quantity X (see p. 45). The trail measured was generally that of the third satellite, sometimes of the second, and the correction was as follows:—

$x_W - x_E$	II.	III.
90	7·39 X sin P	3·67 X sin P
95	7·83 X sin P	3·89 X sin P.

P here denotes the position-angle of the nothern end of Jupiter's axis, and will not be confused with P above, which denotes the orientation.

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The following table contains the measured differences $y_W - y_E$, the three foregoing corrections, and the corrected values of $y_W - y_E$.

TABLE VI.

Measurements of Trails.

Date 1902.	Satellite.	$x_W - x_E$.	Measured $y_W - y_E$.	Refraction.	Correction for $\Delta\delta$ of Jupiter.	Correction for Motion round Jupiter.	Corrected $y_W - y_E$.
July 8	III.	90	+ 0·1121	− 0·0071	+ 0·0084	− 0·0060	+ 0·1074
14	III.	90	0·0434	− 137	+ 92	− 69	320
15	III.	90	0·0529	− 206	+ 93	− 68	348
16	III.	90	0·0621	− 200	+ 94	− 19	496
19	III.	90	0·0653	− 131	+ 98	+ 50	670
20	III.	90	0·0462	− 129	+ 98	− 11	420
24	III.	90	0·0708	− 36	+ 100	+ 40	812
26	III.	95	0·0538	− 117	+ 108	+ 60	589
27	III.	95	0·0760	− 152	+ 108	0	716
28	III.	95	0·1127	− 100	+ 108	− 61	1074
Aug. 3	III.	95	0·0300	− 101	+ 108	+ 10	317
4	III.	95	0·0906	+ 84	+ 108	− 61	1037
6	III.	95	0·0588	− 75	+ 107	− 45	575
7	III.	95	0·0622	− 44	+ 106	+ 22	706
17	J.	95	0·0425	− 102	+ 97		420
18	III.	95	0·0591	− 14	+ 96	− 39	634
19	I.	95	0·0773	− 43	+ 96	− 106	720
	III.	95	0·0836	− 43	+ 96	− 74	815
21	III.	95	0·0622	− 118	+ 92	− 3	593
24	III.	95	0·0362	− 122	+ 88	+ 39	367
25	II.	95	0·0668	− 287	+ 86	− 17	450
27	III.	95	0·0750	− 262	+ 81	− 65	504
28	III.	95	0·0712	− 222	+ 80	− 15	555
31	III.	95	0·0472	− 82	+ 74	+ 45	509
Sept. 2	III.	95	0·0620	− P47	+ 70	− 63	480
3	I.	95	0·1210	− 183	+ 68	+ 60	1155
4	III.	95	0·0443	− 171	+ 66	− 24	314
5	III.	95	0·0931	− 81	+ 64	+ 39	953
6	II.	95	0·0645	− 148	+ 62	− 45	514
7	III.	95	0·0439	− 125	+ 60	+ 52	426
8	III.	75	0·0443	− 74	+ 45	− 3	411
10	II.	95	0·0066	− 112	+ 54	+ 23	31
12	III.	95	0·0497	− 22	+ 49	+ 32	556
13	III.	95	0·0540	− 53	+ 46	+ 67	600
14	III.	95	+ 0·0548	− 58	+ 44	+ 55	+ 0·0589

The value of the orientation was now found by dividing $y_W - y_E$ by the appropriate value of $x_W - x_E$, which it should be remembered is a negative quantity. The

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resulting values of the orientation are given in the following table. But what they represent is the angle between the axis of the plate and a parallel to the earth's equator of the epoch of the plate: what is wanted is the orientation referred to the earth's mean equator of 1902·0. It was necessary, therefore, to apply a correction for precession and nutation, of which the apparent effect is to rotate the axes of the plate. The correction to reduce from the epoch of the plate to the beginning of the year is—

$$-(\gamma' C + \delta' D)$$

where

$$n = 20''{\cdot}05$$
$$\gamma' = n \sin \alpha \sec \delta$$
$$\delta' = \cos \alpha \sec \delta$$

and where C and D are Bessel's day numbers.

A more convenient form for this correction is—

$$-g \sin (G + \alpha) \sec \delta.$$

G and g are given by the Nautical Almanac, and α, δ are the right ascension and declination of Jupiter. When this correction is positive, its effect is to increase observed position-angles, *i.e.* to rotate the zero-line counter-clockwise. It must therefore be added to the observed orientation P, which is the angle between the axis of the plate and the equator of the epoch, the axis of the plate being regarded as the zero-line.

In the next table are collected together the values of the orientation from the trails and from the standards and the scale-value.

TABLE VII.

Orientation and Scale-Value.

Date 1902.	Apparent Orientation from Trails.	Correction for Precession and Nutation.	Orientation from Trails referred to 1902·0.	Orientation from Standards.	Difference.	Residual.	Residual in Seconds of Arc.	Scale-Value from Standards.
July 8	− 0·001193	+ 0·000016	− 0·001177	− 0·001298	+ 121	− 57	− 12″	+ 0·000096
14	− 356	18	− 338	− 406	+ 68	− 4	− 1	312
15	− 387	18	− 369	− 485	+ 116	− 52	− 11	185
16	− 551	19	− 532	− 567	+ 35	+ 29	+ 6	196
19	− 744	20	− 724	− 762	+ 38	+ 26	+ 5	227
20	− 467	20	− 447	− 490	+ 43	+ 21	+ 4	19
24	− 902	22	− 880	− 974	+ 94	− 30	− 6	205
26	− 620	23	− 597	− 655	+ 58	+ 6	+ 1	191
27	− 754	24	− 730	− 807	+ 77	− 13	− 3	294
28	− 1131	24	− 1107	− 1189	+ 82	− 16	− 3	24

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TABLE VII.—*continued.*

Date 1902.	Apparent Orientation of Trails.	Correction for Precession and Nutation.	Orientation from Trails referred to 1902·0.	Orientation from Standards.	Difference.	Residuals.	Residual in Seconds of Arc.	Scale-Value from Standards.
Aug. 3	− 334	27	− 307	− 301	− 6	+ 70	+ 14″	108
4	− 1092	27	− 1065	− 1165	+ 100	− 36	− 7	277
6	− 605	28	− 577	− 645	+ 68	− 4	− 1	203
7	− 743	28	− 715	− 871	+ 156	− 92	− 19	218
17	− 442	32	− 410	− 364	− 46	+ 110	+ 23	173
18	− 667	33	− 634	− 715	+ 81	− 17	− 4	333
19	− 758	33	− 725	− 916	+ 191	− 127	− 26	189
	− 858	33	− 825	− 916	+ 91	− 27	− 6	
21	− 624	34	− 590	− 651	+ 61	+ 3	+ 1	249
24	− 386	35	− 351	− 430	+ 79	− 15	− 3	208
25	− 474	35	− 439	− 492	+ 53	+ 11	+ 2	250
27	− 530	36	− 494	− 440	− 54	+ 118	+ 24	179
28	− 584	36	− 548	− 612	+ 64	0	0	161
31	− 536	38	− 498	− 640	+ 142	− 78	− 16	459
Sept. 1				− 596				193
2	− 505	38	− 467	− 468	+ 1	+ 63	+ 13	204
3	− 1216	39	− 1177	− 1135	− 42	+ 106	+ 22	139
4	− 330	39	− 291	− 440	+ 149	− 85	− 18	32
5	− 1003	39	− 964	− 1038	+ 74	− 10	− 2	160
6	− 541	40	− 501	− 577	+ 76	− 12	− 2	237
7	− 448	40	− 408	− 513	+ 105	− 41	− 8	154
8	− 548	40	− 508	− 609	+ 101	− 37	− 8	284
10	− 33	41	+ 8	+ 100	− 92	+ 156	+ 32	194
12	− 585	41	− 544	− 684	+ 140	− 76	− 16	129
13	− 632	42	− 590	− 532	− 58	+ 122	+ 25	243
14	− 0·000620	+ 0·000042	− 0·000578	− 0·000654	+ 76	− 12	− 2	+ 0·000199

It will be seen that there is a systematic difference between the orientation from the standards and that from the trails, amounting in the mean to 0·000064 or 13″. At first sight this might be attributed to want of coincidence of the origin of co-ordinates on the plate and of the point where the optic axis of the lens cuts the plane of the plate. This, however, will not account for the observed difference.

The following considerations will make this point clear. The axis of symmetry of a star-trail is the projection of the hour-circle which passes through the point where the plane of the plate is cut by the optic axis.

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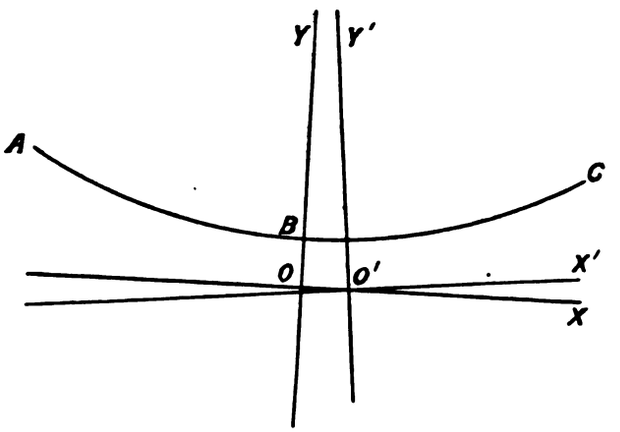
Let ABC be a star-trail supposed free from the effects of refraction, etc., O the point where the optic axis cuts the plate, OBY the projection of the hour-circle through O,



and O′ the origin of co-ordinates on the plate. Let OO′ = p. Then when ξ does not exceed 1°, the equation of the trail referred to the axes OX OY is—

$$\eta = \text{constant} + \tfrac{1}{2}\xi^2 \tan\delta$$

where δ is the declination of the star. Now, rotate these axes through a small angle θ and move the origin to O′. Then referred to the axes O′X′, O′Y′ the equation of the trail is—

$$x \sin\theta + y\cos\theta = \text{constant} + \tfrac{1}{2}\tan\delta\,\{x\cos\theta - y\sin\theta + p\}^2.$$

In the case before us the trails were measured at two points $x_1\ y_1$ and $x_2\ y_2$ such that $x_2 = -x_1$. The orientation is $-\theta$. Each point gives an equation of the form written above and subtracting one from the other, we get to determine θ the equation—

$$\begin{aligned}(x_1 - x_2)\sin\theta + (y_1 - y_2)\cos\theta = {} & \tfrac{1}{2}\tan\delta\,\{x_1\cos\theta - y_1\sin\theta + p\}^2 \\ & - \tfrac{1}{2}\tan\delta\,\{x_2\cos\theta - y_2\sin\theta + p\}^2.\end{aligned}$$

Omitting for the moment the factor $\frac{1}{2}\tan\delta$ and expanding the right-hand side of this, we get—

$$\begin{aligned}(x_1^2 - x_2^2)\cos^2\theta + (y_1^2 - y_2^2)\sin^2\theta - 2\sin\theta\cos\theta\,(x_1 y_1 - x_2 y_2) \\ + 2p\cos\theta\,(x_1 - x_2) + 2p\sin\theta\,(y_1 - y_2).\end{aligned}$$

The first term vanishes in virtue of $x_1 + x_2 = 0$. The second is very small and may be neglected; so too may the last, and we are left with the equation—

$$2x_1\theta + (y_1 - y_2) = \tan\delta\,[2px_1 - \theta x_1 (y_1 + y_2)]$$

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which may be written

$$[2\,x_1 + \tan\delta\, x_1\,(y_1 + y_2)]\;\theta = 2\,p\,x_1 \tan\delta - (y_1 - y_2).$$

y_1 and y_2 are small and their squares may be neglected; hence the equation for θ is—

$$\theta = \left\{ p\tan\delta - \frac{y_1 - y_2}{2\,x_1} \right\} \left\{ 1 - \tfrac{1}{2}\tan\delta\,(y_1 + y_2) \right\}$$

In our case $y_1 + y_2$ was generally about $-4'$ but sometimes reached $-9'$. Hence the maximum value of $-\frac{1}{2}\tan\delta\,(y_1 + y_2)$ was about 0·00041 and could be neglected.

Now the orientation given in Table VII. was derived from the equation—

$$\mathrm{P} = -\theta = +\frac{y_1 - y_2}{2\,x_1}$$

which is true only when $p = 0$. If the foregoing difference between the orientation from the standards and that from the trails is due to the fact that p is not zero, then to account for the difference found the value of p must be about $+0'{\cdot}70$. This corresponds to a linear distance on the plate of only 0·7 millimetre, and mal-adjustment to this extent is certainly not impossible.

But now let us see what effect this has on the orientation derived from the standard stars. It is not difficult to see that the effect on an ordinate of a small displacement, p, of the origin is given by

$$\delta y = x \tan\delta\, p.$$

This may be proved by differentiating the expressions for the conversion of spherical co-ordinates into rectangular or by the consideration that the effect is the same as allowing the whole field to drift across the plate for a time p sec δ and then measuring the star-images in their final positions instead of in their original positions. Hence the equations to determine S and P are

$$\begin{aligned} x + a + \mathrm{S}\,x + \mathrm{P}\,y &= x_0 \\ y + b - \mathrm{P}\,x + \mathrm{S}\,y &= y_0 + x\tan\delta\, p, \end{aligned}$$

and we really have five quantities to determine, a, b, S, P, $(\mathrm{P} - p\tan\delta)$, and only four equations from which to find them. This being impossible, the value of P will be affected by p which will enter with y_0. If P′ denote the value deduced from the measures and P the true value, then

$$\mathrm{P}' = \mathrm{P} + \frac{(x - x')\,(x'\tan\delta' - x\tan\delta)\,p.}{(x - x')^2 + (y - y')^2}$$

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It is sufficient to put $\delta = \delta'$; then substituting the numerical values we get

$$P = P' + 8{\cdot}8.\ 10^{-5}\, p$$

where p is expressed in minutes of arc. From the trails we found

$$\begin{aligned} P &= P' - p \tan \delta \\ &= P' + 9{\cdot}3.\ 10^{-5}\, p. \end{aligned}$$

Thus the effect of non-coincidence of the origin of co-ordinates with the intersection of the optic axis and the plate is practically the same in whichever way the orientation is determined.

It was decided to reduce the measured co-ordinates in the first place with the values of S and P derived from the standards, and then to find later by differential methods the values of the unknowns corresponding to the orientation from the trails. The appropriate corrections for orientation and scale-value are contained in Table VIII.

Of the two values of each co-ordinate of a satellite in this table, the first is the mean of the first set of three exposures and the second is the mean of the second set. Simple means could here be taken since the time occupied in making the three exposures was never more than two minutes.

The position of Jupiter was measured by setting on both limbs and taking the mean. This, needless to say, cannot be a very accurate measurement, but is only required as a first approximation as will be explained later on.

We are now in a position to give the tables which contain the measured co-ordinates of the satellites. The meanings of the various columns are sufficiently explained by their headings. The unit of the figures in the columns containing the corrections for refraction, scale-value, and orientation is one in the fourth decimal, *i.e.* $0'{\cdot}0001$ or $0''{\cdot}006$.

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TABLE VIII.

6942. July 8.

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sx.	Py.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sy.	−Px.	Observed Ordinate.
J.	+ 2′·1189							+ 8′·1825						
I.	+ 4·2786	+ 2′·1597	+ 8	0	+ 2	−10	+ 2′·1597	+ 8·9368	+ 0′·7543	+ 2	+ 2	+ 1	+28	+ 0′·7576
IV.	− 5·1195	− 7·2384	−25	0	− 7	+33	− 7·2383	+ 5·6700	− 2·5125	− 6	− 8	− 2	−94	− 2·5235
J.	+ 2·0588							− 5·5525						
I.	+ 4·2301	+ 2·1713	+ 7	0	+ 2	−10	+ 2·1712	− 4·7955	+ 0·7570	+ 2	+ 2	+ 1	+28	+ 0·7603
II.	+ 5·2760	+ 3·2172	+11	0	+ 3	−15	+ 3·2171	− 4·4190	+ 1·1335	+ 2	+ 4	+ 1	+42	+ 1·1384
III.	+ 5·3884	+ 3·3296	+11	0	+ 3	−15	+ 3·3295	− 4·3844	+ 1·1681	+ 2	+ 4	+ 1	+43	+ 1·1731
IV.	− 5·1511	− 7·2099	−24	0	− 7	+33	− 7·2097	− 8·0559	− 2·5034	− 5	− 8	− 2	−94	− 2·5143

6955. July 14.

(Images of Standards too large.)

J.	+ 1·9102							+ 8·4965						
I.	+ 0·7075	− 1·2027	− 5	0	− 4	+ 2	− 1·2034	+ 8·0778	− 0·4187	− 2	− 1	− 1	− 5	− 0·4196
III.	− 0·3885	− 2·2987	−10	0	− 7	+ 3	− 2·3001	+ 7·6894	− 0·8071	− 4	− 3	− 3	− 9	− 0·8090
IV.	+11·7403	+ 9·8301	+44	+ 1	+31	−14	+ 9·8363	+11·8393	+ 3·3428	+17	+12	+10	+40	+ 3·3507
J.	+ 1·9388							− 5·3042						
I.	+ 0·6519	− 1·2869	− 5	0	− 4	+ 2	− 1·2876	− 5·7518	− 0·4476	− 2	− 1	− 1	− 5	− 0·4485
III.	− 0·3007	− 2·2395	− 9	0	− 7	+ 3	− 2·2408	− 6·0884	− 0·7842	− 3	− 2	− 2	− 9	− 0·7858
IV.	+11·7767	+ 9·8379	+40	+ 1	+31	−14	+ 9·8437	− 1·9598	+ 3·3444	+13	+11	+10	+40	+ 3·3518

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TABLE VIII.—*continued.*

6970. July 15.

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	*Sx.*	*Py.*	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	*Sy.*	*−Px.*	Observed Ordinate.
J.	+ 1·8059							+ 8·4672						
I.	+ 3·4845	+ 1·6786	+10	0	+ 3	− 3	+ 1·6796	+ 9·0444	+ 0·5772	+ 4	+ 2	+ 1	+ 8	+ 0·5787
II.	+ 5·3430	+ 3·5371	+20	+ 1	+ 6	− 6	+ 3·5392	+ 9·7077	+ 1·2405	+ 9	+ 5	+ 2	+17	+ 1·2438
III.	+ 4·1655	+ 2·3596	+13	+ 1	+ 4	− 4	+ 2·3610	+ 9·2830	+ 0·8158	+ 6	+ 3	+ 1	+11	+ 0·8179
IV.	+11·5471	+ 9·7412	+55	+ 2	+18	−16	+ 9·7471	+11·7817	+ 3·3145	+25	+13	+ 6	+47	+ 3·3236
J.	+ 1·6833							− 5·2155						
I.	+ 3·4236	+ 1·7403	+ 9	0	+ 3	− 3	+ 1·7412	− 4·6100	+ 0·6055	+ 4	+ 2	+ 1	+ 8	+ 0·6070
II.	+ 5·2165	+ 3·5332	+18	+ 1	+ 7	− 6	+ 3·5352	− 3·9737	+ 1·2418	+ 7	+ 5	+ 2	+17	+ 1·2449
III.	+ 4·0987	+ 2·4154	+12	0	+ 4	− 4	+ 2·4166	− 4·3772	+ 0·8383	+ 5	+ 3	+ 2	+12	+ 0·8405
IV.	+11·4147	+ 9·7314	+49	+ 1	+18	−16	+ 9·7366	− 1·9001	+ 3·3155	+20	+12	+ 6	+47	+ 3·3240

6984. July 16.

J.	+ 1·8655							+ 8·4609						
I.	− 0·2767	− 2·1422	−11	− 1	− 4	+ 4	− 2·1434	+ 7·7154	− 0·7455	− 5	− 3	− 1	−12	− 0·7476
II.	+ 1·0278	− 0·8377	− 4	0	− 2	+ 2	− 0·8381	+ 8·1433	− 0·3176	− 2	− 1	− 1	− 5	− 0·3185
III.	+ 7·3059	+ 5·4404	+28	+ 1	+11	−11	+ 5·4433	+10·3483	+ 1·8874	+13	+ 7	+ 4	+31	+ 1·8929
IV.	+10·1717	+ 8·3062	+43	+ 1	+16	−16	+ 8·3106	+11·2996	+ 2·8387	+20	+11	+ 6	+47	+ 2·8471
J.	+ 1·7884							− 5·2504						
I.	− 0·3794	− 2·1678	−11	0	− 4	+ 4	− 2·1689	− 6·0044	− 0·7540	− 4	− 3	− 1	−12	− 0·7560
II.	+ 0·8829	− 0·9055	− 4	0	− 2	+ 2	− 0·9059	− 5·5932	− 0·3428	− 2	− 1	− 1	− 5	− 0·3437
III.	+ 7·2476	+ 5·4592	+27	+ 1	+11	−11	+ 5·4620	− 3·3482	+ 1·9022	+11	+ 7	+ 4	+31	+ 1·9075
IV.	+10·0748	+ 8·2864	+41	+ 1	+16	−16	+ 8·2906	− 2·4177	+ 2·8327	+17	+10	+ 6	+47	+ 2·8407

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TABLE VIII.—*continued.*

6990. July 19.

(Last set of three images very diffused.)

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	S*x.*	P*y.*	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	S*y.*	−P*x.*	Observed Ordinate.
J.	+ 1·9646							+ 8·4921						
I.	+ 3·1463	+ 1·1817	+ 5	0	+ 3	− 3	+ 1·1822	+ 8·8966	+ 0·4045	+ 2	+ 1	+ 1	+ 9	+ 0·4058
II.	+ 4·2405	+ 2·2759	+10	0	+ 5	− 6	+ 2·2768	+ 9·2648	+ 0·7727	+ 4	+ 3	+ 2	+17	+ 0·7753
III.	− 2·2325	− 4·1971	−19	0	−10	+11	− 4·1989	+ 7·0407	− 1·4514	− 7	− 5	− 3	−32	− 1·4561
IV.	+ 0·4513	− 1·5133	− 7	0	− 3	+ 4	− 1·5139	+ 8·0183	− 0·4738	− 2	− 2	− 1	−12	− 0·4755
J.	+ 1·8654							− 5·3298						
I.	+ 2·9677	+ 1·1023	+ 4	0	+ 2	− 3	+ 1·1026	− 4·9527	+ 0·3771	+ 1	+ 1	+ 1	+ 8	+ 0·3782
II.	+ 4·0831	+ 2·2177	+ 9	0	+ 5	− 6	+ 2·2185	− 4·5738	+ 0·7560	+ 3	+ 3	+ 2	+17	+ 0·7585
III.	− 2·3641	− 4·2295	−17	0	−10	+11	− 4·2311	− 6·7951	− 1·4653	− 6	− 5	− 3	−32	− 1·4699
IV.	+ 0·3140	− 1·5514	− 6	0	− 4	+ 4	− 1·5520	− 5·8165	− 0·4867	− 2	− 2	− 1	−12	− 0·4884

6991. July 20.

J.	+ 1·9422							+ 8·5340						
II.	− 1·1694	− 3·1116	−14	0	+ 1	+ 5	− 3·1124	+ 7·4361	− 1·0979	− 5	− 4	0	−15	− 1·1003
III.	− 3·6943	− 5·6365	−25	0	+ 1	+10	− 5·6379	+ 6·5898	− 1·9442	− 9	− 7	0	−28	− 1·9486
IV.	− 3·1098	− 5·0520	−22	0	+ 1	+ 8	− 5·0533	+ 6·8623	− 1·6717	− 8	− 6	0	−25	− 1·6756
J.	+ 2·0107							− 5·2536						
II.	− 1·1311	− 3·1418	−12	0	+ 1	+ 5	− 3·1424	− 6·3647	− 1·1111	− 4	− 4	0	−15	− 1·1134
III.	− 3·6116	− 5·6223	−22	0	+ 1	+10	− 5·6234	− 7·1951	− 1·9415	− 7	− 7	0	−28	− 1·9457
IV.	− 3·0790	− 5·0897	−20	0	+ 1	+ 8	− 5·0908	− 6·9386	− 1·6850	− 7	− 6	0	−25	− 1·6888

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TABLE VIII.—*continued.*

6997. July 24.

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction.		Sx.	Py.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction.		Sy.	−Px.	Observed Ordinate.
			ax.	*by.*						*ax.*	*ly.*			
J.	+ 1′9920							+ 8′4909						
I.	+ 4·2204	+ 2′2284	+ 7	0	+ 5	− 7	+ 2′2289	+ 9·2489	+ 0′7580	+ 1	+ 2	+ 2	+22	+ 0′7607
II.	− 1·2698	− 3·2618	−10	0	− 7	+11	− 3·2624	+ 7·3731	− 1·1178	− 1	− 3	− 2	−32	− 1·1216
III.	+ 6·8066	+ 4·8146	+15	0	+10	−16	+ 4·8155	+10·1335	+ 1·6426	+ 2	+ 5	+ 3	+47	+ 1·6483
IV.	− 6·8115	− 8·8035	−27	0	−18	+29	− 8·8051	+ 5·5380	− 2·9529	− 4	− 9	− 6	−86	− 2·9634
J.	+ 1·9333							− 5·2748						
I.	+ 4·1482	+ 2·2149	+ 7	0	+ 5	− 7	+ 2·2154	− 4·5206	+ 0·7542	+ 1	+ 2	+ 2	+22	+ 0·7569
II.	− 1·2900	− 3·2233	−10	0	− 7	+11	− 3·2239	− 6·3802	− 1·1054	− 1	− 3	− 2	−31	− 1·1091
III.	+ 6·7091	+ 4·7758	+14	0	+10	−16	+ 4·7766	− 3·6382	+ 1·6366	+ 2	+ 5	+ 3	+47	+ 1·6423
IV.	− 6·8411	− 8·7744	−26	0	−18	+29	− 8·7759	− 8·2174	− 2·9426	− 4	− 9	− 6	−85	− 2·9530

7000. July 26.

J.	+ 1·9959							+ 8·4506						
I.	+ 3·6945	+ 1·6986	+ 7	0	+ 3	− 4	+ 1·6992	+ 9·0211	+ 0·5705	+ 2	+ 2	+ 1	+11	+ 0·5721
II.	+ 4·7599	+ 2·7640	+11	0	+ 5	− 6	+ 2·7650	+ 9·3796	+ 0·9290	+ 4	+ 3	+ 2	+18	+ 0·9317
III.	− 1·6996	− 3·6955	−15	0	− 7	+ 8	− 3·6969	+ 7·1907	− 1·2599	− 5	− 4	− 2	−24	− 1·2634
IV.	− 1·2781	− 3·2740	−13	0	− 6	+ 7	− 3·2752	+ 7·3231	− 1·1275	− 4	− 4	− 2	−21	− 1·1306
J.	+ 1·9528							− 5·2899						
I.	+ 3·5913	+ 1·6385	+ 6	0	+ 3	− 4	+ 1·6390	− 4·7371	+ 0·5528	+ 2	+ 2	+ 1	+11	+ 0·5544
II.	+ 4·6747	+ 2·7219	+10	0	+ 5	− 6	+ 2·7228	− 4·3729	+ 0·9170	+ 3	+ 3	+ 2	+18	+ 0·9196
III.	− 1·7842	− 3·7370	−14	0	− 7	+ 8	− 3·7383	− 6·5629	− 1·2730	− 4	− 4	− 2	−24	− 1·2764
IV.	− 1·2793	− 3·2321	−12	0	− 6	+ 7	− 3·2332	− 6·4007	− 1·1108	− 4	− 4	− 2	−21	− 1·1139

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TABLE VIII.—*continued.*

7001. July 27.

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sx.	Py.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sy.	−Px.	Observed Ordinate.
J.	+ 1·9609							+ 8·4576						
I.	+ 0·8276	− 1·1333	− 5	0	− 3	+ 3	− 1·1338	+ 8·0765	− 0·3811	− 2	− 1	− 1	− 9	− 0·3824
II.	− 0·7286	− 2·6895	−12	0	− 8	+ 8	− 2·6907	+ 7·5179	− 0·9397	− 5	− 3	− 3	−22	− 0·9430
III.	− 3·7781	− 5·7390	−26	0	−17	+16	− 5·7417	+ 6·5115	− 1·9461	−10	− 7	− 6	−46	− 1·9530
J.	+ 2·1647							− 5·2521						
I.	+ 1·1119	− 1·0528	− 4	0	− 3	+ 3	− 1·0532	− 5·6069	− 0·3548	− 2	− 1	− 1	− 8	− 0·3560
II.	− 0·5678	− 2·7325	−11	0	− 8	+ 8	− 2·7336	− 6·2102	− 0·9581	− 4	− 3	− 3	−22	− 0·9613
III.	− 3·5730	− 5·7377	−24	0	−17	+16	− 5·7402	− 7·2006	− 1·9485	− 8	− 7	− 6	−46	− 1·9552
IV.	+ 2·6496	+ 0·4849	+ 2	0	+ 1	− 1	+ 0·4851	− 5·1350	+ 0·1171	+ 1	0	0	+ 4	+ 0·1176

7002. July 28.

(Images much elongated parallel to axis of *x*.)

J.	+ 2·0999							+ 8·4494						
II.	+ 0·5286	− 1·5713	− 6	0	0	+ 6	− 1·5713	+ 7·9303	− 0·5191	− 2	− 2	0	−19	− 0·5214
III.	− 1·5107	− 3·6106	−14	0	− 1	+15	− 3·6106	+ 7·2210	− 1·2284	− 4	− 4	0	−43	− 1·2335
IV.	+ 6·3103	+ 4·2104	+16	0	+ 1	−16	+ 4·2105	+ 9·8042	+ 1·3548	+ 5	+ 4	0	+50	+ 1·3607
J.	+ 2·1474							− 5·8997						
II.	+ 0·6435	− 1·5039	− 5	0	0	+ 6	− 1·5038	− 6·3938	− 0·4941	− 1	− 2	0	−18	− 0·4962
III.	− 1·4189	− 3·5663	−13	0	− 1	+14	− 3·5663	− 7·1109	− 1·2112	− 3	− 4	0	−42	− 1·2161
IV.	+ 6·3978	+ 4·2504	+15	0	+ 1	−16	+ 4·2504	− 4·5305	+ 1·3692	+ 4	+ 4	0	+51	+ 1·3751

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TABLE VIII.—*continued.*

7005. August 3.

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sx.	Py.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sy.	−Px.	Observed Ordinate.
J.	+ 1·9658							+ 8·4907						
I.	+ 0·3942	− 1·5716	− 6	0	− 2	+ 2	− 1·5722	+ 7·9673	− 0·5234	− 2	− 2	− 1	− 5	− 0·5244
II.	− 0·3184	− 2·2842	− 9	0	− 2	+ 2	− 2·2851	+ 7·6955	− 0·7952	− 3	− 3	− 1	− 7	− 0·7966
III.	− 3·7468	− 5·7126	−22	0	− 6	+ 6	− 5·7148	+ 6·5735	− 1·9172	− 7	− 6	− 2	−17	− 1·9204
IV.	+ 6·4658	+ 4·5000	+17	0	+ 5	− 5	+ 4·5017	+ 9·9952	+ 1·5045	+ 5	+ 5	+ 2	+14	+ 1·5071
J.	+ 1·9191							− 5·2918						
I.	+ 0·4129	− 1·5062	− 5	0	− 2	+ 2	− 1·5067	− 5·7966	− 0·5048	− 1	− 2	− 1	− 5	− 0·5057
II.	− 0·4195	− 2·3386	− 8	0	− 2	+ 2	− 2·3394	− 6·1069	− 0·8151	− 2	− 3	− 1	− 7	− 0·8164
III.	− 3·8001	− 5·7192	−20	0	− 6	+ 6	− 5·7212	− 7·2061	− 1·9143	− 5	− 6	− 2	−17	− 1·9173
IV.	+ 6·3800	+ 4·4609	+16	0	+ 5	− 4	+ 4·4626	− 3·8036	+ 1·4882	+ 4	+ 5	+ 2	+13	+ 1·4906

7007. August 4.

J.	+ 2·5082							+ 9·2689						
II.	+ 1·2380	− 1·2702	− 4	0	− 4	+ 5	− 1·2705	+ 8·8618	− 0·4071	+ 1	− 1	− 1	−15	− 0·4087
III.	+ 1·9688	− 0·5394	− 2	0	− 1	+ 2	− 0·5395	+ 9·0917	− 0·1772	0	− 1	0	− 6	− 0·1779
IV.	− 1·0831	− 3·5913	−12	0	−10	+14	− 3·5921	+ 8·0670	− 1·2019	+ 3	− 4	− 3	−42	− 1·2065
J.	+ 2·0051							− 4·5271						
II.	+ 0·8115	− 1·1936	− 4	0	− 3	+ 4	− 1·1939	− 4·9086	− 0·3815	+ 1	− 1	− 1	−14	− 0·3830
III.	+ 1·3694	− 0·6357	− 2	0	− 2	+ 2	− 0·6359	− 4·7406	− 0·2135	+ 1	− 1	− 1	− 7	− 0·2143
IV.	− 1·5330	− 3·5381	−13	0	−10	+14	− 3·5390	− 5·7093	− 1·1822	+ 4	− 4	− 3	−41	− 1·1866

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TABLE VIII.—*continued.*

7010. August 6.

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sx.	Py.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sy.	−Px.	Observed Ordinate.
J.	+ 1′·9942							+ 8′·4725						
I.	+ 0·9043	− 1′·0899	− 4	0	− 2	+ 2	− 1′·0903	+ 8·1126	− 0′·3599	− 1	− 1	− 1	− 7	− 0′·3609
II.	+ 2·9562	+ 0·9620	+ 3	0	+ 2	− 2	+ 0·9623	+ 8·7705	+ 0·2980	+ 1	+ 1	+ 1	+ 6	+ 0·2989
III.	+ 6·6589	+ 4·6647	+16	0	+ 9	−10	+ 4·6662	+10·0162	+ 1·5437	+ 4	+ 5	+ 3	+30	+ 1·5479
IV.	− 4·3480	− 6·3422	−22	0	−13	+13	− 6·3444	+ 6·4481	− 2·0244	− 6	− 6	− 4	−41	− 2·0301
J.	+ 1·9669							− 5·2770						
I.	+ 0·7965	− 1·1704	− 4	0	− 2	+ 2	− 1·1708	− 5·6650	− 0·3880	− 1	− 1	− 1	− 8	− 0·3891
II.	+ 2·8564	+ 0·8895	+ 3	0	+ 2	− 2	+ 0·8898	− 5·0021	+ 0·2749	+ 1	+ 1	+ 1	+ 6	+ 0·2758
III.	+ 6·6686	+ 4·7017	+16	0	+10	−10	+ 4·7033	− 3·7181	+ 1·5589	+ 4	+ 5	+ 3	+30	+ 1·5631
IV.	− 4·4062	− 6·3731	−21	0	−13	+13	− 6·3752	− 7·3078	− 2·0308	− 5	− 7	− 4	−41	− 2·0365

7013. August 7.

J.	+ 2·0890							+ 8·4777						
I.	+ 3·9730	+ 1·8840	+ 6	0	+ 4	− 5	+ 1·8845	+ 9·1023	+ 0·6246	+ 1	+ 2	+ 1	+16	+ 0·6266
II.	− 1·4877	− 3·5767	−11	0	− 8	+10	− 3·5776	+ 7·2787	− 1·1990	− 2	− 4	− 3	−31	− 1·2030
III.	+ 7·6088	+ 5·5198	+17	0	+12	−16	+ 5·5211	+10·3092	+ 1·8315	+ 3	+ 6	+ 4	+48	+ 1·8376
IV.	− 6·7689	− 8·8579	−28	0	−19	+25	− 8·8601	+ 5·6378	− 2·8399	− 5	− 9	− 6	−77	− 2·8496
J.	+ 1·9677							− 5·2527						
I.	+ 3·9036	+ 1·9359	+ 6	0	+ 4	− 6	+ 1·9363	− 4·6152	+ 0·6375	+ 1	+ 2	+ 1	+17	+ 0·6396
II.	− 1·6042	− 3·5719	−11	0	− 8	+10	− 3·5728	− 6·4520	− 1·1993	− 2	− 4	− 3	−31	− 1·2033
III.	+ 7·4822	+ 5·5145	+17	0	+12	−16	+ 5·5158	− 3·4230	+ 1·8297	+ 3	+ 6	+ 4	+48	+ 1·8358
IV.	− 6·9024	− 8·8701	−27	0	−19	+25	− 8·8722	− 8·0970	− 2·8443	− 4	− 9	− 6	−77	− 2·8539

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TABLE VIII.—*continued.*

7017. August 17.

(Réseau very faint: moonlight: images diffused.)

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sx.	Py.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sy.	−Px.	Observed Ordinate.
J.	+ 1′·9870							+ 8′·4670						
I.	− 0·2615	− 2′·2485	− 9	0	− 4	+ 3	− 2′·2495	+ 7·7405	− 0′·7265	− 3	− 2	− 1	− 8	− 0′·7279
II.	+ 1·0055	− 0·9815	− 4	0	− 2	+ 1	− 0·9820	+ 8·1265	− 0·3405	− 1	− 1	− 1	− 4	− 0·3412
III.	− 3·2950	− 5·2820	−21	0	− 9	+ 6	− 5·2844	+ 6·7614	− 1·7056	− 6	− 6	− 3	−19	− 1·7090
IV.	+12·0618	+10·0748	+39	0	+17	−12	+10·0792	+11·6585	+ 3·1915	+12	+10	+ 6	+37	+ 3·1980
J.	+ 2·0127							− 5·2500						
I.	− 0·2173	− 2·2300	− 8	0	− 4	+ 3	− 2·2309	− 5·9713	− 0·7213	− 2	− 2	− 1	− 8	− 0·7226
II.	+ 0·9608	− 1·0519	− 4	0	− 2	+ 1	− 1·0524	− 5·6153	− 0·3653	− 1	− 1	− 1	− 4	− 0·3660
III.	− 3·2872	− 5·2999	−20	0	− 9	+ 6	− 5·3022	− 6·9664	− 1·7164	− 6	− 6	− 3	−19	− 1·7198
IV.	+12·0872	+10·0745	+37	0	+17	−12	+10·0787	− 2·0619	+ 3·1881	+11	+10	+ 6	+37	+ 3·1945

7018. August 18.

(Réseau extremely faint: moonlight: images diffused.)

J.	+ 2·0337							+ 8·4360						
I.	+ 3·6122	+ 1·5785	+ 5	0	+ 5	− 4	+ 1·5791	+ 8·9437	+ 0·5077	0	+ 2	+ 2	+11	+ 0·5092
II.	− 0·8518	− 2·8855	− 9	0	−10	+ 7	− 2·8867	+ 7·5036	− 0·9324	− 1	− 3	− 3	−21	− 0·9352
III.	− 2·9004	− 4·9341	−15	0	−16	+11	− 4·9361	+ 6·8350	− 1·6010	− 1	− 5	− 5	−35	− 1·6056
IV.	+10·8327	+ 8·7990	+26	0	+29	−20	+ 8·8025	+11·2237	+ 2·7877	+ 2	+ 9	+ 9	+63	+ 2·7960
J.	+ 1·9519							− 5·2828						
I.	+ 3·4538	+ 1·5019	+ 4	0	+ 5	− 3	+ 1·5025	− 4·8005	+ 0·4823	0	+ 2	+ 2	+11	+ 0·4838
II.	− 0·8925	− 2·8444	− 8	0	− 9	+ 7	− 2·8454	− 6·2074	− 0·9246	0	− 3	− 3	−20	− 0·9272
III.	− 2·9614	− 4·9133	−14	0	−16	+11	− 4·9152	− 6·8798	− 1·5970	− 1	− 5	− 5	−35	− 1·6016
IV.	+10·7188	+ 8·7669	+26	0	+29	−20	+ 8·7704	− 2·5062	+ 2·7766	+ 1	+ 9	+ 9	+63	+ 2·7848

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TABLE VIII.—*continued.*

7019. August 19.

(Very bad images and réseau: moonlight.)

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	$Sx.$	$Py.$	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	$Sy.$	$-Px.$	Observed Ordinate.
J.	+ 2·0681							+ 8·4669						
I.	+ 1·0337	− 1·0344	− 3	0	− 2	+ 3	− 1·0346	+ 8·1162	− 0·3507	− 1	− 1	− 1	− 9	− 0·3519
II.	+ 4·6231	+ 2·5550	+ 8	0	+ 5	− 8	+ 2·5555	+ 9·3053	+ 0·8384	+ 1	+ 3	+ 2	+23	+ 0·8413
IV.	+ 8·4784	+ 6·4103	+21	0	+12	−19	+ 6·4117	+10·5021	+ 2·0352	+ 4	+ 7	+ 4	+59	+ 2·0426
J.	+ 2·0876							− 5·2993						
I.	+ 1·1084	− 0·9792	− 3	0	− 2	+ 3	− 0·9794	− 5·6230	− 0·3237	0	− 1	− 1	− 9	− 0·3248
II.	+ 4·6872	+ 2·5996	+ 8	0	+ 5	− 8	+ 2·6001	− 4·4438	+ 0·8555	+ 1	+ 3	+ 2	+24	+ 0·8585
IV.	+ 8·4618	+ 6·3742	+20	0	+12	−19	+ 6·3755	− 3·2647	+ 2·0346	+ 3	+ 7	+ 4	+58	+ 2·0418

7020. August 21.

IV. just emerging from planet.

J.	+ 1·9738							+ 8·4373						
II.	− 1·1077	− 3·0815	−13	0	− 8	+ 7	− 3·0829	+ 7·4290	− 1·0083	− 4	− 3	− 2	−20	− 1·0112
III.	+ 7·6936	+ 5·7198	+23	0	+14	−12	+ 5·7223	+10·2700	+ 1·8327	+ 8	+ 6	+ 5	+37	+ 1·8383
IV.	+ 1·4774	− 0·4964	− 2	0	− 1	+ 1	− 0·4966	+ 8·3332	− 0·1041	− 1	0	0	− 3	− 0·1045
J.	+ 1·9533							− 5·3213						
II.	− 1·1671	− 3·1204	−12	0	− 8	+ 7	− 3·1217	− 6·3466	− 1·0253	− 3	− 3	− 3	−20	− 1·0282
III.	+ 7·6727	+ 5·7194	+21	0	+14	−12	+ 5·7217	− 3·4850	+ 1·8363	+ 6	+ 6	+ 5	+37	+ 1·8417
IV.	+ 1·4295	− 0·5238	− 2	0	− 1	+ 1	− 0·5240	− 5·4413	− 0·1200	− 1	0	0	− 3	− 0·1204

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TABLE VIII.—*continued.*

7023. August 24.

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	*Sx.*	*Py.*	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	*Sy.*	*−Px.*	Observed Ordinate.
J.	+ 2·0140							+ 9·3748						
I.	− 0·1767	− 2·1907	− 9	0	− 5	+ 3	− 2·1918	+ 8·6750	− 0·6998	− 3	− 2	− 1	− 9	− 0·7013
III.	− 2·8409	− 4·8549	−20	0	−10	+ 7	− 4·8572	+ 7·8337	− 1·5411	− 8	− 5	− 3	−21	− 1·5448
IV.	− 7·2576	− 9·2716	−38	0	−19	+12	− 9·2761	+ 6·5155	− 2·8593	−15	− 9	− 6	−40	− 2·8663
J.	+ 2·0011							− 6·3892						
I.	− 0·2094	− 2·2105	− 8	0	− 5	+ 3	− 2·2115	− 7·0927	− 0·7035	− 3	− 2	− 1	−10	− 0·7051
III.	− 2·8843	− 4·8854	−18	0	−10	+ 7	− 4·8875	− 7·9419	− 1·5527	− 6	− 5	− 3	−21	− 1·5562
IV.	− 7·2889	− 9·2900	−35	0	−19	+12	− 9·2942	− 9·2558	− 2·8666	−11	− 9	− 6	−40	− 2·8732

7024. August 25.

(Images and réseau good.)

J.	+ 2·0096							+ 8·6172						
I.	+ 4·2474	+ 2·2378	+15	+ 1	+ 6	− 3	+ 2·2397	+ 9·3260	+ 0·7088	+ 8	+ 3	+ 2	+11	+ 0·7112
II.	− 1·4764	− 3·4860	−24	− 1	− 9	+ 5	− 3·4889	+ 7·4988	− 1·1184	−12	− 5	− 3	−17	− 1·1221
III.	− 3·4842	− 5·4938	−38	− 2	−14	+ 9	− 5·4983	+ 6·8701	− 1·7471	−19	− 7	− 4	−27	− 1·7528
IV.	− 8·0050	−10·0146	−69	− 3	−25	+15	−10·0228	+ 5·5196	− 3·0976	−34	−13	− 8	−49	− 3·1080
J.	+ 2·0076							− 5·1103						
I.	+ 4·2505	+ 2·2429	+13	0	+ 6	− 3	+ 2·2445	− 4·4015	+ 0·7088	+ 6	+ 3	+ 1	+11	+ 0·7109
II.	− 1·4600	− 3·4675	−21	− 1	− 9	+ 5	− 3·4701	− 6·2245	− 1·1142	−10	− 4	− 3	−17	− 1·1176
III.	− 3·4658	− 5·4734	−33	− 1	−14	+ 9	− 5·4773	− 6·8552	− 1·7449	−15	− 7	− 4	−27	− 1·7502
IV.	− 8·0062	−10·0138	−60	− 2	−25	+15	−10·0210	− 8·2098	− 3·0995	−27	−12	− 8	−49	− 3·1091

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TABLE VIII.—*continued.*

7025. August 27.

(Images and réseau good.)

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sx.	Py.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sy.	−Px.	Observed Ordinate.
J.	+ 2·0536							+ 8·5809						
I.	+ 3·6076	+ 1·5540	+10	0	+ 3	− 2	+ 1·5551	+ 9·0721	+ 0·4912	+ 5	+ 2	+ 1	+ 7	+ 0·4927
II.	+ 5·0810	+ 3·0274	+20	+ 1	+ 5	− 4	+ 3·0296	+ 9·5455	+ 0·9646	+ 9	+ 4	+ 2	+13	+ 0·9674
III.	+ 4·5925	+ 2·5389	+16	+ 1	+ 5	− 3	+ 2·5408	+ 9·3702	+ 0·7893	+ 8	+ 3	+ 1	+11	+ 0·7916
IV.	− 5·3965	− 7·4501	−48	− 2	−13	+10	− 7·4554	+ 6·2507	− 2·3302	−23	− 9	− 4	−33	− 2·3371
J.	+ 1·9981							− 5·1698						
I.	+ 3·4839	+ 1·4858	+ 8	0	+ 3	− 2	+ 1·4867	− 4·7000	+ 0·4698	+ 4	+ 2	+ 1	+ 7.	+ 0·4712
II.	+ 4·9930	+ 2·9949	+17	+ 1	+ 5	− 4	+ 2·9968	− 4·2224	+ 0·9474	+ 7	+ 4	+ 2	+13	+ 0·9500
III.	+ 4·5929	+ 2·5948	+15	0	+ 5	− 3	+ 2·5965	− 4·3615	+ 0·8083	+ 6	+ 3	+ 1	+11	+ 0·8104
IV.	− 5·4244	− 7·4225	−42	− 1	−13	+10	− 7·4271	− 7·4888	− 2·3190	−18	− 9	− 4	−33	− 2·3254

7028. August 28.

(Images and réseau very good.)

J.	+ 2·0475							+ 8·4891						
I.	+ 1·2963	− 0·7512	− 4	0	− 1	+ 1	− 0·7516	+ 8·2493	− 0·2398	− 2	− 1	0	− 5	− 0·2406
II.	− 0·4046	− 2·4521	−14	− 1	− 4	+ 5	− 2·4535	+ 7·6891	− 0·8000	− 6	− 3	− 1	−15	− 0·8025
III.	+ 7·5890	+ 5·5415	+32	+ 1	+ 9	−11	+ 5·5446	+10·2328	+ 1·7437	+14	+ 7	+ 3	+34	+ 1·7495
IV.	− 2·3694	− 4·4169	−25	− 1	− 7	+ 9	− 4·4193	+ 7·0870	− 1·4021	−11	− 5	− 2	−27	− 1·4066
J.	+ 2·1337							− 5·2433						
I.	+ 1·4741	− 0·6596	− 3	0	− 1	+ 1	− 0·6599	− 5·4564	− 0·2131	− 1	− 1	0	− 4	− 0·2137
II.	− 0·3745	− 2·5082	−13	0	− 4	+ 5	− 2·5094	− 6·0646	− 0·8213	− 5	− 3	− 1	−15	− 0·8237
III.	+ 7·6833	+ 5·5496	+28	+ 1	+ 9	−11	+ 5·5523	− 3·4969	+ 1·7464	+12	+ 6	+ 3	+34	+ 1·7519
IV.	− 2·2405	− 4·3742	−22	0	− 7	+ 9	− 4·3762	− 6·6342	− 1·3909	− 9	− 5	− 2	−27	− 1·3952

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TABLE VIII.—*continued.*

7029. August 31.

(Good plate.)

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	S*x.*	P*y.*	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	S*y.*	−P*x.*	Observed Ordinate.
J.	+ 2·0343							+ 8·4508						
I.	+ 0·0530	− 1·9813	− 7	0	− 9	+ 4	− 1·9825	+ 7·8335	− 0·6173	− 2	− 2	− 3	−13	− 0·6193
III.	− 2·3825	− 4·4168	−16	0	−20	+ 9	− 4·4195	+ 7·0778	− 1·3730	− 4	− 4	− 6	−28	− 1·3772
IV.	+ 8·4694	+ 6·4351	+23	0	+30	−12	+ 6·4392	+10·3825	+ 1·9317	+ 6	+ 6	+ 9	+41	+ 1·9379
J.	+ 1·9083							− 5·2835						
I.	− 0·1094	− 2·0177	− 7	0	− 9	+ 4	− 2·0189	− 5·9192	− 0·6357	− 1	− 2	− 3	−13	− 0·6376
III.	− 2·5448	− 4·4531	−15	0	−20	+ 9	− 4·4557	− 6·6761	− 1·3926	− 3	− 4	− 6	−28	− 1·3967
IV.	+ 8·3819	+ 6·4736	+22	0	+30	−12	+ 6·4776	− 3·3440	+ 1·9395	+ 5	+ 6	+ 9	+41	+ 1·9456

7030. September 1.

(Fair plate.)

J.	+ 2·0674							+ 8·4332						
I.	+ 4·2920	+ 2·2246	+ 7	0	+ 4	− 4	+ 2·2253	+ 9·1276	+ 0·6944	+ 2	+ 1	+ 1	+13	+ 0·6961
II.	− 1·3914	− 3·4588	−11	0	− 7	+ 7	− 3·4599	+ 7·3347	− 1·0985	− 3	− 2	− 2	−21	− 1·1013
III.	− 3·4385	− 5·5059	−18	0	−11	+10	− 5·5078	+ 6·7016	− 1·7316	− 5	− 3	− 3	−33	− 1·7360
IV.	+10·8998	+ 8·8324	+28	0	+17	−16	+ 8·8353	+11·1038	+ 2·6706	+ 8	+ 5	+ 5	+53	+ 2·6777
J.	+ 2·0102							− 5·2926						
I.	+ 4·2340	+ 2·2238	+ 7	0	+ 4	− 4	+ 2·2245	− 4·5977	+ 0·6949	+ 2	+ 1	+ 1	+13	+ 0·6966
II.	− 1·4336	− 3·4438	−11	0	− 7	+ 6	− 3·4450	− 6·3841	− 1·0915	− 3	− 2	− 2	−21	− 1·0943
III.	− 3·4814	− 5·4916	−17	0	−11	+10	− 5·4934	− 7·0135	− 1·7209	− 5	− 3	− 3	−33	− 1·7253
IV.	+10·8637	+ 8·8535	+28	0	+17	−16	+ 8·8564	− 2·6130	+ 2·6796	+ 8	+ 4	+ 5	+53	+ 2·6866

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TABLE VIII.—*continued.*

7031. September 2.

(Images very poor and diffused: réseau very faint.)

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sx.	Py.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sy.	−Px.	Observed Ordinate.
J.	+ 2·1506							+ 8·4870						
I.	− 0·0322	− 2·1828	−10	0	− 4	+ 3	− 2·1839	+ 7·8159	− 0·6711	− 4	− 2	− 1	−10	− 0·6728
II.	+ 3·0402	+ 0·8896	+ 4	0	+ 2	− 1	+ 0·8901	+ 8·7928	+ 0·3058	+ 2	+ 1	+ 1	+ 4	+ 0·3066
III.	− 0·6685	− 2·8191	−13	0	− 6	+ 4	− 2·8206	+ 7·6036	− 0·8834	− 5	− 3	− 2	−13	− 0·8857
IV.	+12·0356	+ 9·8850	+45	0	+20	−14	+ 9·8901	+11·4990	+ 3·0120	+17	+11	+ 6	+46	+ 3·0200
J.	+ 2·0850							− 5·2538						
I.	− 0·0841	− 2·1691	− 9	0	− 4	+ 3	− 2·1701	− 5·9254	− 0·6716	− 3	− 2	− 1	−10	− 0·6732
II.	+ 3·0447	+ 0·9597	+ 4	0	+ 2	− 1	+ 0·9602	− 4·9339	+ 0·3199	+ 1	+ 1	+ 1	+ 4	+ 0·3206
III.	− 0·6836	− 2·7686	−11	0	− 6	+ 4	− 2·7699	− 6·1320	− 0·8782	− 4	− 3	− 2	−13	− 0·8804
IV.	+11·9722	+ 9·8872	+40	0	+20	−14	+ 9·8918	− 2·2502	+ 3·0036	+14	+10	+ 6	+46	+ 3·0112

7033. September 3.

(Poor plate: réseau faint.)

J.	+ 2·1008							+ 8·4340						
I.	+ 5·3767	+ 3·2759	+17	0	+ 4	−12	+ 3·2768	+ 9·4611	+ 1·0271	+ 7	+ 4	+ 1	+37	+ 1·0320
IV.	+11·7750	+ 9·6742	+49	+ 1	+13	−34	+ 9·6771	+11·3771	+ 2·9431	+21	+10	+ 4	+110	+ 2·9576
J.	+ 2·0785							− 5·2958						
I.	+ 5·3254	+ 3·2469	+15	0	+ 4	−12	+ 3·2476	− 4·2778	+ 1·0180	+ 6	+ 4	+ 1	+37	+ 1·0228
IV.	+11·7433	+ 9·6648	+44	+ 1	+13	−34	+ 9·6672	− 2·3547	+ 2·9411	+17	+10	+ 4	+110	+ 2·9552

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TABLE VIII.—*continued.*

7034. September 4.

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sx.	Py.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sy.	−Px.	Observed Ordinate.
J.	+ 2·0944							+ 8·4684						
I.	+ 0·8092	− 1·2852	− 6	0	0	+ 2	− 1·2856	+ 8·0676	− 0·4008	− 3	− 1	0	− 6	− 0·4018
II.	+ 0·1486	− 1·9458	−10	0	− 1	+ 3	− 1·9466	+ 7·8425	− 0·6259	− 4	− 2	0	− 9	− 0·6274
III.	+ 7·3722	+ 5·2778	+26	+ 1	+ 2	− 7	+ 5·2800	+10·1096	+ 1·6412	+10	+ 6	+ 1	+23	+ 1·6452
IV.	+10·2238	+ 8·1294	+40	+ 1	+ 3	−11	+ 8·1327	+10·9656	+ 2·4972	+16	+ 9	+ 1	+36	+ 2·5034
J.	+ 1·9546							− 5·2954						
I.	+ 0·7464	− 1·2082	− 5	0	0	+ 2	− 1·2085	− 5·6728	− 0·3774	− 2	− 1	0	− 5	− 0·3782
II.	− 0·0494	− 2·0040	− 9	0	− 1	+ 3	− 2·0047	− 5·9381	− 0·6427	− 3	− 2	0	− 9	− 0·6441
III.	+ 7·2521	+ 5·2975	+23	0	+ 2	− 7	+ 5·2993	− 3·6479	+ 1·6475	+ 9	+ 6	+ 1	+23	+ 1·6514
IV.	+10·0576	+ 8·1030	+36	+ 1	+ 3	−11	+ 8·1059	− 2·8055	+ 2·4899	+13	+ 8	+ 1	+36	+ 2·4957

7036. September 5.

J.	+ 2·1220							+ 8·4887						
II.	− 0·0498	− 2·1718	− 8	0	− 3	+ 7	− 2·1722	+ 7·8164	− 0·6723	− 2	− 2	− 1	−23	− 0·6751
III.	+ 6·8355	+ 4·7135	+16	0	+ 8	−15	+ 4·7144	+ 9·9572	+ 1·4685	+ 4	+ 5	+ 2	+49	+ 1·4745
IV.	+ 7·4409	+ 5·3189	+18	0	+ 9	−17	+ 5·3199	+10·1342	+ 1·6455	+ 4	+ 6	+ 3	+55	+ 1·6523
J.	+ 2·1103							− 5·2706						
II.	− 0·0048	− 2·1151	− 7	0	− 3	+ 7	− 2·1154	− 5·9234	− 0·6528	− 2	− 2	− 1	−22	− 0·6555
III.	+ 6·7952	+ 4·6849	+16	0	+ 7	−15	+ 4·6857	− 3·8049	+ 1·4657	+ 4	+ 5	+ 2	+49	+ 1·4717
IV.	+ 7·3925	+ 5·2822	+18	0	+ 8	−17	+ 5·2831	− 3·6318	+ 1·6388	+ 4	+ 5	+ 3	+55	+ 1·6455

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TABLE VIII.—*continued.*

7039. September 6.

(Good plate.)

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sx.	Py.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sy.	−Px.	Observed Ordinate.
J.	+ 2·0767							+ 8·4858						
II.	+ 5·0936	+ 3·0169	+14	0	+ 7	− 5	+ 3·0185	+ 9·4404	+ 0·9546	+ 5	+ 3	+ 2	+17	+ 0·9573
III.	+ 2·9442	+ 0·8675	+ 4	0	+ 2	− 2	+ 0·8679	+ 8·7669	+ 0·2811	+ 1	+ 1	+ 1	+ 5	+ 0·2819
IV.	+ 4·0994	+ 2·0227	+ 9	0	+ 5	− 4	+ 2·0237	+ 9·1440	+ 0·6582	+ 3	+ 2	+ 2	+12	+ 0·6601
J.	+ 2·0964							− 5·2459						
II.	+ 5·1495	+ 3·0531	+12	0	+ 7	− 6	+ 3·0544	− 4·2803	+ 0·9656	+ 4	+ 3	+ 2	+17	+ 0·9682
III.	+ 2·9062	+ 0·8098	+ 3	0	+ 2	− 2	+ 0·8101	− 4·9789	+ 0·2670	+ 1	+ 1	+ 1	+ 5	+ 0·2678
IV.	+ 4·0771	+ 1·9807	+ 8	0	+ 5	− 4	+ 1·9816	− 4·5986	+ 0·6473	+ 3	+ 2	+ 2	+11	+ 0·6491

7043. September 7.

J.	+ 2·0843							+ 8·4852						
I.	+ 0·7175	− 1·3668	− 6	0	− 2	+ 2	− 1·3674	+ 8·0674	− 0·4178	− 2	− 1	− 1	− 7	− 0·4189
II.	+ 3·3235	+ 1·2392	+ 5	0	+ 2	− 2	+ 1·2397	+ 8·8610	+ 0·3758	+ 2	+ 1	+ 1	+ 6	+ 0·3768
III.	− 1·6512	− 3·7355	−15	0	− 6	+ 6	− 3·7370	+ 7·3439	− 1·1413	− 5	− 4	− 2	−19	− 1·1443
IV.	+ 0·3782	− 1·7061	− 7	0	− 3	+ 2	− 1·7069	+ 8·0243	− 0·4609	− 2	− 2	− 1	− 9	− 0·4623
J.	+ 2·1382							− 5·2695						
I.	+ 0·6941	− 1·4441	− 5	0	− 2	+ 2	− 1·4446	− 5·7162	− 0·4467	− 2	− 1	− 1	− 7	− 0·4478
II.	+ 3·3058	+ 1·1676	+ 4	0	+ 2	− 2	+ 1·1680	− 4·9200	+ 0·3495	+ 1	+ 1	+ 1	+ 6	+ 0·3504
III.	− 1·6409	− 3·7791	−14	0	− 6	+ 6	− 3·7805	− 6·4286	− 1·1591	− 4	− 4	− 2	−19	− 1·1620
IV.	+ 0·3875	− 1·7507	− 7	0	− 3	+ 2	− 1·7515	− 5·7477	− 0·4782	− 2	− 2	− 1	− 9	− 0·4796

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TABLE VIII.—*continued.*

7045. September 8.

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sx.	Py.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	Sy.	−Px.	Observed Ordinate.
J.	+ 2'·0707							+ 8'·5063						
I.	+ 4·0464	+ 1'·9757	+ 7	0	+ 6	− 4	+ 1'·9766	+ 9·1213	+ 0'·6150	+ 2	+ 2	+ 2	+12	+ 0'·6168
II.	− 1·3555	− 3·4262	−12	0	−10	+ 6	− 3·4278	+ 7·4420	− 1·0643	− 4	− 3	− 3	−21	− 1·0674
III.	− 3·5002	− 5·5709	−20	0	−16	+10	− 5·5735	+ 6·7938	− 1·7125	− 6	− 5	− 5	−34	− 1·7175
IV.	− 3·1132	− 5·1839	−19	0	−15	+ 9	− 5·1864	+ 7·0009	− 1·5054	− 5	− 5	− 4	−32	− 1·5100
J.	+ 1·9975							− 5·2555						
I.	+ 4·0110	+ 2·0135	+ 7	0	+ 6	− 4	+ 2·0144	− 4·6388	+ 0·6167	+ 2	+ 2	+ 2	+12	+ 0·6185
II.	− 1·4324	− 3·4299	−12	0	−10	+ 7	− 3·4314	− 6·3338	− 1·0783	− 3	− 3	− 3	−21	− 1·0813
III.	− 3·5693	− 5·5668	−19	0	−16	+10	− 5·5693	− 6·9788	− 1·7233	− 5	− 5	− 5	−34	− 1·7282
IV.	− 3·2178	− 5·2153	−18	0	−15	+ 9	− 5·2177	− 6·7808	− 1·5253	− 5	− 5	− 4	−32	− 1·5299

7046. September 10.

J.	+ 2·0702							+ 8·5052						
I.	+ 4·1536	+ 2·0834	+ 8	0	+ 4	− 1	+ 2·0845	+ 9·1449	+ 0·6397	+ 3	+ 2	+ 1	+ 2	+ 0·6405
II.	+ 5·4780	+ 3·4078	+13	0	+ 7	− 1	+ 3·4097	+ 9·5640	+ 1·0588	+ 4	+ 4	+ 2	+ 3	+ 1·0601
III.	+ 3·3067	+ 1·2365	+ 5	0	+ 2	0	+ 1·2372	+ 8·8665	+ 0·3613	+ 2	+ 1	+ 1	+ 1	+ 0·3618
IV.	− 7·3317	− 9·4019	−37	0	−18	+ 3	− 9·4071	+ 5·6907	− 2·8145	−12	−10	− 5	− 9	− 2·8181
J.	+ 2·0564							− 5·2438						
I.	+ 4·1073	+ 2·0509	+ 8	0	+ 4	− 1	+ 2·0520	− 4·6131	+ 0·6307	+ 2	+ 2	+ 1	+ 2	+ 0·6314
II.	+ 5·4420	+ 3·3856	+13	0	+ 7	− 1	+ 3·3875	− 4·1896	+ 1·0542	+ 4	+ 3	+ 2	+ 3	+ 1·0554
III.	+ 3·3437	+ 1·2873	+ 5	0	+ 2	0	+ 1·2880	− 4·8650	+ 0·3788	+ 1	+ 1	+ 1	+ 1	+ 0·3792
IV.	− 7·3687	− 9·4251	−37	0	−18	+ 3	− 9·4303	− 8·0654	− 2·8216	−11	− 9	− 5	− 9	− 2·8250

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TABLE VIII.—*continued.*

7047. September 12.

(Images fair: réseau faint.)

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	S*x.*	P*y.*	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	S*y.*	−P*x.*	Observed Ordinate.
J.	+ 2′1014							+ 8′5092						
II.	− 0·3401	− 2′4415	− 7	0	− 3	+ 5	− 2′4420	+ 7·7535	− 0′7557	− 1	− 2	− 1	−17	− 0′7578
III.	+ 7·0289	+ 4·9275	+14	0	+ 6	−10	+ 4·9285	+10·0229	+ 1·5137	+ 1	+ 5	+ 2	+34	+ 1·5179
IV.	− 6·3135	− 8·4149	−24	0	−11	+17	− 8·4167	+ 5·9611	− 2·5481	− 3	− 8	− 3	−58	− 2·5553
J.	+ 2·0515							− 5·2940						
II.	− 0·3406	− 2·3921	− 7	0	− 3	+ 5	− 2·3926	− 6·0326	− 0·7386	0	− 2	− 1	−16	− 0·7405
III.	+ 6·9509	+ 4·8994	+14	0	+ 6	−10	+ 4·9004	− 3·7801	+ 1·5139	+ 1	+ 5	+ 2	+33	+ 1·5180
IV.	− 6·3467	− 8·3982	−24	0	−11	+17	− 8·4000	− 7·8326	− 2·5386	− 2	− 8	− 3	−57	− 2·5456

7048. September 13.

(Good plate.)

J.	+ 2·0185							+ 8·5188						
II.	+ 4·7952	+ 2·7767	+ 9	0	+ 7	− 5	+ 2·7778	+ 9·3900	+ 0·8712	+ 2	+ 3	+ 2	+15	+ 0·8734
III.	+ 3·3753	+ 1·3568	+ 4	0	+ 3	− 2	+ 1·3573	+ 8·9511	+ 0·4323	+ 1	+ 1	+ 1	+ 7	+ 0·4333
IV.	− 4·0843	− 6·1028	−19	0	−15	+10	− 6·1052	+ 6·6541	− 1·8647	− 4	− 6	− 5	−32	− 1·8694
J.	+ 1·9528							− 5·2266						
II.	+ 4·7689	+ 2·8161	+ 9	0	+ 7	− 5	+ 2·8172	− 4·3427	+ 0·8839	+ 2	+ 3	+ 2	+15	+ 0·8861
III.	+ 3·2578	+ 1·3050	+ 4	0	+ 3	− 2	+ 1·3055	− 4·8089	+ 0·4177	+ 1	+ 1	+ 1	+ 7	+ 0·4187
IV.	− 4·1225	− 6·0753	−19	0	−15	+10	− 6·0777	− 7·0818	− 1·8552	− 3	− 6	− 4	−32	− 1·8597

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TABLE VIII.—*continued.*

7055. September 14.

(Images diffused : réseau faint : very poor plate.)

	Measured Abscissa.	Abscissa referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	S*x*.	P*y*.	Observed Abscissa.	Measured Ordinate.	Ordinate referred to centre of Jupiter.	Refraction. *ax.*	Refraction. *by.*	S*y*.	−P*x*.	Observed Ordinate.
J.	+ 2·0520							+ 8·4824						
I.	+ 1·0478	− 1·0042	− 3	0	− 2	+ 2	− 1·0045	+ 8·1784	− 0·3040	− 1	− 1	− 1	− 7	− 0·3050
II.	+ 3·6990	+ 1·6470	+ 5	0	+ 3	− 3	+ 1·6475	+ 8·9784	+ 0·4960	+ 1	+ 2	+ 1	+11	+ 0·4975
III.	− 1·1597	− 3·2117	−10	0	− 6	+ 6	− 3·2127	+ 7·5110	− 0·9714	− 2	− 3	− 2	−21	− 0·9742
IV.	− 0·8339	− 2·8859	− 9	0	− 6	+ 6	− 2·8868	+ 7·5679	− 0·9145	− 2	− 3	− 2	−19	− 0·9171
J.	+ 1·9022							− 5·3056						
I.	+ 0·8138	− 1·0884	− 3	0	− 2	+ 2	− 1·0887	− 5·6364	− 0·3308	− 1	− 1	− 1	− 7	− 0·3318
II.	+ 3·4812	+ 1·5790	+ 5	0	+ 3	− 3	+ 1·5795	− 4·8283	+ 0·4773	+ 1	+ 2	+ 1	+10	+ 0·4787
III.	− 1·3636	− 3·2658	−10	0	− 6	+ 6	− 3·2668	− 6·2913	− 0·9857	− 2	− 3	− 2	−21	− 0·9885
IV.	− 0·9492	− 2·8514	− 9	0	− 6	+ 6	− 2·8523	− 6·2068	− 0·9012	− 2	− 3	− 2	−19	− 0·9038

The co-ordinates in the foregoing table are the co-ordinates referred to axes parallel and perpendicular to the earth's mean equator of 1902·0; but the computation of all the tabular quantities and the equations of condition are so much simplified by taking Jupiter's equator instead of the earth's as plane of reference, that the axes of reference for the observed co-ordinates were rotated so as to be parallel and perpendicular to Jupiter's equator. The position-angle of the north end of Jupiter's polar axis is the angle P which is given by the ephemeris, and the angle of rotation, θ, is 360° − P. Hence referred to Jupiter's equator, the co-ordinates are—

$$\xi = x \cos P - y \sin P$$
$$\eta = x \sin P + y \cos P.$$

For the sake of convenience the values of P are given here.

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TABLE IX.

VALUES OF P FOR MEAN EPOCH OF EXPOSURES, NOT ANTEDATED BY ABERRATION TIME.

July	8	340°	41′	42″	August	6	341°	43′	20″	September 2	342° 44′ 24″
	14		52	15		7	341	45	49	3	46 10
	15		54	6		17	342	9	46	4	47 59
	16	340	56	2		18		12	17	5	49 49
	19	341	2	8		19		14	33	6	51 34
	20		4	12		21		19	3	7	53 12
	24		12	59		24		25	48	8	54 52
	26		17	19		25		27	52	10	342 57 57
	27		19	33		27		32	11	12	343 0 56
	28		21	55		28		34	19	13	2 16
August	3		36	3		31		40	31	14	343 3 34
	4	341	38	51	September 1		342	42	33		

The observed co-ordinates of the satellites were in this way referred to axes through Jupiter's centre parallel and perpendicular to his equator. These co-ordinates are given in Table XII.

It should be stated that it is unnecessary to apply to P any correction to reduce from the sphere on to the plate, because the mean position of the centre of Jupiter, at which point P is reckoned, is never more than 3′ from the centre of the plate.

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CHAPTER III.

Tabular Co-ordinates.

The observed co-ordinates were compared with co-ordinates calculated from Souillart's theory. An ephemeris had already been constructed for the reduction of the heliometer observations, and the reader is referred to Chapter III. of Vol. XII. Part II. of these Annals for a full account of it. The quantities which are given by the ephemeris and by Marth's Tables of the Inequalities (*Monthly Notices, R.A.S.*, Vol. 51) are the mean and the true longitudes, the radius vectors and all the inequalities, the inclinations and nodes of the orbits; also P the position-angle of the northern end of Jupiter's axis, and L−O + 180° the Jovicentric longitude of the earth reckoned in the plane of Jupiter's equator from O, his vernal equinox. The plane of reference was Jupiter's equator; for as Marth pointed out, the computations were much simplified by adopting this plane, owing to the small inclinations of the orbits when referred to it.

Let a = semi-major axis at distance unity.
$a'' = a/\sin 1''$.
ρ = radius vector, $a = 1$.
v = true longitude.
Ω = ascending node, ι = inclination } referred to Jupiter's equator.
Δ = jovicentric distance of the earth.
B = ,, latitude ,,
L + 180° = ,, longitude ,,
Δ' = geocentric distance of the satellite.

Then

$$\Delta' = \Delta + a\rho \cos (v - \mathrm{L}) \cos \mathrm{B}.$$

$$x = \frac{a''\rho}{\Delta'} [\sin (v - \mathrm{L}) - \sin (v - \Omega) \sin \iota \tan \tfrac{1}{2} \iota \cos (\mathrm{L} - \Omega)]$$

$$y = \frac{a''\rho}{\Delta'} [\cos (v - \mathrm{L}) \sin \mathrm{B} + \sin (v - \Omega) \sin \iota \cos \mathrm{B}].$$

x and y are the co-ordinates of the satellite in seconds of arc as seen from the centre

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of the earth, the origin being at Jupiter's centre, and the axes parallel and perpendicular respectively to the plane of Jupiter's equator. The second term in x is insensible for I. and III., and was neglected.

The longitudes and radius vectors were calculated for the Greenwich Mean Time corresponding to the mean sideral time of the six exposures on Jupiter, corrected for the aberration of light, that is antedated by the amount of the light time. In converting Cape Sideral Time into Greenwich Mean Time the adopted longitude of the Cape Observatory was $1^h\,13^m\,54^s{\cdot}8$ East.

Δ in the above formula is the distance between the centre of the earth at the moment of observation and the centre of Jupiter at the moment the light rays left it.

This value of Δ was found from the *Nautical Almanac* by interpolation for the Greenwich mean epoch of observation corrected for aberration and by then applying the correction denoted by δ_2 (log Δ) on pp. 62, 64 of Vol. XII. Part II.

The quantities in x and y to which corrections were sought, were a'', ρ, v, ι, Ω. Taking them in this order, we have—

$$\frac{\delta x}{\delta a} = \frac{x}{a}, \qquad \frac{\delta y}{\delta a} = \frac{y}{a},$$

$$\frac{\delta x}{\delta \rho} = \frac{x}{\rho}, \qquad \frac{\delta y}{\delta \rho} = \frac{y}{\rho},$$

$$\frac{\delta x}{\delta v} = \mathrm{X} = \frac{a''\rho}{\Delta'} \sin 1^\circ \cos (v - \mathrm{L}).$$

$$\frac{\delta y}{\delta v} = \mathrm{Y} = \frac{a''\rho}{\Delta'} \sin 1^\circ \left\{ - \sin (v - \mathrm{L}) \sin \mathrm{B} + \sin \iota \cos \mathrm{B} \cos (v - \Omega) \right\}.$$

Instead of ι and Ω it is convenient to take two other variables and to follow Marth and put—

$$p = \iota \sin (-\Omega),$$
$$q = \iota \cos (-\Omega);$$

so that the expression for y now is

$$y = \frac{a''\rho}{\Delta'} \left\{ \cos (v - \mathrm{L}) \sin \mathrm{B} + p \cos \mathrm{B} \cos v + q \cos \mathrm{B} \sin b \right\}$$

and

$$\frac{\delta y}{\delta p} = \frac{a''\rho}{\Delta'} \sin 1^\circ \cos \mathrm{B} \cos v,$$

$$\frac{\delta y}{\delta q} = \frac{a''\rho}{\Delta'} \sin 1^\circ \cos \mathrm{B} \sin v.$$

$\frac{\delta x}{\delta p}$ and $\frac{\delta x}{\delta q}$ were inappreciable and were neglected.

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These co-efficients and $\frac{\delta x}{\delta v}$, $\frac{\delta y}{\delta v}$ were multiplied by sin 1° so that the unknowns Δp, Δq and Δv might be expressed in degrees instead of in circular measure. We have thus—

$$\Delta x = x \frac{\Delta (a''\rho)}{a''\rho} + \mathrm{X}\,\Delta v$$

$$\Delta y = y \frac{\Delta (a''\rho)}{a''\rho} + \mathrm{Y}\,\Delta v + \frac{a''\rho}{\Delta'} \sin 1^\circ \cos \mathrm{B}\,\{\cos v\,\Delta p + \sin v\,\Delta q\}$$

Of these variables a, p and q are fundamental constants of the theory, but ρ and v are not and must be expressed in terms of such constants. The most important terms in the longitude and radius vector are :

$$v = nt + \epsilon + k \sin l - h \cos l + x \sin \psi$$
$$\rho = 1 \; - \tfrac{1}{2} \sin 1^\circ \{k \cos l + h \sin l + x \cos \psi\}$$

in which

$$l = nt + \epsilon = \text{the mean longitude}$$
$$h = 2\,e \sin \omega$$
$$k = 2\,e \cos \omega$$

e being the eccentricity and ω the perijove. $x \frac{\sin}{\cos} \psi$ represents the "great Inequality" which arises from the near commensurability of the mean motions of the first three satellites. For the fourth satellite this term does not exist. The mean motion n must be considered as known since the observations do not cover nearly sufficient time to determine a reliable correction to it. For further details reference may be made to pp. 79, 80 of Vol. XII. Part II. : here it must suffice to say that the following were the expressions for Δv, $\Delta \rho$:

$$\Delta v = \Delta\epsilon - \cos l\,\Delta h + \sin l\,\Delta k + \sin \psi\,\Delta x$$
$$\Delta \rho = -\lambda \sin l\,\Delta h - \lambda \cos l\,\Delta k - \lambda \cos \psi\,\Delta x$$

where

$$\lambda = \tfrac{1}{2} \sin 1^\circ.$$

If we put

$$\mathrm{Z} = \frac{a''\rho}{\Delta'} \sin 1^\circ \cos \mathrm{B}$$

the equations of condition take the following form :

$$\Delta x = \mathrm{X}\,\Delta\epsilon + x \frac{\Delta a}{a} + \left(-\mathrm{X} \cos l - \frac{\lambda x}{\rho} \sin l\right) \Delta h$$
$$+ \left(\mathrm{X} \sin l - \frac{\lambda x}{\rho} \cos l\right) \Delta k$$
$$+ \left(\mathrm{X} \sin \psi - \frac{\lambda x}{\rho} \cos \psi\right) \Delta x,$$

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$$\begin{aligned}\Delta y = {} & \mathrm{Y}\,\Delta\epsilon + y\frac{\Delta a}{a} + \left(-\mathrm{Y}\cos l - \frac{\lambda y}{\rho}\sin l\right)\Delta h \\ & + \left(\mathrm{Y}\sin l - \frac{\lambda y}{\rho}\cos l\right)\Delta k \\ & + \left(\mathrm{Y}\sin\psi - \frac{\lambda y}{\rho}\cos\psi\right)\Delta r \\ & + \mathrm{Z}\cos v\,\Delta p + \mathrm{Z}\sin v\,\Delta q\,.\end{aligned}$$

From the measurements of a satellite on every plate a pair of equations of this form were derived, the coefficients in the equations referring to an epoch which is the mean epoch of the six exposures on Jupiter. But as the first set of three exposures was separated by an interval of 17–20 minutes from the second set, it was not safe to assume that throughout that period the change in the abscissæ was a linear function of the time. The abscissæ were therefore calculated for two epochs, one the mean of the first three exposures, the other the mean of the second three and the two corresponding absolute terms were found. The mean of these two values of O – C was the value used in the equations of condition.

In the following tables are given the longitudes and radius vectors of the tabular co-ordinates of the satellites and their rates of change for 1^{s} of longitude. The Greenwich Mean Time is the mean epoch of the six exposures on Jupiter, corrected for aberration. This is the epoch for which the equations of condition were calculated. The values of the co-ordinates at the epochs of the first and second set of three exposures are given in Table XII. These were calculated by finding first the longitudes and radius vectors for these two epochs and then by means of differential formulæ deriving the corresponding values of the co-ordinates from the values for the mean epoch given in these tables.

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TABLE X.

Tabular Co-ordinates of I.

Date.		Greenwich Mean Time. h	m	Longitude.	Radius Vector.	x.	y.	X.	Y.
July	8	10	38·716	263°·2544	0·998021	+138″·156	+ 0″·022	+ 0·289	− 0·002
	14	9	15·905	33·2719	0·999096	− 78·319	− 0·104	− 2·043	+ 0·001
	15	8	33·584	229·9305	0·999852	+108·885	+ 0·081	+ 1·564	− 0·002
	16	8	31·236	73·8898	1·001791	−136·503	− 0·039	− 0·655	+ 0·002
	19	8	57·401	327·7919	0·996382	+ 73·111	− 0·090	− 2·113	− 0·002
	24	10	12·388	275·4830	0·996865	+141·078	− 0·004	− 0·310	− 0·002
	26	8	41·844	309·9218	0·996262	+106·059	− 0·053	− 1·658	− 0·001
	27	8	13·056	149·1594	1·003484	− 69·040	+ 0·072	+ 2·194	+ 0·001
Aug.	3	8	18·356	134·3995	1·003672	− 97·448	+ 0·031	+ 1·846	+ 0·001
	4	12	13·948	11·6875	0·998725	− 36·599	− 0·044	− 2·418	0·000
	6	8	24·024	26·1918	0·999765	− 70·735	− 0·031	− 2·179	0·000
	7	8	58·239	233·6886	0·998411	+120·267	+ 0·022	+ 1·354	0·000
	17	7	12·240	94·1244	1·003597	−141·519	− 0·016	+ 0·386	− 0·001
	18	8	52·764	311·8502	0·996462	+ 97·452	+ 0·042	− 1·807	0·000
	19	8	8·519	148·6657	1·002950	− 63·357	− 0·055	+ 2·235	− 0·001
	24	6	26·530	72·2012	1·003123	−138·916	+ 0·011	− 0·525	− 0·002
	25	5	5·660	263·8615	0·996477	+140·994	+ 0·008	+ 0·020	+ 0·002
	27	5	5·220	311·1313	0·996659	+ 95·894	+ 0·090	− 1·797	+ 0·001
	28	5	14·340	155·3755	1·002305	− 45·044	− 0·119	+ 2·334	0·000
	31	6	29·638	57·1191	1·002719	−125·761	+ 0·056	− 1·097	− 0·002
Sept.	1	6	58·983	264·3393	0·996352	+139·396	+ 0·113	− 0·032	+ 0·003
	2	5	30·103	95·4251	1·003757	−137·150	− 0·042	+ 0·506	− 0·003
	4	5	7·060	138·8287	1·002847	− 78·938	− 0·148	+ 2·007	− 0·002
	7	5	25·120	32·5114	1·001694	− 88·108	+ 0·142	− 1·867	− 0·002
	8	5	43·558	237·9597	0·996976	+124·655	− 0·074	+ 1·014	+ 0·003
	10	5	19·474	281·8563	0·996288	+129·402	+ 0·077	− 0·780	+ 0·003
	14	5	55·350	21·2228	1·001330	− 65·467	+ 0·199	− 2·087	− 0·002

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TABLE X.—*continued.*

TABULAR CO-ORDINATES OF II.

Date.		Greenwich Mean Time.		Longitude.	Radius Vector.	x.	y.	X.	Y.
		h	m						
July	8	10	38·716	292·8875	1·008023	+206″·250	− 0″·171	− 1·513	− 0·034
	15	8	33·584	273·9074	1·006639	+225·285	+ 0·474	− 0·312	− 0·034
	16	8	31·236	13·5151	1·004958	− 55·672	− 1·983	− 3·825	− 0·004
	19	8	57·401	320·2902	1·009134	+142·318	− 1·074	− 3·109	− 0·029
	20	8	56·347	60·6686	0·997420	−198·871	− 1·439	− 1·857	+ 0·023
	24	10	12·388	112·1786	0·991348	−205·640	+ 0·152	+ 1·588	+ 0·034
	26	8	41·844	308·9303	1·009126	+173·601	− 0·712	− 2·621	− 0·032
	27	8	13·056	47·2146	0·998740	−172·266	− 1·695	− 2·589	+ 0·017
	28	8	45·388	152·3096	0·991651	− 97·102	+ 1·349	+ 3·551	+ 0·024
Aug.	3	8	18·356	37·2052	0·999521	−147·192	− 1·797	− 3·045	+ 0·011
	4	12	13·948	156·7658	0·992417	− 77·848	+ 1·406	+ 3·707	+ 0·022
	6	8	24·024	342·0723	1·006840	+ 57·907	− 1·521	− 3·885	− 0·020
	7	8	58·239	85·9435	0·992528	−226·466	− 0·707	− 0·039	+ 0·030
	17	7	12·240	11·8253	1·001917	− 64·964	− 1·747	− 3·810	− 0·004
	18	8	52·764	121·4257	0·991072	−181·104	+ 0·356	+ 2·321	+ 0·030
	19	8	8·519	220·5026	1·002848	+162·013	+ 1·623	+ 2·779	− 0·011
	21	6	43·166	55·4321	0·994781	−196·285	− 1·384	− 1·926	+ 0·017
	25	5	5·660	94·6743	0·991249	−219·489	− 0·449	+ 0·700	+ 0·028
	27	5	5·220	297·4149	1·008785	+189·363	− 0·188	− 2·173	− 0·029
	28	5	14·340	38·7031	0·996361	−157·244	− 1·539	− 2·772	+ 0·009
Sept.	1	6	58·983	92·3265	0·991192	−218·058	− 0·516	+ 0·586	+ 0·026
	2	5	30·103	188·6377	0·999491	+ 58·002	+ 1·511	+ 3·739	+ 0·006
	3	5	4·439	287·0741	1·008874	+205·042	+ 0·136	− 1·570	− 0·027
	4	5	7·060	27·7817	0·997204	−125·100	− 1·518	− 3·179	+ 0·003
	5	6	9·616	135·1180	0·992663	−135·184	+ 0·580	+ 3·009	+ 0·024
	6	5	11·204	232·5505	1·006026	+190·893	+ 1·310	+ 1·968	− 0·013
	7	5	25·120	333·3273	1·005170	+ 74·683	− 0·975	− 3·638	− 0·020
	8	5	43·558	76·4575	0·991676	−216·358	− 0·873	− 0·430	+ 0·021
	10	5	19·474	277·8264	1·008888	+213·059	+ 0·392	− 0·998	− 0·025
	12	6	52·466	127·7230	0·992509	−152·388	+ 0·344	+ 2·667	+ 0·024
	13	6	5·658	226·0186	1·005855	+175·220	+ 1·322	+ 2·269	− 0·009
	14	5	55·350	325·1125	1·005525	+100·338	− 0·731	− 3·376	− 0·021

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TABLE X.—*continued.*

Tabular Co-ordinates of III.

Date.		Greenwich Mean Time. h	m	Longitude.	Radius Vector.	x.	y.	X.	Y.
July	8	10	38·716	216°·7980	0·998874	+210″·985	+ 0″·302	+ 4·940	+ 0·011
	14	9	15·905	155·6903	0·999460	−143·933	− 0·405	+ 5·698	+ ·010
	15	8	33·584	204·6172	0·998846	+152·269	+ 0·155	+ 5·637	+ ·012
	16	8	31·236	254·9670	0·999015	+346·985	+ 0·622	+ 1·544	+ ·006
	19	8	57·401	46·7286	1·001292	−267·408	− 0·389	− 4·231	− ·010
	20	8	56·347	96·9002	1·000769	−357·054	− 0·703	+ 0·887	− ·001
	24	10	12·388	301·1154	0·999749	+304·463	+ 0·694	− 3·426	− ·003
	26	8	41·844	38·4338	1·001412	−235·450	− 0·257	− 4·835	− ·012
	27	8	13·056	87·6207	1·000998	−363·172	− 0·681	− 0·036	− ·005
	28	8	45·388	139·0234	0·999802	−227·105	− 0·635	+ 4·932	+ ·006
Aug.	3	8	18·356	80·0207	1·001127	−361·670	− 0·630	− 0·778	− ·008
	4	12	13·948	138·5213	0·999644	−225·551	− 0·712	+ 4·972	+ ·005
	6	8	24·024	231·3550	0·998447	+296·312	+ 0·314	+ 3·662	+ ·013
	7	8	58·239	282·9824	0·999414	+348·982	+ 0·766	− 1·800	+ ·004
	17	7	12·240	62·1916	1·001294	−333·958	− 0·372	− 2·482	− ·014
	18	8	52·764	115·9404	1·000125	−310·568	− 0·887	+ 3·233	− ·003
	21	6	43·166	262·6801	0·999064	+360·256	+ 0·631	+ 0·230	+ ·012
	24	6	26·530	52·8496	1·001399	−307·037	− 0·167	− 3·294	− ·017
	25	5	5·660	100·2435	1·000591	−345·365	− 0·832	+ 1·720	− ·010
	27	5	5·220	200·9834	0·998544	+161·438	− 0·417	+ 5·558	+ ·016
	28	5	14·340	251·7620	0·998808	+348·878	+ 0·457	+ 1·320	+ ·016
	31	6	29·638	45·2118	1·001480	−279·257	+ 0·042	− 3·872	− ·019
Sept.	1	6	58·983	96·4338	1·000752	−346·320	− 0·820	+ 1·380	− ·013
	2	5	30·103	143·6319	0·999529	−176·109	− 1·102	+ 5·357	+ ·002
	4	5	7·060	243·7140	0·998619	+332·065	+ 0·279	+ 2·062	+ ·019
	5	6	9·616	296·3100	0·999715	+295·408	+ 1·053	− 3·361	+ ·008
	6	5	11·204	344·5513	1·000959	+ 52·546	+ 1·061	− 6·085	− ·008
	7	5	25·120	35·2154	1·001655	−236·339	+ 0·315	− 4·553	− ·020
	8	5	43·558	86 0256	1·001144	−350·080	− 0·678	+ 0·328	− ·017
	10	5	19·474	185·8649	0·998516	+ 78·552	− 0·867	+ 5·910	+ ·014
	12	6	52·466	290·0405	0·999477	+308·098	+ 1·038	− 2·786	+ ·012
	13	6	5·658	338·7021	1·000897	+ 83·310	+ 1·197	− 5·880	− ·006
	14	5	55·350	28·5189	1·001747	−203·596	+ 0·524	− 4·893	− 0·020

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TABLE X.—*continued.*

TABULAR CO-ORDINATES OF IV.

Date.		Greenwich Mean Time.		Longitude.	Radius Vector.	x.	y.	X.	Y.
		h	m	°		″	″		
July	8	10	38·716	132·0503	0·993258	−458·806	+ 1·394	+ 7·201	− 0·053
	14	9	15·905	261·3389	1·002084	+623·541	− 3·297	+ 1·558	+ ·015
	15	8	33·584	282·1373	1·004452	+617·388	− 2·786	− 2·440	+ ·035
	16	8	31·236	303·4426	1·006279	+525·977	− 1·875	− 6·233	+ ·050
	19	8	57·401	7·6580	1·006049	− 97·078	+ 1·829	−11·030	+ ·052
	20	8	56·347	28·9954	1·004101	−321·010	+ 2·787	− 9·629	+ ·036
	24	10	12·388	116·6015	0·994131	−556·527	+ 2·153	+ 5·228	− ·047
	26	8	41·844	158·9611	0·992717	−206·642	− 0·274	+10·410	− ·061
	27	8	13·056	180·3977	0·993473	+ 27·026	− 1·546	+11·031	− ·055
	28	8	45·388	202·7082	0·995225	+266·740	− 2·642	+10·030	− ·040
Aug.	3	8	18·356	331·1417	1·007146	+282·416	− 0·046	−10·150	+ ·063
	6	8	24·024	35·1753	1·003326	−401·037	+ 3·309	− 8·792	+ ·028
	7	8	58·239	57·1636	1·000721	−559·220	+ 3·656	− 5·492	+ ·004
	17	7	12·240	272·8353	1·003366	+633·969	− 3·039	− 1·473	+ ·039
	18	8	52·764	295·6989	1·005555	+551·961	− 1·909	− 5·693	+ ·058
	19	8	8·519	316·3439	1·006777	+400·984	− 0·597	− 8·746	+ ·066
	24	6	26·530	61·5956	1·000376	−583·799	+ 3·820	− 4·290	− ·010
	25	5	5·660	81·9937	0·997845	−629·827	+ 3·367	− 0·446	− ·033
	27	5	5·220	125·5200	0·993636	−468·344	+ 1·085	+ 7·211	− ·065
	28	5	14·340	147·5288	0·992762	−277·448	− 0·410	+ 9·727	− ·067
	31	6	29·638	214·1829	0·996257	+403·971	− 3·758	+ 8·254	− ·019
Sept.	1	6	58·983	236·2975	0·998898	+553·741	− 3·907	+ 4·984	+ ·008
	2	5	30·103	256·5191	1·001469	+619·590	− 3·519	+ 1·320	+ ·032
	3	5	4·439	277·5965	1·003953	+606·167	− 2·631	− 2·677	+ ·052
	4	5	7·060	298·9922	1·005947	+508·375	− 1·340	− 6·374	+ ·066
	5	6	9·616	321·2061	1·007139	+331·684	+ 0·212	− 9·264	+ ·070
	6	5	11·204	341·6047	1·007268	+124·763	+ 1·626	−10·690	+ ·064
	7	5	25·120	3·0886	1·006373	−108·726	+ 2·883	−10·710	+ ·049
	8	5	43·558	24·6986	1·004558	−326·375	+ 3·727	− 9·194	+ ·027
	10	5	19·474	67·2980	0·999474	−591·113	+ 3·756	− 2·863	− ·025
	12	6	52·466	112·1077	0·994493	−527·780	+ 1·604	+ 5·202	− ·064
	13	6	5·658	133·2366	0·993145	−382·615	+ 0·153	+ 8·143	− ·070
	14	5	55·350	154·9588	0·992690	−181·510	− 1·358	+10·010	− 0·067

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The co-ordinates in the foregoing table are the co-ordinates as seen from the centre of the earth; they must now be projected on to the plate. The usual formulæ for converting from spherical to rectangular co-ordinates are—

$$x = \tan(\alpha - \alpha_0)\cos\phi\sec(\phi - \delta_0)$$
$$y = \tan(\phi - \delta_0)$$

where

$$\cot\phi = \cot\delta\cos(\alpha - \alpha_0)$$

and $\alpha_0\ \delta_0$, $\alpha\ \delta$ are the right ascension and declination of the plate-centre and the star respectively. But in this investigation the plane of reference was Jupiter's equator, so that in these formulæ 'right ascension' and 'declination' refer to Jupiter's equator and not to the earth's. Referred to axes parallel and perpendicular to Jupiter's equator, the co-ordinates of Jupiter and the maximum co-ordinates of a satellite were—

(1) For the first set of three exposures,

Jupiter	$x = +\ 4'\cdot8$	$y = +\ 7'\cdot3$
Satellite	$x = +\ 14\cdot8$	$y = +\ 7\cdot4$

(2) For the second set of three exposures,

Jupiter	$x = +\ 0'\cdot3$	$y = -\ 5'\cdot6$
Satellite	$y = +\ 10\cdot3$	$y = -\ 5\cdot5$.

Hence in the transformation formulæ, we have—

(1) For Jupiter . . .	$\alpha - \alpha_0 = +\ 4'\cdot8$	$\delta = \ 0$	$\delta_0 = -\ 7'\cdot3$
Satellite . . .	$\alpha - \alpha_0 = +\ 14\cdot8$	$\delta = +0'\cdot1$	$\delta_0 = -\ 7\cdot3$
(2) For Jupiter . . .	$\alpha - \alpha_0 = +\ 0'\cdot3$	$\delta = \ 0$	$\delta_0 = +\ 5'\cdot6$
Satellite . . .	$\alpha - \alpha_0 = +\ 10\cdot3$	$\delta = +0'\cdot1$	$\delta_0 = +\ 5\cdot6$.

Thus as we are working to $0'\cdot0001$ we have in all cases $\phi = \delta$ and

$$x = \alpha - \alpha_0 \qquad y = \delta - \delta_0$$

that is to say, we can use the calculated co-ordinates as they stand.

We can now give the equations of condition. In the following tables, the first co-ordinate for each day is the mean of the first set of three exposures on Jupiter. Next to this is the corresponding calculated co-ordinate and then O–C. The second

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co-ordinate is the mean of the second set of three exposures and in the same way gives a value of O–C. In the fourth column is the mean value of O–C. In the case of the ordinates these were the values of the absolute terms used for the first approximation to the values of the unknowns; but for the abscissæ rough estimates of the effect of phase on the measured position of Jupiter were arrived at by substituting in the equations of condition the values of the unknowns derived from the heliometer investigation and finding the residuals. The following are the corrections which were applied on this account to the values of O–C.

TABLE XI.

Rough Corrections for Phase.

July	8	+0″·500	Aug.	6	0″·000	Sept.	2	−0″·500
	14	+ ·400		7	− ·100		3	− ·500
	15	+ ·400		17	− ·200		4	− ·500
	16	+ ·400		18	− ·200		5	− ·500
	19	+ ·400		19	− ·200		6	− ·500
	20	+ ·300		21	− ·200		7	− ·500
	24	+ ·200		24	− ·300		8	− ·600
	26	+ ·200		25	− ·300		10	− ·600
	27	+ ·200		27	− ·400		12	− ·600
	28	+ ·200		28	− ·400		13	− ·600
Aug.	3	+ ·100		31	− ·400		14	−0·600
	4	·000	Sept.	1	−0·400			

The coefficient of $\frac{\Delta a}{a}$ is x or y, and to avoid large numbers and make all the coefficients of the same order of magnitude, a new unknown was introduced such that—

$$\Delta a = 100 \frac{\Delta a}{a}.$$

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TABLE XII.

Equations of Condition.

First Satellite, I x.

Date.	Observed Abscissa.	Calculated Abscissa.	O—C.	Mean O—C.	First Second Approximation.	Coefficients of Δa_1	Δh_1	Δk_1	Δx_1	Δe_1
	"	"	"	"	"					
July 8	+ 137·323	+ 137·757	− 0·434							
				− 0·448	+ 0·052	+1·38	+1·24	−0·15	−0·89	+0·29
	+ 138·028	+ 138·489	− ·461		+ ·111					
14	− 76·467	− 75·597	− ·870							
				− ·840	− ·440	−0·78	+2·09	−0·53	−1·82	−2·04
	− 81·808	− 80·999	− ·809		− ·560					
15	+ 106·590	+ 106·850	− ·260							
				− ·243	+ ·157	+1·09	+1·73	−0·60	−1·63	+1·56
	+ 110·638	+ 110·864	− ·226		+ ·177					
16	− 136·202	− 135·693	− ·509							
				− ·534	− ·134	−1·37	+1·33	−0·29	−1·14	−0·66
	− 137·812	− 137·253	− ·559		− ·104					
19	+ 74·995	+ 75·633	− ·638							
				− ·628	− ·228	+0·73	+2·13	+0·59	−1·15	−2·11
	+ 69·940	+ 70·557	− ·617		− ·407					
24	+ 141·308	+ 141·496	− ·188							
				− ·142	+ ·058	+1·41	+1·26	+0·18	−0·86	−0·31
	+ 140·468	+ 140·564	− ·096		+ ·056					
26	+ 107·575	+ 107·965	− ·390							
				− ·344	− ·144	+1·06	+1·78	+0·68	−0·99	−1·67
	+ 103·814	+ 104·111	− ·297		− ·250					
27	− 71·793	− 71·531	− ·262							
				− ·222	− ·022	−0·69	+2·19	+0·60	−1·38	+2·19
	− 66·704	− 66·521	− ·183		+ ·254					
Aug. 3	− 99·441	− 99·536	+ ·095							
				+ ·030	+ ·130	−0·97	+1·90	+0·72	−1·23	+1·85
	− 95·358	− 95·322	− ·036		+ ·053					

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TABLE XII.—*continued.*

FIRST SATELLITE, I x.

Date.	Observed Abscissa.	Calculated Abscissa.	O—C	Mean O—C.	First / Second Approximation.	Coefficients of Δa_1	Δh_1	Δk_1	Δx_1	Δe_1
Aug. 6	− 68″·909	− 68″·237	− 0″·672							
				− 0·746	− 0·746	−0·71	+2·23	−0·39	−2·13	−2·18
	− 74·026	− 73·205	− ·821		− ·675					
7	+ 119·156	+ 119·034	+ ·122							
				+ ·324	+ ·224	+1·20	+1·65	−0·48	−1·67	+1·35
	+ 122·352	+ 121·827	+ ·525		+ ·232					
17	− 141·860	− 141·955	+ ·095							
				+ ·208	+ ·008	−1·42	+1·26	−0·47	−1·06	+0·39
	− 140·700	− 141·021	+ ·321		+ ·069					
18	+ 99·549	+ 99·519	+ ·030							
				− ·303	− ·503	+0·97	+1·84	+0·78	−1·44	−1·81
	+ 94·707	+ 95·343	− ·636		− ·487					
19	...	...	...							
				− 1·076	− 1·276	−0·63	+2·20	+0·68	−1·83	+2·24
	− 61·908	− 60·832	− 1·076							
24	− 138·075	− 138·278	+ ·203							
				+ ·214	− ·086	−1·39	+1·31	−0·12	−1·29	−0·53
	− 139·271	− 139·496	+ ·225		− ·211					
25	+ 140·995	+ 140·942	+ ·053							
				+ ·164	− ·136	+1·41	+1·23	+0·11	−1·15	+0·02
	+ 141·264	+ 140·988	+ ·276		+ ·066					
27	+ 97·878	+ 98·060	− ·182							
				− ·150	− ·550	+0·96	+1·81	+0·81	−1·55	−1·80
	+ 93·576	+ 93·693	− ·117		− ·352					
28	− 47·350	− 47·923	+ ·573							
				+ ·534	+ ·134	−0·45	+2·29	+0·60	−2·08	+2·33
	− 41·616	− 42·112	+ ·496		+ ·158					

Note :—Aug. 19 rejected.

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TABLE XII.—*continued.*

FIRST SATELLITE I x.

Date.	Observed Abscissa.	Calculated Abscissa.	O—C.	Mean O—C.	First / Second Approximation.	Coefficients of Δa_1	Δh_1	Δk_1	Δx_1	Δe_1
	"	"	"	"	"					
Aug. 31	− 124·619	− 124·471	− 0·148							
				− 0·089	− 0·489	−1·26	+1·51	−0·32	−1·55	−1·10
	− 127·031	− 127·001	− ·030		− ·477					
Sept. 1	+ 139·898	+ 139·402	+ ·496							
				+ ·518	+ ·118	+1·39	+1·21	+0·15	−1·17	−0·03
	+ 139·861	+ 139·320	+ ·541		− ·075					
2	− 137·112	− 137·770	+ ·658							
				+ ·396	− ·104	−1·37	+1·24	+0·39	−1·15	+0·51
	− 136·328	− 136·462	+ ·134		− ·130					
4	− 80·815	− 81·159	+ ·344							
				+ ·524	+ ·024	−0·79	+1·97	+0·79	−1·83	+2·01
	− 75·977	− 76·681	+ ·704		+ ·054					
7	− 85·807	− 85·514	− ·293							
				− ·192	− ·692	−0·88	+1·99	−0·34	−2·01	−1·87
	− 90·744	− 90·652	− ·092		− ·679					
8	+ 124·235	+ 123·535	+ ·700							
				+ ·700	+ ·100	+1·25	+1·46	−0·29	−1·48	+1·01
	+ 126·433	+ 125·733	+ ·700		+ ·172					
10	+ 130·841	+ 130·278	+ ·563							
				+ ·452	− ·148	+1·29	+1·27	+0·53	−1·23	−0·78
	+ 128·816	+ 128·476	+ ·340		− ·276					
14	− 62·988	− 62·893	− ·095							
				− ·186	− ·786	−0·65	+2·15	−0·21	−2·16	−2·09
	− 68·289	− 68·011	− ·278		− ·828					

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TABLE XII—*continued.*

SECOND SATELLITE, II x.

Date.	Observed Abscissa.	Calculated Abscissa.	O—C.	Mean O—C.	First / Second Approximation.	Coefficients of Δa_2	Δh_2	Δk_2	Δx_2	$\Delta \epsilon_2$
July 8	″ ...	″ ...	″ ...	″	″					
				− 0·529	− 0·029	+2·06	+2·23	+0·72	+0·84	−1·51
	+ 204·753	+ 205·282	− 0·529		+ ·030					
15	+ 225·082	+ 225·471	− ·389							
				− ·292	+ ·108	+2·25	+1·97	+0·20	+1·22	−0·31
	+ 224·877	+ 225·071	− ·194		+ ·128					
16	− 53·770	− 53·399	− ·371							
				− ·270	+ ·130	−0·56	+3·82	−0·48	+3·03	−3·82
	− 58·108	− 57·939	− ·169		+ ·160					
19	+ 144·310	+ 144·169	+ ·141							
				+ ·183	+ ·583	+1·42	+3·18	+1·04	+1·62	−3·11
	+ 140·674	+ 140·449	+ ·225		+ ·404					
20	− 198·062	− 197·638	− ·424							
				− ·182	+ ·118	−1·99	+2·41	−0·81	+2·33	−1·86
	− 200·019	− 200·078	+ ·059		+ ·157					
24	− 206·987	− 206·807	− ·180							
				− ·151	+ ·049	−2·06	+2·28	+0·77	+1·27	+1·59
	− 204·559	− 204·437	− ·122		+ ·047					
26	+ 175·065	+ 175·109	− ·044							
				+ ·158	+ ·358	+1·74	+2·82	+1·10	+1·55	−2·62
	+ 172·434	+ 172·075	+ ·359		+ ·252					
27	− 171·059	− 170·784	− ·275							
				− ·197	+ ·003	−1·72	+2·85	−0·93	+2·83	−2·59
	− 173·849	− 173·730	− ·119		+ ·279					
28	− 99·332	− 99·194	− ·138							
				− ·074	+ ·126	−0·97	+3·54	+0·92	+2·30	+3·55
	− 95·011	− 95·000	− ·011		+ ·132					
Aug. 3	− 145·183	− 145·454	+ ·271							
				+ ·268	+ ·368	−1·47	+3·19	−0·88	+3·17	−3·04
	− 148·650	− 148·916	+ ·266		+ ·291					
4	− 80·074	− 80·267	+ ·193							
				+ ·192	+ ·192	−0·78	+3·67	+0·87	+2·69	+3·71
	− 75·227	− 75·419	+ ·192		+ ·091					

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TABLE XII—*continued.*

SECOND SATELLITE, II x.

Date.	Observed Abscissa.	Calculated. Abscissa.	O—C.	Mean O—C.	First / Second Approximation.	Coefficients of Δa_2	Δl_2	Δk_2	Δx_2	$\Delta \epsilon_2$
Aug. 6	+ 60″.450	+ 60″.109	+ 0″.341							
				+ 0″.264	+ 0″.264	+0″.58	+3″.86	+0″.67	+3″.02	−3″.88
	+ 55″.885	+ 55″.699	+ ″.186		+ ″.335					
7	− 226″.462	− 226″.436	− ″.026							
				+ ″.128	+ ″.028	−2″.26	+1″.99	+0″.08	+1″.70	−0″.04
	− 226″.193	− 226″.476	+ ″.283		+ ″.036					
17	− 62″.359	− 62″.669	+ ″.310							
				+ ″.364	+ ″.164	−0″.65	+3″.84	−0″.29	+3″.68	−3″.81
	− 66″.836	− 67″.253	+ ″.417		+ ″.225					
18	− 182″.062	− 182″.431	+ ″.369							
				+ ″.285	+ ″.085	−1″.81	+2″.57	+1″.16	+1″.99	+2″.32
	− 179″.558	− 179″.759	+ ″.201		+ ″.101					
19	+ 161″.420	+ 160″.450	+ ″.970							
				+ ″.847	+ ″.647	+1″.62	+3″.04	−0″.68	+3″.09	+2″.78
	+ 164″.284	+ 163″.560	+ ″.724		+ ″.143					
21	− 194″.663	− 195″.127	+ ″.464							
				+ ″.348	+ ″.148	−1″.96	+2″.50	−0″.65	+2″.58	−1″.93
	− 197″.190	− 197″.421	+ ″.231		+ ″.034					
25	− 219″.893	− 219″.891	− ″.002							
				+ ″.163	− ″.137	−2″.19	+1″.98	+0″.53	+1″.77	+0″.70
	− 218″.735	− 219″.063	+ ″.328		+ ″.065					
27	+ 190″.817	+ 190″.669	+ ″.148							
				− ″.132	− ″.532	+1″.89	+2″.46	+1″.16	+2″.10	−2″.17
	+ 188″.627	+ 189″.040	− ″.413		− ″.334					
28	− 154″.874	− 155″.526	+ ″.652							
				+ ″.581	+ ″.181	−1″.57	+3″.01	−0″.71	+3″.09	−2″.77
	− 158″.454	− 158″.964	+ ″.510		+ ″.205					
Sept. 1	− 217″.852	− 218″.420	+ ″.568							
				+ ″.682	+ ″.282	−2″.18	+1″.94	+0″.50	+1″.82	+0″.59
	− 216″.874	− 217″.670	+ ″.796		+ ″.089					
2	+ 56″.459	+ 55″.591	+ ″.868							
				+ ″.594	+ ″.094	+0″.58	+3″.77	0″.00	+3″.72	+3″.74
	+ 60″.726	+ 60″.405	+ ″.321		+ ″.068					

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TABLE XII—*continued.*

SECOND SATELLITE, II *x*.

Date.	Observed Abscissa.	Calculated Abscissa.	O—C.	Mean O—C.	First Second Approximation.	Coefficients of Δa_2	Δh_2	Δk_2	Δx_2	Δe_2
Sept. 3	+ 206″·126	+ 205″·988	+ 0″·138							
				+ 0·178	− 0·322	+2·05	+2·16	+0 98	+1·97	−1·57
	+ 204·290	+ 204·072	+ ·218		+ ·064					
4	− 122·704	− 123·169	+ ·465							
				+ ·576	+ ·076	−1·25	+3·31	−0·57	+3·36	−3·18
	− 126·331	− 127·017	+ ·686		+ ·106					
5	− 136·481	− 136·860	+ ·379							
				+ ·498	− ·002	−1·35	+2·96	+1·31	+2·76	+3·01
	− 132·878	− 133·496	+ ·618		+ ·124					
6	+ 189·996	+ 189·708	+ ·288							
				+ ·238	− ·262	+1·91	+2·52	−0·52	+2·56	+1·97
	+ 192·246	+ 192·058	+ ·188		− ·180					
7	+ 77·741	+ 77·168	+ ·573							
				+ ·774	+ ·274	+0·75	+3·56	+1·00	+3·43	−3·64
	+ 73·164	+ 72·188	+ ·976		+ ·287					
8	− 215·408	− 216·117	+ ·709							
				+ ·716	+ ·116	−2·16	+1·95	+0·02	+1·94	−0·43
	− 215·859	− 216·581	+ ·722		+ ·188					
10	+ 214·239	+ 213·623	+ ·616							
				+ ·512	− ·088	+2·13	+1·96	+0·74	+1·90	−1·00
	+ 212·883	+ 212·475	+ ·408		− ·216					
12	− 153·410	− 153·887	+ ·477							
				+ ·542	− ·058	−1·52	+2·67	+1·32	+2·62	+2·67
	− 150·269	− 150·875	+ ·606		− ·024					
13	+ 174·706	+ 173·984	+ ·722							
				+ ·736	+ ·136	+1·75	+2·68	−0·54	+2·69	+2·27
	+ 177·190	+ 176·440	+ ·750		+ ·204					
14	+ 103·259	+ 102·395	+ ·864							
				+ ·811	+ ·211	+1·00	+3·28	+1·17	+3·26	−3·38
	+ 99·027	+ 98·269	+ ·758		+ ·169					

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TABLE XII.—*continued.*

THIRD SATELLITE, III *x*.

Date.	Observed Abscissa.	Calculated Abscissa.	O—C.	Mean O—C	First / Second Approximation.	Coefficients of Δa_3	Δh_3	Δk_3	Δx_3	Δe_3
July 8	″ ...	″ ...	″ ...	″	″					
				− 0·725	− 0·225	+2·11	+5·06	−1·48	−4·33	+4·94
	+ 211·806	+ 212·531	− 0·725		− ·166					
14	− 146·292	− 145·794	− ·498							
				− ·452	− ·052	−1·44	+5·71	+1·19	−3·05	+5·70
	− 142·474	− 142·068	− ·406		− ·172					
15	+ 149·919	+ 150·479	− ·560							
				− ·550	− ·150	+1·52	+5·68	−1·14	−4·76	+5·64
	+ 153·515	+ 154·055	− ·540		− ·130					
16	+ 345·781	+ 346·525	− ·744							
				− ·526	− ·126	+3·47	+3·33	−0·70	−2·84	+1·54
	+ 347·128	+ 347·435	− ·307		− ·096					
19	− 266·652	− 266·148	− ·504							
				− ·296	+ ·104	−2·67	+4·60	−1·48	−4·37	−4·23
	− 268·748	− 268·660	− ·088		− ·075					
20	− 357·909	− 357·337	− ·572							
				− ·422	− ·122	−3·57	+3·20	+0·50	−2·03	+0·89
	− 357·030	− 356·759	− ·271		− ·083					
24	+ 305·386	+ 305·724	− ·338							
				− ·234	− ·034	+3·04	+4·04	+1·57	−2·13	−3·43
	+ 303·060	+ 303·190	− ·130		− ·036					
26	− 234·409	− 234·058	− ·351							
				− ·264	− ·064	−2·35	+5·06	−1·39	−4·85	−4·84
	− 237·012	− 236·836	− ·176		− ·170					
27	− 363·884	− 363·158	− ·726							
				− ·694	− ·494	−3·63	+3·16	+0·09	−2·47	−0·04
	− 363·841	− 363·178	− ·663		− ·218					
28	− 228·929	− 228·548	− ·381							
				− ·401	− ·201	−2·27	+5·03	+1·73	−3·02	+4·93
	− 226·077	− 225·656	− ·421		− ·195					
Aug. 3	− 361·728	− 361·446	− ·282							
				− ·215	− ·115	−3·62	+3·24	−0·22	−2·87	−0·78
	− 362·034	− 361·886	− ·148		− ·192					

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TABLE XII.—*continued.*

THIRD SATELLITE, III x.

Date.	Observed Abscissa.	Calculated Abscissa.	O—C.	Mean O—C.	First / Second Approximation.	Coefficients of Δa_3	Δh_3	Δk_3	Δx_3	$\Delta \epsilon_3$
	"	"	"	"	"					
Aug. 4	− 227·357	− 227·166	− 0·191							
				− 0·106	− 0·106	−2·26	+5·04	+1·80	−3·38	+4·97
	− 223·957	− 223·936	− ·021		− ·207					
6	+ 294·974	+ 295·280	− ·306							
				− ·138	− ·138	+2·96	+4·31	−1·24	−4·36	+3·66
	+ 297·375	+ 297·344	+ ·031		− ·067					
7	+ 349·129	+ 349·374	− ·245							
				+ ·046	− ·054	+3·49	+3·77	+1·08	−2·42	−1·80
	+ 348·794	+ 348·456	+ ·338		− ·046					
17	− 333·232	− 333·212	− ·020							
				+ ·114	− ·086	−3·34	+3·73	−0·84	−3·78	−2·48
	− 334·448	− 334·696	+ ·248		− ·025					
18	− 311·436	− 311·488	+ ·052							
				− ·238	− ·438	−3·11	+3·86	+1·71	−3·00	+3·23
	− 310·169	− 309·640	− ·529		− ·422					
21	+ 360·617	+ 360·183	+ ·434							
				+ ·380	+ ·180	+3·60	+3·15	+0·18	−2·93	+0·23
	+ 360·645	+ 360·319	+ ·326		+ ·066					
24	− 305·815	− 306·088	+ ·273							
				+ ·248	− ·052	+3·07	+4·12	−1·01	−4·24	−3·29
	− 307·756	− 307·978	+ ·222		− ·177					
25	− 346·255	− 345·866	− ·389							
				− ·270	− ·570	−3·45	+3·28	+1·15	−2·84	+1·72
	− 345·006	− 344·856	− ·150		− ·368					
27	+ 159·676	+ 159·764	− ·088							
				+ ·006	− ·394	+1·61	+5·69	−0·68	−5·68	+5·56
	+ 163·202	+ 163·102	+ ·100		− ·196					
28	+ 348·843	+ 348·470	+ ·373							
				+ ·208	− ·192	+3·49	+3·31	−0·29	−3·29	+1·32
	+ 349·326	+ 349·283	+ ·043		− ·168					
31	− 277·746	− 278·150	+ ·404							
				+ ·298	− ·102	−2·79	+4·46	−1·03	−4·57	−3·87
	− 280·167	− 280·358	+ ·191		− ·090					

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TABLE XII.—*continued.*

THIRD SATELLITE, III *x*.

Date.	Observed Abscissa.	Calculated Abscissa.	O—C.	Mean O—C.	First / Second Approximation.	Coefficients of Δa_3	Δl_3	Δk_3	Δx_3	$\Delta \epsilon_3$
	"	"	"	"	"					
Sept. 1	− 346·493	− 346·753	+ 0·260		− 0·070					
				+ 0·330		−3·46	+3·16	+1·03	−2·92	+1·38
	− 345·477	− 345·877	+ ·400		− ·263					
2	− 177·395	− 177·816	+ ·421							
				+ ·216	− ·284	−1·76	+5·23	+1·93	−4·82	+5·36
	− 174·385	− 174·396	+ ·011		− ·310					
4	+ 331·823	+ 331·441	+ ·382							
				+ ·370	− ·130	+3·32	+3·52	−0·56	−3·56	+2·06
	+ 333·039	+ 332·681	+ ·358		− ·100					
5	+ 296·373	+ 296·337	+ ·036							
				+ ·122	− ·378	+2·95	+3·80	+1·88	−3·52	−3·36
	+ 294·679	+ 294·471	+ ·208		− ·252					
6	+ 54·746	+ 54·247	+ ·499							
				+ ·420	− ·080	+0·53	+5·98	+1·20	−5·80	−6·08
	+ 51·183	+ 50·843	+ ·340		+ ·002					
7	− 234·495	− 234·786	+ ·291							
				+ ·436	− ·064	−2·36	+4·91	−0·94	−4·98	−4·55
	− 237·302	− 237·884	+ ·582		− ·051					
8	− 349·929	− 350·164	+ ·235							
				+ ·173	− ·427	−3·50	+3·02	+0·53	−2·96	+0·33
	− 349·877	− 349·988	+ ·111		− ·355					
10	+ 77·334	+ 76·864	+ ·470							
				+ ·394	− ·206	+0·79	+5·95	+0·07	−5·93	+5·91
	+ 80·555	+ 80·238	+ ·317		− ·334					
12	+ 309·415	+ 308·873	+ ·542							
				+ ·516	− ·084	+3·08	+3·48	+1·71	−3·40	−2·79
	+ 307·805	+ 307·315	+ ·490		− ·050					
13	+ 85·480	+ 84·889	+ ·591							
				+ ·557	− ·043	+0·83	+5·74	+1·48	−5·69	−5·88
	+ 82·252	+ 81·729	+ ·523		+ ·025					
14	− 201·430	− 202·109	+ ·679							
				+ ·486	− ·114	−2·04	+5·15	−0·77	−5·16	−4·89
	− 204·785	− 205·077	+ ·292		− ·156					

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TABLE XII—*continued.*

FOURTH SATELLITE IV *x*.

Date.	Observed Abscissa.	Calculated Abscissa.	O—C.	Mean O—C.	First / Second Approximation.	Coefficients of $\Delta\alpha_4$	Δh_4	Δk_4	$\Delta\epsilon_4$
	"	"	"	"	"				
July 8	− 459·934	− 459·775	− 0·159						
				− 0·227	+ 0·273	− 4·59	+ 7·83	+ 2·61	+ 7·20
	− 458·132	− 457·837	− ·295		+ ·332				
14	+ 623·470	+ 623·323	+ ·147						
				+ ·150	+ 0·550	+ 6·24	+ 5·61	+ 0·64	+ 1·56
	+ 623·911	+ 623·759	+ ·152		+ ·430				
15	+ 617·884	+ 617·720	+ ·164						
				+ ·202	+ 0·602	+ 6·17	+ 5·74	+ 1·32	− 2·44
	+ 617·297	+ 617·056	+ ·241		+ ·622				
16	+ 527·083	+ 526·764	+ ·319						
				+ ·476	+ 0·876	+ 5·26	+ 7·22	+ 2·74	− 6·23
	+ 525·824	+ 525·190	+ ·634		+ ·906				
19	− 95·175	− 95·674	+ ·499						
				+ ·696	+ 1·096	− 0·97	+ 11·04	− 0·72	− 11·03
	− 97·589	− 98·482	+ ·893		+ 0·917				
20	− 319·417	− 319·664	+ ·247						
				+ ·400	+ ·700	− 3·21	+ 9·75	− 2·34	− 9·63
	− 321·802	− 322·356	+ ·554		+ ·739				
24	− 557·419	− 557·356	− ·063						
				+ ·038	+ ·238	− 5·57	+ 6·73	+ 2·43	+ 5·23
	− 555·559	− 555·698	+ ·139		+ ·236				
26	− 207·889	− 207·925	+ ·036						
				+ ·108	+ ·308	− 2·07	+ 10·36	+ 2·06	+ 10·41
	− 205·180	− 205·359	+ ·179		+ ·202				
27	...	...	...						
				+ 1·472	+ 1·672	+ 0·27	+ 11·03	+ 0·24	+ 11·03
	+ 29·833	+ 28·361	+ 1·472						
28	+ 265·474	+ 265·480	− ·006						
				+ ·006	+ ·206	+ 2·67	+ 10·18	− 1·60	+ 10·03
	+ 268·019	+ 268·000	+ ·019		+ ·212				
Aug. 3	+ 284·836	+ 283·644	+ 1·192						
				+ 1·151	+ 1·251	+ 2·82	+ 10·07	+ 2·76	− 10·15
	+ 282·298	+ 281·188	+ 1·110		+ 1·174				

Note :—July 27 rejected.

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TABLE XII—*continued.*

FOURTH SATELLITE IV x.

Date.	Observed Abscissa.	Calculated Abscissa.	O—C.	Mean O—C.	First Second Approximation.	Coefficients of Δa_4	Δh_4	Δk_4	$\Delta \epsilon_4$
	"	"	"	"	"				
Aug. 6	− 399·660	− 399·971	+ 0·311						
				+ 0·436	+ ·436	− 4·01	+ 9·17	− 2·33	− 8·79
	− 401·535	− 402·095	+ ·560		+ ·507				
7	− 558·410	− 558·702	+ ·292						
				+ ·508	+ ·408	− 5·59	+ 7·05	− 2·07	− 5·49
	− 559·179	− 559·903	+ ·724		+ ·416				
17	+ 634·456	+ 634·158	+ ·298						
				+ ·440	+ ·240	+ 6·34	+ 5·57	+ 1·27	− 1·47
	+ 634·363	+ 633·780	+ ·583		+ ·301				
18	+ 554·149	+ 552·658	+ 1·491						
				+ 1·168	+ ·968	+ 5·52	+ 6·75	+ 3·11	− 5·69
	+ 552·110	+ 551·264	+ 0·846		+ ·984				
19	+ 403·754	+ 402·026	+ 1·728						
				+ 1·728	+ 1·528	+ 4·01	+ 8·71	+ 3·57	− 8·75
	+ 401·671	+ 399·942	+ 1·729		+ 1·024				
24	− 582·517	− 583·271	+ 0·754						
				+ ·701	+ ·401	− 5·84	+ 6·50	− 1·44	− 4·29
	− 583·679	− 584·327	+ ·648		+ ·276				
25	− 629·609	− 629·771	+ ·162						
				+ ·260	− ·040	− 6·30	+ 5·52	+ 0·25	− 0·45
	− 629·526	− 629·883	+ ·357		+ ·162				
27	− 468·789	− 469·274	+ ·485						
				+ ·472	+ ·072	− 4·68	+ 7·56	+ 3·41	+ 7·21
	− 466·958	− 467·417	+ ·459		+ ·270				
28	− 278·262	− 278·724	+ ·462						
				+ ·515	+ ·115	− 2·77	+ 9·52	+ 3·15	+ 9·73
	− 275·589	− 276·157	+ ·568		+ ·139				
31	+ 403·448	+ 402·962	+ ·486						
				+ ·646	+ ·246	+ 4·04	+ 8·84	− 1·59	+ 8·25
	+ 405·786	+ 404·980	+ ·806		+ ·258				
Sept. 1	+ 553·914	+ 553·063	+ ·851						
				+ ·856	+ ·456	+ 5·54	+ 6·81	− 1·36	+ 4·98
	+ 555·281	+ 554·419	+ ·862		+ ·263				

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TABLE XII—*continued.*

FOURTH SATELLITE IV x_0

Date.	Observed Abscissa.	Calculated Abscissa.	O—C.	Mean O—C.	First / Second Approximation.	Coefficients of Δa_4	Δh_4	Δk_4	Δe_4
Sept. 2	+ 620″.449	+ 619″.409	+ 1″.040	″	″				
				+ 0.830	+ 0.330	+ 6.20	+ 5.46	+ 0.06	+ 1.32
	+ 620.390	+ 619.771	+ .619		+ .304				
3	+ 607.134	+ 606.515	+ .619						
				+ 0.662	+ .162	+ 6.06	+ 5.55	+ 2.02	− 2.68
	+ 606.523	+ 605.819	+ 0.704		+ .548				
4	+ 510.557	+ 509.196	+ 1.361						
				+ 1.346	+ .846	+ 5.08	+ 6.92	+ 3.50	− 6.37
	+ 508.884	+ 507.554	+ 1.330		+ .876				
5	+ 334.234	+ 332.787	+ 1.447						
				+ 1.434	+ 0.934	+ 3.32	+ 9.01	+ 3.60	− 9.26
	+ 332.003	+ 330.581	+ 1.422		+ 1.060				
6	+ 127.702	+ 126.120	+ 1.582						
				+ 1.636	+ 1.136	+ 1.25	+ 10.49	+ 2.33	− 10.69
	+ 125.095	+ 123.406	+ 1.689		+ 1.218				
7	− 106.041	− 107.163	+ 1.122						
				+ 1.253	+ 0.753	− 1.09	+ 10.75	+ 0.29	− 10.71
	− 108.905	− 110.289	+ 1.384		+ .766				
8	− 324.069	− 325.319	+ 1.250						
				+ 1.232	+ .632	− 3.26	+ 9.52	− 1.37	− 9.19
	− 326.216	− 327.431	+ 1.215		+ .704				
10	− 589.196	− 590.762	+ 1.566						
				+ 1.191	+ .591	− 5.91	+ 5.86	− 0.73	− 2.86
	− 590.648	− 591.464	+ 0.816		+ .463				
12	− 527.760	− 528.405	+ .645						
				+ 0.584	− .016	− 5.28	+ 6.28	+ 3.02	+ 5.20
	− 526.632	− 527.155	+ .523		+ .018				
13	− 383.100	− 383.553	+ .453						
				+ .389	− .211	− 3.83	+ 8.04	+ 3.59	+ 8.14
	− 381.352	− 381.677	+ 0.325		− .143				
14	− 181.726	− 182.811	+ 1.085						
				+ .890	+ .290	− 1.82	+ 9.74	+ 2.80	+ 10.01
	− 179.313	− 180.209	+ 0.696		+ .248				

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TABLE XII.—*continued*

FIRST SATELLITE, I *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of	
						Δp_1	Δq_1
July 8	+ 0″·060	+ 0″·022	+ 0″·038	0″·000	+ 0″·101	− 0″·29	− 2″·41
	− 0·015	+ 0·022	− ·037				
14	− 0·125	− 0·104	− ·021	− ·012	+ ·077	+ 2·06	+ 1·35
	− 0·108	− 0·104	− ·004				
15	− 0·162	+ 0·081	− ·243	− ·045	+ ·133	− 1·58	− 1·88
	+ 0·234	+ 0·081	+ ·153				
16	− 0·386	− 0·039	− ·347	− ·335	− ·325	+ 0·69	+ 2·37
	− 0·362	− 0·039	− ·323				
19	− 0·025	− 0·090	+ ·065	+ ·058	+ ·100	+ 2·09	− 1·32
	− 0·039	− 0·090	+ ·051				
24	+ 0·150	− 0·004	+ ·154	+ ·176	+ ·185	+ 0·24	− 2·47
	+ 0·195	− 0·004	+ ·199				
26	− 0·194	− 0·053	− ·141	− ·065	+ ·139	+ 1·60	− 1·91
	− 0·042	− 0·053	+ ·011				
27	+ 0·046	+ 0·072	− ·026	− ·050	− ·067	− 2·15	+ 1·28
	− 0·002	+ 0·072	− ·074				
Aug. 3	− 0·081	+ 0·031	− ·112	− ·200	− ·052	− 1·76	+ 1·79
	− 0·257	+ 0·031	− ·288				
4	+ 0·061	− 0·044	+ ·105	− ·021	− ·017	+ 2·45	+ 0·51
	− 0·191	− 0·044	− ·147				

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TABLE XII.—*continued.*

FIRST SATELLITE, I *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of	
						Δp_1	Δq_1
Aug. 6	− 0″·046	− 0″·031	− 0″·015				
				− 0·060	− 0·036	+ 2·25	+ 1·10
	− 0·137	− 0·031	− ·106				
7	+ 0·323	+ 0·022	+ ·301				
				+ ·185	+ ·155	− 1·48	− 2·01
	+ 0·091	+ 0·022	+ ·069				
17	− 0·232	− 0·016	− ·216				
				− ·236	− ·260	− 0·18	+ 2·49
	− 0·271	− 0·016	− ·255				
18	+ 0·134	+ 0·042	+ ·092				
				+ ·068	+ ·246	+ 1·66	− 1·85
	+ 0·087	+ 0·042	+ ·045				
19	...	...	...				
				− ·583	− ·062	− 2·13	+ 1·30
	− 0·638	− 0·055	− ·583				
24	− 0·417	+ 0·011	− ·428				
				− ·358	− ·265	+ 0·76	+ 2·36
	− 0·277	+ 0·011	− ·288				
25	+ 0·200	+ 0·008	+ ·192				
				+ ·142	+ ·146	− 0·26	− 2·45
	+ 0·101	+ 0·008	+ ·093				
27	+ 0·198	+ 0·090	+ ·108				
				+ ·108	+ ·142	+ 1·62	− 1·85
	+ 0·199	+ 0·090	+ ·109				
28	− 0·267	− 0·119	− ·148				
				− ·202	− ·025	− 2·24	+ 1·03
	− 0·374	− 0·119	− ·255				
31	− 0·050	+ 0·056	− ·106				
				− ·305	− ·274	+ 1·33	+ 2·06
	− 0·448	+ 0·056	− ·504				

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TABLE XII.—*continued.*

FIRST SATELLITE, I *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of Δp_1	Coefficients of Δq_1
				"	"		
Sept. 1	+ 0".194	+ 0".113	+ 0".081	+ 0".102	+ 0".136	− 0".24	− 2".42
	+ 0".237	+ 0".113	+ ".124				
2	+ 0".329	− 0".042	+ ".371	+ ".236	− ".072	− 0".23	+ 2".44
	+ 0".060	− 0".042	+ ".102				
4	− 0".219	− 0".148	− ".071	− ".079	− ".177	− 1".83	+ 1".60
	− 0".235	− 0".148	− ".087				
7	+ 0".121	+ 0".142	− ".021	− ".168	− ".156	+ 2".04	+ 1".30
	− 0".174	+ 0".142	− ".316				
8	+ 0".530	− 0".075	+ ".605	+ ".321	+ ".074	− 1".27	− 2".03
	− 0".038	− 0".075	+ ".037				
10	+ 0".106	+ 0".077	+ ".029	+ ".054	+ ".212	+ 0".49	− 2".34
	+ 0".155	+ 0".077	+ ".078				
14	+ 0".055	+ 0".199	− ".144	− ".176	− ".079	+ 2".22	+ 0".86
	− 0".010	+ 0".199	− ".209				

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TABLE XII.—*continued.*

SECOND SATELLITE, II *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of						
						Δa_2	Δh_2	Δk_2	Δx_2	Δp_2	Δq_2	Δe_2
July 8	″ ...	″ ...	″ ...	″	″							
	+0·649	−0·171	+0·820	+0·820	+0·921	0·00	+0·01	+0·03	−0·02	+1·52	−3·60	−0·03
15	+1·042	+0·474	+ ·568									
	+1·182	+0·474	+ ·708	+ ·638	+ ·816	0·00	0·00	+0·03	−0·02	+0·27	−3·94	−0·03
16	−1·636	−1·983	+ ·347									
	−1·736	−1·983	+ ·247	+ ·297	+ ·307	−0·02	+0·01	+0·02	0·00	+3·84	+0·92	0·00
19	−0·402	−1·074	+ ·672									
	−0·218	−1·074	+ ·856	+ ·764	+ ·806	−0·01	+0·02	+0·03	0·00	+3·06	−2·54	−0·03
20	−1·866	−1·439	− ·427									
	−2·025	−1·439	− ·586	− ·506	− ·446	−0·01	0·00	+0·03	−0·02	+1·93	+3·43	+0·02
24	−0·684	+0·152	− ·836									
	−0·718	+0·152	− ·870	− ·853	− ·844	0·00	+0·01	+0·03	−0·01	−1·48	+3·63	+0·03
26	−0·273	−0·712	+ ·439									
	−0·149	−0·712	+ ·563	+ ·501	+ ·705	−0·01	+0·02	+0·02	+0·01	+2·52	−3·12	−0·03
27	−1·909	−1·695	− ·214									
	−2·125	−1·695	− ·430	− ·322	− ·339	−0·02	0·00	+0·02	−0·01	+2·70	+2·91	+0·02
28	+0·481	+1·349	− ·868									
	+0·619	+1·349	− ·730	− ·799	− ·671	+0·01	+0·02	+0·02	0·00	−3·48	+1·83	+0·02
Aug. 3	−2·078	−1·797	− ·281									
	−2·177	−1·797	− ·380	− ·330	− ·182	−0·02	0·00	+0·02	−0·01	+3·17	+2·41	+0·01
4	+0·727	+1·406	− ·679									
	+0·743	+1·406	− ·663	− ·671	− ·667	+0·01	+0·02	+0·02	0·00	−3·63	+1·56	+0·02

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TABLE XII.—*continued.*

SECOND SATELLITE, II *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of						
						Δa_2	Δh_2	Δk_2	Δx_2	Δp_2	Δq_2	$\Delta \epsilon_2$
Aug. 6	−1″·078	−1″·521	+0″·443									
				+0·466	+0·490	−0·02	+0·02	+0·02	0·00	+3·82	−1·24	−0·02
	−1·031	−1·521	+0·490									
7	−1·380	−0·707	− ·673									
				− ·726	− ·756	−0·01	+0·01	+0·03	−0·01	+0·28	+3·94	+0·03
	−1·487	−0·707	− ·780									
17	−1·440	−1·747	+ ·307									
				+ ·246	+ ·222	−0·02	+0·01	+0·02	0·00	+3·89	+0·82	0·00
	−1·563	−1·747	+ ·184									
18	−0·493	+0·356	− ·849									
				− ·999	− ·821	0·00	+0·01	+0·03	0·00	−2·04	+3·35	+0·03
	−0·793	+0·356	−1·149									
19	+1·309	+1·623	− ·314									
				− ·230	+ ·116	+0·02	0·00	+0·02	0·00	−3·01	−2·57	−0·01
	+1·476	+1·623	− ·147									
21	−1·621	−1·384	− ·237									
				− ·370	− ·410	−0·01	0·00	+0·02	−0·01	+2·23	+3·24	+0·02
	−1·886	−1·384	− ·502									
25	−1·124	−0·449	− ·675									
				− ·716	− ·712	0·00	+0·01	+0·03	0·00	−0·32	+3·88	+0·03
	−1·206	−0·449	− ·757									
27	+0·818	−0·188	+1·006									
				+ ·804	+ ·838	0·00	+0·01	+0·02	+0·01	+1·82	−3·51	−0·03
	+0·413	−0·188	+0·601									
28	−1·848	−1·539	− ·309									
				− ·414	− ·237	−0·02	0·00	+0·02	0·00	+3·04	+2·44	+0·01
	−2·058	−1·539	− ·519									
Sept. 1	−1·391	−0·516	− ·875									
				− ·807	− ·773	−0·01	+0·01	+0·03	0·00	−0·16	+3·85	+0·03
	−1·255	−0·516	− ·739									
2	+1·722	+1·511	+ ·211									
				− ·012	− ·320	+0·02	+0·01	+0·01	0·00	−3·83	−0·58	+0·01
	+1·276	+1·511	− ·235									

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TABLE XII.—*continued.*

SECOND SATELLITE, II *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of						
						Δa_2	Δh_2	Δk_2	Δr_2	Δp_2	Δq_2	$\Delta \epsilon_2$
	"	"	"	"	"							
Sept. 3	+0·902	+0·095	+0·807	+0·803	+0·811	0·00	+0·01	+0·03	0·00	+1·15	−3·74	−0·03
	+0·894	+0·095	+ ·799									
4	−1·422	−1·518	+ ·096	+ ·133	+ ·035	−0·02	0·00	+0·01	0·00	+3·41	+1·80	0·00
	−1·348	−1·518	+ ·170									
5	−0·226	+0·580	− ·806	− ·748	− ·682	−0·01	+0·02	+0·01	+0·02	−2·71	+2·70	+0·02
	−0·109	+0·580	− ·689									
6	+1·510	+1·310	+ ·200	+ ·196	+ ·335	+0·01	0·00	+0·02	0·00	−2·35	−3·07	−0·01
	+1·501	+1·310	+ ·191									
7	−0·282	−0·975	+ ·693	+ ·570	+ ·582	−0·01	+0·01	+0·02	+0·01	+3·45	−1·74	−0·02
	−0·529	−0·975	+ ·446									
8	−0·793	−0·873	+ ·080	− ·286	− ·533	−0·01	0·00	+0·02	0·00	+0·89	+3·70	+0·02
	−1·526	−0·873	− ·653									
10	+0·885	+0·392	+ ·493	+ ·554	+ ·712	0·00	+0·01	+0·02	0·00	+0·52	−3·82	−0·02
	+1·006	+0·392	+0·614									
12	−0·684	+0·344	−1·028	− ·964	− ·739	0·00	+0·01	+0·02	+0·01	−2·30	+2·98	+0·02
	−0·557	+0·344	−0·901									
13	+1·500	+1·322	+ ·178	+ ·198	+ ·227	+0·01	0·00	+0·02	0·00	−2·64	−2·74	−0·01
	+1·539	+1·322	+ ·217									
14	−0·248	−0·731	+ ·483	+ ·538	+ ·635	−0·01	+0·01	+0·02	+0·01	+3·12	−2·18	−0·02
	−0·138	−0·731	+ ·593									

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TABLE XII.—*continued.*

THIRD SATELLITE, III *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of						
						Δa_3	Δh_3	Δk_3	Δx_3	Δp_3	Δq_3	Δa_3
July 8	″ ...	″ ...	″ ...	″ +0·082	″ +0·183	0·00	+0·01	0·00	−0·01	−4·93	−3·69	+0·01
	+0·384	+0·302	+0·082									
14	−0·635	−0·405	− ·230	− ·156	− ·067	0·00	+0·01	0·00	−0·01	−5·68	+2·56	+0·01
	−0·486	−0·405	− ·081									
15	+0·023	+0·155	− ·132	− ·037	+ ·141	0·00	+0·01	0·00	−0·01	−5·67	−2·60	+0·01
	+0·213	+0·155	+ ·058									
16	+0·656	+0·622	+ ·034	+ ·265	+ ·275	+0·01	+0·01	0·00	−0·01	−1·62	−6·04	+0·01
	+1·118	+0·622	+ ·496									
19	−0·750	−0·389	− ·361	− ·438	− ·396	0·00	+0·01	0·00	−0·01	+4·32	+4·59	−0·01
	−0·905	−0·389	− ·516									
20	−0·854	−0·703	− ·151	− ·210	− ·150	−0·01	+0·01	0·00	−0·01	−0·76	+6·25	0·00
	−0·972	−0·703	− ·269									
24	+0·597	+0·694	− ·097	+ ·108	+ ·117	+0·01	+0·01	0·00	0·00	+3·27	−5·41	0·00
	+1·008	+0·694	+ ·314									
26	−0·640	−0·257	− ·383	− ·354	− ·150	0·00	+0·01	0·00	−0·01	+4·97	+3·94	−0·01
	−0·581	−0·257	− ·324									
27	−0·704	−0·681	− ·023	− ·100	− ·117	−0·01	+0·01	0·00	−0·01	+0·26	+6·33	0·00
	−0·858	−0·681	− ·177									
28	−0·909	−0·635	− ·274	− ·204	− ·076	−0·01	+0 01	0·00	−0 01	−4·78	+4·15	+0·01
	−0·768	−0·635	− ·133									
Aug. 3	−1·108	−0·630	− ·478	− ·329	− ·181	−0·01	+0·01	−0·01	−0·01	+1·10	+6·26	−0·01
	−0·810	−0·630	− ·180									

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TABLE XII.—*continued.*

THIRD SATELLITE, III *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of						
						Δa_3	Δh_3	Δk_3	Δx_3	Δp_3	Δq_3	$\Delta \epsilon_3$
Aug. 4	−0″·847	−0″·712	−0″·135	−0·070	−0·066	−0·01	+0·01	0·00	−0·01	−4·75	+4·20	0·00
	−0·718	−0·712	− ·006									
6	+0·383	+0·314	+ ·069	+ ·153	+ ·177	0·00	+0·01	−0·01	−0·01	−3·96	−4·95	+0·01
	+0·551	+0·314	+ ·237									
7	+1·052	+0·766	+ ·286	+ ·284	+ ·254	+0·01	+0·01	0·00	−0·01	+1·43	−6·19	0·00
	+1·048	+0·766	+ ·282									
17	−0·487	−0·372	− ·115	− ·262	− ·286	0·00	+0·01	−0·01	−0·01	+2·96	+5·60	−0·01
	−0·781	−0·372	− ·409									
18	−1·212	−0·887	− ·325	− ·402	− ·224	−0·01	+0·01	−0·01	−0·01	−2·76	+5·68	0·00
	−1·367	−0·887	− ·480									
21	+0·802	+0·631	+ ·171	+ ·274	+ ·234	+0·01	+0·01	−0·01	−0·01	−0·80	−6·24	+0·01
	+1·007	+0·631	+ ·376									
24	−0·390	−0·167	− ·223	− ·274	− ·181	0·00	+0·01	−0·01	−0·02	+3·80	+5·01	−0·02
	−0·493	−0·167	− ·326									
25	−0·883	−0·832	− ·051	− ·166	− ·162	−0·01	0·00	−0·01	−0·01	−1·11	+6·17	−0·01
	−1·114	−0·832	− ·282									
27	−0·443	−0·417	− ·026	+ ·010	+ ·044	0·00	+0·02	−0·01	−0·02	−5·82	−2·23	+0·02
	−0·370	−0·417	+ ·047									
28	+0·511	+0·457	+ ·054	+ ·054	+ ·231	0·00	+0·01	−0·01	−0·01	−1·95	−5·92	+0·02
	+0·511	+0·457	+ ·054									
31	+0·082	+0·042	+ ·040	− ·195	− ·164	0·00	+0·01	−0·01	−0·02	+4·38	+4·42	−0·02
	−0·388	+0·042	− ·430									

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TABLE XII.—*continued.*

THIRD SATELLITE, III *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of						
						Δa_3	Δl_3	Δk_3	Δx_3	Δp_3	Δq_3	$\Delta \epsilon_3$
Sept. 1	−1″·231	−0″·820	−0″·411									
				−0·233	−0·199	−0·01	+0·01	−0·01	−0·01	−0·70	+6·16	−0·01
	−0·875	−0·820	− ·055									
2	−0·535	−1·102	+ ·567									
				+ ·268	− ·040	−0·01	+0·01	−0·01	−0·01	−4·97	+3·66	0·00
	−1·134	−1·102	− ·032									
4	+0·616	+0·279	+ ·337									
				+ ·344	+ ·246	0·00	+0·01	−0·02	−0·01	−2·72	−5·51	+0·02
	+0·630	+0·279	+ ·351									
5	+1·025	+1·053	− ·028									
				+ ·146	+ ·212	+0·01	0 00	−0·01	−0·01	+2·73	−5·52	+0·01
	+1·373	+1·053	+ ·320									
6	+0·816	+1·061	− ·245									
				− ·138	+ ·001	+0·01	+0·01	−0·01	−0·02	+5·93	−1·64	−0·01
	+1·030	+1·061	− ·031									
7	+0·361	+0·315	+ ·046									
				− ·078	− ·066	0·00	+0·01	−0·01	−0·02	+5·02	+3·54	−0·02
	+0·114	+0·315	− ·201									
8	−0·252	−0·678	+ ·426									
				+ ·082	− ·165	−0·01	+0·01	−0·02	−0·01	+0·42	+6·10	−0·02
	−0·940	−0·678	− ·262									
10	−0·990	−0·867	− ·123									
				− ·070	+ ·088	−0·01	+0·01	−0·01	−0·01	−6·04	−0·62	+0·01
	−0·884	−0·867	− ·017									
12	+0·721	+1·038	− ·317									
				− ·068	+ ·157	+0·01	0·00	−0·01	0·00	+2·08	−5·69	+0·01
	+1·220	+1·038	+ ·182									
13	+1·108	+1·197	− ·089									
				− ·054	− ·025	+0·01	+0·01	−0·01	−0·01	+5·64	−2·20	−0·01
	+1·177	+1·197	− ·020									
14	+0·252	+0·524	− ·272									
				− ·210	− ·113	+0·01	+0·02	−0·01	−0·02	+5·31	+2·89	−0·02
	+0·377	+0·524	− ·147									

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TABLE XII.—*continued.*

FOURTH SATELLITE, IV *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of					
						Δa_4	Δh_4	Δk_4	Δp_4	Δq_4	$\Delta \epsilon_4$
July 8	+ 0″·683	+ 1″·394	− 0″·711								
				− 0·734	− 0·633	+0·01	−0·04	−0·03	− 7·21	+ 8·00	−0·05
	+ 0·636	+ 1·394	− ·758								
14	− 3·461	− 3·297	− ·164								
				− ·206	− ·117	−0·03	−0·03	−0·02	− 1·66	− 10·86	+0·02
	− 3·544	− 3·297	− ·247								
15	− 2·909	− 2·786	− ·123								
				− ·009	+ ·169	−0·03	−0·03	−0·03	+ 2·32	− 10·80	+0·04
	− 2·681	− 2·786	+ ·105								
16	− 1·430	− 1·875	+ ·445								
				+ ·460	+ ·470	−0·02	−0·04	−0·03	+ 6·12	− 9·26	+0·05
	− 1·400	− 1·875	+ ·475								
19	+ 2·538	+ 1·829	+ ·709								
				+ ·714	+ ·756	+0·02	−0·05	−0·01	+ 11·05	+ 1·49	+0·05
	+ 2·548	+ 1·829	+ ·719								
20	+ 3·261	+ 2·787	+ ·474								
				+ ·464	+ ·524	+0·03	−0·04	0·00	+ 9·74	+ 5·40	+0·04
	+ 3·242	+ 2·787	+ ·455								
24	+ 1·775	+ 2·153	− ·378								
				− ·364	− ·355	+0·02	−0·04	−0·03	− 4·94	+ 9·86	−0·05
	+ 1·802	+ 2·153	− ·351								
26	− 1·209	− 0·274	− ·935								
				− ·865	− ·661	0·00	−0·06	−0·02	− 10·28	+ 3·96	−0·06
	− 1·069	− 0·274	− ·795								
27	...	...	...								
				− 1·089	...	−0·02	−0·06	−0·01	− 11·04	− 0·08	−0·06
	− 2·635	− 1·546	− 1·089								

Note:—July 27 rejected.

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TABLE XII.—*continued.*

FOURTH SATELLITE, IV *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of					
						Δa_4	Δh_4	Δk_4	Δp_4	Δq_4	$\Delta \epsilon_4$
July 28	− 3″361	− 2″642	− 0″719								
				− 0·692	− 0·564	−0·03	−0·05	−0·01	− 10·21	− 4·27	−0·04
	− 3·307	− 2·642	− ·665								
Aug. 3	+ 0·551	− 0·046	+ ·597								
				+ ·498	+ ·646	0·00	−0·06	−0·03	+ 9·88	− 5·45	+0·06
	+ 0·352	− 0·046	+ ·398								
6	+ 3·724	+ 3·309	+ ·415								
				+ ·522	+ ·546	+0·03	−0·04	−0·01	+ 9·18	+ 6·47	+0·03
	+ 3·938	+ 3·309	+ ·629								
7	+ 3·973	+ 3·656	+ ·317								
				+ ·308	+ ·278	+0·04	−0·03	−0·01	+ 6·07	+ 9·41	0·00
	+ 3·955	+ 3·656	+ ·299								
17	− 2·586	− 3·039	+ ·453								
				+ ·358	+ ·334	−0·03	−0·03	−0·04	+ 0·55	− 11·15	+0·04
	− 2·776	− 3·039	+ ·263								
18	− 1·680	− 1·909	+ ·229								
				+ ·204	+ ·382	−0·02	−0·04	−0·04	+ 4·85	− 10·08	+0·06
	− 1·731	− 1·909	+ ·178								
19	− 0·612	− 0·597	− ·015								
				+ ·294	+ ·640	−0·01	−0·05	−0·04	+ 8·10	− 7·73	+0·07
	+ 0·005	− 0·597	+ ·602								
21	+ 3·076	+ 2·114	+ ·962								
				+ ·758	+ ·718	+0·02	−0·06	−0·02	+ 11·18	− 0·46	+0·06
	+ 2·667	+ 2·114	+ ·553								
24	+ 4·056	+ 3·820	+ ·236								
				+ ·200	+ ·293	+0·04	−0·02	−0·02	+ 5·26	+ 9·72	−0·01
	+ 3·983	+ 3·820	+ ·163								
25	+ 3·378	+ 3·367	+ ·011								
				− ·036	− ·032	+0·03	−0·02	−0·04	+ 1·53	+ 10·90	−0·03
	+ 3·283	+ 3·367	− ·084								

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TABLE XII.—*continued.*

FOURTH SATELLITE, IV *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of					
						Δa_4	Δh_4	Δk_4	Δp_4	Δq_4	$\Delta \epsilon$
Aug. 27	+ 0.″479	+ 1.″085	− 0.″606								
				− 0.526	− 0.492	+0.01	−0.05	−0.05	− 6.33	+ 8.87	−0.06
	+ 0.639	+ 1.085	− .446								
28	− 1.104	− 0.410	− .694								
				− .755	− .578	0.00	−0.05	−0.04	− 9.17	+ 5.83	−0.07
	− 1.226	− 0.410	− .816								
31	− 4.053	− 3.758	− .295								
				− .418	− .387	−0.04	−0.03	−0.02	− 8.98	− 6.10	−0.02
	− 4.298	− 3.758	− .540								
Sept. 1	− 4.160	− 3.907	− .253								
				− .186	− .152	−0.04	−0.02	−0.02	− 6.03	− 9.05	+0.01
	−4.026	− 3.907	− .119								
2	− 3.030	− 3.519	+ .489								
				+ .222	− .086	−0.03	−0.02	−0.04	− 2.54	− 10.59	+0.03
	− 3.564	− 3.519	− .045								
3	− 2.501	− 2.631	+ .130								
				+ .149	+ .157	−0.03	−0.03	−0.05	+ 1.44	− 10.81	+0.05
	− 2.463	− 2.631	+ .168								
4	− 0.808	− 1.340	+ .532								
				+ .548	+ .450	−0.01	−0.04	−0.05	+ 5.30	− 9.55	+0.07
	− 0.775	− 1.340	+ .565								
5	+ 0.494	+ 0.212	+ .282								
				+ .413	+ .479	0.00	−0.05	−0.05	+ 8.51	− 6.84	+0.07
	+ 0.756	+ 0.212	+ .544								
6	+ 2.062	+ 1.626	+ .436								
				+ .492	+ .631	+0.02	−0.06	−0.03	+ 10.35	− 3.44	+0.06
	+ 2.175	+ 1.626	+ .549								
7	+ 3.627	+ 2.883	+ .744								
				+ .642	+ .654	+0.03	−0.05	−0.02	+ 10.86	+ 0.59	+0.05
	+ 3.423	+ 2.883	+ .540								

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TABLE XII.—*continued.*

FOURTH SATELLITE, IV *y*.

Date.	Observed Ordinate.	Calculated Ordinate.	O—C.	Mean O—C.	Second Approximation.	Coefficients of					
						Δa_4	Δh_4	Δk_4	Δp_4	Δq_4	$\Delta \epsilon_4$
Sept. 8	+ 4″·824	+ 3″·727	+ 1″·097	″	″						
				+ 0·802	+ 0·555	+0·04	−0·04	−0·02	+ 9·82	+ 4·52	+0·03
	+ 4·235	+ 3·727	+ 0·508								
10	+ 3·675	+ 3·756	− ·081								
				− ·076	+ ·082	+0·04	−0·02	−0·04	+ 4·13	+ 9·88	−0·02
	+ 3·686	+ 3·756	− ·070								
12	+ 0·886	+ 1·604	− ·718								
				− ·586	− ·361	+0·02	−0·04	−0·05	− 3·98	+ 9·80	−0·06
	+ 1·149	+ 1·604	− ·455								
13	− 0·417	+ 0·153	− ·570								
				− ·532	− ·503	0·00	−0·05	−0·05	− 7·21	+ 7·67	−0·07
	− 0·341	+ 0·153	− ·494								
14	− 2·169	− 1·358	− ·811								
				− ·731	− ·634	−0·01	−0·06	−0·04	− 9·51	+ 4·44	−0·07
	− 2·009	− 1·358	− ·651								

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CHAPTER IV.

Solution of Equations of Condition.

Normal equations were formed from the equations of condition in the usual way. It was decided not to weight the equations according to the quality of the images on the plate; for in the case of visual observations experience had shown that nothing was gained by weighting according to the seeing for reasons stated in the writer's former paper (Vol. XII. Part II. pp. 134–5), and it seemed likely that the same reasons would hold in the case of photographic measurements. Half weight was assigned to one equation, namely Aug. 19 I.x, because the two values of O – C were very discordant: the observer's note on this plate was "Very bad images and *réseau*: moonlight." Half weight ought to have been, but by an oversight was not, assigned to both equations for II. and III. July 8, and to both equations for IV. July 27; in these cases only the second set of three images was visible, the satellites having been occulted during the first set.

In forming the normals all the multiplications were done out in full and checked to the last figure. The two unnecessary figures were not discarded till all the necessary additions had been made and checked.

The normals for satellite I. are very simple; there are no terms which are common to the normals in x and the normals in y, so that they could be solved separately.

In the case of the other satellites, there are terms common to both sets of equations. On the following pages, the normal in x is printed first, immediately under it is the normal for the same unknown in y, and beneath is the sum of the two. The last are the normals to be solved.

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NORMALS I x.

Δa_1	Δh_1	Δk_1	Δx_1	$\Delta \epsilon_1$	
+ 31·22	− 0·82	+ 2·20	+ 1·90	− 1·98	= A_1
	+ 76·65	+ 6·26	− 65·19	− 8·11	= H_1
		+ 6·38	− 3·56	+ 2·37	= K_1
			+ 57·25	+ 7·10	= X_1
				+ 59·09	= E_1

NORMALS I y.

Δp_1	Δq_1	
+ 58·73	− 0·70	= P_1
	+ 95·51	= Q_1

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Normals II.

Δa_2	Δh_2	Δk_2	Δx_2	$\Delta \epsilon_2$	Δp	Δq	
+ 88·44	− 17·26	+ 6·93	− 20·62	− 10·57	0·00	0·00	$= {}_xA_2$
0·00	0·00	0·00	0·00	0·00	− 0·88	− 0·40	$= {}_yA_2$
+ 88·44	− 17·26	+ 6·92	− 20·62	− 10·57	− 0·88	− 0·40	$= A_2$
	+ 265·65	+ 24·79	+ 230·35	− 58·42	0·00	0·00	$= {}_xH_2$
	0·00	+ 0·01	0·00	0·00	+ 0·09	+ 0·03	$= {}_yH_2$
	+ 265·65	+ 24·79	+ 230·35	− 58·42	+ 0·09	+ 0·03	$= H_2$
		+ 20·95	+ 15·62	+ 6·70	0·00	0·00	$= {}_xK_2$
		+ 0·02	0·00	0·00	+ 0·45	+ 0·26	$= {}_yK_2$
		+ 20·96	+ 15·62	+ 6·70	+ 0·45	+ 0·26	$= K_2$
			+ 207·20	− 51·49	0·00	0·00	$= {}_xX_2$
			0·00	0·00	− 0·11	− 0·10	$= {}_yX_2$
			+ 207·20	− 51·49	− 0·11	− 0·10	$= X_2$
				+ 219·08	0·00	0·00	$= {}_xE_2$
				+ 0·02	− 0·62	+ 1·93	$= {}_yE_2$
				+ 219·10	− 0·62	+ 1·93	$= E_2$
					0·00	0·00	$= {}_xP_2$
					+ 217·51	− 14·23	$= {}_yP_2$
					+ 217·51	− 14·23	$= P_2$
						0·00	$= {}_xQ_2$
						271·46	$= {}_yQ_2$
						271·46	$= Q_2$

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Normals III.

Δa_3	Δh_3	Δk_3	Δx_3	$\Delta \epsilon_3$	Δp_3	Δq_3	
+258·53	− 57·53	− 2·34	+ 46·92	+ 28·13	0·00	0·00	$= {}_xA_3$
0·00	0·00	0·00	0·00	0·00	+ 0·48	− 0·90	$= {}_yA_3$
+258·53	− 57·53	− 2·34	+ 46·92	+ 28·13	+ 0·48	− 0·90	$= A_3$
	+645·84	+ 28·04	−571·89	+ 47·12	0·00	0·00	$= {}_xH_3$
	0·00	0·00	0·00	0·00	− 0·10	+ 0·29	$= {}_yH_3$
	+645·84	+ 28·04	−571·90	+ 47·12	− 0·10	+ 0·29	$= H_3$
		+ 45·79	− 12·46	+ 16·10	0·00	0·00	$= {}_xK_3$
		0·00	0·00	0·00	− 0·06	− 0·16	$= {}_yK_3$
		+ 45·79	− 12·46	+ 16·10	− 0·06	− 0·16	$= K_3$
			+521·44	− 8·95	0·00	0·00	$= {}_xX_3$
			0·00	0·00	− 0·08	− 0·46	$= {}_yX_3$
			+521·44	− 8·95	− 0·08	− 0·46	$= X_3$
				+494·40	0·00	0·00	$= {}_xE_3$
				0·00	− 1·11	− 1·29	$= {}_yE_3$
				+494·40	− 1·11	− 1·29	$= E_3$
					0·00	0·00	$= {}_xP_3$
					+501·34	+ 69·48	$= {}_yP_3$
					+501·34	+ 69·48	$= P_3$
						0·00	$= {}_xQ_3$
						+781·11	$= {}_yQ_3$
						+781·11	$= Q_3$

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Normals IV.

Δa_4	Δh_4	Δk_4	Δe_4	Δp_4	Δq_4	
+ 678·07	− 13·05	+ 48·14	− 163·67	0·00	0·00	$= {}_xA_4$
+ 0·02	0·00	0·00	0·00	+ 3·15	+ 5·43	$= {}_yA_4$
+ 678·09	− 13·05	+ 48·14	− 163·68	+ 3·15	+ 5·43	$= A_4$
	+2244·59	+ 266·50	− 234·53	0·00	0·00	$= {}_xH_4$
	+ 0·06	+ 0·04	− 0·01	− 1·51	+ 0·17	$= {}_yH_4$
	+2244·66	+ 266·54	− 234·53	− 1·51	+ 0·17	$= H_4$
		+ 171·72	+ 41·83	0·00	0·00	$= {}_xK_4$
		+ 0·04	0·00	− 0·55	+ 0·74	$= {}_yK_4$
		+ 171·76	+ 41·82	− 0·55	+ 0·74	$= K_4$
			+1901·44	0·00	0·00	$= {}_xE_4$
			+ 0·08	+ 10·81	− 8·54	$= {}_yE_4$
			+1901·52	+ 10·81	− 8·54	$= E_4$
				0·00	0·00	$= {}_xP_4$
				1980·93	− 267·85	$= {}_yP_4$
				1980·93	− 267·85	$= P_4$
					0·00	$= {}_xQ_4$
					2109·72	$= {}_yQ_4$
					2109·72	$= Q_4$

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It is necessary now to explain the method of solution of these normals. It is clear that if an error had been made in setting on Jupiter or if the correction for phase were erroneous, the co-ordinates of all four satellites would be affected by this error; and if the observation of the satellite's position had been perfect and exact, this error would appear as the residual common to all four equations of condition. We ought therefore to introduce into the equations the unknown corrections to the assumed centre of Jupiter and solve for these unknowns as well as for the corrections to the elements of the orbits. The corrections to the centre would vary from plate to plate, and since there are thirty-five plates and two co-ordinates, the rigorous method of treatment would involve the solution of over seventy normals. This would mean a quite unnecessary amount of labour and some other method had to be found. Another plan would have been to treat the satellites in pairs and to eliminate the position of Jupiter altogether by subtracting the equation of every satellite from the mean equation of all four or by subtracting the equations in pairs. The first alternative would have involved the formation and solution of twenty-seven normals. The second would have involved the formation of six sets of normals consisting of fourteen unknowns and the solution of twenty-seven normals. Such heavy labour did not seem to be justified and another procedure was adopted.

The equations were therefore solved in the following manner. The position of Jupiter on the plate was measured and rough corrections for phase were applied, as already explained. The resulting absolute terms were used as first approximations; the normals for each satellite were separately solved and the values of the unknowns found. Substituting in the equations of condition we get a series of residuals thus :—

	Sat. I.	Sat. II.	Sat. III.	Sat. IV.	Mean.
Plate 1	v_1	v_2	v_3	v_4	v
2	v_1'	v_2'	v_3'	v_4'	v'
3	v_1''	v_2''	v_3''	v_4''	v''
4	v_1'''	v_2'''	v_3'''	v_4'''	v'''
5	v_1''''	v_2''''	v_3''''	v_4''''	v''''
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

The mean residual for each plate was regarded as the correction to the adopted centre of Jupiter. The absolute terms were corrected by adding the residuals

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v v' v'' . . . . and a second solution was made with the new absolute terms. The values of the unknowns from this solution were second approximations and clearly the process of approximation could be carried as far as desired.

The four equations from one plate are not really independent, though no unknown is common to them; they are bound together by the fact that they were all derived from measurements made on one plate and they are all affected by any peculiarity that plate may possess. Moreover we might have expressed the corrections to the axes of the orbits in terms of corrections to the mass of Jupiter and to the constant of polar compression; and in that case all the equations would have contained the unknowns ΔM and ΔJ, so that they should not, strictly speaking, have been solved separately. Thus the most rigorous method of procedure would have been one which involved combination of the normals before solution. The combined normals should then have been solved by least squares, the object of which method is to find those values of the unknowns which make the sum of the squares of the residuals a minimum. Now in the method adopted in this paper the sum of the squares of the residuals for Satellite I. denoted by $[v_1\, v_1]$ was made a minimum; so also were $[v_2\, v_2]$, $[v_3\, v_3]$ and $[v_4\, v_4]$. This step constitutes the first approximation. Then by taking the mean residuals v, v', v'', ... we made

$$\begin{array}{llllllll}
v_1v_1 & + & v_2v_2 & + & v_3v_3 & + & v_4v_4 & = \text{ a minimum} \\
v_1'v_1' & + & v_2'v_2' & + & v_3'v_3' & + & v_4'v_4' & = \text{ a minimum} \\
v_1''v_1'' & + & v_2''v_2'' & + & v_3''v_3'' & + & v_4''v_4'' & = \text{ a minimum} \\
\cdot & & \cdot & & \cdot & & \cdot & \\
\cdot & & \cdot & & \cdot & & \cdot &
\end{array}$$

and the corresponding values of the unknowns were found (second approximation). Proceeding in this way we can go on till the sum of the squares of the residuals can no longer be reduced, *i.e.*, is a minimum. We shall then have arrived at the same values of the unknowns as we should have obtained had we introduced corrections to the adopted centre of Jupiter and solved the resulting eighty or ninety normals. The two methods are analytically the same for the reason that they both reduce the sum of the squares of the residuals to a minimum.

The normals were solved in the symbolical form, which was found so convenient in solving the much larger set of normals in the heliometer work (*Cape Annals* XII. Part II. pp. 162–3). The solutions are given on the pages following. The unit is one in the sixth place of decimals except for the first satellite where it is one in the fifth

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place. These solutions have the form of normals, so that, for instance, the value of the unknown $\Delta\epsilon_2$ is given by

$$\Delta\epsilon_2 \times 10^6 = 1163\,A_2 + 3518\,H_2 - 4597\,K_2 - 2159\,X_2 + 5192\,E_2 + 24\,P_2 - 31\,Q_2$$

where A_2, H_2, etc., are the absolute terms in the normals for Δa_2, Δh_2, etc.; and the weight of any unknown Δx is the reciprocal of the coefficient of X in the value of Δx. Thus the weight of $\Delta\epsilon_2$ is the reciprocal of 0·005192 or 192·6. The numerical values of A, H, K, etc., will be given presently.

After the values of the unknowns had been found, they were substituted in the equations of condition and the residuals were formed. There were three outstanding, viz.

I x	Aug. 19 (+0″·862)
IV x and y	July 27 (−1″·241 and +0″·381).

In the case of I. the separate pointings were very discordant; half weight had already been assigned to this observation (see page 79), and the very large residual showed that it should never have been included. It was therefore rejected and the normals were solved again, this equation having weight zero. This is the solution given below.

In the case of IV. examination showed that the measured abscissa of IV. was apparently in error. At my request Mr Hough remeasured at the Cape the position of this satellite; his ordinate agreed closely with mine but the abscissa differed considerably, and Mr Hough wrote, "The separate pointings are very erratic, and I consider them of little or no value in defining the position of the satellite, since at each exposure its image is more than half obscured by the disc of the planet and it is pure guess-work where its centre should be." The two equations for July 27 were accordingly taken out of the normals and the corrected normals were solved again. The result is given on p. 87.

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SOLUTION OF NORMALS I.

(Rejecting Aug. 19, x.)
(Unit one in *fifth* place of decimals.)

	A_1	H_1	K_1	X_1	E_1	P_1	Q_1
Δa_1 =	3362	−927	− 1099	−1214	131	0	0
Δh_1 =		63071	−22977	70339	1042	0	0
Δk_1 =			25894	−24549	−1085	0	0
Δx_1 =				80360	819	0	0
$\Delta \epsilon_1$ =					1862	0	0
Δp_1 =						1703	12
Δq_1 =							1047

SOLUTION OF NORMALS II.

(Unit one in *sixth* place of decimals.)

	A_2	H_2	K_2	X_2	E_2	P_2	Q_2
Δa_2 =	12479	−3691	− 4576	5979	1163	69	21
Δh_2 =		151388	−57166	−163478	3518	− 38	−59
Δk_2 =			76426	56192	−4597	−139	−25
Δx_2 =				182387	−2159	68	64
$\Delta \epsilon_2$ =					5192	24	−31
Δp_2 =						4614	242
Δq_2 =							3697

SOLUTION OF NORMALS III.

(Unit one in *sixth* place of decimals.)

	A_3	H_3	K_3	X_3	E_3	P_3	Q_3
Δa_3 =	4038	2576	− 562	2442	− 413	− 5	5
Δh_3 =		85486	−24958	92830	−5800	12	10
Δk_3 =			29526	− 26598	967	− 3	1
Δx_3 =				102768	−6259	13	12
$\Delta \epsilon_3$ =					2454	4	2
Δp_3 =						2020	− 180
Δq_3 =							1296

SOLUTION OF NORMALS IV.

(Rejecting July 27, x and y.)
(Unit one in *sixth* place of decimals.)

	A_4	H_4	K_4	E_4	P_4	Q_4
Δa_4 =	1572	128	− 679	188	− 5	− 4
Δh_4 =		628	−1038	161	− 1	1
Δk_4 =			7710	−441	5	− 2
$\Delta \epsilon_4$ =				621	− 4	2
Δp_4 =					548	70
Δq_4 =						483

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For the first approximation to the values of the unknowns the values of A, H, K, etc., were those derived from the absolute terms printed with the equations of condition, Table XII.; they are given in the following table, where the value from the equations in x is given first, under it is the value from the equations in y and then the sum. These sums were formed and checked before discarding unnecessary figures.

TABLE XIII.

Absolute Terms in Normals.

First Approximation.

	I.	II.	III.	IV.
A_x	+ 2".019	− 1".503	+ 5".425	+ 24".881
A_y	.000	− .006	+ .020	+ .120
A	+ 2.019	− 1.509	+ 5.446	+ 25.001
H_x	− 8.195	+ 11.184	− 22.221	+ 138.264
H_y	.000	− .014	− .020	− .047
H	− 8.195	+ 11.170	− 22.242	138.216
K_x	− 0.108	− 0.063	− 2.570	+ 19.628
K_y	.000	− .054	+ .007	+ .014
K	− 0.108	− 0.117	− 2.562	+ 19.642
X_x	+ 7.467	+ 9.224	+ 19.660	
X_y	.000	+ .006	+ .029	
X	+ 7.467	+ 9.230	+ 19.689	
E_x	+ 10.008	− 3.988	− 8.557	+ 90.821
E_y	.000	− .398	+ .045	− .728
E	+ 10.008	− 4.386	− 8.512	− 90.094
P_x	0.000	0.000	0.000	0.000
P_y	− 0.500	+ 23.110	− 7.936	+ 117.158
P	− 0.500	+ 23.110	− 7.936	+ 117.158
Q_x	0.000	0.000	0.000	0.000
Q_y	− 6.242	− 48.810	− 25.681	− 40.206
Q	− 6.242	− 48.810	− 25.681	− 40.206

The first approximations to the values of the unknowns were derived by substituting the above values of A, H, etc., in the solutions of the normals. The results of this substitution are given in the second column of Table XVII., p. 94. Residuals

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were then formed by substituting these values of the unknowns in the equations of condition, and were tabulated as follows.

TABLE XIV.

Residuals in x.

First Approximation.

Date.		I.	II.	III.	IV.	Mean.
July	8	+ 0″·061	+ 0″·216	+ 0″·114	− 0″·156	+ 0″·059
	14	− 71		− 132	− 156	− 120
	15	+ 54	+ 28	− 11	+ 11	+ 20
	16	− 144	+ 170	+ 108	− 16	+ 30
	19	− 92	− 363	− 146	− 116	− 179
	20		+ 75	− 5	+ 46	+ 39
	24	− 65	+ 108	+ 87	− 139	− 2
	26	− 108	− 202	+ 14	− 126	− 106
	27	+ 257	+ 207	+ 365		+ 276
	28		+ 73	− 45	− 9	+ 6
August	3	+ 9	− 155	− 15	− 148	− 77
	4		− 25	− 177		− 101
	6	+ 130	− 83	+ 11	+ 225	+ 71
	7	− 81	+ 88	+ 46	− 23	+ 8
	17	− 83	+ 25	− 27	+ 329	+ 61
	18	+ 79	− 28	+ 151	− 136	+ 16
	19		− 512		− 497	− 504
	21		+ 6	− 234		− 114
	24	− 242		− 55	− 78	− 125
	25	+ 93	+ 191	+ 322	+ 203	+ 202
	27	+ 83	+ 531	+ 124	+ 52	+ 198
	28	− 64	− 6	+ 105	+ 63	+ 24
	31	+ 53		+ 2	− 18	+ 12
September	1	− 184	− 240	− 179	− 168	− 193
	2	− 73	+ 15	− 131	+ 86	− 26
	3		+ 303		+ 468	+ 386
	4	− 33	+ 97	+ 35	+ 22	+ 30
	5		− 6	+ 268	+ 117	+ 126
	6		+ 366	− 77	− 43	+ 82
	7	+ 103	− 245	− 36	+ 229	+ 13
	8	− 7	− 37	+ 216	+ 114	+ 72
	10	− 100	+ 64	− 136	− 339	− 128
	12		+ 18	− 33	− 118	+ 34
	13		− 20	− 129	+ 354	+ 68
	14	+ 0·144	− 0·228	+ 0·013	− 0·097	− 0·042

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TABLE XIV.—*continued.*

RESIDUALS IN y.

First Approximation.

Date.		I.	II.	III.	IV.	Mean.
July	8	+ 0".160	− 0".044	+ 0".090	+ 0".198	+ 0".101
	14	− 95		+ 139	+ 223	+ 89
	15	+ 183	+ 76	+ 183	+ 270	+ 178
	16	+ 174	− 94	− 56	+ 17	+ 10
	19	+ 10	− 30	+ 242	− 55	+ 42
	20		+ 89	+ 19	+ 72	+ 60
	24	− 17	+ 79	+ 27	− 52	+ 9
	26	+ 175	+ 284	+ 171	+ 185	+ 204
	27	− 14	+ 68	− 106		− 17
	28		+ 151	+ 125	+ 109	+ 128
August	3	+ 99	+ 209	+ 118	+ 167	+ 148
	4	− 35	+ 56	− 10		+ 4
	6	− 33	+ 115	+ 49	− 33	+ 24
	7	− 40	+ 65	− 104	− 42	− 30
	17	+ 75	− 20	+ 50	− 202	− 24
	18	+ 38	+ 219	+ 253	+ 204	+ 178
	19	+ 362	+ 391		+ 286	+ 346
	21		+ 15	− 66	− 69	− 40
	24	+ 197		+ 70	+ 12	+ 93
	25	+ 20	+ 6	− 18	+ 6	+ 4
	27	− 2	− 19	+ 127	+ 32	+ 34
	28	+ 155	+ 273	+ 157	+ 123	+ 177
	31	+ 157		+ 3	− 67	+ 31
September	1	+ 58	+ 117	+ 45	− 85	+ 34
	2	− 393	− 248	− 328	− 262	− 308
	3		− 41		+ 56	+ 8
	4	− 9	− 126	− 137	− 120	− 98
	5		+ 19	− 1	+ 181	+ 66
	6		+ 117	+ 121	+ 180	+ 139
	7	+ 65	+ 59	− 92	+ 15	+ 12
	8	− 176	− 280	− 281	− 252	− 247
	10	+ 95	+ 163	+ 159	+ 217	+ 158
	12		+ 222	+ 226	+ 228	+ 225
	13		+ 29	+ 60	− 1	+ 29
	14	+ 0·099	+ 0·138	+ 0·057	+ 0·093	+ 0·097

As already explained, the mean residual for each day is the correction to the measured co-ordinate of the planet's disc; it was accordingly added to the absolute term for the date and with new absolute terms formed in this way, new values of the unknowns were found. These constitute the second approximation. It need hardly be said that the actual procedure was not exactly as here described: what was actually

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done was to treat the daily mean residuals as absolute terms and to find the corresponding *corrections* to the unknowns. The absolute terms in the normals found in this way are given here in Table XV.

TABLE XV.

Absolute Terms in Normals.

Providing Corrections to First Approximation.

	I.	II.	III.	IV.
A_x	− 0″·011	− 0″·642	+ 1″·226	− 2″·097
A_y	·000	− ·002	+ ·004	− ·001
A	− 0·011	− 0·644	+ 1·230	− 2·098
H_x	+ 0·248	+ 0·517	+ 0·539	− 1·087
H_y	·000	·014	+ ·013	− ·092
H	+ 0·248	0·531	+ 0·552	− 1·179
K_x	+ 0·027	+ 0·476	+ 0·383	+ 1·058
K_y	·000	+ ·042	− ·004	− ·061
K	+ 0·027	+ 0·519	+ 0·380	+ 0·997
X_x	− 0·257	+ 0·605	− 0·703	
X_y	·000	·000	− ·017	
X	− 0·257	+ 0·605	− 0·720	
E_x	+ 1·187	− 2·227	+ 0·272	+ 2·706
E_y	·000	− ·018	+ ·006	− ·019
E	+ 1·187	− 2·246	+ 0·278	+ 2·687
P_x	0·000	0·000	0·000	0·000
P_y	+ ·607	− ·001	+ ·878	− 1·317
P	+ 0·607	− 0·001	+ 0·878	− 1·317
Q_x	0·000	0·000	0·000	0·000
Q_y	− 1·011	− 2·579	− ·877	+ 1·722
Q	− 1·011	− 2·579	− 0·877	+ 1·722

By substituting these values of A, H, K, etc., in the solution of the normals, corrections to the values of the unknowns were determined; they are given in column 3 of Table XVII. By adding them to the previously found first approximation we get the second approximation (column 4).

A second set of residuals was then formed by substituting these last values (column 4) in the equations of condition, in which the absolute terms are now the original ones

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plus the mean residual for the date. The new absolute terms are printed beneath the first values in the table containing the equations of condition (Table XII.). The residuals resulting from this second approximation are tabulated in the following table :—

TABLE XVI.

Residuals in x.

Second Approximation.

Date.		I.	II.	III.	IV.	Mean.
July	8	+ 0.″013	+ 0.″121	+ 0.″046	− 0.″184	− 0.″001
	14	+ 18		− 50	− 73	− 35
	15	+ 85	− 46	− 40	− 28	− 7
	16	− 181	+ 129	+ 84	− 55	− 6
	19	+ 37	− 181	+ 2	+ 37	− 26
	20		+ 33	− 73	− 24	− 21
	24	− 64	+ 100	+ 76	− 102	+ 2
	26	− 40	+ 89	+ 95	− 3	+ 35
	27	+ 28	− 69	+ 66		+ 8
	28		+ 13	− 68	− 49	− 35
August	3	+ 126	− 72	+ 43	− 78	+ 5
	4		+ 29	− 80		− 26
	6	+ 31	− 117	− 41	+ 128	0
	7	− 39	+ 95	+ 52	− 45	+ 16
	17	− 129	− 3	− 106	+ 248	+ 2
	18	+ 34	− 21	+ 144	− 158	0
	19		− 101		+ 6	− 48
	21		+ 138	− 97		+ 18
	24	− 117		+ 56	+ 41	− 7
	25	− 95	+ 24	+ 126	+ 16	+ 18
	27	− 140	+ 387	− 45	− 106	+ 24
	28	− 19	− 10	+ 114	+ 68	+ 38
	31	+ 33		− 26	− 69	− 21
September	1	+ 22	− 7	+ 24	− 15	+ 6
	2	− 29	− 22	− 63	+ 81	− 8
	3		− 42		+ 69	+ 14
	4	− 3	+ 95	+ 30	− 8	+ 28
	5		− 98	+ 186	− 9	+ 26
	6		+ 207	− 129	− 131	− 18
	7	+ 68	− 167	− 61	+ 198	+ 8
	8	− 37	− 77	+ 148	+ 20	+ 13
	10	+ 26	+ 219	+ 30	− 207	+ 17
	12		+ 35	− 19	+ 124	+ 47
	13		− 171	− 160	+ 325	− 2
	14	+ 0.162	− 0.086	+ 0.046	− 0.031	+ 0.023

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TABLE XVI.—*continued.*

RESIDUALS IN y.

Second Approximation.

Date.		I.	II.	III.	IV.	Mean.
July	8	+ 0″.083	− 0″.113	− 0″.015	+ 0″.106	+ 0″.015
	14	− 177		+ 35	+ 127	− 5
	15	+ 8	− 64	− 2	+ 84	+ 6
	16	+ 145	− 115	− 61	− 3	− 8
	19	+ 4	− 48	+ 202	− 103	+ 14
	20		− 3	− 51	+ 10	− 15
	24	+ 3	+ 37	+ 31	− 53	+ 4
	26	+ 9	+ 108	− 28	− 10	+ 20
	27	− 34	+ 57	− 95		− 24
	28		+ 9	− 16	− 16	− 8
August	3	− 88	+ 36	− 38	+ 9	− 20
	4	− 18	+ 39	− 29		− 3
	6	− 45	+ 99	+ 24	− 57	+ 5
	7	− 4	+ 57	− 64	− 10	− 5
	17	+ 70	− 7	+ 72	− 187	− 13
	18	− 102	+ 12	+ 61	+ 16	− 3
	19	− 9	+ 74		− 71	− 2
	21		+ 24	− 20	− 37	− 11
	24	+ 86		− 22	− 77	− 4
	25	+ 41	− 33	− 32	+ 7	− 4
	27	+ 1	− 21	+ 85	+ 8	+ 18
	28	− 57	+ 73	− 17	− 45	− 12
	31	+ 119		− 25	− 99	− 2
September	1	+ 49	+ 49	+ 1	− 122	− 6
	2	− 114	+ 69	− 35	+ 39	− 10
	3		− 14		+ 40	+ 13
	4	+ 52	− 45	− 38	− 32	− 16
	5		− 70	− 56	+ 105	− 7
	6		+ 9	− 5	+ 31	+ 12
	7	+ 60	+ 63	− 100	− 3	+ 5
	8	+ 79	− 66	− 41	− 7	− 9
	10	− 32	+ 40	− 10	+ 63	+ 15
	12		− 27	+ 11	+ 11	− 2
	13		+ 29	+ 43	− 21	+ 17
	14	+ 0·016	+ 0·060	− 0·035	+ 0·004	+ 0·011

The daily mean residuals in this second approximation are so small that they are practically without any real signification, and though it is clear that by proceeding to further approximations we could cause them to vanish entirely, yet it is equally clear that any further corrections to the values of the unknowns that might be found in this way, would be less than their probable errors and would be quite meaningless. Accordingly the values adopted were those given in column 4 of Table XVII.

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TABLE XVII.

Values of Unknowns.

1	2 First Approximation.	3 Corrections to First Approximation.	4 Second Approximation.	1	2 First Approximation.	3 Corrections to First Approximation.	4 Second Approximation.
Δa_1	+ ·06745	+ ·00171	+ ·06916	Δa_3	+ ·017643	+ ·004294	+ ·021937
Δh_1	+ ·19390	− ·01809	+ ·17581	Δh_3	+ ·053247	− ·027576	+ ·025671
Δk_1	− ·10892	+ ·00034	− ·10858	Δk_3	− ·055485	+ ·016168	− ·039317
Δx_1	+ ·32023	− ·02887	+ ·29136	Δx_3	+ ·092884	− ·031594	+ ·061290
$\Delta \epsilon_1$	+ ·16589	+ ·02228	+ ·18817	$\Delta \epsilon_3$	− ·019922	+ ·001846	− ·018076
Δp_1	− ·009295	+ ·010210	+ ·000915	Δp_3	− ·011473	+ ·001924	− ·009549
Δq_1	− ·065422	− ·010511	− ·075933	Δq_3	− ·031833	− ·001291	− ·033124
Δa_2	− ·008861	− ·011434	− ·020295				
Δh_2	+ ·180771	− ·053555	+ ·127216	Δa_4	+ ·026377	− ·003621	+ ·022756
Δk_2	− ·103704	+ ·056648	− ·047056	Δh_4	+ ·055049	− ·001608	+ ·053441
Δx_2	− ·150116	+ ·053527	− ·096589	Δk_4	+ ·031415	+ ·009141	+ ·040556
$\Delta \epsilon_2$	− ·002550	− ·014155	− ·016705	$\Delta \epsilon_4$	− ·038230	+ ·000653	− ·037577
Δp_2	+ ·094832	− ·000776	+ ·094056	Δp_4	+ ·061608	− ·000596	+ ·061012
Δq_2	− ·174818	− ·009484	− ·184302	Δq_4	− ·011448	+ ·000750	− ·010698

To find the probable errors of the unknowns, the sums of the squares of the residuals in the second approximation were found. These sums were checked by means of the well-known relation—

$$(vv) = (nn) - (an)x - (bn)y - (cn)z - etc.,$$

which serves as a control on the whole of the solution.

The magnitude of the quantities (nn) shows clearly that the theory of the motions of the satellites is much in need of revision.

The probable errors in the following table are the probable errors of one observation of weight unity. It will be noticed that in the case of all four satellites the probable error of a measurement of an ordinate is smaller than that of a measurement of an abscissa. This is probably quite accidental and due to the fact that a few large residuals happen to occur in the equations in x, though it is possible that both Mr Löwinger and

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the writer in making the measurements, may unconsciously have found it easier to measure in the vertical than in the horizontal direction. In deriving the probable errors in y, the number of unknowns was taken as two, since only Δp and Δq enter these equations with practically appreciable effect.

The sum of the squares of all the residuals is $1''{\cdot}813$, the total number of equations is 247, and there are in all 27 unknowns. From these figures we derive a probable error of $\pm 0''{\cdot}061$, which may be taken as a measure of the accuracy of one observation in this investigation. No further elaboration of this point was made, for no real advantage could be gained by doing so. Indeed, the probable error is not a very satisfactory measure of the accuracy of a result; it is too small and the mean error

TABLE XVIII.

Probable Errors.

	I.	II.	III.	IV.
$(nn)_x$	3·121	1·149	1·229	12·424
$(nn)_y$	0·691	11·758	1·053	7·592
Sum	3·812	12·907	2·282	20·016
$(rr)_x$	0·163	0·482	0·242	0·425
$(rr)_y$	0·141	0·107	0·106	0·147
Sum	0·304	0·589	0·348	0·572
Check	0·302	0·588	0·348	0·570
Number of Equations x	25	32	33	32
Number of Equations y	27	32	33	33
Probable Error r_x	± 0·061	± 0·090	± 0·063	± 0·083
Probable Error r_y	± 0·051	± 0·040	± 0·039	± 0·046
Probable Error x and y.	± 0·055	± 0·068	± 0·052	± 0·066

would give a better idea of the limits within which one could place some reliance on a result. But for a practical judgment of the accuracy of a result, neither need be determined with minute elaboration, though it must be confessed that it sometimes

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happens that more attention is paid to the method of determining a probable error than to the probable error itself.

There is a point with regard to the residuals in y which must be mentioned here. The heliometer observations seemed to indicate the existence of a periodic inequality in the latitude of satellite IV. The period was half the period of revolution of IV., the amplitude was 0″·150, and the phase was such that the inequality was at its maximum when the satellite was at its greatest elongation from Jupiter. No explanation of this could be found, but the fact remained that the observations were much better represented by the inclusion of this purely empirical correction (*Heliometer*, pp. 171–3), and it was of much interest to see whether it was confirmed by the photographs. Not a trace of it can be detected. If it did exist, it would enter into the residuals in y with its full effect in the first approximation and with three-fourths of its effect in the second approximation; but in neither is there any sign of it. It can only be concluded that there was something in the appearance of the fourth satellite which affected visual observations but not photographic; moreover, since no certain trace of it is to be found in Sir David Gill's observations with the heliometer in 1891, it is not of a permanent character. There are many records[1] of surface markings having been seen on the satellites, and there are also notes in the writer's observing-book of variability in the relative brightness of the satellites; for instance, under date 27 August 1902, there is the entry "Satellite III. variable — IV. often noted also." This was noticed before the inequality showed itself in the course of the reductions, and it is unfortunate that more attention was not paid to their photometric variability[2]. No attempt at a detailed explanation of the observed inequality by the aid of surface markings need be given here, for such an explanation can only be purely speculative; but it would seem that it is in the distribution of surface markings that the explanation of the phenomenon is to be sought.

The values of the unknowns were also determined after correcting the co-ordinates with the value of the orientation derived from the trails.

The abscissæ are the same as with the orientation from the standard stars, but the ordinates are slightly changed. The corrections to be applied to the absolute terms (first approximation) in the normals are as follows:—

[1] *e.g.* Lowell, Vol. II. of Lowell Observatory Publications.

[2] Their variability has long been known. See, *e.g.*, Burton, *M.N.R.A.S.* XL. p. 169: Engelmann, Ueber die Helligkeits Verhältnisse der Jupiter's Trabanten, 1871.

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TABLE XIX.

Corrections to Absolute Terms in Normals Reducing to Orientation of Trails.

	I.	II.	III.	IV.
A	0".000	− 0".002	− 0".003	+ 0".017
H	·000	+ ·001	·000	·000
K	·000	+ ·003	·000	+ ·003
X	·000	·000	− ·002	
E	·000	+ ·006	− ·006	− ·021
P	+ ·098	− ·035	+ ·312	− ·286
Q	+ 0·297	+ 0·871	+ 3·060	+ 6·010

If these values are substituted in the solution of the normals (pp. 79-82), we get the corrections to the unknowns to reduce them to the orientation from the trails. The only corrections that need be given here, are those to Δp, Δq; all the others are very small and quite meaningless. The corrections to Δp, Δq are as follows:—

Corrections to Unknowns to Reduce to Orientation of Trails.

	I.	II.	III.	IV.
Δp	+ 0·001706	+ 0·000049	+ 0·000079	+ 0·000234
Δq	+ ·003122	+ ·003212	+ ·003910	+ ·002877

These corrections were added to the values in the second approximation (column 4 of Table XVII.), and the resulting values of the unknowns were substituted in the equations of condition, in which the absolute terms were now those previously used in the second approximation plus the corrections necessary to reduce to the orientation of the trails, *i.e.*, the corrections from which the small absolute terms in Table XIX. were formed.

The residuals in x are the same as before (p. 90); those in y are given in the next table. These, it may be repeated, are the residuals in the second approximation, the scale-value being derived from the standard stars and the orientation from the trails.

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TABLE XX.

RESIDUALS IN y.

Orientation of Trails.

1902.		I.	II.	III.	IV.	Mean.
July	8	+ 0".092	− 0".100	− 0".003	+ 0".072	+ 0".015
	14	− 174		+ 35	+ 137	− 1
	15	+ 12	− 50	+ 5	+ 124	+ 23
	16	+ 149	− 113	− 73	− 11	− 12
	19	+ 5	− 50	+ 210	− 101	+ 16
	20		− 1	− 42	+ 14	− 10
	24	+ 7	+ 30	+ 39	− 77	0
	26	+ 10	+ 108	− 27	− 13	+ 19
	27	− 38	+ 53	− 98		− 28
	28		+ 6	− 19	− 8	− 7
August	3	− 84	+ 45	− 11	− 7	− 14
	4	− 17	+ 35	− 36		− 6
	6	− 43	+ 92	+ 25	− 63	3
	7	+ 6	+ 34	− 34	− 69	− 16
	17	+ 85	0	+ 109	− 248	− 14
	18	− 98	+ 8	+ 58	+ 33	0
	19	− 17	+ 88		− 34	+ 12
	21		+ 22	− 22	− 38	− 13
	24	+ 85		− 26	− 94	− 12
	25	+ 39	− 33	− 26	+ 6	− 4
	27	− 8	− 42	+ 67	+ 58	+ 19
	28	− 59	+ 72	− 18	− 48	− 13
	31	+ 110		− 47	− 62	0
September	1	+ 41	+ 61	+ 25	− 149	− 6
	2	− 105	+ 66	− 21	+ 7	− 13
	3		− 35		− 16	− 25
	4	+ 43	− 56	− 9	+ 12	− 2
	5		− 71	− 55	+ 112	− 4
	6		+ 14	− 7	+ 33	+ 13
	7	+ 59	+ 67	− 111	− 10	+ 1
	8	+ 83	− 76	− 53	− 25	− 18
	10	− 51	+ 8	− 19	+ 147	+ 21
	12		− 38	+ 32	− 36	− 14
	13		+ 10	+ 29	+ 23	+ 21
	14	+ 0.017	+ 0.062	− 0.040	+ 0.001	+ 10

The sums of the squares of the foregoing residuals are :—

I.	II.	III.	IV.
0".141	0".106	0".115	0".209

These are rather larger than the values found before—as might be expected by comparing the mean residuals. But here again any further corrections to the values of the unknowns would be very small and without signification. No further approximation was therefore carried out.

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CHAPTER V.

Results and Comparison with Heliometer Results.

It will be convenient here to collect the values of the unknowns with their probable errors and to set beside them the values found from the writer's observations with the Cape heliometer. In the following table are given the values of the unknowns from the two investigations together with their probable errors and weights. In deriving the probable errors of the unknowns from the photographs, the probable error of one observation was taken as $\pm 0''\cdot 061$, which is the value found by taking all the observations together. This seemed the best measure available of the accuracy of an observation; the probable error of a quantity derived from some thirty equations is not worthy of very much confidence, and there seemed no *a priori* reason why the measurements of one satellite should be any better than those of another. Hence the probable error of one observation was taken to be the same for all four satellites and was that derived by taking the same of the squares of all the residuals.

For the photographs, then, weight unity corresponds to a probable error of

$$\pm 0''\cdot 061$$

or

$$\epsilon_P^2 = 0\cdot 00824,$$

ϵ_P being the mean error.

For the heliometer observations, weight unity corresponds to a probable error of

$$\pm 0''\cdot 107$$

or

$$\epsilon_H^2 = 0\cdot 02507$$

and in order to compare the two systems of weights, the photographic weights were increased in the ratio of 0·02507 : 0·00824. These are given in the last column but one.

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TABLE XXI.

Comparison of Photographic and Visual Values of Unknowns.

	Values of Unknowns.		Difference. P—H.	Probable Errors.		Weights. Photographs. $\epsilon^2 = \cdot00824$.	Weights. Photographs. $\epsilon^2 = \cdot02507$.	Weights. Heliometer. $\epsilon^2 = \cdot02507$.
	Photographs.	Heliometer.		Photographs.	Heliometer.			
Δh_1	$+0^{\circ}\cdot1758$	$-0^{\circ}\cdot0438$		$\pm0^{\circ}\cdot0484$	$\pm0^{\circ}\cdot0378$	1·6	5	8
Δk_1	− ·1086	− ·0087		·0310	·0276	3·9	12	15
Δx_1	+ ·2914	− ·0606		·0555	·0378	1·2	4	8
F_1	− ·0772	+ ·0069	−0·0841	·0071	·0169			
Δh_2	+ ·1272	− ·0412		·0237	·0193	6·6	20	31
Δk_2	− ·0471	+ ·0344		·0169	·0169	13·1	40	40
Δx_2	− ·0966	+ ·1447		·0260	·0205	5·5	17	27
F_2	+ ·0426	+ ·0827	− ·0401	·0041	·0139			
Δh_3	+ ·0257	− ·0221		·0178	·0125	11·7	36	73
Δk_3	− ·0393	− ·0165		·0105	·0112	33·9	103	91
Δx_3	+ ·0613	− ·0289		·0196	·0133	9·7	29	65
F_3	− ·0383	+ ·0012	− ·0395	·0027	·0090			
Δh_4	+ ·0534	+ ·0844		·0015	·0037	1592·4	4844	824
Δk_4	+ ·0406	+ ·0226		·0054	·0062	129·7	395	301
F_4	+ ·0555	+ ·0855	− ·0300	·0014	·0036			
$\Delta\epsilon_1$	+ ·1882	+ ·1338	+ ·0542	·0083	·0075	53·7	163	201
$\Delta\epsilon_2$	− ·0167	− ·0254	+ ·0087	·0044	·0046	192·6	586	541
$\Delta\epsilon_3$	− ·0181	− ·0223	+ ·0047	·0030	·0029	407·5	1240	1344
$\Delta\epsilon_4$	− ·0376	− ·0343	− ·0033	·0015	·0017	1610·3	4899	3811
Δp_1	+ ·0009	+ ·0137	− ·0128	·0080	·0072	58·7	178	223
Δq_1	− ·0759	− ·0755	− ·0004	·0062	·0065	95·5	290	269
Δp_2	+ ·0941	+ ·0922	+ ·0019	·0042	·0044	216·7	659	587
Δq_2	− ·1843	− ·1860	+ ·0017	·0037	·0040	270·5	823	728
Δp_3	− ·0095	− ·0072	− ·0023	·0027	·0028	495·0	1506	1465
Δq_3	− ·0331	− ·0334	+ ·0003	·0022	·0023	771·6	2347	2120
Δp_4	+ ·0610	+ ·0658	− ·0048	·0014	·0015	1824·8	5551	5236
Δq_4	−0·0107	−0·0070	−0·0037	$\pm0^{\circ}\cdot0013$	$\pm0^{\circ}\cdot0014$	2070·4	6298	5493

This table shows large differences between the two determinations of the unknowns Δh, Δk, and Δx. The explanation of this has been given in the writer's former paper, but for the sake of completeness may be reproduced here.

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If we neglect the inclinations and eccentricities, we have—

$$\frac{\partial x}{\partial v} = \frac{a''\rho}{\Delta'} \sin 1^\circ \cos (l - \mathrm{L}).$$

$$\frac{\partial x}{\partial \rho} = \frac{a''\rho}{\Delta'} \sin (l - \mathrm{L}).$$

Put

$$\frac{a''\rho}{\Delta'} \sin 1^\circ = \kappa;$$

then

$$\frac{\partial x}{\partial v} = \kappa \cos (l - \mathrm{L}),$$

$$\frac{\partial x}{\partial \rho} = \frac{\kappa}{\sin 1^\circ} \sin (l - \mathrm{L}).$$

But

$$\frac{\partial x}{\partial h} = \frac{\partial x}{\partial v}\frac{\partial v}{\partial h} + \frac{\partial x}{\partial \rho}\frac{\partial \rho}{\partial h}$$

and

$$\frac{\partial v}{\partial h} = -\cos l \text{ and } \frac{\partial \rho}{\partial h} = -\tfrac{1}{2} \sin 1^\circ \sin l.$$

Hence

$$\frac{\partial x}{\partial h} = -\kappa \left\{ \cos l \cos (l - \mathrm{L}) + \tfrac{1}{2} \sin l \sin (l - \mathrm{L}) \right\}.$$

In a similar way we find—

$$\frac{\partial x}{\partial \kappa} = \kappa \left\{ \sin l \cos (l - \mathrm{L}) - \tfrac{1}{2} \cos l \sin (l - \mathrm{L}) \right\},$$

$$\frac{\partial x}{\partial x} = \kappa \left\{ \sin \psi \cos (l - \mathrm{L}) - \tfrac{1}{2} \cos \psi \sin (l - \mathrm{L}) \right\}.$$

x in the denominator is the coefficient of the great inequality and will not be confounded with x in the numerator which is the abscissa of the satellite. Let suffixes 1, 2, 3 refer to satellites I., II., and III., and let

$$c = l_1 - 2\, l_2 = l_2 - 2\, l_3 + 180^\circ;$$

then

$$\psi_1 = 2\,(l_1 - l_2) = l_1 + c$$
$$\psi_2 = l_1 - l_2 + 180^\circ = l_2 + c + 180^\circ$$
$$\psi_3 = l_2 - l_3 + 180^\circ = l_3 + c.$$

Now

$$3\,\frac{\partial x}{\partial h} = -3\kappa \left\{ \cos \mathrm{L} - \tfrac{1}{2} \sin l \sin (l - \mathrm{L}) \right\},$$

$$\frac{\partial x}{\partial h} = -\kappa \left\{ \cos (2\, l - \mathrm{L}) + \tfrac{3}{2} \sin l \sin (l - \mathrm{L}) \right\}.$$

Hence

$$\frac{\partial x}{\partial h} = -\frac{\kappa}{4} \left\{ 3 \cos \mathrm{L} + \cos (2\, l - \mathrm{L}) \right\}.$$

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In the same way we find—

$$\frac{\partial x}{\partial \kappa} = + \frac{\kappa}{4} \left\{ 3 \sin \mathrm{L} + \sin (2l - \mathrm{L}) \right\},$$

$$\frac{\partial x}{\partial x} = \pm \frac{\kappa}{4} \left\{ 3 \sin (\mathrm{L} + c) + \sin (2l + c - \mathrm{L}) \right\},$$

where the lower sign in the last refers to the second satellite. For the fourth satellite the last equation does not exist. The second terms within the brackets are terms of short period; neglecting them, we get—

$$\frac{\partial x}{\partial h} = - \tfrac{3}{4} \kappa \cos \mathrm{L}$$

$$\frac{\partial x}{\partial \kappa} = + \tfrac{3}{4} \kappa \sin \mathrm{L}$$

$$\frac{\partial x}{\partial x} = \pm \tfrac{3}{4} \kappa \sin (\mathrm{L} + c)$$

Now L changes very slowly; its change during the period of observation was 8°, whilst c varied progressively from 37° to 91°. If there had been no change at all, it would not have been possible to separate Δh, Δk, Δx, and all that could have been determined would have been a linear function of these unknowns, viz. :—

$$\mathrm{F} = - \cos \mathrm{L}\, \Delta h + \sin \mathrm{L}\, \Delta \kappa \pm \sin (\mathrm{L} + c)\, \Delta x.$$

These functions F were calculated with the mean values of L and c, namely—

$$\mathrm{L} = 177^\circ \text{ and } c = 61^\circ$$

and are tabulated in Table XXI. The signs of these functions as given in the discussion of the heliometer observations (p. 160) have been changed; $p - \mathrm{P}$ was there put equal to $+90^\circ$, but to conform with the signs in the present paper, $p - \mathrm{P}$ must be taken equal to 270°.

The probable errors of the functions F were calculated from the formula on p. 162 of the heliometer paper.

The differences between the two determinations of the functions F appear at first sight rather large, but their probable errors are large. These quantities may be given here.

	P—H.	Probable Error.	Ratio.
F_1	− 0·0841	± 0·0183	5
F_2	− ·0401	± ·0145	3
F_3	− ·0395	± ·0094	4
F_4	− ·0300	± ·0039	8

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It is doubtful if any real significance can be attached to the differences P−H, even though the signs of all four are the same.

The agreement between the two determinations of all the other unknowns is very satisfactory excepting in the case of $\Delta\epsilon_1$, where the difference P−H amounts to five times its probable error. This, however, can only be attributed to purely accidental error.

In Table XXI. the values of the unknowns Δp, Δq are those derived with the orientation of the Standards; in the next table, Table XXII, both sets of values are given and a comparison is made of the values derived with the orientation of the trails and the heliometer values.

TABLE XXII.

	Values of unknowns.			Difference P−H.	Probable Errors.		Probable Error of Difference.
	Photographs.		Heliometer.		Photographs.	Heliometer.	
	Standards.	Trails.					
Δp_1	+0".0009	+0".0026	+0".0137	−0".0111	±".0080	±".0072	±".0108
Δq_1	− ·0759	− ·0728	− ·0755	+ ·0027	·0062	·0065	·0090
Δp_2	+ ·0941	+ ·0941	+ ·0922	+ ·0019	·0042	·0044	·0061
Δq_2	− ·1843	− ·1812	− ·1860	+ ·0048	·0037	·0040	·0055
Δp_3	− ·0095	− ·0095	− ·0072	− ·0023	·0027	·0028	·0039
Δq_3	− ·0331	− ·0292	− ·0334	+ ·0042	·0022	·0023	·0032
Δp_4	+ ·0610	+ ·0612	+ ·0658	− ·0046	·0014	·0015	·0021
Δq_4	− ·0107	− ·0078	− ·0070	− ·0008	±·0013	±·0014	±·0019

The probable error in the last column is the square root of the sum of the squares of the probable errors in the two preceding columns.

Practically it is a matter of indifference whether the orientation is derived from the standard stars or from the trails, but there are slight *a priori* reasons for giving the preference to the latter.

Corrections to the Axes of the Orbits.

The most uncertain element of reduction in this investigation is the scale-value. The two most prominent sources of error are perhaps error in the adopted places of the standard stars and optical distortion of the lens. As to the first of these, the error in

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the places of the standards cannot be large; it is very unlikely that their distance apart is in error by as much as 0″·15 (see *Heliometer*, p. 203). If this were the error of their distance, the error of scale-value would be 3 in 10^5. As to the second, no special investigation of the distortion of the lens could be undertaken, but in his discussion of photographs of circumpolar stars, Mr Hough found a small radial distortion of the field (*Cape Annals*, Vol. XI., Part II. p. 227). He found that it was expressed analytically as follows:—

$$S = S_0 + \frac{0{\cdot}171}{1000}(r^2 - 2500)$$

where S is the scale-value at points at a distance r from the origin. The unit of r is a minute of arc, and of S one in the fourth place of decimals. According to this expression the mean scale-value for distances up to 10′ is

$$S = S_0 - 0{\cdot}422.$$

This is the average scale-value for the satellites. The standard stars are distant 33′ and 42′ respectively from the origin, and the mean scale-value for the two may be taken as

$$S' = S_0 - 0{\cdot}346.$$

Hence the difference between the scale-value for the short distances of the satellites, and that for the longer distances of the standard stars is 0·076 or only 8 in 10^6, a quantity which is practically negligible.

It appears then that as far as the places of the standard and optical distortion are concerned, the scale-value must be regarded as practically free from error.

The observed corrections to the axes of the orbits are as follows:—

$$\frac{\Delta a}{a} = 100\,\Delta a = \begin{cases} +692 \pm 112, & \text{Satellite I.} \\ -203 \pm 68, & \text{,, II.} \\ +219 \pm 39, & \text{,, III.} \\ +228 \pm 24, & \text{,, IV.} \end{cases}$$

the unit being one in the sixth place of decimals.

The same values of α were adopted in this investigation as in Heliometer work; and as pointed out in that paper (p. 202), the second and higher order terms were accidentally applied twice to the tabular values of α. These terms are therefore included in the observed values of $\Delta a/a$, and their values are—

I. . . .	$\frac{\partial a}{a} = +10 \times 10^{-6}$
II. . . .	$= +42 \times 10^{-6}$
III. . . .	$= +1 \times 10^{-6}$
IV. . . .	$= +26 \times 10^{-6}$

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Hence the values of $\Delta a/a \times 10^6$ corrected on this account are—

I.	. . .	+ 692 + 10	= + 702
II.	. . .	− 203 + 42	= − 161
III.	. . .	+ 219 + 1	= + 220
IV.	. . .	+ 228 + 26	= + 254

If we neglect the correction to the adopted constant of polar compression of Jupiter, we may put

$$\frac{\Delta M}{M} = -3\frac{\Delta a}{a},$$

where M is the reciprocal of the mass of Jupiter (*Heliometer*, p. 81). The adopted value of M was 1047·7; substituting this value, we get the following values and probable errors of the corrections to M.

I.	. . .	− 2·21 ± 0·35	Weight 30
II.	. . .	+ 0·51 ± 0·21	„ 80
III.	. . .	− 0·69 ± 0·12	„ 248
IV.	. . .	− 0·80 ± 0·08	„ 636
Weighted Mean	.	− 0·71 ± 0·06	

The weights are the weights of the determinations of the unknowns Δa. The value of the mass then corresponding to the corrected values of the axes of the satellites' orbits is

$$1046{\cdot}99 \pm 0{\cdot}07$$

whereas the value found from the Heliometer observations was

$$1047{\cdot}30 \pm 0{\cdot}06$$

and the value adopted by Newcomb was

$$1047{\cdot}35 \pm 0{\cdot}04.$$

The probable error ±0·07 was found by regarding the weighted mean as a function of the four values of δM. namely,

$$\text{Mean} = \frac{\Sigma p\,\delta M}{\Sigma p}$$

so that the probable error of the mean was

$$\frac{\Sigma p^2 r^2_M}{(\Sigma p)^2}$$

This is the value given above and not that from the interagreement of the four results, since a probable error derived from only four determinations is in fact meaningless.

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After correction to reduce to the value 1047·0 of the mass, the residual errors of the observed values of $\Delta a/a$ are

I. . . .	$\frac{\Delta a}{a} = +479 \pm 112$
II. . . .	-384 ± 68
III. . . .	-3 ± 39
IV. . . .	$+31 \pm 24$

or if a'' is the apparent semi major axis at opposition expressed in seconds of arc, then

I. . . .	$\Delta a'' = +0''{\cdot}068 \pm 0''{\cdot}016$
II. . . .	$-0{\cdot}088 \pm 0{\cdot}016$
III. . . .	$-0{\cdot}001 \pm 0{\cdot}014$
IV. . . .	$+0{\cdot}020 \pm 0{\cdot}015$.

The first two are not satisfactory and are difficult to account for.

General Summary.

The values of the unknowns found from the heliometer observations are in general confirmed quite satisfactorily by the present investigation, and the conclusions reached in the former paper are not invalidated. In a few cases, however, they differ; and it will be best to point out where there is disagreement, not where there is agreement.

Longitudes of the Epoch.—From the values of the unknowns $\Delta\epsilon$, we find that the values of

$$\theta = \epsilon_1 - 3\epsilon_2 + 2\epsilon_3 - 180^\circ$$

are—

Heliometer	$\theta = +0^\circ{\cdot}164 \pm 0^\circ{\cdot}017$
Photographs	$+0{\cdot}202 \pm 0{\cdot}017$.

This difference of $0^\circ{\cdot}038$ is almost entirely due to the disagreement of the two values of $\Delta\epsilon_1$ but it only amounts to little over one and a half times its probable error and must therefore be ascribed to accidental error. The mean value of θ from the two investigations is

$$\theta = +0^\circ{\cdot}183 \pm 0^\circ{\cdot}012$$

at the epoch 1902·6.

Coefficients of great Inequalities.—It was concluded from the heliometer observations that they indicated a rather large positive correction to the coefficient

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x_2 of the great inequality of the second satellite. This, however, is not at all borne out by the photographs; the mean of all would show that x_2 was in need of no correction amounting to as much as $\pm 0^{\circ}\cdot 05$. The general conclusion must be that it is not possible to fix the values of these coefficients or of the unknowns Δh, Δk within closer limits than these by means of the observation of either the polar or the rectangular co-ordinates of the satellites; the only way it can be done is by the discussion of an extensive series of accurate eclipse observations. It is chiefly for this reason that Professor Sampson's reduction and discussion of the Harvard photometric observations of eclipses is of such importance to the subject of Jupiter's satellites. By a combination of the results of his work with the results of Dr de Sitter's discussion of Sir David Gill's heliometer observations and of Cape photographs and with those of the present writer, it may confidently be hoped that our knowledge of the motions of the four Galilean satellites will be greatly increased.

Comparison of the two Methods.—In convenience and ease of reduction the photographic method is much superior to the heliometer. The time spent in getting a complete set of observations with the heliometer was about two hours, whilst a photograph took less than half-an-hour; so that the time spent in the observatory was much less—a considerable gain in uncertain weather. To take and measure a photograph occupied more than two hours, but the time of measurement could be chosen by the observer, who was not thus compelled at the end of the day to observe at the limit of his power in a strained position of body. Hence other things being equal, it was reasonable to expect that one plate would contribute more weight in the solution of the equations than one night's measurement with the heliometer. Another advantage that photographic methods in general have over visual, is that in the photograph one has a permanent record of the state of the sky, and discordances subsequently found during the reductions may generally be traced, whether to errors of measurement, or to defects in the plate, or to refraction irregularities, etc.

But it was principally in the reductions that in the present instance the photographic method, to reach the same end, involved so much less labour. In the first place, the epochs of observation of all four satellites were the same, so that the co-ordinates and coefficients had to be calculated for one epoch on each night instead of for three. Secondly, by the use of rectangular co-ordinates, the conversion of rectangular into polar co-ordinates was avoided, and also the calculation of the coefficients $\frac{\partial s}{\partial v}$, $\frac{\partial s}{\partial \rho}$, $\frac{\partial p}{\partial q}$, $\frac{\partial v}{\partial \rho}$, etc., from the coefficients $\frac{\partial x}{\partial v}$, $\frac{\partial x}{\partial \rho}$, $\frac{\partial y}{\partial q}$, $\frac{\partial y}{\partial \rho}$, etc. In these two ways the numerical work was much reduced.

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If, then, the work were to be repeated, there could be little hesitation in adopting the photographic rather than the heliometric method. It should also be pointed out that the measurements discussed in this paper were made with the instrument made for measuring the plates for the *Astrographic Catalogue*, and that it was never intended that this instrument should be used for making the most refined measurements. By using a more delicate instrument, it would be possible still further to increase the accuracy of measurement.

At the same time the heliometer is better suited for investigations in which the problem depends on the determination of angular distances in absolute measure. The difficulty with a photographic instrument is to find a numerical expression for the optical distortion of the lens. It is true that the same kind of difficulty exists in the case of the heliometer but in smaller degree. It is comparatively easy to determine the variations of the scale-value of a heliometer, and, in the writer's investigation of the mass of Jupiter, this was a vital point. The procedure there was to find whether the direct measurement of two points was equal to the sum of a series of smaller distances measured along the line joining the points. This method, however, does not lend itself well to the case of a photographic lens: the weights of the unknowns come out very small, and, up to the present, no means of determining accurately the distortion of a lens without excessive labour have been suggested.

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APPENDIX.

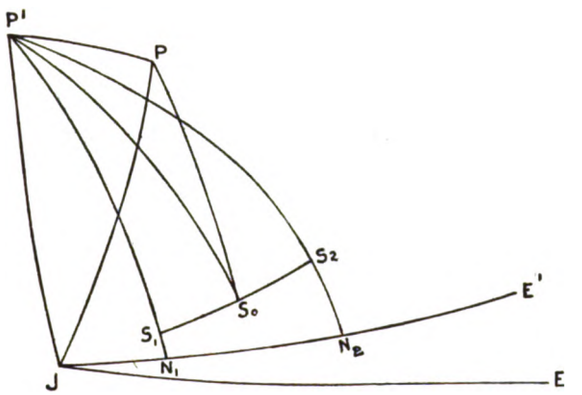
Corrections to Values of Nodes and Inclinations found from the Heliometer Observations.

After the foregoing pages had been printed, it was pointed out, in a letter from Professor Becker to the writer, that the inequality in the apparent motion of the Fourth Satellite, which appeared in the heliometer observations and is referred to on p. 96 of the present paper and on p. 172 of Vol. XII. Part II., might be due to the neglect of a correction to the value of the angle P, which occurs in the formulæ giving the distance and position-angle of a pair of satellites, namely :—

$$s \sin (p - P) = x_2 - x_1,$$
$$s \cos (p - P) = y_2 - y_1.$$

(See Vol. XII. Part II. p. 77.)

The value of P, which was adopted in these formulæ, was that interpolated from the ephemeris, pp. 63–4, where P denotes the position-angle of the northern end of Jupiter's axis of rotation reckoned at Jupiter's centre. This requires a correction to reduce it to the value reckoned at the point midway between the pair of satellites under consideration.



In the diagram, let J be the centre of Jupiter, JE′ his equator, JE a parallel to the earth's equator: let P′, P be the corresponding poles. Then PP′ is 360°−P.

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Let S_0 be the point midway between two satellites, S_1 and S_2 : S_1N_1 and S_2N_2 are the latitudes denoted by y_1, y_2. Put

$$y_0 = \tfrac{1}{2}(y_1 + y_2) = \tfrac{1}{2}(S_1N_1 + S_2N_2)$$
$$\delta_0 = \tfrac{1}{2}(\delta_1 + \delta_2) = 90^\circ - \tfrac{1}{2}(PS_1 + PS_2)$$

then from the triangle $P'PS_0$ we have

$$\cos P = \sin \delta_0 \sin y_0 + \cos \delta_0 \cos y_0 \cos P'S_0P.$$

The position-angle of the line S_1S_2 joining the satellites is PS_0S_2; and in the formulæ for finding it the angle PS_0P' should have been used instead of the angle P or PJP'. If we put $PS_0P' = P + \Delta P$, we find

$$\Delta P \sin P \cos \delta_0 = y_0 \sin \delta_0 - \cos P (1 - \cos \delta_0)$$

since y_0 is always very small. Instead of δ_0, we may put into sufficient accuracy

$$\delta_J - x_0 \sin P + y_0 \cos P$$

where δ_J is the declination of Jupiter's centre and $x_0 = \frac{1}{2}(x_1 + x_2) = \frac{1}{2}(JN_1 + JN_2)$. Whence we find

$$\Delta P = \left(\frac{x_1 + x_2}{2} \cos P + \frac{y_1 + y_2}{2} \sin P\right) \tan \delta.$$

The correction to be applied to an absolute term taken in the sense O – C and reckoned in arc of great circle is

$$- \Delta P \, s \sin 1''.$$

This correction was not applied in the writer's former paper, but has since been calculated and applied to the observations of position-angle in both years 1901 and 1902. The following tables contain the value of the correction :—

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Corrections to Absolute Terms, 1901.

1901.		I.–II.	I.–III.	I.–IV.	II.–III.	II.–IV.	III.–IV.
		"	"	"	"	"	"
June	24				+ 0·082	+ 0·348	+ 0·267
	25						+ 254
	29				+ 10	+ 40	− 48
July	2	− 0·009	+ 0·089	− 0·396	+ 81	− 385	− 307
	8	− 10	+ 96	+ 112	+ 84	+ 111	+ 28
	10	+ 32	+ 10	+ 373	+ 26	+ 341	+ 369
	11	− 9	− 34	+ 357	− 40	+ 375	+ 320
	12	− 30	− 108	+ 270	− 79	+ 243	+ 160
	14	+ 32		+ 16		+ 19	
	16	− 32	+ 117	− 78	+ 85	− 49	+ 35
	18	− 15			− 4	− 331	
	22	− 11	+ 47	− 48			0
	23	− 34	+ 118		+ 83		
	27	− 5	− 83	+ 367	− 92	+ 374	+ 283
	29		+ 27	+ 214			+ 184
August	10	− 21	− 110	+ 76	− 91	+ 59	− 32
	13	− 31	+ 101	+ 341		+ 308	+ 237
	14		+ 67	+ 281			+ 215
	15				+ 69	+ 113	
	19	− 13	0	− 191	+ 16		
	23				− 43	− 184	− 139
	24	− 24	− 107	− 67	− 85	− 45	− 42
	26	− 3	− 7	+ 13		+ 21	
	27	− 21	+ 69	+ 108	+ 49	+ 87	+ 39
	28		+ 74	+ 230			+ 159
	29	+ 35	0	+ 312	+ 35	+ 278	+ 312
	31				− 72		
September	1	+ 14	− 43	+ 98	− 33	+ 82	+ 49
	3	− 16	+ 56	+ 6	+ 40	− 24	+ 64
	4	− 12	+ 78	− 64	+ 90	− 80	+ 12
	9	+ 23		− 139		− 117	
	10	− 12	+ 44		+ 32	− 22	
	14	− 31	− 77	+ 239	− 46	+ 139	+ 162
	16	+ 28	0	+ 261	+ 27	+ 233	+ 260
	17	− 8	+ 32	+ 169	+ 24	+ 158	+ 132
	19	+ 13	+ 26	+ 9	+ 0·017	+ 22	+ 0·035
	23	+ 34					
	25	− 18	+ 0·082				
	27	0·000		− 0·013		− 0·012	

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Corrections to Absolute Terms, 1902.

1902.		I.–II.	I.–III.	I.–IV.	II.–III.	II.–IV.	III.–IV.
June	10	+ 0".003	+ 0".057	+ 0".127			+ 0".132
	11				+ 0".032	+ 0".202	+ 172
	16	+ 2	− 9	− 7	+ 12	− 4	− 17
	19				+ 8	− 228	− 221
	21		− 70				
	24	+ 8	+ 36	+ 4	+ 31	+ 10	+ 42
	30	− 8		+ 103 + 98		+ 110 + 105	
July	1		+ 27			+ 5	+ 5
	3	− 8	+ 25				
	7	− 8	0	+ 226	− 4	− 232	− 230
	8	+ 10	+ 35	− 117	+ 27	− 106	− 74
	14		− 16	− 270			+ 257
	15	+ 28	+ 8	+ 259	+ 16	+ 232	+ 250
	16	− 11	− 72	+ 181	+ 83		
	18	+ 9		+ 4		0 + 1	
	19	+ 11	− 44				
	20				− 64	− 45	− 16 − 14
	24	− 21	+ 60	− 219	+ 38	− 197	− 159
	26	+ 15	− 28	− 23	− 17	− 10	− 4 − 10
	27	− 15	− 92		− 76		
August	3	− 3	− 87	+ 53	− 82	+ 45	− 30 − 31
	4	0	− 29		− 31		
	7	− 32	+ 89	− 220	+ 57	− 190	− 135 − 136
	16	+ 26	+ 6		+ 32		+ 285
	17		− 67	+ 295			+ 226 + 224
	18	− 18	− 69	+ 230	− 51	+ 214	+ 163 + 163
	19		− 2		+ 14	+ 117 + 114	
	20	+ 17					
	21	− 31	+ 99	0	+ 73 + 72 + 71	− 29	+ 100
	22	− 2	− 44	− 50	+ 42	− 52	− 5
	24	− 3	− 73	− 268	− 80	− 273	− 194
	25	− 21	− 76	− 295 − 296	− 55	− 210	− 220
	27	+ 21	+ 16	− 162 − 160	+ 3	− 140	− 133
	28	− 19	+ 94	− 59	+ 75	− 38	+ 39
	30		+ 11		+ 36	+ 6	+ 43
September	1						+ 135
	2	− 13	− 11	+ 288	− 23	+ 302	− 276
	3	+ 22	+ 2	+ 281	+ 20	+ 259	+ 276
	4				+ 68	+ 153	+ 85
	6	+ 29	0	+ 9	+ 26	+ 17 + 20	+ 10
	7	+ 2	− 38	− 2	− 38	− 3	− 36
	8	− 26	− 87	− 70	− 62	− 46	− 14
	10	+ 22		− 288	+ 5	− 267	− 274
	12	− 21	+ 13				
	16		− 19				
	26	− 22	+ 80	− 224	+ 59	− 208	− 145 − 151
October	4	+ 0·010	+ 0·022	+ 0·188	+ 7	+ 0·181	− 170
	5				+ 0·010		

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Corrections to the previously found values of the unknowns were determined by taking the foregoing corrections as absolute terms in the equations of condition. From the manner in which the normals were solved, this new solution was easily made. The following are the corresponding corrections to the values of the unknowns: the probable errors of the unknowns which were derived from the sums of the squares of the residuals presently to be given are given alongside these corrections, and it will be seen that in nearly all cases the corrections are less than the probable errors; as was expected, the corrections to all the unknowns in longitude are without any practical meaning, and in 1902 the values of Δp and Δq are hardly affected, but the corrections to Δp_4 and Δq_4 in 1901 are larger.

CORRECTIONS TO VALUES OF UNKNOWNS.

	1901.	1902.		1901.	1902.
μ	−0·00053 ±0·00274	−0·00006 ±0·00183	ϵ_1	+0·00012 ±0·00917	−0·00037 ±0·00683
			ϵ_2	+0·00056 ±0·00586	0·00000 ±0·00417
			ϵ_3	+0·00077 ±0·00432	0·00000 ±0·00265
			ϵ_4	+0·00034 ±0·00182	+0·00003 ±0·00157
h_1	+0·0157 ±0·0282	+0·0025 ±0·0342			
k_1	−0·0177 ±0·0540	+0·0005 ±0·0250			
x_1	−0·0022 ±0·0468	+0·0004 ±0·0342			
			p_1	+0·01684 ±0·00713	−0·00465 ±0·00650
			q_1	−0·01058 ±0·00786	+0·00777 ±0·00592
h_2	+0·0040 ±0·0187	+0·0012 ±0·0175			
k_2	+0·0037 ±0·0312	+0·0022 ±0·0153			
x_2	−0·0037 ±0·0296	+0·0002 ±0·0186	p_2	+0·00053 ±0·00518	+0·00009 ±0·00400
			q_2	+0·01051 ±0·00452	+0·00563 ±0·00360
h_3	−0·0019 ±0·0137	+0·0005 ±0·0113			
k_3	+0·0101 ±0·0174	−0·0001 ±0·0101	p_3	+0·00437 ±0·00330	−0·00259 ±0·00253
x_3	+0·0045 ±0·0199	−0·0004 ±0·0120	q_3	−0·00160 ±0·00304	−0·00052 ±0·00210
h_4	−0·00411 ±0·00509	+0·00029 ±0·00337	p_4	+0·01075 ±0·00172	−0·00217 ±0·00133
k_4	+0·00596 ±0·00593	+0·00112 ±0·00558	q_4	+0·01606 ±0·00164	−0·00105 ±0·00130

The first thing that was done with the new values of the unknowns was to substitute them in the equations of condition in order to see if they caused the systematic run of the residuals to vanish. The new residuals are contained in the next two tables.

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Corrected Residuals, 1901.

1901.		I.–II.	I.–III.	I.–IV.	II.–III.	II.–IV.	III.–IV.
June	24				− 0".021	+ 0".082	+ 0".062
	25						+ 7
	29				− 149	− 8	+ 68
July	2	+ 0".105	+ 0".088	+ 0".017	+ 82	− 90	− 183
	8	− 82	− 36	+ 75	+ 246	− 89	+ 168
	10	− 154	+ 44	+ 69	+ 113	+ 2	+ 70
	11	− 89	+ 18	− 4	− 79	− 25	− 93
	12	− 104	− 24	− 4	+ 485	− 182	− 388
	14	− 34		+ 2		− 22	
	16	− 116	− 137	+ 18	+ 96	+ 158	− 397
	18	− 14			+ 41	− 58	
	22	− 47	− 276	+ 98			+ 257
	23	− 162	− 244		− 139		
	27	− 89	+ 164	+ 132	+ 4	+ 5	+ 122
	29		− 208	− 68			+ 36
August	10	+ 145	− 212	+ 122	+ 52	− 34	− 68
	13	+ 33	− 190	+ 341		− 138	+ 189
	14		− 291	− 189			− 49
	15				− 27	− 306	
	19	− 159	− 51	− 193	− 71		
	23				+ 6	− 23	+ 18
	24	+ 171	− 4	+ 50	− 314	+ 1	+ 12
	26	+ 63	− 42	− 176		− 31	
	27	+ 51	− 183	− 138	+ 408	− 115	+ 85
	28		+ 63	− 74			+ 31
	29	− 84	− 27	+ 47	− 339	− 101	+ 108
	31				+ 92		
September	1	− 66	+ 120	+ 90	+ 99	+ 140	− 51
	3	+ 311	− 60	+ 71	− 189	+ 82	+ 245
	4	− 90	+ 77	− 22	0	− 36	− 93
	9	+ 70		+ 49		+ 97	
	10	+ 34	+ 133		+ 60	− 162	
	14	+ 57	+ 8	+ 84	− 158	+ 123	+ 40
	16	− 36	− 118	+ 110	+ 39	+ 84	+ 22
	17	+ 191	+ 215	− 70	+ 35	+ 85	+ 51
	19	− 9	− 390	− 91	+ 0.070	+ 19	− 0.217
	23	− 40					
	25	− 16	− 0.053				
	27	− 0.109		− 0.005		+ 0.052	

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Corrected Residuals, 1902.

1902.		I.–II.	I.–III.	I.–IV.	II.–III.	II.–IV.	III.–IV.
June	10	− 0″·097	− 0″·113	− 0″·025			− 0″·096
	11				+ 0″·057	+ 0″·106	− 214
	16	− 12	− 11	− 250	− 7	− 177	+ 244
	19				− 22	+ 79	+ 139
	21		+ 86				
	24	− 7	− 196	− 95	+ 118	+ 130	− 138
	30	− 104		{ − 77 } { − 131 }		{ − 211 } { − 40 }	
July	1		− 111			− 89	+ 81
	3	+ 325	+ 150				
	7	− 83	+ 122	+ 57	− 91	− 20	− 316
	8	− 366	+ 55	+ 54	− 35	− 7	− 77
	14		− 22	+ 63			− 26
	15	− 103	+ 156	+ 175	− 60	+ 3	+ 69
	16	+ 91	+ 24	+ 97	− 72		
	18	− 29		− 392		{ − 101 } { − 74 }	
	19	− 15	0				
	20				+ 96	+ 152	{ − 197 } { + 3 }
	24	+ 12	− 129	+ 81	− 222	+ 113	− 57
	26	+ 48	+ 1	+ 51	+ 106	− 150	{ − 42 } { + 4 }
	27	− 9	− 104		− 141		
August	3	− 103	+ 107	+ 123	+ 43	− 51	{ + 20 } { + 252 }
	4	+ 194	+ 172		− 41		
	7	− 124	− 67	− 46	− 2	− 37	{ − 191 } { − 33 }
	16	− 22	+ 20		+ 113		− 52
	17		+ 1	− 22			{ + 95 } { + 83 }
	18	− 259	+ 139	+ 280	− 404	+ 42	{ − 260 } { − 168 }
	19		+ 130		+ 292	{ + 4 } { + 213 }	
	20	+ 377					
	21	+ 8	+ 20	+ 11	{ + 159 } { + 134 } { − 290 }	+ 14	− 126
	22	− 53	+ 270	− 44	− 32	+ 129	− 221
	24	+ 12	+ 233	− 18	− 38	+ 140	− 14
	25	− 82	+ 42	{ − 84 } { + 75 }	− 20	0	+ 73
	27	+ 150	+ 158	{ + 104 } { + 107 }	+ 11	+ 43	− 39
	28	+ 477	+ 161	+ 149	− 110	− 69	+ 18
	30		+ 159		+ 264	− 63	− 4
September	1						− 296
	2	+ 28	+ 38	− 109	+ 59	− 86	+ 230
	3	− 332	+ 140	− 55	+ 197	+ 188	− 10
	4				− 61	− 244	+ 92
	6	+ 155	+ 300	+ 26	+ 127	{ + 258 } { − 116 }	− 57
	7	− 36	+ 6	− 2	− 115	+ 10	+ 169
	8	+ 139	+ 43	+ 160	+ 41	+ 33	+ 326
	10	− 104		− 23	− 38	− 1	− 61
	12	+ 142	− 26				
	16		− 5				
	26	+ 28	+ 83	− 68	− 32	+ 245	{ + 68 } { + 114 }
October	4	− 0·193	− 0·111	− 0·153	+ 0·060	− 0·029	+ 0·087
	5				+ 0·063		

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The new residuals show a considerable improvement on the old ones, and they give no evidence of any systematic run: no such distribution of them as was found before (see Vol. XII. Part II. p. 172) can now be detected. For the sake of comparison it is convenient to give in the table below the sums of the squares of the new residuals (2) alongside of the previously found sums (1). It was assumed that the residuals in distance remained unchanged; it is true that they are slightly altered, but, as can easily be seen, the effect on them is without any real signification. In the next table the sums of the squares of the residuals and the squares of the mean errors are given for the various pairs of satellites. The mean error is given by

$$\epsilon^2 = \frac{[pvv]}{n-m}$$

where n is the number of equations and m the number of unknowns for the pair under consideration. Found in this way the mean error will be rather too large, because the distance equations are well separated from the position-angle equations, and several unknowns included in m in the denominator have almost no effect in reducing the residuals.

			I.–II.	I.–III.	I.–IV.	II.–III.	II.–IV.	III.–IV.
1901 [vv]	Distances . .		0".319	0".569	0".586	0".678	0".440	0".249
	Position-Angles	1	0·385	0·617	0·774	0·898	0·678	0·833
		2	0·372	0·702	0·355	0·853	0·320	0·660
1902 [vv]	Distances . .		0·534	0·558	0·774	0·588	0·913	0·966
	Position-Angles	1	1·037	0·640	1·384	0·786	0·993	1·382
		2	1·047	0·585	0·533	0·689	0·537	0·925
1901 ϵ^2	Distances . .		0·0189	0·0406	0·0366	0·0484	0·0259	0·0178
	Position-Angles	1	0·0214	0·0386	0·0455	0·0599	0·0376	0·0520
		2	0·0207	0·0439	0·0209	0·0569	0·0178	0·0413
1902 ϵ^2	Distances . .		0·0243	0·0223	0·0387	0·0245	0·0351	0·0322
	Position-Angles	1	0·0451	0·0246	0·0629	0·0314	0·0382	0·0446
		2	0·0455	0·0225	0·0242	0·0276	0·0207	0·0298

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The resulting probable errors of one observation of weight unity are then as follows :—

1901	In Distance	r_s	$\pm$ 0".095
	In Position-Angle	r_p	$\pm$ 0 .100
	In both together	r	$\pm$ 0 .094
1902	In Distance	r_s	$\pm$ 0 .100
	In Position-Angle	r_p	$\pm$ 0 .100
	In both together	r	$\pm$ 0 .097

There is thus nothing to choose in point of accuracy between the observations of distance and position-angle, and we have seen that neither the residuals in distance nor in position-angle show any systematic run.

Thus the unexplained systematic run of residuals in position-angle is accounted for and is now removed. The period originally found before the application of the correction to P, namely, one equal to half the period of the fourth satellite, was correct as far as it went. For the correction that has just been applied is

$$\frac{x_1 + x_2}{2} \tan \delta \cos \mathrm{P}\, s \sin 1''$$

which varies with

$$(x_1 + x_2)(x_2 - x_1) \text{ or } x^2_2 - x^2_1.$$

Hence, considering only the fourth satellite the correction varies with x^2_4 and clearly has a period of half its period of revolution; so that omission of the correction to P would give rise to a periodic run in the residuals with this period. This is what was found.

In the following table the corrected values of the unknowns are given together with the values derived from the photographs. These are the values as given directly by the solutions, and are referred to the epochs and fundamental planes adopted at the beginning of the reductions. The quantities denoted by F are the functions given on p. 102, viz.,

$$\mathrm{F} = -\Delta h \cos \mathrm{L} + \Delta k \sin \mathrm{L} \pm \Delta x \sin (\mathrm{L} + c)$$

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Values of Unknowns.

	1901.		1902.			
	Heliometer.		Heliometer.		Photographs.	
	Value of Unknowns.	Probable Error.	Value of Unknowns.	Probable Error.	Value of Unknowns.	Probable Error.
Δh_1	+0°·0913	±0°·0282	−0°·0413	±0°·0342	+0°·1758	±0°·0484
Δk_1	− ·1020	·0540	− ·0082	·0250	− ·1086	·0310
Δx_1	+ ·0490	·0468	− ·0602	·0342	+ ·2914	·0555
F_1	− ·0450	·0193	+ ·0090	·0153	− ·0762	·0071
Δh_2	− ·0148	·0187	− ·0401	·0175	+ ·1272	·0237
Δk_2	+ ·0297	·0312	+ ·0366	·0153	− ·0471	·0169
Δx_2	+ ·0628	·0296	+ ·1450	·0186	− ·0966	·0260
F_2	+ ·0665	·0120	+ ·0843	·0126	+ ·0428	·0041
Δh_3	+ ·0069	·0137	− ·0216	·0113	+ ·0257	·0178
Δk_3	− ·0688	·0174	− ·0166	·0101	− ·0393	·0105
Δx_3	− ·0144	·0199	− ·0293	·0120	+ ·0613	·0196
F_3	− ·0276	·0080	+ ·0022	·0081	− ·0282	·0027
Δh_4	+ ·0653	·0051	+ ·0847	·0034	+ ·0534	·0015
Δk_4	+ ·0390	·0059	+ ·0237	·0056	+ ·0406	·0054
F_4	+ ·0747	·0055	+ ·0859	·0033	+ ·0557	·0014
$\Delta \epsilon_1$	+ ·1360	·0092	+ ·1335	·0068	+ ·1882	·0083
$\Delta \epsilon_2$	+ ·0201	·0059	− ·0254	·0042	− ·0167	·0044
$\Delta \epsilon_3$	− ·0373	·0043	− ·0228	·0026	− ·0181	·0030
$\Delta \epsilon_4$	− ·0361	·0018	− ·0342	·0016	− ·0376	·0015
Δp_1	+ ·0338	·0071	+ ·0091	·0065	+ ·0009	·0080
Δq_1	− ·0800	·0079	− ·0677	·0059	− ·0759	·0062
Δp_2	+ ·1119	·0052	+ ·0923	·0040	+ ·0941	·0042
Δq_2	− ·1665	·0045	− ·1804	·0036	− ·1843	·0037
Δp_3	− ·0105	·0033	− ·0097	·0025	− ·0095	·0027
Δq_3	− ·0376	·0030	− ·0339	·0021	− ·0331	·0022
Δp_4	+ ·0564	·0017	+ ·0636	·0013	+ ·0610	·0014
Δq_4	− ·0198	±0·0016	− ·0080	±0·0013	− ·0107	±0·0013

Since the values here given are not in exact agreement with those from which the value of ΔJ was derived in the writer's former paper, pp. 189–200, it follows that the value found for J is in need of small correction. The corrected values of p_2 and q_2 are now

	1901·61	1902·60
p_2	− 0°·2929	− 0°·3501
q_2	− 0 ·4033	− 0 ·3282

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Whence the corrected node and inclination referred to the fundamental planes of the epochs are :—

	1901·61.	1902·60.
Node	144°·01	133°·16
Inclination	0 ·4984	0 ·4800.

The values of the node differ very little from those found before : hence the value of J can only differ very little.

Further experience has shown that the whole of the theory of Jupiter's satellites is much in need of revision, and with the theory in its present state little advantage can be gained by deriving values of isolated constants. Hence it did not seem worth while to find the small correction to the value of J, which corresponds to the small corrections to the unknowns found in this appendix. It need hardly be stated that the value found for the mass of Jupiter is not appreciably affected.

It is worth while to put together in this place the chief results of the writer's investigation.

Mass of Jupiter.

$$1 : 1047{\cdot}30 \pm 0{\cdot}06.$$

Constant of Compression of Jupiter.

$$J = 0{\cdot}01950 \left(\frac{39{\cdot}00}{d}\right)^2$$

where d is Jupiter's equatorial diameter.

Eccentricities and Perijoves.

The values of

$$h = 2\,E \sin \Omega$$
$$k = 2\,E \cos \Omega$$

for the four satellites are :—

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VALUES OF h, k.

	1901·61.	1902·60.			1901·61.	1902·60.	
	Heliometer.	Heliometer.	Photographs.		Heliometer.	Heliometer.	Photographs.
h_1	+ 0·0923	− 0·0391	+ 0·1780	h_3	− 0·0283	− 0·0618	− 0·0146
k_1	− ·1025	− ·0106	− ·1110	k_3	− ·2395	− ·1848	− ·2076
h_2	− ·0255	− ·0545	+ ·1128	h_4	+ ·4402	+ ·4516	+ ·4203
k_2	− ·0116	− ·0027	− ·0864	k_4	− ·6938	− ·7134	− ·6965

It will be remembered that Ω is measured from the point O, the vernal equinox of Jupiter. An explanation of the differences between the two sets of values for 1902 has been given on p. 101: it may be worth while to give here the values of the functions

$$G = -h \cos L + k \sin L \pm x \sin (L + c)$$

where h, k have the values given in the foregoing table and $x = x_0 + \Delta x$, x_0 being the adopted value, namely :—

Satellite I. . . .	x_0	= + 1555″	= + 0·4320
Satellite II. . . .		= + 3693	= + 1·0259
Satellite III. . . .		= + 228	= + 0·0633

VALUES OF FUNCTIONS G.

	1901·61.	1902·60.	
	Heliometer.	Heliometer.	Photographs.
I.	− 0·4411	− 0·3524	− 0·4377
II.	+ 0·9728	+ 0·9308	+ 0·8894
III.	− 0·2256	− 0·1013	− 0·1314
IV.	+ 0·1343	+ 0·4085	+ 0·3782

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Nodes and Inclinations.

When the nodes are reckoned from the point O and the inclinations are referred to the reference plane of the epoch, the values of

$$p = i \sin(-\Omega)$$
$$q = i \cos(-\Omega)$$

are as follow :—

VALUES OF p, q REFERRED TO REFERENCE PLANES OF EPOCHS AND THE POINT O.

	1901·61.	1902·60.		
			Photographs.	
	Heliometer.	Heliometer.	Orientation from Standards.	Orientation from Trails.
p_1	+ 0·0266	+ 0·0004	− 0·0078	− 0·0061
q_1	− ·0840	− ·0707	− ·0789	− ·0758
p_2	− ·2929	− ·3501	− ·3483	− ·3483
q_2	− ·4033	− ·3282	− ·3322	− ·3290
p_3	+ ·0566	+ ·0521	+ ·0523	+ ·0524
q_3	− ·1420	− ·1442	− ·1434	− ·1395
p_4	+ ·2169	+ ·2272	+ ·2246	+ ·2248
q_4	+ ·2685	+ ·2760	+ ·2734	+ ·2763

The planes to which these quantities are referred are defined relatively to the ecliptic and the earth's mean equinox by the following :—

	Node.	Long. of O.	Inclination.
1901·61	336° 27′ 6″	135° 29′ 20″	2° 9′ 10″
1902·60	336 33 33	135 30 15	2 11 38

and relatively to Jupiter's orbit by the quantities :—

	Node.	Long. of O.	Inclination.
1901·61	315° 30′ 36″	135° 30′ 36″	3° 4′ 5″
1902·60	315 51 5	135 31 30	3 6 22

If the longitudes of the nodes are measured from the earth's mean equinox instead of from the point O, the values of p_i' and q_i' are as follow, where now—

$$p_i' = i_i \sin(+\Omega_i')$$
$$q_i' = i_i \cos(+\Omega_i')$$

the planes of reference being the same as before.

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VALUES OF p', q' REFERRED TO REFERENCE PLANES OF EPOCH AND EARTH'S MEAN EQUINOX.

	1901·61.	1902·60.		
	Heliometer.	Heliometer.	Photographs.	
			Orientation from Standards.	Orientation from Trails.
p_1	− 0°·0399	− 0°·0492	− 0°·0609	− 0°·0575
q_1	+ ·0785	+ ·0508	+ ·0508	+ ·0498
p_2	− ·4916	− ·4797	− ·4812	− ·4790
q_2	+ ·0824	− ·0111	− ·0069	− ·0092
p_3	− ·0591	− ·0638	− ·0632	− ·0603
q_3	+ ·1410	+ ·1394	+ ·1389	+ ·1362
p_4	+ ·3429	+ ·3555	+ ·3518	+ ·3540
q_4	− ·0395	− ·0378	− ·0377	− ·0397

Whence by applying the formulæ given on p. 192 of Vol. XII. Part II., we find the following values of the nodes and inclinations of the instantaneous orbits referred to the earth's mean equinox and Jupiter's orbit at the epoch.

VALUES OF NODES AND INCLINATIONS REFERRED TO JUPITER'S ORBIT OF EPOCH AND EARTH'S MEAN EQUINOX.

		1901·61.	1902·60.		
		Heliometer.	Heliometer.	Photographs.	
				Orientation from Standards.	Orientation from Trails.
I.	Node	315° 59′ 33″	315° 51′ 12″	315° 42′ 9″	315° 44′ 0″
	Inclination	3 9 9	3 10 36	3 11 6	3 10 55
II.	Node	310 41 12	309 59 50	310 2 3	310 1 42
	Inclination	3 29 3	3 27 1	3 27 14	3 27 3
III.	Node	316 31 16	316 45 22	316 45 29	316 45 44
	Inclination	3 12 40	3 15 3	3 15 0	3 14 46
IV.	Node	320 2 2	320 28 16	320 24 53	320 25 24
	Inclination	2 48 17	2 50 26	2 50 35	2 50 24

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ANNALS OF THE CAPE OBSERVATORY.

VOL. XII. PART V.

DERIVATION OF FINAL INCLINATIONS AND NODES

OF THE

ORBITAL PLANES OF JUPITER'S SATELLITES

FROM THE

CAPE OBSERVATIONS OF 1891, 1901, 1902, 1903 AND 1904.

BY

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PROFESSOR OF ASTRONOMY IN THE UNIVERSITY OF LEIDEN.

Published by Order of the Lords Commissioners of the Admiralty in obedience to His Majesty's Command.

EDINBURGH:
PRINTED UNDER THE AUTHORITY OF HIS MAJESTY'S STATIONERY OFFICE
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DERIVATION OF FINAL INCLINATIONS AND NODES

OF THE

ORBITAL PLANES OF JUPITER'S SATELLITES.

In Part III. of this volume a discussion of the inclinations and nodes of the orbits of the satellites based on the observations of 1891, 1901, 1902, 1903, and 1904 was given. For several reasons this discussion could only be considered as provisional, as has been duly stated at the time.

Firstly, only provisional results of Mr Cookson's work were available at that time, and the assumption was necessarily made that these were referred to the same fundamental plane as those of the other epochs. Mr Cookson has since found corrections to the adopted places of his standard stars, used in 1902, and in consequence of this the fundamental plane to which his observed values of p and q are referred is not the same as the one used in other years (see Part IV., page 118, footnote).

Secondly, in the discussion in Part III. I was compelled to reject the observations of the Satellites III. and IV. in 1901 and 1902 on account of an inexplicable apparent periodic term found by Mr Cookson in the latitude of Satellite IV. (see Part III., pages 93 and 107). Mr Cookson has since found the true explanation of the anomalous residuals of Satellite IV. [which were due to the neglect of a certain small correction in the reduction of the position-angles], and has corrected his results accordingly (see Part IV., Appendix). The corrected results must now be used in the discussion.

Thirdly, as has been explained in Part III. (page 95), the theoretical basis of the solution depends to a certain extent on the adopted values of the masses. Souillart's values were used in Part III. I have in the *Publications of the Astronomical Laboratory at Groningen, No. 17*, derived new values of the masses, which, though still subject to considerable uncertainty, are doubtlessly more trustworthy than Souillart's. These must now be substituted for those of Souillart in the coefficients σ_{ij} (Part III., page 94) and in the theoretical motions of the nodes. The formulas by which this substitution is to be effected have also been derived by me in *Gron. Publ. 17* (pages 86–87).

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The present discussion was made in the last months of the year 1907, and the account of it which is now presented was drawn up and written out for press at that time. The results have been incorporated in the summary of all my work on Jupiter's satellites which was communicated to the Academy of Sciences at Amsterdam in March 1908.*

The notations employed here are the same as those used in Part III., to which the reader is referred for explanation.

The Fundamental Plane.—The adopted fundamental plane has been defined in Part III., pages 95–97. For the several epochs we find the following comparison between the longitudes of the node and the inclinations of this plane on the ecliptic, and the longitude of the zero 0 of longitudes in the plane, and the values of these quantities as actually used in the computation of the tabular places.

1891·75.		1901·61.		1902·62.		1903·72.		1904·89.	
Fundamental Plane.	Tabular Places.	Fundamental Plane.	Tabular Places.	Fundamental Plane.	Tabular Places.	Fundamental Plane.	Tabular Places.	Fundamental Plane.	Tabular Places.
$J = 2^\circ\ 9'\ 1''{\cdot}1$	$1''{\cdot}1$	$9'\ 4''{\cdot}5$	$9'\ 10''$	$9'\ 4''{\cdot}8$	$8'\ 38''$	$9'\ 5''{\cdot}2$	$5''{\cdot}3$	$9'\ 5''{\cdot}6$	$5''{\cdot}6$
$N = 336\ 15\ 6{\cdot}5$	$6{\cdot}5$	$22\ 43{\cdot}2$	$27\ 6$	$23\ 28{\cdot}7$	$33\ 33$	$24\ 20{\cdot}6$	$20{\cdot}5$	$25\ 15{\cdot}1$	$35{\cdot}0$
$O = 135\ 20\ 31{\cdot}3$	$31{\cdot}3$	$28\ 19{\cdot}2$	$29\ 20$	$29\ 5{\cdot}8$	$30\ 15$	$29\ 58{\cdot}9$	$58{\cdot}9$	$30\ 54{\cdot}8$	$54{\cdot}8$

The degrees of the values employed for the tabular places are the same as for the adopted fundamental plane; where the minutes are not repeated, they also are the same. The values employed for the tabular places have been taken from the published results, with the exception of the tabular value of the inclination in 1902. This is given by Mr Cookson as $2^\circ\ 11'\ 38''$, which is evidently a mistake, as it is not consistent with the inclination and nodes referred to the equator. The tabular value of N in 1904 contains a small error of computation, which was detected too late to be corrected in Part III.

We have thus the following excesses of the tabular values over those corresponding to the adopted plane:—

	1891.	1901.	1902.	1903.	1904.
ΔJ	$0''$	$+\ 6''$	$-\ 27''$	$0''$	$0''$
ΔN	0	+263	+604	0	+20
ΔO	0	+ 61	+ 69	0	0.

* "On the Masses and Elements of Jupiter's Satellites and the Mass of the System," by Dr W. de Sitter, *Proceedings of the Royal Academy of Sciences at Amsterdam*, vol. x. pp. 653–673 and 710–729.

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The corresponding excesses of the tabular values of ω_0 and ψ_0 over the correct ones are

	1901.	1902.	1904.
$\Delta\omega_0$	$+0^\circ\cdot0007$	$-0^\circ\cdot0092$	$-0^\circ\cdot0001$
$\Delta\psi_0 \sin 1^0$	$+0^\circ\cdot0010$	$+0^\circ\cdot0010$	$+0^\circ\cdot0001$.

Now p_i and q_i were computed by the formulas*

$$p_i = \Sigma_j b_{ij} \sin(\psi_0 - \theta_j) + \mu_i\omega \sin(\psi_0 - \theta_0),$$
$$q_i = \Sigma_j b_{ij} \cos(\psi_0 - \theta_j) + \mu_i\omega \cos(\psi_0 - \theta_0) + \omega_0,$$
$$(j = 1, 2, 3, 4)$$

where the values of $\psi_0 - \theta_i$ and $\psi_0 - \theta_0$, *and not those of* ψ_0, θ_i *and* θ_0 *separately*, were taken from Souillart's theory. Consequently the angles $\psi_0 - \theta_i$ and $\psi_0 - \theta_0$ require no correction, and the only correction necessary to reduce all observations to one and the same fundamental plane is

$$\Delta q_i = \Delta\omega_0 .$$

Observed Values, Weights.—The corrections so derived were applied. For 1901 and 1902 Mr Cookson's definitive results were used. For 1891 and 1902 the results from the heliometer and the photographic plates were combined, giving double weight to the heliometer in each case. For the plates of 1902 the results from the solution in which the orientation was determined from the trails were adopted. We thus find the observed corrections Δp_i and Δq_i which are contained in Table I. The values for 1891 differ slightly from those given in Part III. This is due to some corrections which were applied to the results for 1891, and which have been explained in Part I. of this volume, Chapter II.

The "computed probable errors" are those directly derived from the observations of each epoch. The "adopted probable errors" correspond to the adopted weights and a probable error for weight unity of $\pm0^\circ\cdot0100$. These adopted weights were assigned as follows.

In deriving the computed probable errors from the solution of the equations for each opposition, equal weight was given to one observation of any one satellite. The necessary consequence of this is that the unknowns Δp and Δq belonging to the outer satellites receive a very much higher weight than those of the inner satellites. This is evident from the computed probable errors, which are inversely proportional to the square roots of those weights. It was found in Part III. that the observations of

* A small mistake has been made by Marth in the computation of these formulas. He has for ω_0 used the value stated by himself (*M.N.*, 1891, p. 510). For ω in the terms $\mu_i\omega \begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(\psi_0 - \theta_0)$, however, he takes Souillart's value. Now these are not identical, as was pointed out by me (Part III., p. 96). The effect of this error on the values of q_i is negligible.

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Satellite IV., and also, though in a minor degree, those of III. and I., leave much larger residuals than are consistent with the assumption that the observations of all satellites are of the same accuracy. It thus appeared desirable to assign a lower weight to the results relating to these satellites. The adopted weights were derived from the residuals of a preliminary solution. They are :—

Satellite	I.	II.	III.	IV.
Weight	1	5	7	10.

The weights used in Part III., *i.e.* those depending on the solution with equal weight to each observation, were

Weight formerly used . . .	1·2	2·8	6·7	25·0.

TABLE I.

Observed Corrections and Probable Errors.

Epoch.	Observed Corrections.	Computed p.e.	Adopted p.e.	Observed Corrections.	Computed p.e.	Adopted p.e.	Observed Corrections.	Computed p.e.	Adopted p.e.	Observed Corrections.	Computed p.e.	Adopted p.e.
	Δp_1.			Δp_2.			Δp_3.			Δp_4.		
1891·75	+0·0364	±·0034	±·0045	+0·0746	±·0024	±·0020	−0·0027	±·0015	±·0018	+0·0633	±·0008	±·0014
1901·61	+ ·0338	± 71	± 100	+ ·1119	± 52	± 45	− ·0105	± 33	± 37	+ ·0564	± 17	± 31
1902·62	+ ·0069	± 51	± 59	+ ·0929	± 30	± 26	− ·0096	± 19	± 21	+ ·0628	± 10	± 19
1903·72	+ ·0021	± 60	± 71	+ ·0526	± 33	± 31	− ·0199	± 22	± 26	+ ·0583	± 12	± 21
1904·89	− ·0028	± 78	± 100	+ ·0158	± 44	± 45	− ·0104	± 28	± 37	+ ·0648	± 13	± 31
	Δq_1.			Δq_2.			Δq_3.			Δq_4.		
1891·75	−0·0268	±·0038	±·0045	+0·0849	±·0023	±·0020	−0·0703	±·0010	±·0018	−0·0130	±·0008	±·0014
1901·61	− ·0793	± 79	± 100	− ·1658	± 45	± 45	− ·0369	± 30	± 37	− ·0191	± 16	± 31
1902 62	− ·0786	± 44	± 59	− ·1899	± 27	± 26	− ·0415	± 16	± 21	− ·0171	± 10	± 19
1903·72	− ·0597	± 48	± 71	− ·2120	± 32	± 31	− ·0442	± 20	± 26	− ·0209	± 11	± 21
1904·89	− ·0336	± 77	± 100	− ·2253	± 48	± 45	− ·0477	± 26	± 37	− ·0210	± 17	± 31

The weights assigned to the different epochs have been derived directly from the computed probable errors, found in the different solutions for the same unknowns. They are :—

Opposition . . .	1891	1901	1902	1903	1904
Weight	5	1	3	2	1.

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The adopted weights are given in Table II.

TABLE II.

Weights.

Opposition.	Satellite I.	Satellite II.	Satellite III.	Satellite IV.
1891	5	25	35	50
1901	1	5	7	10
1902	3	15	21	30
1903	2	10	14	20
1904	1	5	7	10

These weights correspond to a probable error of weight unity of

$$\pm 0^{\circ}\cdot 0100.$$

Those used in Part III., and given there in Table XXIII., correspond to $\pm 0^{\circ}\cdot 0050$. The latter must, therefore, be multiplied by four to make them comparable with the weights given above.

Theoretical Basis of the Solution.—We must now for the masses used in Souillart's theory substitute those derived in *Gron. Publ. 17*, and determine the corresponding values of the coefficients σ_{ij} and μ_i (see Part III., pages 94, 95).

These masses are (*Gron. Publ. 17*, pages 116, 118):—

$m_1 = 0\cdot 0000256$
$m_2 = 0\cdot 0000231$
$m_3 = 0\cdot 0000820$

$Jb^2 = 0\cdot 02125$ ($b = 1$ for $d = 39''\cdot 0$)
$= 0\cdot 000000005141$ (astron. units).

For m_4 Souillart's value has been retained. The adopted value of Jb^2 is very nearly that found in Part III., Solution VI. The corresponding values of the factors ν_i and κ' which occur in the formulas of *Gron. Publ. 17*, are given there on page 81, viz.:—

$\nu_1 = -0\cdot 360$ $\quad$ $\nu_3 = +0\cdot 025$
$\nu_2 = +0\cdot 050$ $\quad$ $\kappa' = +0\cdot 025$.

We find then by means of the formulas of *Gron. Publ. 17*, pages 86–87, the following values of the coefficients, which must replace those given in Part III., Table XX.

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TABLE III.

Theoretical Coefficients.

i.	σ_{i1}.	σ_{i2}.	σ_{i3}.	σ_{i4}.	μ_i.
1	1	+ ·0204	+ ·0059	− ·0018	0·9994
2	− ·019	1	+ ·1524	+ ·0189	0·9942
3	− ·001	− ·0348	1	+ ·1216	0·9725
4	·000	− ·0010	− ·1829	1	0·8631

The corresponding values of σ_{ij}' and μ_i' which are employed in the solution are given in Table IX., which now replaces Table XXI. of Part III.

TABLE IV.

Theoretical Coefficients of Solution.

i.	σ_{i1}'.	σ_{i2}'.	σ_{i3}'.	σ_{i4}'.	μ_i'.
1	+0·9996	−0·0205	−0·0023	+0·0025	0·9786
2	+ ·0188	+ ·9944	− ·1517	− ·0004	0·8595
3	+ ·0016	+ ·0337	+ ·9732	− ·1190	0·8788
4	+ ·0004	+ ·0072	+ ·1779	+ ·9783	1·0250

The values of Δx_i and Δy_i as computed by the formulas

$$\Delta x_i = \Sigma_j \sigma_{ij}' \Delta p_j$$
$$\Delta y_i = \Sigma_j \sigma_{ij}' \Delta q_j$$

are given in Table V., which replaces Table XXII. of Part III.

TABLE V.

Transformed Right-Hand Members of Equations of Condition.

Epoch.	Satellite I.		Satellite II.		Satellite III.		Satellite IV.	
	Δx_1.	Δy_1.	Δx_2.	Δy_2.	Δx_3.	Δy_3.	Δx_4.	Δy_4.
1891·75	+ ·0351	− ·0283	+ ·0751	+ ·0946	− ·0075	− ·0640	+ ·0619	− ·0246
1901·61	+ ·0316	− ·0758	+ ·1135	− ·1608	− ·0130	− ·0393	+ ·0541	− ·0265
1902·62	+ ·0052	− ·0746	+ ·0940	− ·1839	− ·0137	− ·0449	+ ·0604	− ·0255
1903·72	+ ·0011	− ·0554	+ ·0553	− ·2052	− ·0245	− ·0477	+ ·0539	− ·0298
1904·89	− ·0029	− ·0290	+ ·0172	− ·2174	− ·0173	− ·0516	+ ·0616	− ·0306

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The motions of the nodes corresponding to the adopted masses are, by the formulas of *Gron. Publ. 17*—

$$\frac{d\Gamma_1}{dt} = +0^{\circ}\cdot13517 \;+0^{\circ}\cdot1357\;\kappa'',$$
$$\frac{d\Gamma_2}{dt} = +0\cdot032218+0\cdot02662\kappa'',$$
$$\frac{d\Gamma_3}{dt} = +0\cdot006849+0\cdot00506\kappa'',$$
$$\frac{d\Gamma_4}{dt} = +0\cdot001816+0\cdot00079\kappa'',$$

where, in accordance with *Gron. Publ. 17*, page 81,

$$\kappa'' = \kappa' - 0\cdot025 = \kappa + 0\cdot030$$
$$\kappa = \frac{\Delta(Jb^2)_0}{(Jb^2)_0} \qquad (Jb^2)_0 = 0\cdot0219087 \qquad (b=1 \text{ for } d = 39''\cdot0).$$

The meaning of κ is thus here different from what it was in Part III., owing to the adoption of a different value for $(Jb^2)_0$.

The coefficients of κ'' differ slightly from the approximate values (derived from Souillart's theory) used in Part III. (page 100).

The values of p_i and q_i are found from the formulas*

$$p_i = \Sigma_j \sigma_{ij}\gamma_j \sin\Gamma_j + \text{periodic inequalities}$$
$$q_i = \Sigma_j \sigma_{ij}\gamma_j \cos\Gamma_j + (1-\mu_i)\omega_0 + \text{periodic inequalities.}$$

Now, if we wish to adopt the above values of σ_{ij} and μ_i instead of Souillart's, at the same time retaining the same tabular values of γ_i and Γ_i, *i.e.* of x_i and y_i, then the tabular values of p_i and q_i (*i.e.* those to which the observations give the corrections Δp_i and Δq_i contained in Table I.) must be corrected accordingly. The required corrections, which must be subtracted from the observed values of Δp_i and Δq_i, are

$$\delta p_i = \Sigma_j \delta\sigma_{ij} \cdot x_j$$
$$\delta q_i = \Sigma_j \delta\sigma_{ij} \cdot y_j - \delta\mu_i \cdot \omega_0,$$

where $\delta\sigma_{ij}$ and $\delta\mu_i$ are: adopted value *minus* Souillart.

It is, however, preferable to continue to use the same values of p_i and q_i and to correct the tabular values of x_i and y_i, *i.e.* the values denoted by x_{i0} and y_{i0} in Part III. so as to make them correspond to the tabular values of p_i and q_i and the adopted coefficients. The corrections which must be applied to the values of x_{i0} and y_{i0} as used in Part III. are then

(a)
$$\delta x_{i0} = \Sigma_j \delta\sigma_{ij}' \cdot p_j$$
$$\delta y_{i0} = \Sigma \delta\sigma_{ij}' \cdot q_j + [\delta\mu_i' - \Sigma_j \delta\sigma_{ij}']\omega_0.$$

* Part III., p. 98. The periodic inequalities are not mentioned there, and they play no part in the discussions either of Part III. or of the present pages, since they are too small to be subject to appreciable corrections. They have, however, been correctly applied in the tabular places of all epochs, Souillart's values being adopted.

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The two terms in the coefficient of ω_0 in δy_{i0} practically cancel each other, except for Satellite IV., where $\delta\mu_i'$ becomes very large owing to the large difference of μ_4 from unity.

If, now, we replace x_{i0} and y_{i0} of Part III. by $x_{i0}+\delta x_{i0}$ and $y_{i0}+\delta y_{i0}$, we get the following values of $x_{i0}+\Delta x_i$ and $y_{i0}+\Delta y_i$, which replace those given in Part III., Table XXIV.

TABLE VI.

Right-Hand Members of Equations (8).

Epoch.	Satellite I.		Satellite II.		Satellite III.		Satellite IV.	
	$x_{10}+\Delta x_1$.	$y_{10}+\Delta y_1$.	$x_{20}+\Delta x_2$.	$y_{20}+\Delta y_2$.	$x_{30}+\Delta x_3$.	$y_{30}+\Delta y_3$.	$x_{40}+\Delta x_4$.	$y_{40}+\Delta y_4$.
1891·75	+ ″·0341	− ″·0269	+ ″·4712	− ″·1596	+ ″·0986	− ″·2090	+ ″·2479	− ″·1658
1901·61	+ ·0331	− ·0762	− ·2988	− ·3857	+ ·0208	− ·2157	+ ·2243	− ·1888
1902·62	+ ·0058	− ·0758	− ·3563	− ·3167	+ ·0119	− ·2226	+ ·2288	− ·1898
1903·72	+ ·0005	− ·0563	− ·4131	− ·2314	− ·0076	− ·2264	+ ·2205	− ·1962
1904·89	− ·0038	− ·0285	− ·4432	− ·1280	− ·0098	− ·2311	+ ·2260	− ·1994

New Solution from Modern Observations alone.—As a first approximation, the following values of x_0 and y_0 were assumed :—

$$x_{00} = -0^{\circ}\cdot0200$$
$$y_{00} = +0\ \cdot0452.$$

The corrections to be applied to the quantities of Table VI. are then :—

$\mu_1'x_{00} = -^{\circ}\cdot0196$ $\quad \mu_2'x_{00} = -^{\circ}\cdot0172$ $\quad \mu_3'x_{00} = -^{\circ}\cdot0176$ $\quad \mu_4'x_{00} = -^{\circ}\cdot0205$

$\mu_1'y_{00} = +\ \cdot0442$ $\quad \mu_2'y_{00} = +\ \cdot0388$ $\quad \mu_3'y_{00} = +\ \cdot0397$ $\quad \mu_4'y_{00} = +\ \cdot0463$

The following auxiliary quantities were computed by the formula (11) [Part III., page 102].

TABLE VII.

Auxiliary Quantities. Solution VIII.

Epoch.	Satellite I.		Satellite II.		Satellite III.		Satellite IV.	
	g_1.	G_1.	g_2.	G_2.	g_3.	G_3.	g_4.	G_4.
1891·75	″·0225	39·9	″·4698	104·90	″·1877	154·43	″·2570	117·76
1901·61	·0348	147·1	·4692	222·33	·1760	178·96	·2488	125·00
1902·62	·0345	203·6	·4656	233·35	·1830	181·79	·2531	124·60
1903·72	·0226	237·6	·4715	245·89	·1884	187·69	·2500	126·89
1904·89	·0282	303·9	·4689	259·03	·1934	188·15	·2564	126·72

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The solution was effected in exactly the same way as those of Part III. The result was :—

Solution VIII.

$$x_0 = -0''.0197 \pm .0015$$
$$y_0 = +\ .0454 \pm .0029$$
$$\kappa'' = +0''.0026 \pm .0058, \text{ from which } \kappa = -.0274$$

$\gamma_1 = 0^\circ.0270 \pm .0019$	$\Gamma_{10} = 74^\circ.8 \pm 6^\circ.3$
$\gamma_2 = .4688 \pm 17$	$\Gamma_{20} = 202\ .21 \pm 0\ .18$
$\gamma_3 = .1860 \pm 28$	$\Gamma_{30} = 175\ .65 \pm 0\ .48$
$\gamma_4 = .2543 \pm 24$	$\Gamma_{40} = 123\ .35 \pm 0\ .62.$

The residuals from this solution are :—

TABLE VIII.

Residuals of Solution VIII.

Epoch.	Satellite I.			Satellite II.			Satellite III.			Satellite IV.		
	$\Delta\gamma_1$.	$\sin\gamma_1\Delta\Gamma_1$.	Probable Error.	$\Delta\gamma_2$.	$\sin\gamma_2\Delta\Gamma_2$.	Probable Error.	$\Delta\gamma_3$.	$\sin\gamma_3\Delta\Gamma_3$.	Probable Error.	$\Delta\gamma_4$.	$\sin\gamma_4\Delta\Gamma_4$.	Probable Error.
1891·75	$-.0042$	$+.0062$	$\pm.0045$	$+.0013$	$-.0007$	$\pm.0020$	$+.0017$	$-.0022$	$\pm.0018$	$+.0029$	$-.0007$	$\pm.0014$
1901·61	+ 78	− 38	± 100	+ 1	+ 91	± 45	− 101	− 27	± 37	− 53	+ 22	± 31
1902·62	+ 72	− 7	± 59	− 35	+ 15	± 26	− 31	− 18	± 21	− 10	− 25	± 19
1905·72	− 44	− 97	± 71	+ 23	− 15	± 31	+ 23	+ 87	± 26	− 41	+ 44	± 21
1904·89	+ 11	− 58	± 100	− 2	− 74	± 45	+ 73	+ 5	± 37	+ 23	+ 2	± 31

The probable errors added for comparison are the adopted probable errors of Table I.

The probable error of weight unity derived from these residuals is

$$\pm 0^\circ.0101.$$

This solution is extremely satisfactory. Nothing has been rejected, and still the representation is better than in Part III., where the Satellites III. and IV. were rejected in 1901 and 1902. This is, of course, due to the corrections applied by Mr Cookson to his results for these years, to the reduction of all observations to one and the same fundamental plane, and also, but in a much smaller degree, to the improvement in the masses and consequently in the coefficients σ_{ij}. But the most satisfactory point of the solution, which does not appear from the residuals, is the enormous improvement of the representation of the values of the nodes in 1750. The Solution VIII. is based on the modern observations alone, and is thus comparable with Solution VI. of Part III.,

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and not with Solution VII. The values of θ_i and their motions corresponding to the values of Γ_{i0} and of κ'' found from Solution VIII. are

$$\theta_1 = 60^\circ 6 - 0^\circ 13548\ t,$$
$$\theta_2 = 293^\circ 24 - 0^\circ 032251\ t,$$
$$\theta_3 = 319^\circ 80 - 0^\circ 006826\ t,$$
$$\theta_4 = 12^\circ 10 - 0^\circ 001782\ t,$$

where t is the time counted in days from 1900 Jan. 0·0.

For 1750·0 we find, thus :—

	Solution VI.	Solution VIII.	Delambre.	Damoiseau.
θ_2	235°·4	260°·0	283°·3	282°·0
θ_3	329·2	333·8	352·5	353·5
θ_4	114·0	109·6	105·0	98·3.

The expectation expressed in Part III. (page 122), that the change introduced into the results by the adoption of better data "may be confidently expected to be in the direction of reconciling the two solutions," is thus entirely fulfilled.

Derivation of New Masses.—From the comparison of the values of θ_i in 1750 derived from Solution VIII. with those of Delambre, we find the following corrections to the motions of the nodes :—

$$\delta \frac{d\theta_2}{dt} = -0^\circ 000425 \pm 000200$$
$$\delta \frac{d\theta_3}{dt} = - 000341 \pm 200$$
$$\delta \frac{d\theta_4}{dt} = + 000084 \pm 500.$$

The probable errors are only rough estimates.

On the other hand, we can derive similar corrections from the residuals $\sin \gamma_i\ \Delta\Gamma_i$ given in Table VIII. From these we find

$$\delta \frac{d\theta_1}{dt} = +0^\circ 00935 \pm 00291$$
$$\delta \frac{d\theta_2}{dt} = -0^\circ 000011 \pm 000088$$
$$\frac{d\theta_3}{dt} = -0^\circ 000484 \pm 000232$$
$$\delta \frac{d\theta_4}{dt} = -0^\circ 000127 \pm 000111.$$

The probable errors in this case have been derived in the usual way from the residuals remaining after substitution of these values.

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Now, the expressions of the motions of the nodes in terms of the masses (*Gron. Publ. 17*, page 87) give the following equations of condition :—

							Delambre.	Modern.
θ_1	$+\cdot1357\delta\kappa''$	$+\cdot0001\delta\nu_1$	$+\cdot0022\delta\nu_2$	$+\cdot0010\delta\nu_3$	$+\cdot0001\nu_4$	$=$		-0094 ± 0029
θ_2	$+0266$	$+\cdot0030$	$+\cdot0001$	$+\cdot0040$	$+\cdot0002$	$=$	$+\cdot00042\pm00020$	$+\cdot00001\pm00009$
θ_3	$+\cdot0051$	$+\cdot0003$	$+\cdot0007$	$\cdot0000$	$+\cdot0007$	$=$	$+\cdot00034\pm\ 20$	$+\cdot00048\pm\ 23$
θ_4	$+\cdot0008$	$\cdot0000$	$+\cdot0001$	$+\cdot0007$	$-\cdot0001$	$=$	$-\cdot00008\pm\ 50$	$+\cdot00013\pm\ 11$.

The results from the Satellites I. and IV. are too uncertain to be of much value for the derivation of the masses. The two results from Satellite III. are in excellent agreement with each other, but they are inconsistent with the values derived from Satellite II., unless we assume a very large positive value of ν_4.

The above equations of condition have been combined with similar equations derived from the motion of the perijove of IV., from the large inequalities, and from the period of the libration. The discussion has been published in the *Proceedings of the Academy of Sciences at Amsterdam*, March 1908 (vol. x. pages 718–719), and need not be repeated here. The finally adopted values with their probable errors are (*loc. cit.*, page 719) :—

$$\begin{aligned}
\delta\kappa' &= +0\cdot005 \pm \cdot0075\\
\delta\nu_1 &= +0\cdot010 \pm \cdot030\\
\delta\nu_2 &= \ \ 0 \ \ \pm \cdot050\\
\delta\nu_3 &= -0\cdot020 \pm \cdot020\\
\nu_4 &= \ \ 0 \ \ \pm \cdot25.
\end{aligned}$$

The corresponding masses are* :—

$$\begin{aligned}
Jb^2 &= 0\cdot02142 \pm \cdot00015 \qquad (b = 1 \text{ for } d = 39''\cdot0)\\
&= 0\cdot000000005182 \pm \cdot000000000040 \text{ (astron. units)}\\
m_1 &= 0\cdot0000261 \pm \cdot0000012\\
m_2 &= 0\cdot0000231 \pm \ \ 011\\
m_3 &= 0\cdot0000804 \pm \ \ 016\\
m_4 &= 0\cdot0000425 \pm \ \ 106.
\end{aligned}$$

* By a later investigation (1910), which has not been published, I have been led to adopt the following values :—

$$\begin{aligned}
Jb^2 &= 0\cdot000000000500\\
m_1 &= 0\cdot0000390\\
m_2 &= 0\cdot0000250\\
m_3 &= 0\cdot0000820\\
m_4 &= 0\cdot0000450
\end{aligned}$$

These values, while leaving the motions of the nodes of II., III. and IV. practically unaltered, increase the motion of θ_1 by about $+0\cdot0034$, *i.e.* about one-third of the correction required by the modern observations.

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The corresponding motions of the nodes are

$$\frac{d\theta_1}{dt} = -0^\circ\cdot13614 \pm 0^\circ\cdot00100$$
$$\frac{d\theta_2}{dt} = -0\cdot032335 \pm 0\cdot000240$$
$$\frac{d\theta_3}{dt} = -0\cdot006854 \pm 0\cdot000180$$
$$\frac{d\theta_4}{dt} = -0\cdot001772 \pm 0\cdot000030.$$

The probable errors correspond to the adopted probable errors of the masses and Jb^2.

Final Solution based on Adopted Masses.—If, now, the masses which have been derived above are adopted, the values of σ_{ij} and μ_i, and therefore those of σ_{ij}' and μ_i', are altered. The new values are given in the Tables IX. and X.

TABLE IX.

Final Theoretical Coefficients.

i.	σ_{i1}.	σ_{i2}.	σ_{i3}.	σ_{i4}.	μ_i.
1	1	+ ·0203	+ ·0056	− ·0018	0·9994
2	− ·019	1	+ ·1488	+ ·0183	0·9943
3	− ·001	− ·0347	1	+ ·1203	0·9727
4	·000	− ·0010	− ·1772	1	0·8625

TABLE X.

Final Theoretical Coefficients of Solution.

i.	σ_{i1}'.	σ_{i2}'.	σ_{i3}'.	σ_{i4}'.	μ_i'.
1	+0·9996	−0·0205	−0·0021	+0·0024	0·9786
2	+ ·0188	+ ·9944	− ·1483	− ·0004	0·8629
3	+ ·0016	+ ·0336	+ ·9739	− ·1178	0·8804
4	+ ·0004	+ ·0070	+ ·1726	+ ·9791	1·0193

Now the values of Δx_i and Δy_i of Table V. must be corrected by the addition of

$$\delta\Delta x_i = \Sigma_j \delta\sigma'_{ij} \cdot \Delta p_j,$$
$$\delta\Delta y_i = \Sigma_j \delta\sigma'_{ij} \cdot \Delta q_j,$$

where $\delta\sigma_{ij}'$ are the differences between the values given in Table X. and those of Table

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IV. (in the sense X.–IV.) At the same time, the values of x_{i0} and y_{i0} must be corrected again by the formulas (a) given above (page 9). The sums of these two corrections may be denoted by dx_i and dy_i. They are given in Table XI.

TABLE XI.

CORRECTIONS TO BE APPLIED TO VALUES OF $x_i + \Delta x_i$ AND $y_i + \Delta y_i$ CONTAINED IN TABLE VI.

Epoch.	dx_1.	dy_1.	dx_2.	dy_2.	dx_3.	dy_3.	dx_4.	dy_4.
1891·75	·0000	− ·0003	+ ·0004	− ·0004	+ ·0003	+ ·0006	− ·0006	− ·0039
1901·61	0	− 3	+ 2	− 5	+ 1	+ 5	− 1	− 38
1902·62	0	− 3	+ 2	− 5	+ 2	+ 5	− 1	− 37
1903·72	0	− 3	+ 1	− 6	+ 1	+ 5	0	− 37
1904·89	0	− 3	+ 1	− 6	+ 2	+ 5	0	− 37

The consequent corrections to the auxiliary quantities g_i and G_i are computed by the equations

$$dg_i = \sin \Gamma_i \, dx_i + \cos \Gamma_i \, dy_i$$
$$\sin g_i \, dG_i = \cos \Gamma_i \, dx_i - \sin \Gamma_i \, dy_i .$$

From the equations of condition thus corrected, the ten unknowns γ_i, Γ_{i0}, x_0 and y_0 must be determined. The motions of the nodes are now considered as being no longer subject to correction; this solution is therefore entirely analogous to Solution VII. in Part III., and the unknown κ'' is not introduced.

Since, however, the corrections dx_i and dy_i, and therefore also dg_i and $\sin g_i \, dG_i$ are extremely small, it is not necessary to make a new solution, but it is sufficient to find corrections to the Solution VIII. The equations of condition giving these corrections are

$$d\gamma_i - \mu_i' \sin G_i \, dx_0 - \mu_i' \cos G_i \, dy_0 = dg_i$$
$$\sin \gamma_i \, d\Gamma_{i0} - \mu_i' \cos G_i \, dx_0 + \mu_i' \sin G_i \, dy_0 = \sin g_i \, dG_i - (t - t_0) \sin \gamma_i \, d\frac{d\Gamma_i}{dt},$$

where dg_i and $\sin g_i \, dG_i$ are determined by the equations given above, and $d\frac{d\Gamma_i}{dt}$ are the differences between the adopted motions of Γ_i (corresponding to the adopted masses) and those resulting from the value of κ'' found in Solution VIII.

The corrections $d\gamma_i$, $\sin \gamma_i \, d\Gamma_{i0}$, dx_0 and dy_0 were determined from the solution of these equations of condition in the usual way. Adding them to the values found in Solution VIII., we get the following results :—

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Solution IX.

$$x_0 = -0^{\circ}\cdot0202 \pm \cdot0014$$
$$y_0 = +0\cdot0473 \pm 14$$

$\gamma_1 = 0^{\circ}\cdot0272 \pm {}^{\circ}\cdot0028$	$\Gamma_{10} = 75^{\circ}\cdot2 \pm 7^{\circ}\cdot0$
$\gamma_2 = \cdot4683 \pm 16$	$\Gamma_{20} = 202\cdot27 \pm 0\cdot19$
$\gamma_3 = \cdot1839 \pm 26$	$\Gamma_{30} = 175\cdot72 \pm 0\cdot52$
$\gamma_4 = \cdot2536 \pm 23$	$\Gamma_{40} = 123\cdot47 \pm 0\cdot67$.

The residuals from this solution were computed, but they need not be given here, as they are practically identical to those contained in Table VIII.

The values of the nodes in 1750·0 now become

	Solution IX.	Delambre.	Damoiseau.
θ_2	$264^{\circ}\cdot7 \pm 13^{\circ}\cdot2$	283·3	282·0
θ_3	$355\cdot2 \pm 10\cdot0$	352·5	353·5
θ_4	$109\cdot1 \pm 1\cdot8$	105·0	98·3.

It seems very probable that the values computed from Solution IX. are actually correct within the limits given by the probable errors, and that the remaining differences are due to errors in the old determinations. It may be remembered that the agreement between the values found by Delambre and by Damoiseau is no measure of the accuracy of their respective determinations, since it is entirely unknown in how far Damoiseau is independent of Delambre. The differences may, of course, also partly be due to defects in the theory * by which the motions of the nodes were derived, and the longitudes carried back from 1900 to 1750. The node of II. has completed nearly five revolutions in the interval. The differences between the motions of the nodes, found from the same masses by Souillart's two methods of integration, would amount to $1^{\circ}\cdot4$ in the interval from 1750 to 1900. In both methods terms of the second and higher orders were neglected, and the effect of these may easily reach an amount exceeding the difference between the two methods. Considering all this, the agreement between the extrapolated values and those of Delambre and Damoiseau appears to be as close as can reasonably be expected.

From the residuals $\Delta\gamma_i$ and $\sin\gamma_i\Delta\Gamma_i$ we find residuals Δx_i and Δy_i by the formulas

$$\Delta x_i = \sin\Gamma_i\Delta\gamma_i + \cos\Gamma_i \sin\gamma_i\Delta\Gamma_i ,$$
$$\Delta y_i = \cos\Gamma_i\Delta\gamma_i - \sin\Gamma_i \sin\gamma_i\Delta\Gamma_i ,$$

and from these

$$\Delta p_i = \Sigma_j \sigma_{ij}\Delta x_j$$
$$\Delta q_i = \Sigma_j \sigma_{ij}\Delta y_j .$$

* The probable errors stated for 1750 include, of course, the probable errors of the motions, and these again take account of the uncertainty of the masses, as was explained above. But they do not take account of possible defects in the analytical theory by which the motions were derived from the masses.

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The residuals Δp_i and Δq_i can also be derived directly from the formulas

$$\Gamma_i = \Gamma_{i0} + (t - t_0)\frac{d\Gamma_i}{dt}$$

$$(p_i)\text{ comp.} = \Sigma_j \sigma_{ij}\gamma_j \sin \Gamma_j - \mu_i x_0 + \text{periodic inequalities},$$
$$(q_i)\text{ comp.} = \Sigma_j \sigma_{ij}\gamma_j \cos \Gamma_j - \mu_i y_0 + (1-\mu_i)\omega_0 + \text{periodic inequalities}.$$

$$\Delta p_i = (p_i)\text{ comp.} - (p_i)\text{ obs.}$$
$$\Delta q_i = (q_i)\text{ comp.} - (q_i)\text{ obs.}$$

The residuals Δp_i and Δq_i have been computed by both methods. The results of the two computations agreed in all cases within one or two units of the last decimal place, which is a very severe check on the numerical accuracy, not only of the last solution, but also of all previous reductions.

The residuals computed in the second manner are given in Table XII., together with the observed values of p_i and q_i (reduced to one and the same fundamental plane) and the "computed probable errors," *i.e.* the probable errors as derived by the discussion of each opposition separately.

TABLE XII.

Observed Values and Final Residuals.

Epoch	and Instrument.	Observed Values.	Probable Error.	Residual.	Observed Values.	Probable Error.	Residual.	Observed Values.	Probable Error.	Residual.	Observed Values.	Probable Error.	Residual.
		p_1.			p_2.			p_3.			p_4.		
1891	Hel^r.	+0°0436	±°0045	+°0023	+0°4914	±°0031	+°0025	+0°1137	±°0020	+°0032	+0°2324	±°0010	+°0022
,,	Plates	+ ·0448	± 50	+ 35	+ ·4895	± 36	+ 6	+ ·1142	± 19	+ 37	+ ·2332	± 12	+ 30
1901	Hel^r.	+ ·0266	± 71	+ 20	− ·2928	± 52	− 64	+ ·0565	± 33	+ 18	+ ·2170	± 17	− 54
1902	Hel^r.	+ ·0004	± 65	+ 1	− ·3501	± 40	+ 19	+ ·0521	± 25	+ 18	+ ·2272	± 13	+ 16
,,	Plates	− ·0061	± 80	− 64	− ·3483	± 42	+ 37	+ ·0524	± 27	+ 21	+ ·2248	± 14	− 8
1903	Plates	− ·0080	± 60	+ 75	− ·4093	± 33	− 33	+ ·0354	± 22	− 97	+ ·2259	± 12	− 30
1904	Plates	− ·0132	± 78	− 58	− ·4400	± 44	+ 8	+ ·0344	± 28	− 11	+ ·2309	± 13	+ 27
		q_1.			q_2.			q_3.			q_4.		
1891	Hel^r.	−0°0299	±°0049	−°0063	−0°1747	±°0029	+°0015	−0°1378	±°0017	+°0016	+0°2891	±°0010	−°0005
,,	Plates	− ·0284	± 61	− 48	− ·1801	± 35	− 39	− ·1445	± 23	− 51	+ ·2911	± 11	+ 15
1901	Hel^r.	− ·0832	± 79	− 57	− ·4026	± 45	+ 76	− ·1413	± 30	+ 103	+ ·2692	± 16	0
1902	Hel^r.	− ·0799	± 59	− 37	− ·3375	± 36	+ 41	− ·1534	± 21	− 4	+ ·2669	± 13	+ 22
,,	Plates	− ·0850	± 62	− 88	− ·3383	± 37	+ 33	− ·1487	± 22	+ 43	+ ·2671	± 13	+ 24
1903	Plates	− ·0599	± 48	− 58	− ·2508	± 32	− 29	− ·1584	± 20	− 4	+ ·2631	± 11	− 6
1904	Plates	− ·0301	± 77	− 34	− ·1481	± 48	− 90	− ·1645	± 26	− 54	+ ·2657	± 17	0

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The probable error of weight unity derived from these residuals is

$$\pm 0^{\circ}\cdot 0097.$$

This is extremely satisfactory. The weights were given to correspond to a probable error of weight unity of

$$\pm 0^{\circ}\cdot 0100.$$

The agreement of these two values points to the freedom of systematic errors of the values of Δp and Δq derived from the different oppositions. The same conclusion is reached by comparing the distribution of the residuals with the distribution according to the theory of errors. If ρ be the probable error of any observed correction Δp or Δq as given by the reductions of the separate epochs (*i.e.* the "computed probable error"), we find the following numbers of residuals:—

	Actual Number.	Theoretical Number.
Smaller than ρ	$30\frac{1}{2}$	28·0
Between ρ and 2ρ	$16\frac{1}{2}$	18·1
Between 2ρ and 3ρ	6	7·5
Exceeding 3ρ	3	2·4.

Residuals exactly equal to the limit between two divisions have been counted as $\frac{1}{2}$ in each.

Remembering that the corrections Δp and Δq for each epoch are the result of a separately made and separately reduced series of observations, made by different observers and with different instruments, and discussed entirely independently of one another by different methods, this agreement between the actual and the adopted probable error of unit weight and between the actual and theoretical distribution is eminently satisfactory and entitles us to assign a very high standard of accuracy and reliability to the results. The resulting values of the inclinations and nodes can confidently be assumed to be entirely correct within the limits indicated by the probable errors as derived from the solution.

Summary of Results.—From the results of the last solution we derive the following elements for the epoch 1900 *January* 0·0, *Greenwich mean noon.*

Plane of Jupiter's Equator referred to Leverrier's Orbit of 1900·0.

		Souillart.
$\omega =$	$3^{\circ}\cdot 1153 \pm 0^{\circ}\cdot 0014$	$3^{\circ}\cdot 0680$
$\theta' =$	$315\cdot 779 \pm 0\cdot 025$	$315\cdot 410.$

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θ' is the node counted "in the equator." When counted "in the orbit" it becomes

$$\theta = 315^{\circ}\cdot 800 .$$

From these we derive

Plane of Jupiter's Equator referred to Ecliptic and Mean Equinox.

		Adopted in Computation of Tabular Places.
$J =$	$2^{\circ}\ 12'\ 8''\cdot 7$	$2^{\circ}\ 9'\ 3''\cdot 9$
$N =$	$336\ 24\ 24$	$336\ 21\ 28$.

Inclinations ("Inclinaisons propres") *of the Satellites.*

		Souillart.
$\gamma_1 =$	$0^{\circ}\cdot 0272 \pm 0^{\circ}\cdot 0028$	$0^{\circ}\cdot 0013$
$\gamma_2 =$	$0\cdot 4683 \pm 0\cdot 0016$	$0\cdot 4694$
$\gamma_3 =$	$0\cdot 1839 \pm 0\cdot 0026$	$0\cdot 1789$
$\gamma_4 =$	$0\cdot 2536 \pm 0\cdot 0023$	$0\cdot 2254$.

Nodes ("Noeuds propres") *counted from Terrestrial Equinox.*

$\theta_1 =$	$60^{\circ}\cdot 2 \pm 7^{\circ}\cdot 0$	$99^{\circ}\cdot 8$
$\theta_2 =$	$293\cdot 16 \pm 0\cdot 19$	$273\cdot 32$
$\theta_3 =$	$319\cdot 71 \pm 0\ 52$	$330\cdot 59$
$\theta_4 =$	$11\cdot 96 \pm 0\cdot 67$	$5\cdot 79$.

Daily Motions of Nodes referred to Mean Equinox.

$\frac{d\theta_1}{dt} =$	$-0\cdot 13614 \pm 0\cdot 00100$	$-0\cdot 14105$
$\frac{d\theta_2}{dt} =$	$-0\cdot 032335 \pm 0\cdot 000240$	$-0\cdot 032974$
$\frac{d\theta_3}{dt} =$	$-0\cdot 006854 \pm 0\cdot 000180$	$-0\cdot 006983$
$\frac{d\theta_4}{dt} =$	$-0\cdot 001772 \pm 0\cdot 000030$	$-0\cdot 001863$.

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